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Optimization of feedforward controllers to minimize sensitivity to model inaccuracies

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Abstract—**The common view on feedforward control is that it needs an accurate model in order to accurately predict a future state of the system. However, in this paper we show that there are model inaccuracies that do not affect the final position of a motion, when using the right feedforward controller. Having an accurate final position is the main requirement in the task we consider: a pick-and-place task. We optimized the feedforward controllers such that the effect of model inaccuracies on the final position was minimized. The system we studied is a one DOF robotic arm in the horizontal plane, of which we show simulation and hardware results. The results show that the errors in the final position can be reduced to approximately zero for an inaccurate Coulomb, viscous or torque dependent friction. Furthermore, errors in the final position can be reduced, but not to zero, for an inaccurate inertia or motor constant. In conclusion, we show that for certain model inaccuracies, no feedback is required to eliminate the effect of an inaccurate model on the final position of a motion.**

I. Introduction

There is a difference between traditional robot control and the way humans control their body: humans use feedforward extensively while traditional robot control is mainly based on feedback.

Humans use both feedback and feedforward when sending out motor commands [1]. However, for fast motions, humans cannot rely on feedback at all, due to the large time delays (typically 150 ms for humans [2, 3]). Therefore, they have to rely on feedforward, in which control signals are generated based on the prediction of an (inaccurate) internal model [4]. In feedforward control, humans make use of the fact that most tasks can be executed in multiple or an infinite number of ways, the so-called task redundancy. Experiments on eye movements indicate that humans exploit this task redundancy such that the error in final position due to the influence of uncertainty is minimized [5]. Similar error-minimizing human feedforward motions have been reported for the games of darts and skittles [6, 7, 8, 9]. We will focus on pick-and-place tasks of robotic arms, which also possess task redundancy; only the initial and final positions matter and the path in between can be chosen freely. The knowledge from the field of human motion control suggests that some feedforward motions are more sensitive to uncertainty than others. Therefore, we study the sensitivity of feedforward controlled motions to an inaccurate model.

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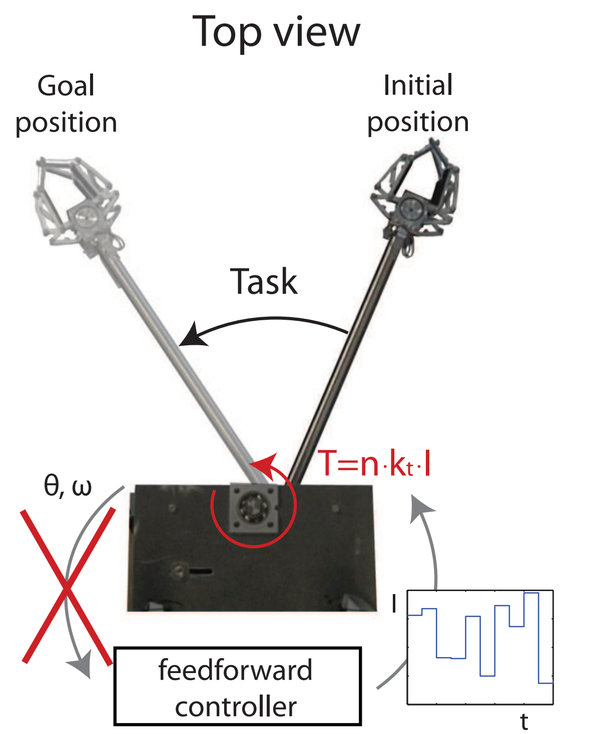

Fig. 1. A schematic representation of the content of this paper. The one DOF robotic arm has to perform a pick and place task. In this task, the arm has to move from the initial to the goal position. The controller is a feedforward controller, which means that the state (i.e. position θ and velocity ω) is not used to determine the control signal. The control signal is a current I, which is only a function of time. In this paper, we investigate the sensitivity of feedforward motions to parametric model inaccuracies. These model inaccuracies cause the arm to end up in a different position than the goal state. We aim to minimize this error in the final position.

Traditionally, robots mainly use feedback to perform their motions (e.g. PID control). The main reason that robots can rely more on feedback than humans, is their smaller time delays (typically <10 ms for robots). However, feedforward still has advantages over fast feedback since it incorporates a prediction of the behavior of the system, which means that the controller is able to anticipate future states. Therefore, the use of feedforward control has been investigated before by robot researchers.

There are multiple examples of robotic systems where the implementation of feedforward control (in combination with feedback or not) already led to positive results. Schaal and Atkeson [10] showed that robot juggling can be performed by an open loop controller. In their case, open loop means that there is no feedback about the position of the ball. Seyfarth et al. [11] showed that the right feedforward control scheme for the swing leg retraction improves the stability of running in a humanoid robot. And finally, Mombaur et al. [12, 13] showed that stable walking and running are possible by creating open loop stable periodic motions.

The work most strongly related to this study is that of Becker and Bretl, since they also investigated feedforward control under the presence of model inaccuracies. They researched the influence of an uncertain wheel diameter [14] on the motion of a differential drive robot and found feedforward motions for which these model inaccuracies did not influence the final position.

In this paper we will research a pick-and-place task of a robotic arm, where the initial and goal positions and the time to move are fixed and the path in between is only constrained by the maximum current (see Fig. 1). We demonstrate that by choosing the right feedforward controller, the error in the final position due to parametric model inaccuracies is reduced. For certain inaccuracies, it is even possible to reduce the error to approximately zero. We performed the simulations on a model of a one DOF robotic arm, while optimizing a feedforward controller. We also performed hardware experiments to confirm the results from simulation.

The rest of this paper is structured as follows. Section II explains the methods we used, including the simulation model and the optimization method. Sections III and IV show results of respectively the simulations and the hardware experiments. Finally, the paper ends with a discussion in Section V and a conclusion in Section VI, where we will conclude that for certain model inaccuracies, no feedback is required to eliminate the effect of the model inaccuracy on the final position.

II. Methods

We studied the sensitivity of feedforward controllers by optimizing the controller in order to minimize the error in the final position of the arm due to a variation of the model parameters. For all model parameters, we simulated the model using five different values for the parameters (i.e. the nominal value and four deviating values) while keeping the feedforward controller the same for all five simulations. This results in four motions that deviate from the nominal motion. We chose to use four deviating values to capture non-linear effects around the nominal parameter value. We did not use more deviating values because this would unnecessarily increase the computational time.

In the remainder of this section, we will discuss the configurations we studied, the simulation model, which model inaccuracies we considered, the implementation of the feed-forward control, the task the arm has to perform and the optimization method.

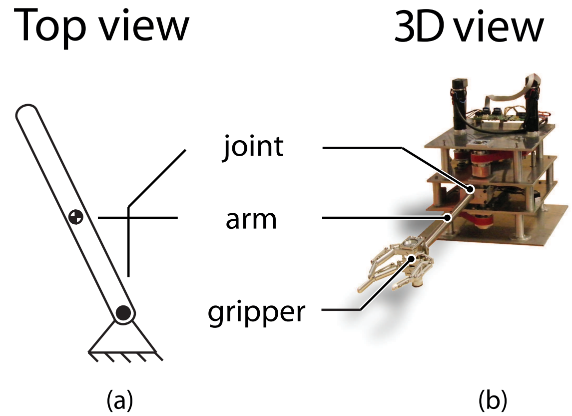

Fig. 2. The two configurations we studied: (a) A simulation model of a one DOF robotic arm to obtain theoretical results. (b) Hardware experiments on a one DOF robotic arm to confirm the results of the simulation model optimizations on a real-world system.

A. Configurations

We studied two configurations: a one DOF simulation model and a one DOF robotic arm (see Fig. 2):

(a) **A simulation model of a one DOF robotic arm.** We obtained the feedforward controller which is least sensitive for inaccuracies in the simulation model.

(b) **Hardware experiments on a one DOF robotic arm.** We performed hardware experiments on a one DOF robotic arm in the horizontal plane (see Fig. 2b) to confirm the results from the simulations.

B. Simulation model

The simulation model consists of a rotating inertia in the horizontal plane. In the simulation model, we included three types of frictional losses: Coulomb friction, viscous friction and torque dependent gearbox friction. Torque dependent gearbox friction is less commonly used than the other two. The way we implemented it is similar to the force dependent friction term in [15]. We obtained the values of the parameters of the friction model by performing a system identification on the robotic arm. The parameters of the simulation models are listed in Table I. It might seem strange that the viscous friction coefficient is negative, but in the velocity range in which the arm operates, the total friction in the model does not become negative. Apparently, this value for the viscous friction coefficient (together with the values for the Coulomb and torque dependent friction coefficients) captures the nonlinear friction effects best for the tested range of torques and velocities. The equations of motion are:

$$x = \begin{bmatrix} \theta \\ \omega \end{bmatrix} \tag{1}$$

$$\dot{x} = \begin{bmatrix} \omega \\ \left(\dfrac{T - \mu_v \cdot \omega - \text{sign}(\omega) \cdot (\mu_c + \mu_t \cdot |T|)}{J_{joint}} \right) \end{bmatrix} \tag{2}$$

TABLE I
THE MODEL PARAMETERS OF THE ONE DOF ARM. THE VALUES ARE OBTAINED THROUGH A SYSTEM IDENTIFICATION OF THE ROBOTIC ARM. ALL INERTIAL TERMS ARE COMBINED IN THE INERTIA ABOUT THE JOINT (J_{joint}).

Parameter arm	Symbol	Value	Varied?
Coulomb friction	μ_c	0.19 Nm	yes
Viscous friction	μ_v	-0.05 Nms/rad	yes
Torque dependent friction	μ_t	22 %	yes
Inertia	J_{joint}	0.17 kgm^2	yes
Motor constant	k_t	26.7 mNm/A	yes
Gearbox ratio	n	1:54	no

for $\omega \neq 0$

$$\dot{x} = \left[\begin{array}{c} 0 \\ \left(\frac{T - \min(\mu_c + \mu_t \cdot |T|; |T|) \cdot \text{sign}(T)}{J_{joint}} \right) \end{array} \right] \quad (3)$$

for $\omega = 0$.

θ is the angle of the joint, ω is the velocity of the joint, T is the torque exerted by the motor on the joint, μ_v is the viscous friction coefficient, μ_c is the Coulomb friction coefficient, μ_t is the torque dependent friction coefficient and J_{joint} is the mass moment of inertia about the joint.

The simulation model includes a DC motor. The torque applied by the DC motor on the joint is equal to:

$$T = n \cdot k_t \cdot I \quad (4)$$

Where k_t is the motor constant, n is the gearbox ratio and I is the current through the motor.

C. Model inaccuracies

The simulation model includes six model parameters. From these parameters, the gearbox ratio can easily be determined accurately and therefore the gearbox ratio is not varied in this study. The other parameter values in Table I are potentially inaccurate and will be varied. For all inaccurate parameters, the simulation is performed with five parameter values: the estimated (nominal) value and deviations of -20%, -10%, +10% and +20% of this value. ±20% was chosen as a realistic value for parameter inaccuracies. An alternative would be to calculate the derivatives of the final position to the individual parameters. However, this only gives information about infinitely small variations in the parameters.

D. Feedforward control

We implemented the feedforward control as a feedforward current controller (see eq. (4)). One could argue that this current controller is a feedback controller since it controls the current in a feedback loop. However, since we have no task specific feedback (i.e. the state is not fed back into the controller), the total controller is a feedforward controller. An alternative would be to use voltage control implemented as pulse width modulation control. We use current control because in control theory, torque control (implemented as current control) is more common than voltage control. In Section V, we will show that the choice for current control or voltage control does not influence the results of our optimization qualitatively.

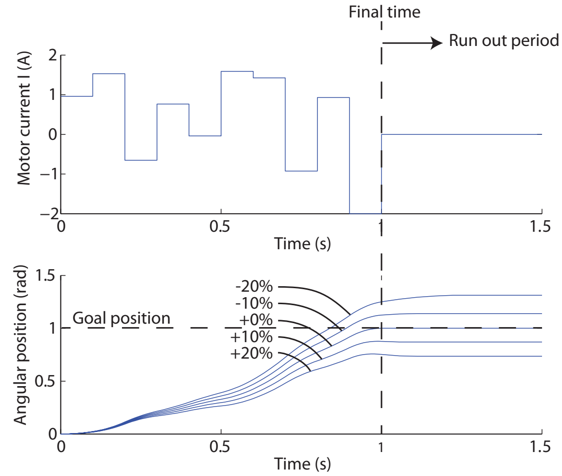

Fig. 3. An example of a feedforward control signal and the five motions that result from the signal. The signal represents the current through the motor as function of the time. There are 10 set points in total ($t_0 \dots t_9$) and the current is constant in between the set points. The duration of the signal is 1 second, after which there is a run out period. This run out period stops when the velocities are reduced to zero by the friction. In this example, the difference between the five motions is the result of a variation of the Coulomb friction (the nominal value -20%, -10%, +0%, +10% and +20%). The objective of the optimization is to minimize the RMS error of the final positions.

E. Task

The robotic arm has to perform a pick-and-place task. The important task parameters are the distance and time per stroke. We chose the time per stroke to be 1 s and the distance per stroke to be 1 rad. In Section V, we will show that these arbitrary choices do not influence the results of our optimization qualitatively.

F. Optimization method

The goal is to minimize the error in the position at the end of the stroke. The error in the final position is caused by a variation in the model parameters. In general, there is also an error in the final velocity. To include this in the optimization, we let the system run with a control signal of zero, until the velocity is zero. During this time (the run out period, see Fig. 3), the velocity is reduced to zero by the friction. The error in the final position is evaluated after this run out period.

As mentioned earlier, we picked four values for which we evaluate the final position (-20%, -10%, +10% and +20% of the nominal value of the parameters), while the feedforward controller is the same as in the simulation with the nominal value of the parameters. The goal of the optimizations is to find the feedforward controllers that minimize the errors in these final positions. We weigh the four errors equally, which assumes a uniform distribution of the inaccurate parameter. This can be seen as the worst case scenario for the distribution of the parameter values [16, 17].

In this optimization with only four values of the model parameter, there is a chance of over fitting the objective function (i.e. obtaining results for which the error is only small at the four deviating values of the model parameter and is large in between). Therefore, in Section III, we will show the error in the final position as function of the change in the parameters and we will show that the choice to use four values leads to good results that are not over fitted.

The cost function of the optimization is the RMS of the error in the final positions of the four movements with deviating parameter values. The movement is constrained by the maximum current through the motor and constrained such that the error in the final position of the movement with the nominal value for the model parameter is zero:

$$E = \sqrt{\frac{\epsilon_{-20\%}^2 + \epsilon_{-10\%}^2 + \epsilon_{+10\%}^2 + \epsilon_{+20\%}^2}{4}} \tag{5}$$

$$\underset{I(t)}{\text{minimize}} \quad E(I(t)) \tag{6}$$

$$\begin{aligned} \text{subject to} \quad & |I(t)| && \leq I_{max} \forall t \\ & \epsilon_{+0\%} && = 0 \\ & \omega_{+0\%} && = 0 \end{aligned} \tag{7}$$

with

$$\epsilon_i = |\theta_i - \theta_{goal}| \tag{8}$$

where E is the error function, ϵ_i is the error in the final position of motion i, $I(t)$ is the feedforward signal as function of the time, I_{max} is the maximum current through the motor and $\omega_{+0\%}$ is the angular velocity of the nominal motion at $t = t_f$. The maximum current in the optimizations is 2A. Since this is a system specific value, in Section V we will show that the choice for this value does not influence the results of our optimization qualitatively.

Since the robotic arm has to perform a pick-and-place task, the arm should end up close enough to the goal position to pick or place an object. As a reasonable maximum deviation, we chose 1 cm. Since the arm we use has a length of 0.4 m, the maximum allowable error is E_{max} = 0.025 rad. We will evaluate the performance of the feedforward controllers by comparing this value to the error values of the controllers.

To have something to compare the minimization results against, we also performed a maximization of the RMS of the error in the final positions:

$$\underset{I(t)}{\text{maximize}} \quad E(I(t)) \tag{9}$$

also subject to (7). We also calculated the factor of improvement f:

$$f = 1 - \frac{E_{minimized}}{E_{maximized}} \tag{10}$$

which is a measure for the improvement that can be achieved by optimization. An f of 1 means that the error can be reduced to zero (or can be increased to infinity) by optimizing the feedforward controller, an f of 0 means that the error does not depend on the chosen feedforward controller at all.

The feedforward control signal $I(t)$ is parameterized as a piecewise constant function, an example of which is shown in Fig. 3. The profile is defined by N set points at $t_0, t_1, \ldots, t_{N-1}$, the time step between the set points is constant. This means that we have N free variables for the feedforward profile. For our simulations, we chose $N = 10$.

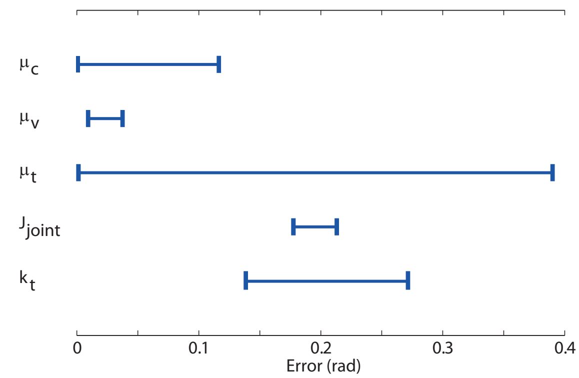

Fig. 4. This figure shows bars from the minimal to the maximal errors of the simulation model, that are caused by the model inaccuracies on the vertical axis. The error of any feedforward signal that satisfies the constraints lies on those bars. We clearly see that the optimizations of the feedforward signal had an effect for all model inaccuracies. However, only the errors due to inaccurate friction model parameters are close to zero. Note that we did not prove that the errors are dense sets so the bars shown here might be an overestimation of the set of all possible errors.

TABLE II
THE RESULTS OF THE ONE DOF FEEDFORWARD OPTIMIZATION ON THE SIMULATION MODEL.

Parameter	$E_{minimized}$ (rad)	$E_{maximized}$ (rad)	f
μ_c	0.0002	0.1164	0.9987
μ_v	0.0092	0.0374	0.7548
μ_t	0.0013	0.3901	0.9967
J_{joint}	0.1775	0.2131	0.1667
k_t	0.1385	0.2715	0.4898

In Section V, we will show that the results do not change significantly when we change N. Because the optimization problem is a non-convex constrained optimization problem, the optimization is performed with a multi start of the MATLAB function `fmincon`.

III. Simulation Results

In this section, we show the results of the optimizations on the simulation model. Fig. 4 visualizes the minimum and maximum error that can be caused by model inaccuracies. Table II shows the minimized and maximized errors and the factors of improvement f.

We see that the various parameter inaccuracies have different effects on the final position of the arm:

- The effect of the **Coulomb friction** on the error highly depends on the feedforward controller (f = 0.9987). There are feedforward controllers for which the error is the smallest of all parameter inaccuracies (E = 0.0002 rad). However, there are also feedforward controllers for which the error is larger than the maximum allowable error (E = 0.1164 rad $> E_{max}$).
- The effect of the **viscous friction** on the error depends on the feedforward controller (f = 0.7548) and the minimum error is allowable (E = 0.0092 rad $< E_{max}$). However, since the maximum error is also relatively small (E = 0.0374 rad), the factor of improvement is not close to one.
- The effect of the **torque dependent friction** on the error is similar to that of the Coulomb friction: it depends

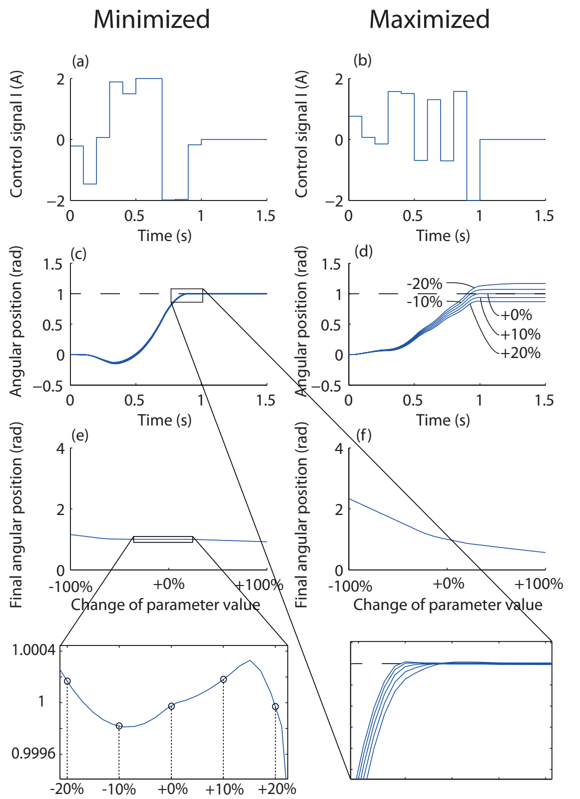

Fig. 5. The results from the optimization with an inaccurate Coulomb friction coefficient. The plots on the left show the results of the minimization of the error, the plots on the right show the results of the maximization of the error. The plots show the optimized current profiles (a and b), five different trajectories as function of time due to five different values for the Coulomb friction coefficient (c and d) and the error in final position as function of change of the parameter value of the Coulomb friction (e and f). The graph also shows enlargements of the end of the position profile of the minimization (c) and the error in position around the nominal value of the Coulomb friction (e). The latter also shows five circles, which represent the final positions of the arm with the nominal value of the Coulomb friction and the four deviating values.

highly on the feedforward controller (f = 0.9967). There are feedforward controllers for which a change in the torque dependent friction results in an error that is significantly smaller than the maximum allowable error (E = 0.0013 rad $< E_{max}$). However, there are also feedforward controllers for which the error is larger than the errors of all other parameter inaccuracies. Therefore, the factor of improvement is close to one.

- The effect of the **inertia** on the error only slightly depends on the feedforward controller (f = 0.1667). Also, the minimal error is significantly larger than the maximum allowable error (E = 0.1775 rad $> E_{max}$).
- The effect of the **motor constant** on the error can be reduced by the choice of the feedforward controller (f = 0.4898). However, the minimum error is too large (E = 0.1385 rad $> E_{max}$).

In general, we conclude that inaccuracies in the friction model can be compensated for by the choice of the feedforward controller. Inaccuracies in the inertia and motor constant cannot be compensated for.

Fig. 5 shows the results for the Coulomb friction optimization in more detail. We clearly see how two different current profiles (Fig. 5a and 5b) lead to different errors (Fig. 5c and 5d), while the nominal motion reaches the goal state. In Fig. 5e, we see that in the motion with minimized error, there is a range of values of the Coulomb friction for which the arm ends up in approximately the same position. This shows that the optimization did not over fit the objective function at the four values of the inaccurate parameter. Similar graphs were obtained for the other optimizations.

The optimized current profiles in Fig. 5a and 5b have no clear structure. This is due to the fact that the optimization with 10 controller set points is redundant. This means that there are multiple current profiles that lead to the same (optimal) error value. This redundancy makes it hard to detect a structure in the current profiles.

Interestingly, the motion that results from the minimization of the error first moves in the negative direction before moving towards the goal position. We observed such behavior in many of our results. Probably such behavior reduces the effect of the parameter inaccuracy on the final position by canceling out effects in positive and negative direction.

IV. Hardware Results

In this section, we show the results of the hardware experiments. First, we will explain the test set up and second, we will show the test results.

A. The robotic arm

Fig. 2b shows a picture of the one DOF robotic arm [18]. The DOF is created by an 18x1.5mm stainless steel tube, connected with a joint. A weight of 1 kg is connected to the end of the tube, which represents the weight of a gripper plus a product. The motor is placed on a housing and AT3-gen III 16mm timing belts are used to transfer torques within the housing. The joint is actuated by a Maxon 60W RE30 motor with a gearbox ratio of 18:1. The timing belts provide an additional transfer ratio of 3:1. The model parameters as shown in Table I are based on a system identification of this robotic arm.

We tested the results of the optimizations of two parameters: the Coulomb friction and the inertia, because these two parameter inaccuracies give qualitatively different results in simulation (see Fig. 4 and Table II)

To change the Coulomb friction, we designed a mechanism that adds Coulomb friction by clamping a nylon sleeve bearing on the motor axis. The Coulomb friction can be increased by tightening the screw of the clamping mechanism. Before each experiment, we ran a system identification to determine the amount of Coulomb friction we added. To change the inertia, we added extra weights at the end point of the arm. Table III shows the values of the parameter changes.

B. Results

On the robotic arm we did not perform an optimization of the torque profile. Instead, we tested a grid of possible feedforward controllers. Per parameter value for the Coulomb

TABLE III
THE VALUES OF THE CHANGED PARAMETERS IN THE HARDWARE EXPERIMENTS.

Parameter	Nominal value	Values in experiments	Units
μ_c	0.19	0.19, 0.22, 0.25	Nm
J_{joint}	0.17	0.17, 0.19, 0.21	kgm^2

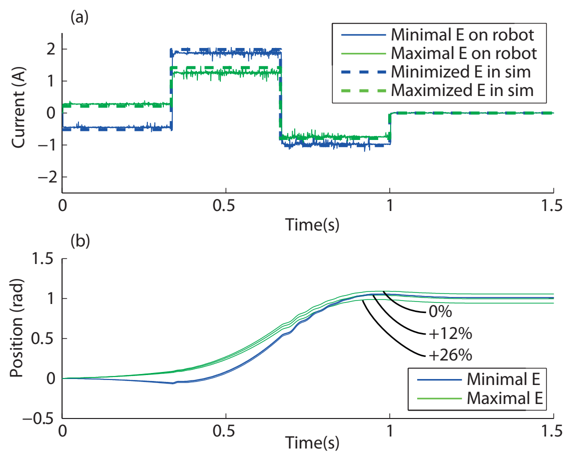

Fig. 6. A typical example of the data obtained from hardware experiments. This figure shows the minimized and maximized motions for a changing Coulomb friction. a) The feedforward current as function of the time. The solid lines correspond to the motions with minimized and maximized error in hardware experiments. The dashed lines are optimized current profiles in simulation. This graph shows that the current profiles optimized in simulation are the same as the current profiles with minimal and maximal error in hardware experiments. b) The position of the arm as function of the time. We clearly see that the spread in the final position of the minimized motion is smaller than that of the maximized motion. For the minimized motion, it is hard to distinguish the three lines, for the maximized motion the spread is clearly visible.

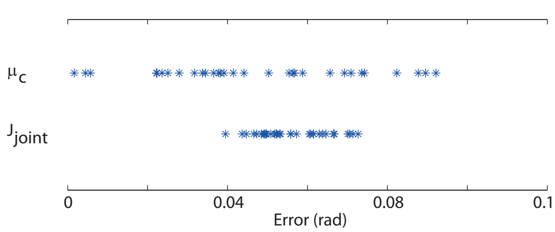

Fig. 7. This figure shows the results of hardware experiments. In these experiments, we tested a grid of 32 possible feedforward controllers. Every star represents the error value as a result of one current profile while varying the Coulomb friction μ_c or the inertia J_{joint}. We see that the results in this figure are comparable to the results from simulation.

friction and inertia, we repeated the experiment with 32 different current profiles. Each current profile had three controller set points of which the first one was determined by a grid of 32 set points. The other two set points were determined by the constraints on the final position and final velocity in simulation. There are two reasons why we use three set points in the hardware experiments instead of the ten we used in simulation. First of all, using three set points makes it feasible to test a grid of controllers since the grid is one dimensional. Second, due to the switches in the controller, the influence of the unmodeled backlash increases and using only three controller set points minimizes the influence of this backlash. In Section V we will show that by limiting ourselves to three controller set points instead of ten, the factors of improvement will change but will still show the same trend (e.g. 10% change for an inaccurate Coulomb friction).

TABLE IV
THE RESULTS OF THE ONE DOF HARDWARE EXPERIMENTS.

Parameter	$E_{minimized}$ (rad)	$E_{maximized}$ (rad)	f
μ_c	0.001	0.092	0.982
J_{Joint}	0.040	0.073	0.457

Fig. 6 shows the current and the position as function of the time of a typical experimental run. It also shows that the current profiles with minimal and maximal error in hardware experiments correspond to the current profiles with minimized and maximized error in simulation. For systems with more DOFs, the current profiles from optimization should be used instead of a less feasible grid search. Per value of the first setpoint, we calculated the RMS of the error in the final position with respect to the final position of the motion with the nominal parameter value. Fig. 7 and Table IV show the error values that were obtained from the hardware experiments. From these results, we see that:

- The results of the hardware experiments with an inaccurate **Coulomb friction** are comparable to the results from simulation. The maximum error in simulation and hardware experiments are respectively 0.1164 rad and 0.092 rad and in both cases, the minimum error is approximately zero. Furthermore, the error due to an inaccurate Coulomb friction has a larger spread than the error due to an inaccurate inertia.
- The results of the hardware experiments with an inaccurate **inertia** differs from the results from simulation. The main difference is that the errors are smaller in the hardware experiments. Since the difference between the minimum and maximum error is the same in simulation and the hardware experiments (respectively 0.0356 rad and 0.033 rad), the factor of improvement is larger in the hardware experiments.

We conclude that although there clearly are differences between the simulations and the hardware results, in general the hardware experiments confirm the conclusions from the simulation study: the errors due to inaccuracies in the Coulomb friction can be reduced to approximately zero, while the errors due to inaccuracies in the inertia are always larger than the maximum allowable error E_{max}.

V. DISCUSSION

In this study we researched motions of a one DOF robotic arm, controlled by feedforward control. The task consisted of fixed initial and goal positions and a fixed time per stroke. The motion in between was only constrained by the maximum current. We showed that the choice of the motions in between the initial and goal positions is important for the error in the final position due to parametric model inaccuracies. For the one DOF system, the error in the final position can even be reduced to approximately zero when the Coulomb friction, viscous friction or torque dependent friction are not accurately known.

A. *Implications*

The results of this study are important to consider when implementing feedforward control, even in combination with feedback control. The correct use of feedforward control improves the performance of the system and this study shows that the performance can even be improved in such a way that certain model parameters do not have to be known accurately. An interesting result was that most optimized motions do not move from the initial to the goal position directly, but first move away from the goal position.

This study also has implications on the field of human motion control. Recent studies in the field of human motion control focused on the uncertainty (i.e. noise) in the control signals [5]. It would be interesting to research the accuracy of the internal models of humans and the influence of this accuracy on the motions humans choose. Another interesting topic for future research would be the influence of noise on the performance of feedforward control in robotic systems.

B. *Choice of optimization parameters*

In Section II, we introduced the task parameters time and distance per stroke and their values. We also introduced values for the maximum current and the number of set points of the feedforward controller. Since these values are arbitrary, we analyzed how the results in this paper depend on the chosen parameter values. We varied those parameters and ran the optimizations again to evaluate their influence. Fig. 8 shows the result of this analysis. From these results, we conclude that:

- The **number of controller set points** does not influence the optimizations, as long as there are enough set points. For all optimizations, six set points appears to suffice.
- The effects of the choice of the **distance per stroke**, **time per stroke** and **maximum current** are related. A too low maximum current, too large distance per stroke or too short time per stroke lead to a decreased performance. This is intuitive since the maximum current should be high enough to move the distance per stroke within the time per stroke.
- The factors of improvement increase when we increase the time per stroke or the maximum current. This suggests that the system is approximately controllable [14, 19, 20]. The approximate controllability of robotic arms is an interesting topic for future research.

In general, we conclude that the specific results depend on the choice for the distance per stroke, time per stroke and maximum current. However, the qualitative results are the same for reasonable values for those parameters.

C. *Voltage control*

In this paper we considered torque control, which we implemented as current control. However, the lowest level of control in the electronics is voltage control (i.e. controlling the pulse width modulation). Fig. 9 shows the results of the one DOF optimization using voltage feedforward control. By comparing Fig. 4 and Fig. 9, we conclude that the conclusions in this paper would have been the same had we used voltage control.

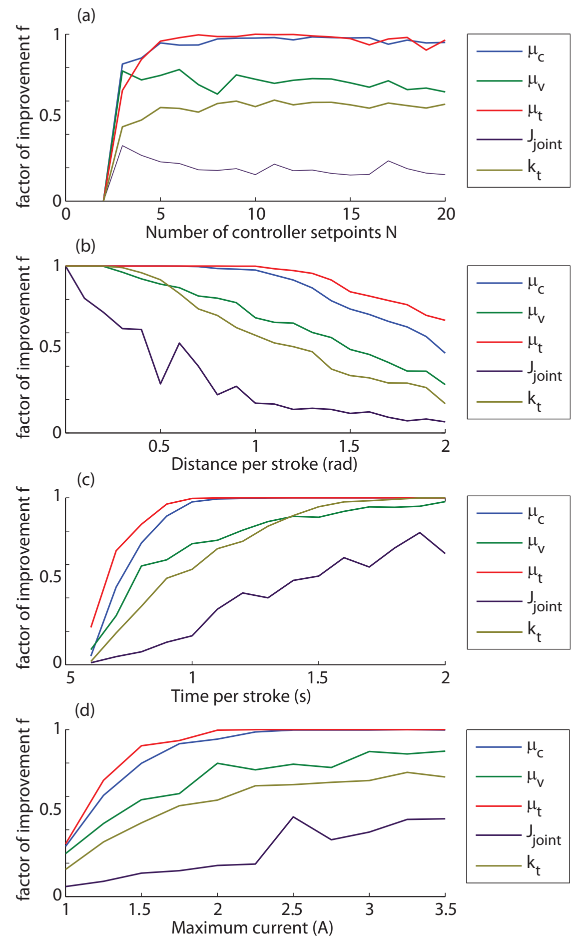

Fig. 8. This figure shows four analyses of the influence of the choice of four parameters on the results in this paper. The four graphs show the factors of improvements we obtain through optimization for various number of controller set points (a), distance per stroke (b), time per stroke (c) and maximum current (d).

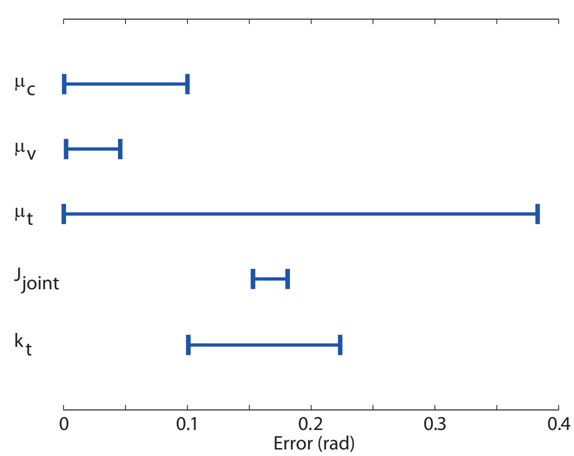

Fig. 9. This figure shows bars from the minimal to the maximal errors of the one DOF model using voltage feedforward control instead of current feedforward control.

D. Future work

There are four topics that need to be addressed in future work in order to make feedforward control fully applicable. Firstly, a more analytical approach, could lead to a deeper understanding of paths that are insensitive to model inaccuracies. Secondly, in practice it is likely to have multiple parameter inaccuracies. Therefore, future research should include optimization of feedforward controllers for multiple parameter inaccuracies simultaneously. Thirdly, more complex systems (e.g. system with more DOFs) have to be researched. Finally, in order to prevent the error from accumulating after multiple motions, the stability of feedfoward motions needs to be addressed. A promising approach could be to analyze cyclic pick-and-place tasks with the use of limit cycle theory.

VI. Conclusions

In this paper, we showed that by optimizing the feedforward controller for a one DOF robotic arm, we can reduce the sensitivity for an inaccuracy in all model parameters. For the three friction parameters (Coulomb friction, viscous friction and torque dependent friction), the errors in the final positions can even be reduced to approximately zero. For the inertia and the motor constant, the errors in the final positions can be reduced, but not to zero.

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References

[1] M. Desmurget and S. Grafton, "Forward modeling allows feedback control for fast reaching movements," *Trends in cognitive sciences*, vol. 4, no. 11, pp. 423–431, 2000.

[2] S. Thorpe, D. Fize, and C. Marlot, "Speed of processing in the human visual system," *Nature*, vol. 381, no. 6582, pp. 520–522, 1996.

[3] P. Cordo, L. Carlton, L. Bevan, M. Carlton, and G. K. Kerr, "Proprioceptive coordination of movement sequences: role of velocity and position information," *Journal of Neurophysiology*, vol. 71, no. 5, pp. 1848–1861, 1994.

[4] M. Kawato, "Internal models for motor control and trajectory planning," *Current Opinion in Neurobiology*, vol. 9, no. 6, pp. 718 – 727, 1999.

[5] C. M. Harris and D. M. Wolpert, "Signal-dependent noise determines motor planning," *Nature*, vol. 394, no. 6695, pp. 780–784, 1998.

[6] J. Smeets, M. Frens, and E. Brenner, "Throwing darts: timing is not the limiting factor," *Experimental Brain Research*, vol. 144, pp. 268–274, 2002, 10.1007/s00221-002-1072-2.

[7] H. Müller and E. Loosch, "Functional variability and an equifinal path of movement during targeted throwing," *Journal of Human Movement Studies*, vol. 36, pp. 103–126, 1999.

[8] H. Müller and D. Sternad, "Decomposition of variability in the execution of goal-oriented tasks: Three components of skill improvement." *Journal of Experimental Psychology: Human Perception and Performance*, vol. 30, no. 1, pp. 212–233, 2004.

[9] R. G. Cohen and D. Sternad, "State space analysis of timing: exploiting task redundancy to reduce sensitivity to timing," *Journal of Neurophysiology*, vol. 107, no. 2, pp. 618–627, 2012.

[10] S. Schaal and C. Atkeson, "Open loop stable control strategies for robot juggling," in *Robotics and Automation, 1993. Proceedings., 1993 IEEE International Conference on*, may 1993, pp. 913 –918 vol.3.

[11] A. Seyfarth, H. Geyer, and H. Herr, "Swing-leg retraction: a simple control model for stable running," *Journal of Experimental Biology*, vol. 206, no. 15, pp. 2547–2555, 2003.

[12] K. Mombaur, R. Longman, H. Bock, and J. Schlöder, "Open-loop stable running," *Robotica*, vol. 23, no. 1, pp. 21–33, 2005.

[13] K. D. Mombaur, H. G. Bock, J. P. Schlöder, and R. W. Longman, "Open-loop stable solutions of periodic optimal control problems in robotics," *ZAMM-Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik*, vol. 85, no. 7, pp. 499–515, 2005.

[14] A. Becker and T. Bretl, "Approximate steering of a unicycle under bounded model perturbation using ensemble control," *IEEE Transactions on Robotics*, vol. 28, no. 3, pp. 580–591, 2012.

[15] P. E. Dupont, "The effect of coulomb friction on the existence and uniqueness of the forward dynamics problem," in *Robotics and Automation, 1992. Proceedings., 1992 IEEE International Conference on*. IEEE, 1992, pp. 1442–1447.

[16] B. Barmish and C. Lagoa, "The uniform distribution: A rigorous justification for its use in robustness analysis," *Mathematics of Control, Signals, and Systems (MCSS)*, vol. 10, no. 3, pp. 203–222, 1997.

[17] E. Bai, R. Tempo, and M. Fu, "Worst-case properties of the uniform distribution and randomized algorithms for robustness analysis," *Mathematics of Control, Signals, and Systems (MCSS)*, vol. 11, no. 3, pp. 183–196, 1998.

[18] M. Plooij and M. Wisse, "A novel spring mechanism to reduce energy consumption of robotic arms," in *Proceedings of the 2012 International Conference on Intelligent Robots and Systems*, 10 2012, pp. 2901–2908.

[19] K. Beauchard, J. Coron, and P. Rouchon, "Controllability issues for continuous-spectrum systems and ensemble controllability of bloch equations," *Communications in Mathematical Physics*, vol. 296, no. 2, pp. 525–557, 2010.

[20] J. Li, "Control of inhomogeneous ensembles," Ph.D. dissertation, Harvard University Cambridge, Massachusetts, 2006.