DARSim
Delft Advanced Reservoir Simulation

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Darsim Lecture Series – 14/October/2016 – 12:45-13:45 – Room F

Dr. Phil Vardon is an Assistant Professor in the Geo-Engineering Section of the Faculty of Civil Engineering and Geosciences, TU Delft. He focuses his research on coupled processes in geo-materials and novel computational methods to characterise geotechnical behaviour.

Topics of interest include thermo-hydro-mechanical behaviour, uncertain processes, the material point method, radioactive waste disposal and shallow geothermal energy.

The Material Point Method

One of the disadvantages of the use of methods using meshes, such as the finite element method, is that for problems with large deformations the meshes tend to tangle and, without extensive numerical treatment, become unsolvable. The material point method, or MPM, provides a method to deal with this problem, by using two levels of discretization: (i) a material level discretisation and (ii) a computational grid. The material properties and state variables are maintained on the material level discretization, or material points, and mapped in each computational step to the computational grid where the governing equations are solved. The results are mapped back, and the computational grid reset, so that the material moves through the mesh. This presentation will outline the resent developments made in MPM at TU Delft in relation to soil slope failure simulations.

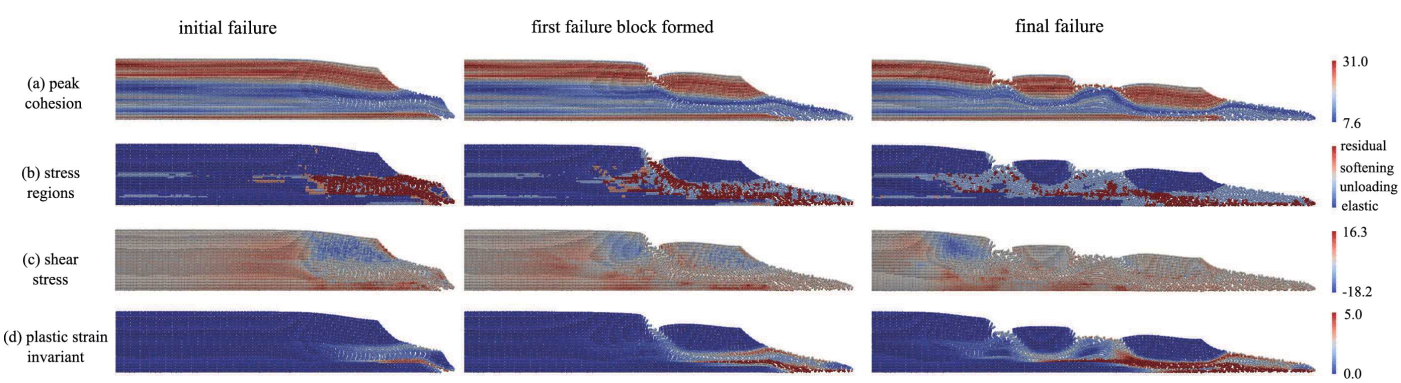
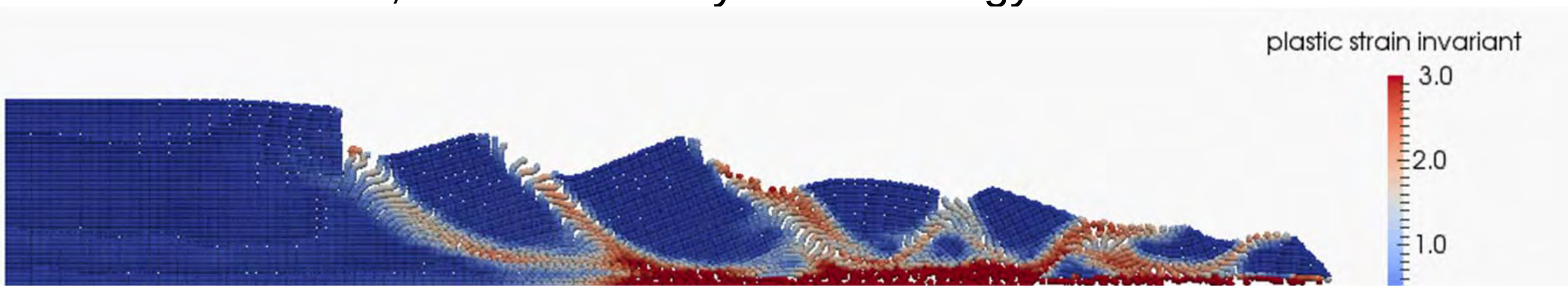


Fig 1. A slope failing process with multiple failures.

The material point method

Phil Vardon, Michael Hicks, Bin Wang, Leon Acosta Gonzalez, Noor Pruijn, Ivaylo Pantev
Section of Geo-Engineering, Faculty of Civil Engineering and Geosciences, Delft University of technology



Contents

- MPM
- Examples – with key features explained
- Conclusions

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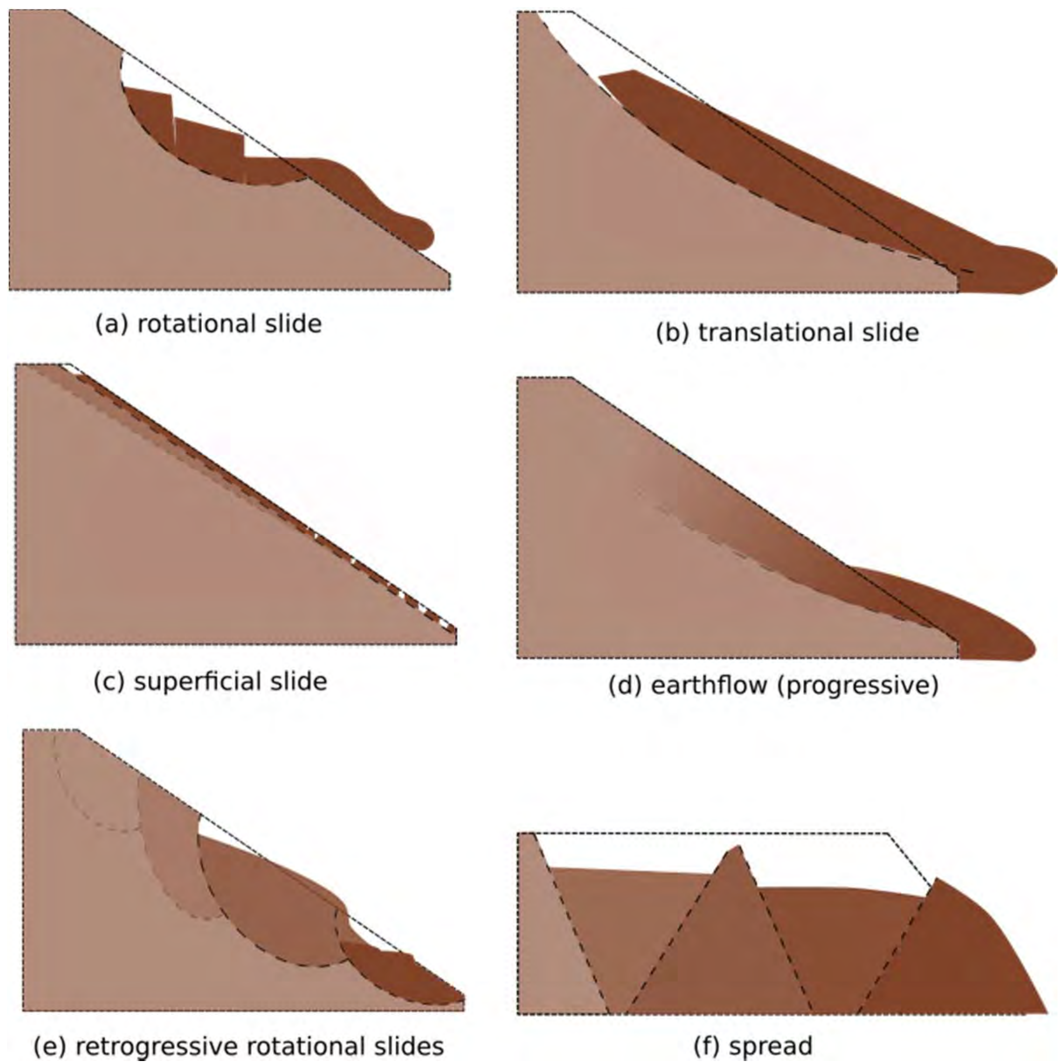
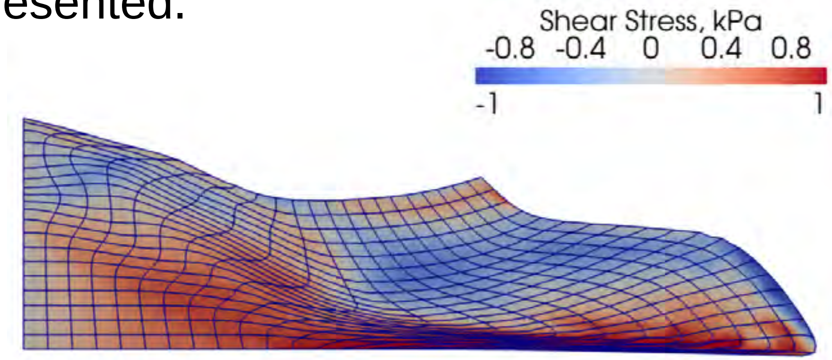
Geo-Engineering slopes

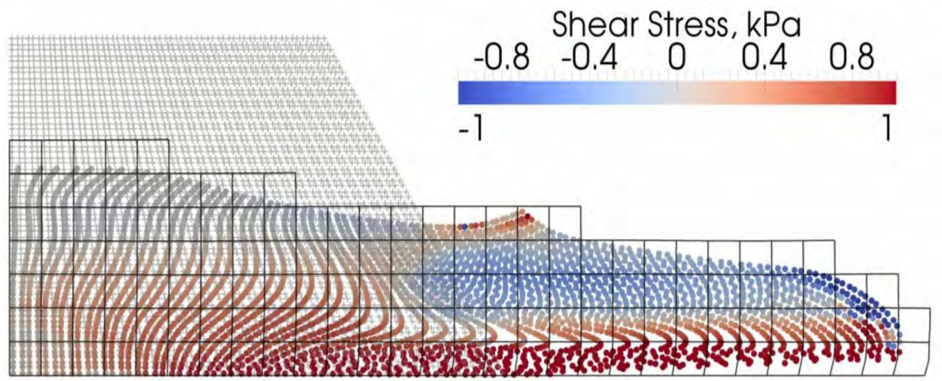


FEM

- Mesh tangling means that only initial failures can be well represented.



- Options:
  - Remeshing
  - Material is uncoupled from mesh



MPM

- The Material Point Method uses points to represent material mass.
- The points are allowed to move through the mesh.
- The calculation mesh stays in the same position.

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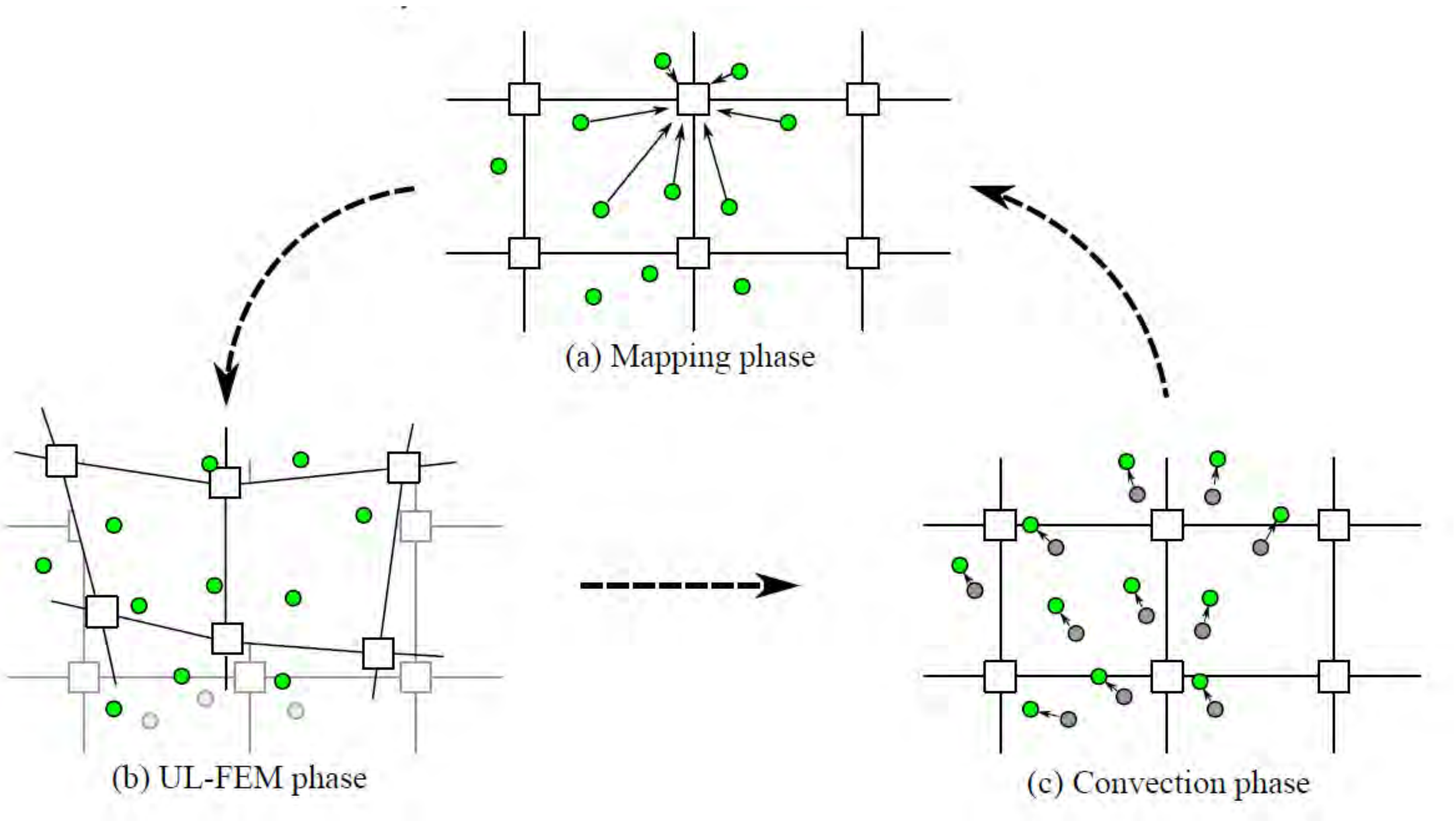
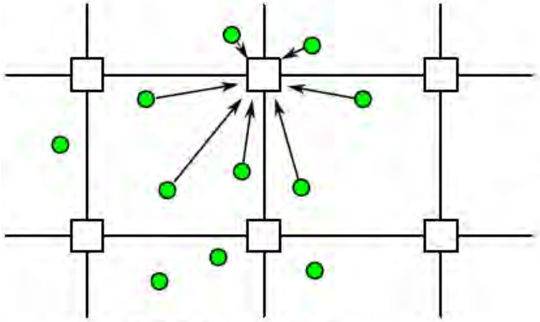
MPM



Wang, Vardon, Hicks, 2016a, Computers and Geotechnics

MPM



(a) Mapping phase

- All state variables are mapped to the nodes of the mesh.
- Various options – using the shape functions here:

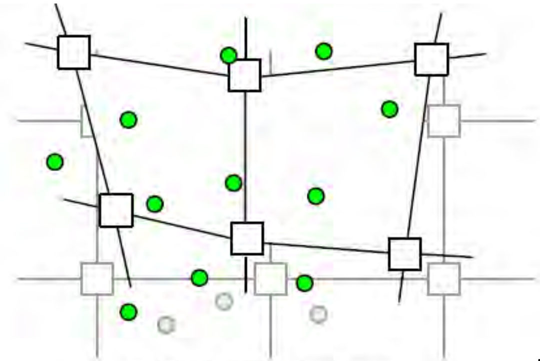
$$u_i = \sum_{p=1}^{n} u_p N_i(x_{\boldsymbol{p}}, y_{\boldsymbol{p}}) \frac{m_p}{m_i}$$

MPM

- A standard FEM calculation is undertaken.

- The area integration is carried out on the material points, instead of the Gauss Points.

- The mesh moves and the material points move with it.



(b) UL-FEM phase

- Integration is undertaken on the material points – this can makes an error:

$$\int f(x,y)dA \approx \sum_{i=1}^{N} w_i f(x_i, y_i) = \sum_{i=1}^{nmp} W_i\, f(\xi,\eta)_i \det|\mathbf{J}|_i$$

$$W_i = \frac{1}{nmp} \qquad [\mathbf{B}] = \left[\mathbf{B}(x_{\boldsymbol{p}}, y_{\boldsymbol{p}})\right]$$

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MPM

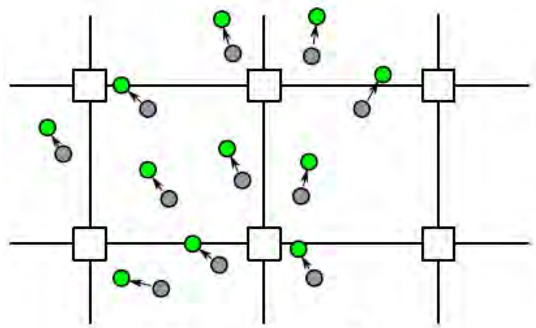
- The mesh is reset and the material points remain in the deformed locations.

- Any state variables are mapped from the nodes to the material points.

$$u_p^{t+\Delta t} = u_p^t + \delta u_p$$

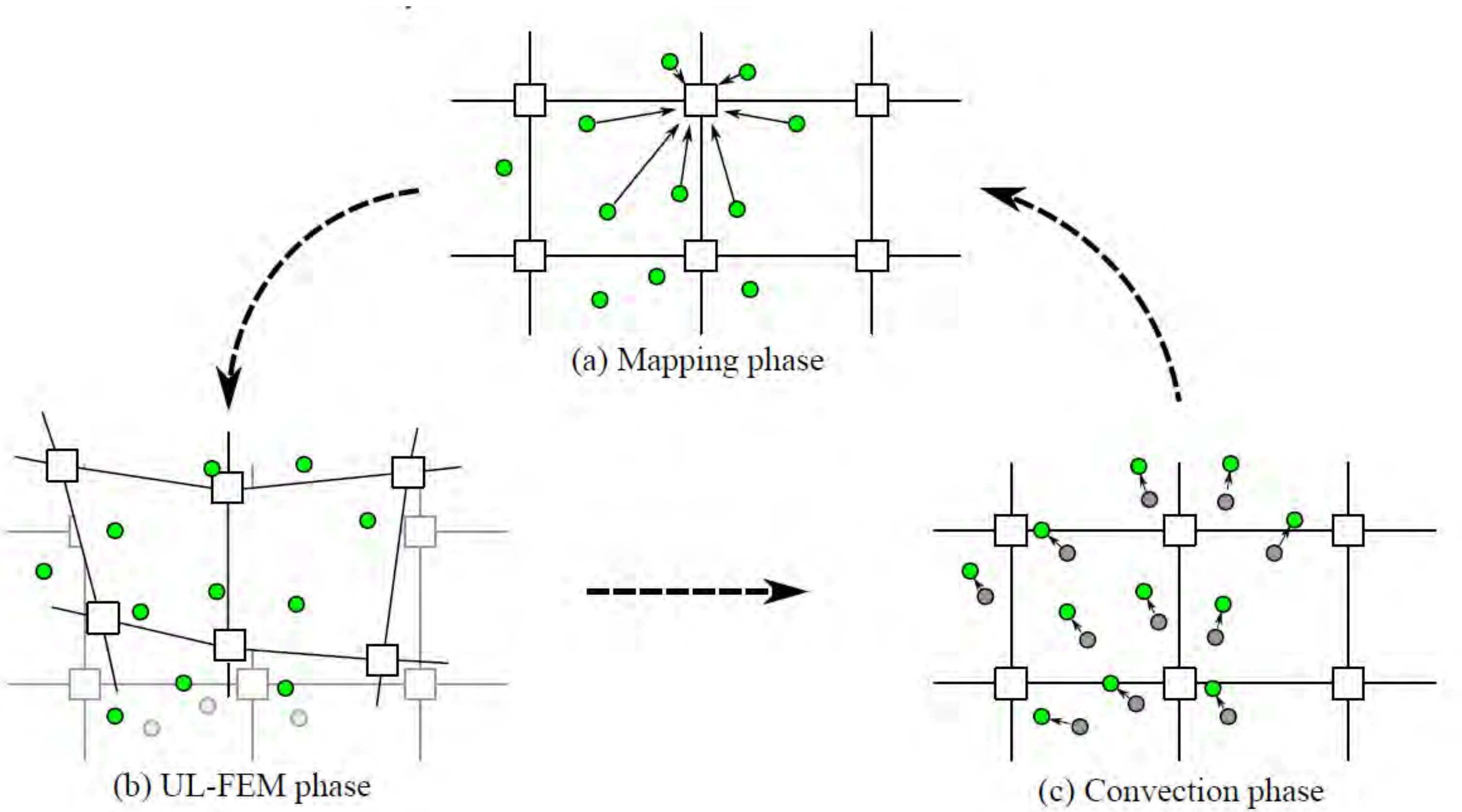
$$\delta u_p = \sum_{i=1}^{nn} u_i N_i(x_{\boldsymbol{p}}, y_{\boldsymbol{p}})$$

- Need to know in which elements the MPs
    - Housekeeping



(c) Convection phase

MPM



Differences and similarities to FEM

In essence FEM is the engine of the method.

Major differences:

- Integration is undertaken on the material points – this makes a rather large numerical approximation or error.
- Mapping backwards and forwards can lead to error accumulation.
- Boundaries and boundary conditions can be difficult to apply.
- Material volumes can be more difficult to interpret (points ≠ particles).

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Things to consider

- Is the method accurate?
    - Mesh dependant?
    - Material behaviour? As deformation progresses
    - Does it match reality?
    - Strain / stress measures – conjugate pairs
- Is the method usable?
    - Times to run? Mapping?
    - Contact?

Things to consider

- Is my formulation appropriate?
    - Explicit/implicit time integration
    - Quasi-static vs. dynamic
    - Total stress vs. hydro-mechanical
    - Etc...

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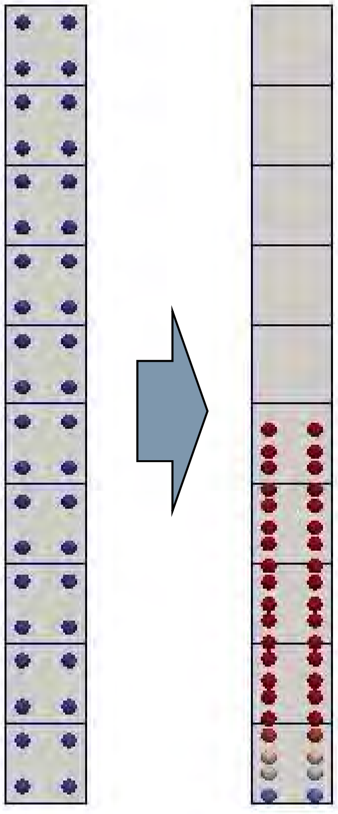
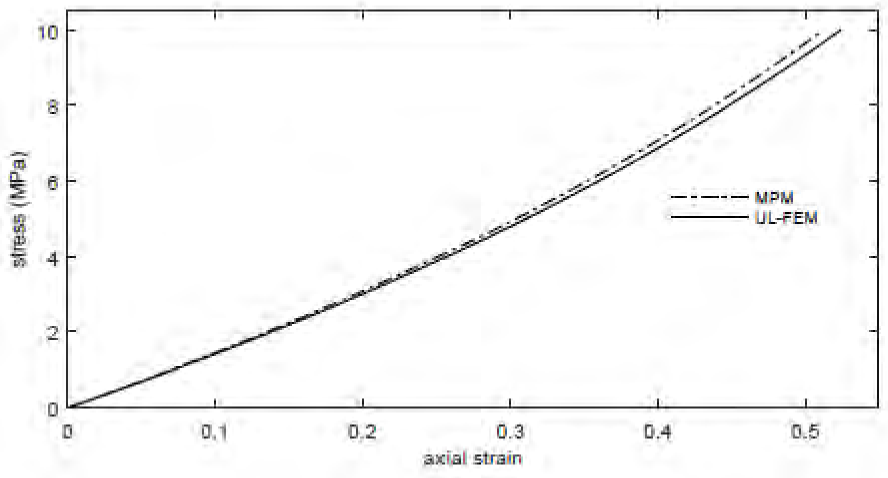
Examples

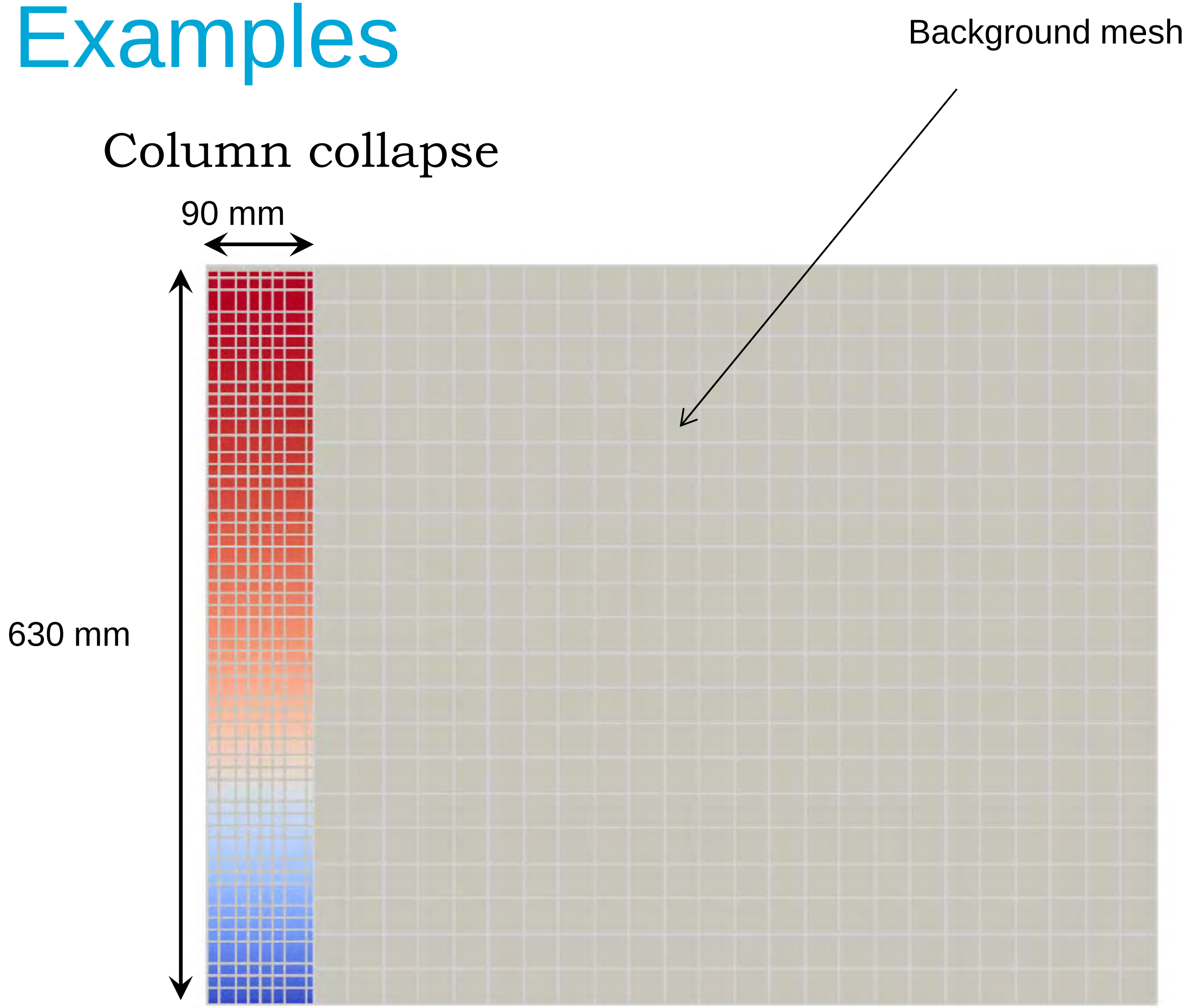
1D column compression





Examples

Column collapse



- 4 noded squares
- 2268 material points
- Mohr-Coulomb material model

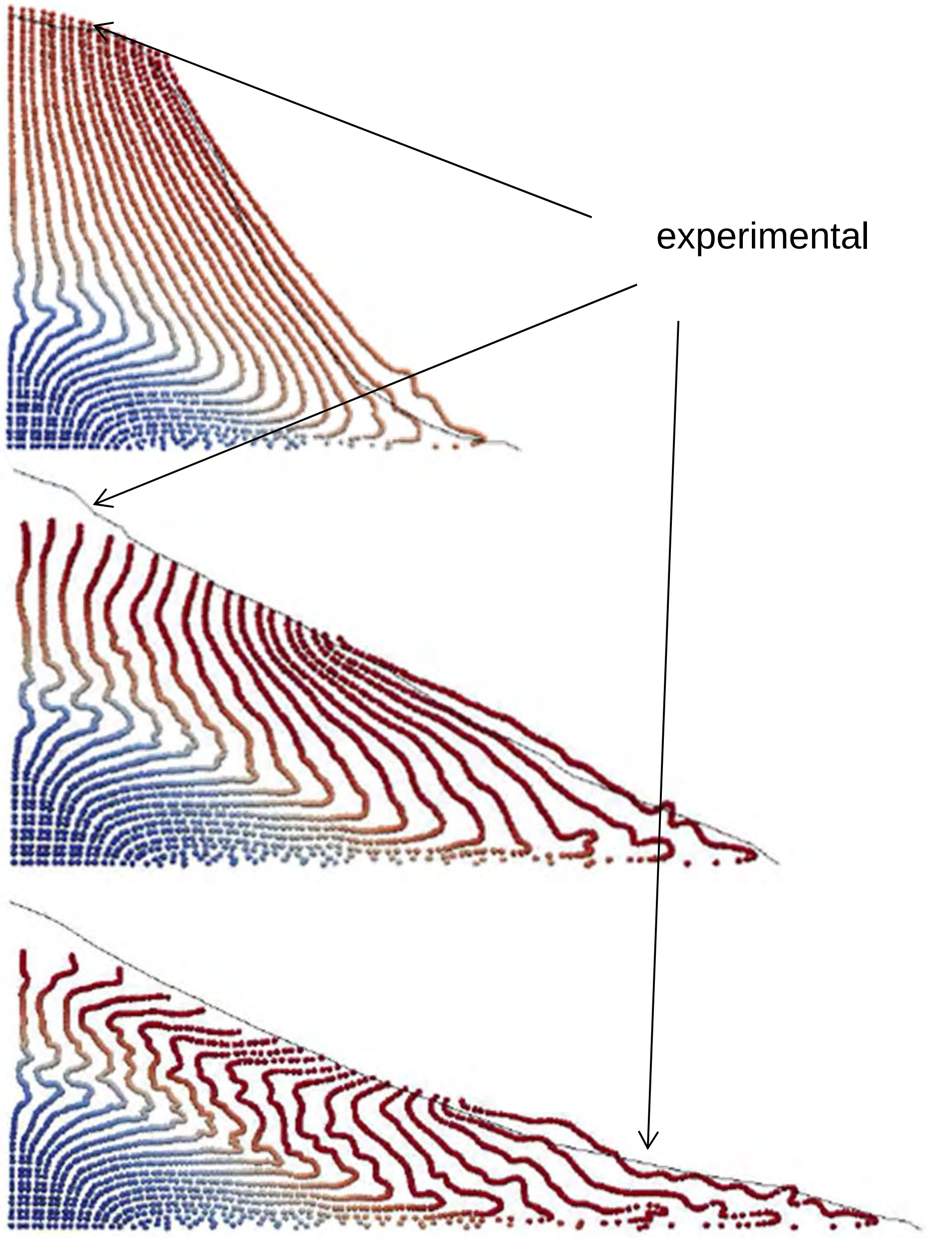
Examples

Column collapse

0.25 seconds (0.30 exp.)

0.375 seconds (0.42 exp.)

0.50 seconds (0.55 exp.)



Examples

Slope failure

- Slope failure is a major geotechnical problem.
- Initial failure, i.e. Factor of Safety is typically calculated.
- This does not tell us anything about what happens afterwards
  - Small deformation, or
  - Huge landslide

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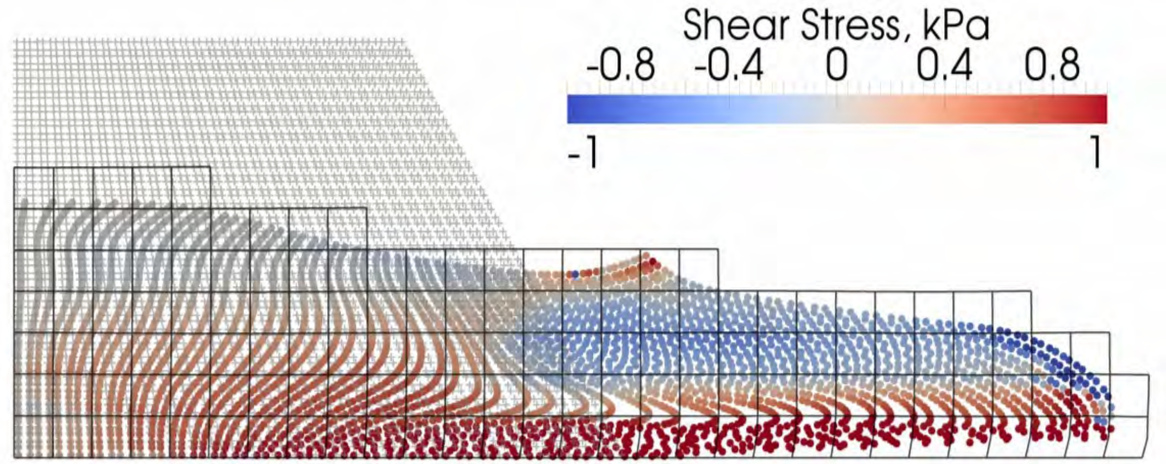
Examples

Slope failure – progressive and retrogressive

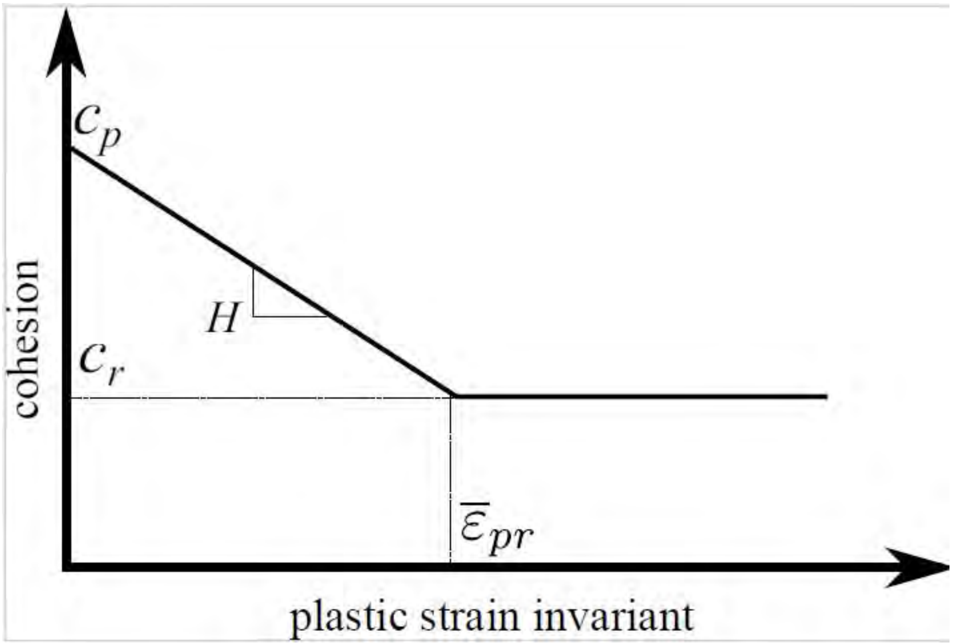
Need to have constitutive behaviour which incorporates post-failure behaviour – Mohr Coulomb model gives:



Examples

Slope failure – progressive and retrogressive

Need to have constitutive behaviour which incorporates post-failure behaviour – in this case strain softening.

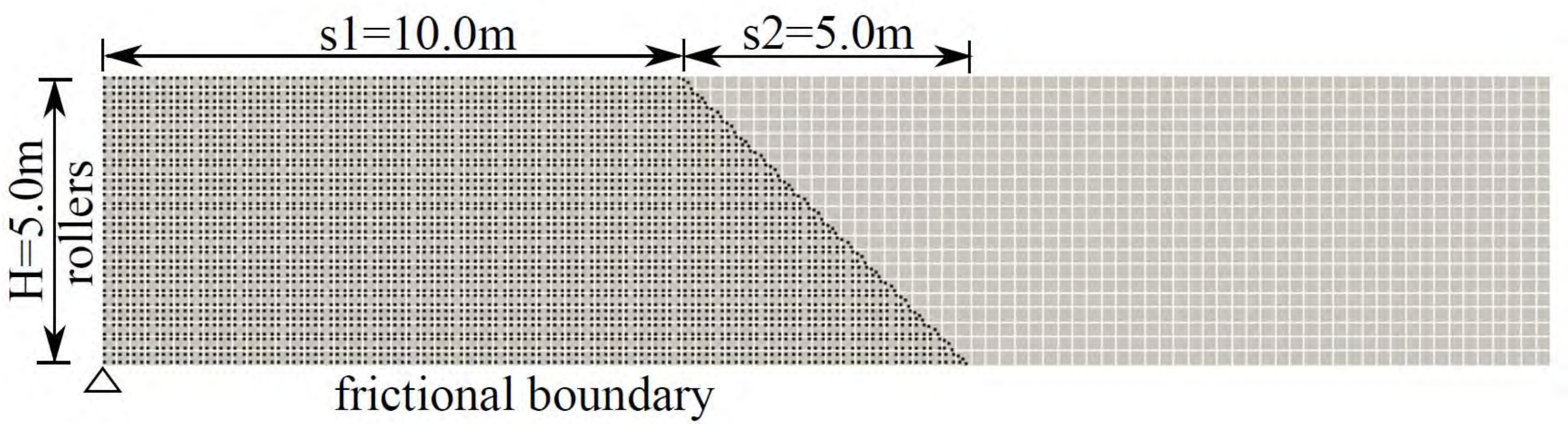


Examples

Slope failure – progressive and retrogressive

- Short slope – embankment

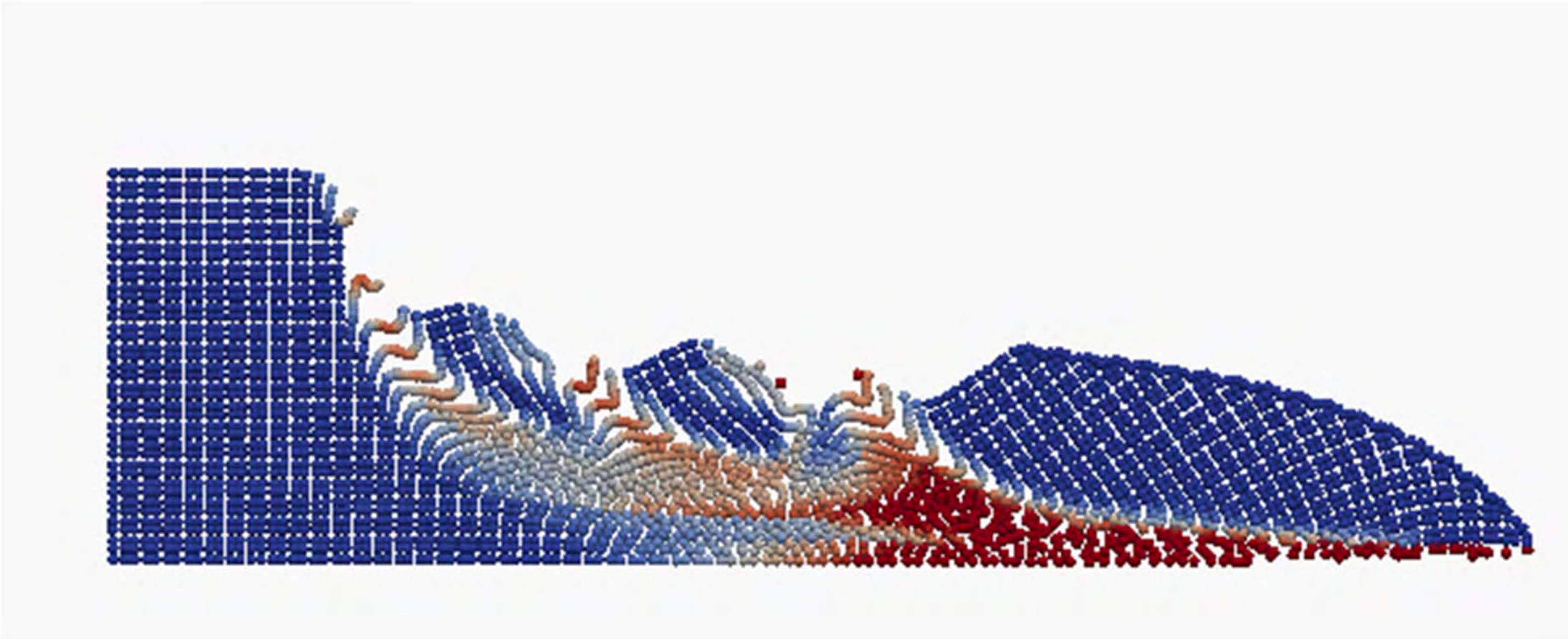
- Undrained clay behaviour
- Initially unstable slope (FoS = 0.96)
- Mesh 40 x 80 4 noded elements, 4 mps per element



Wang, Vardon, Hicks, 2016b, Computers and Geotechnics

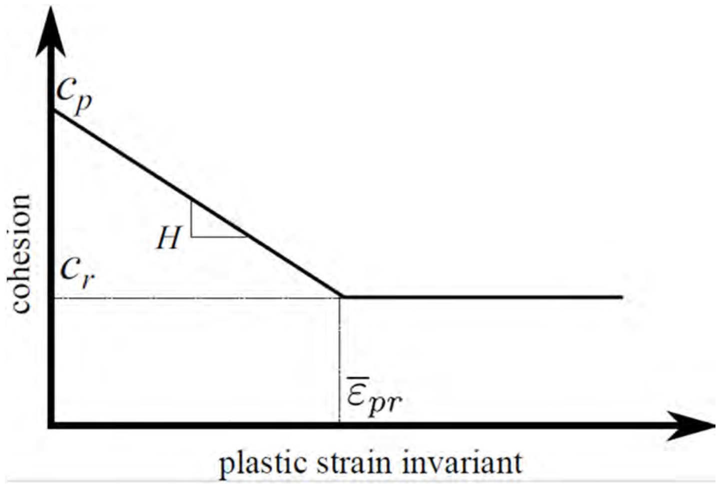
Examples

Slope failure – progressive and retrogressive



Examples

Slope failure – progressive and retrogressive



(a) $t = 1.5$ s

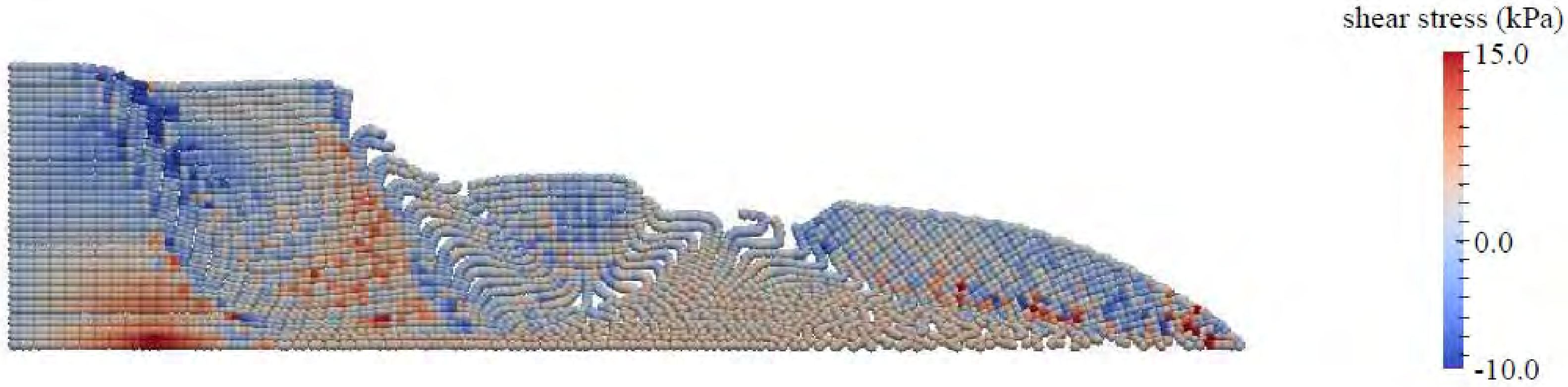
(b) $t = 2.5$ s

(c) $t = 40.5$ s

Examples

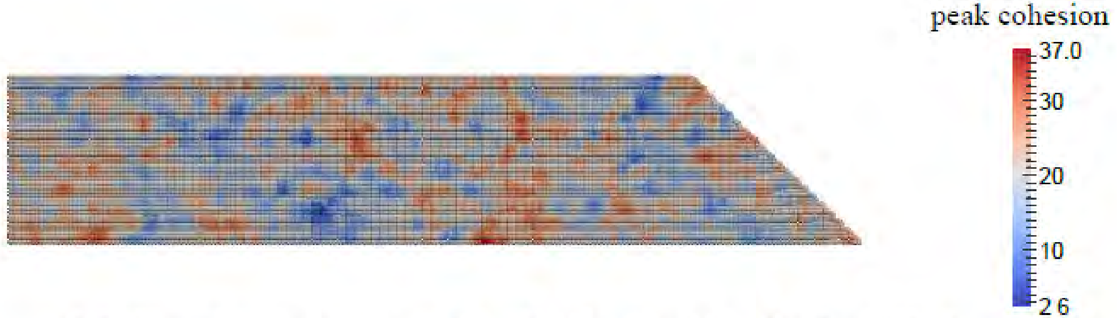
Slope failure – progressive and retrogressive

- Stresses can oscillate due to non-ideal stress locations.

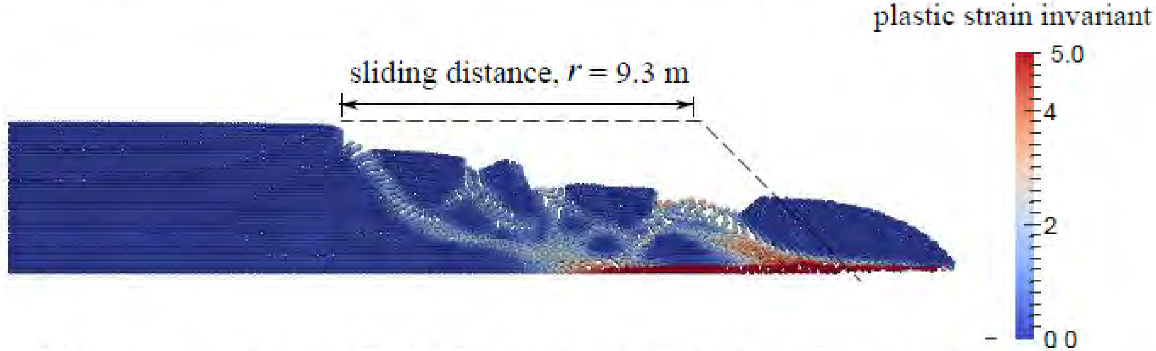


Examples

Slope failure – spatially variable materials



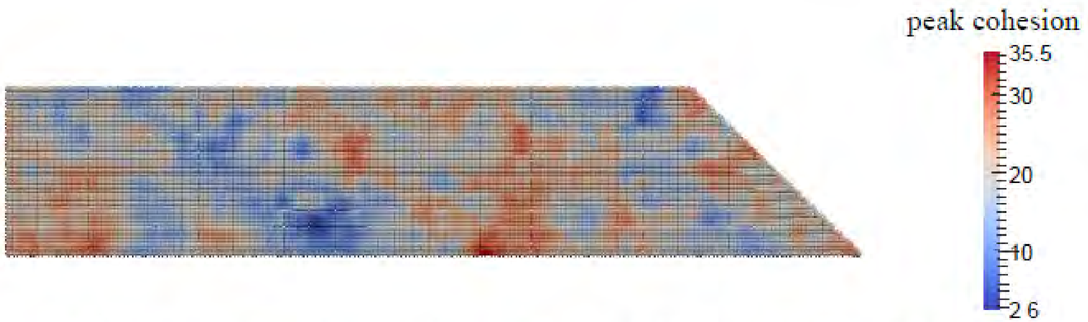
(a) $\theta_h = \theta_v = 1.0$ m, random field of peak cohesion (kPa)



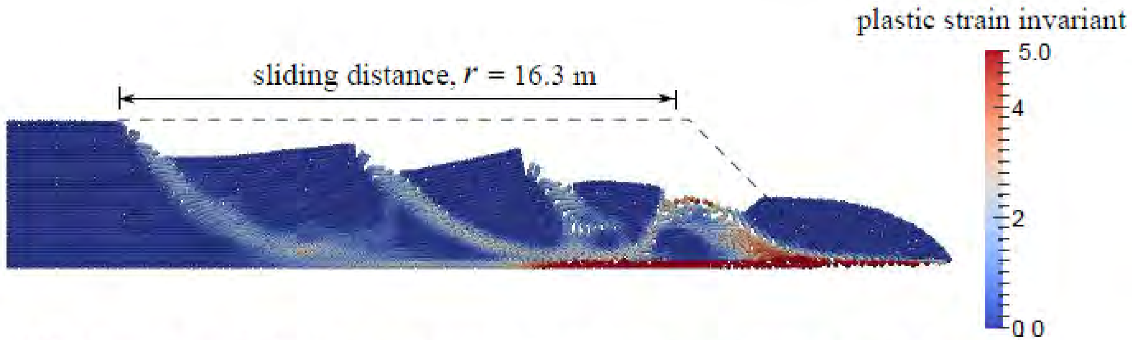
(b) $\theta_h = \theta_v = 1.0$ m, final plastic shear strain invariant contours

Examples

Slope failure – spatially variable materials



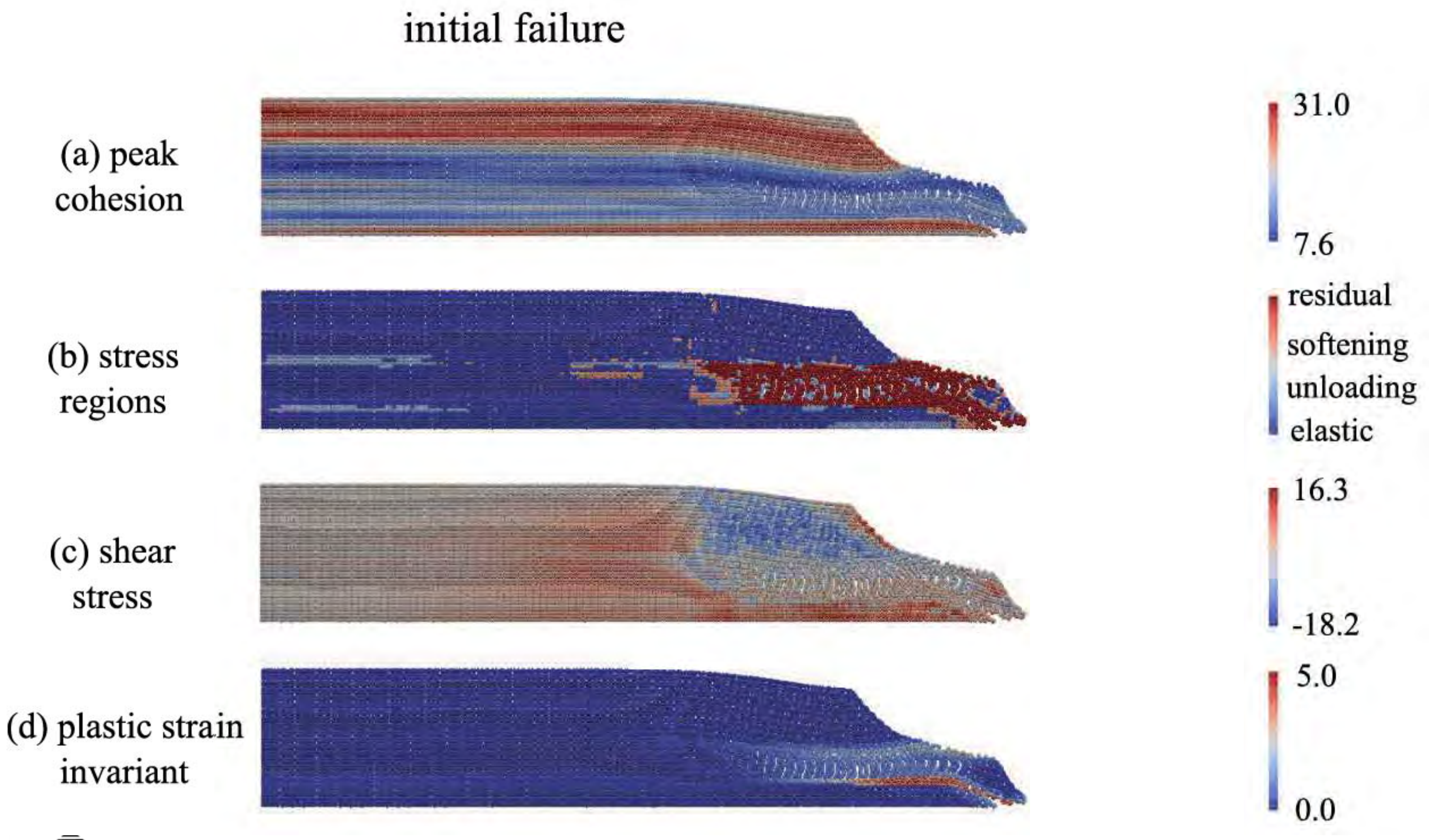
(c) $\theta_h = \theta_v = 2.5$ m, random field of peak cohesion (kPa)



(d) $\theta_h = \theta_v = 2.5$ m, final plastic shear strain invariant contours

Examples

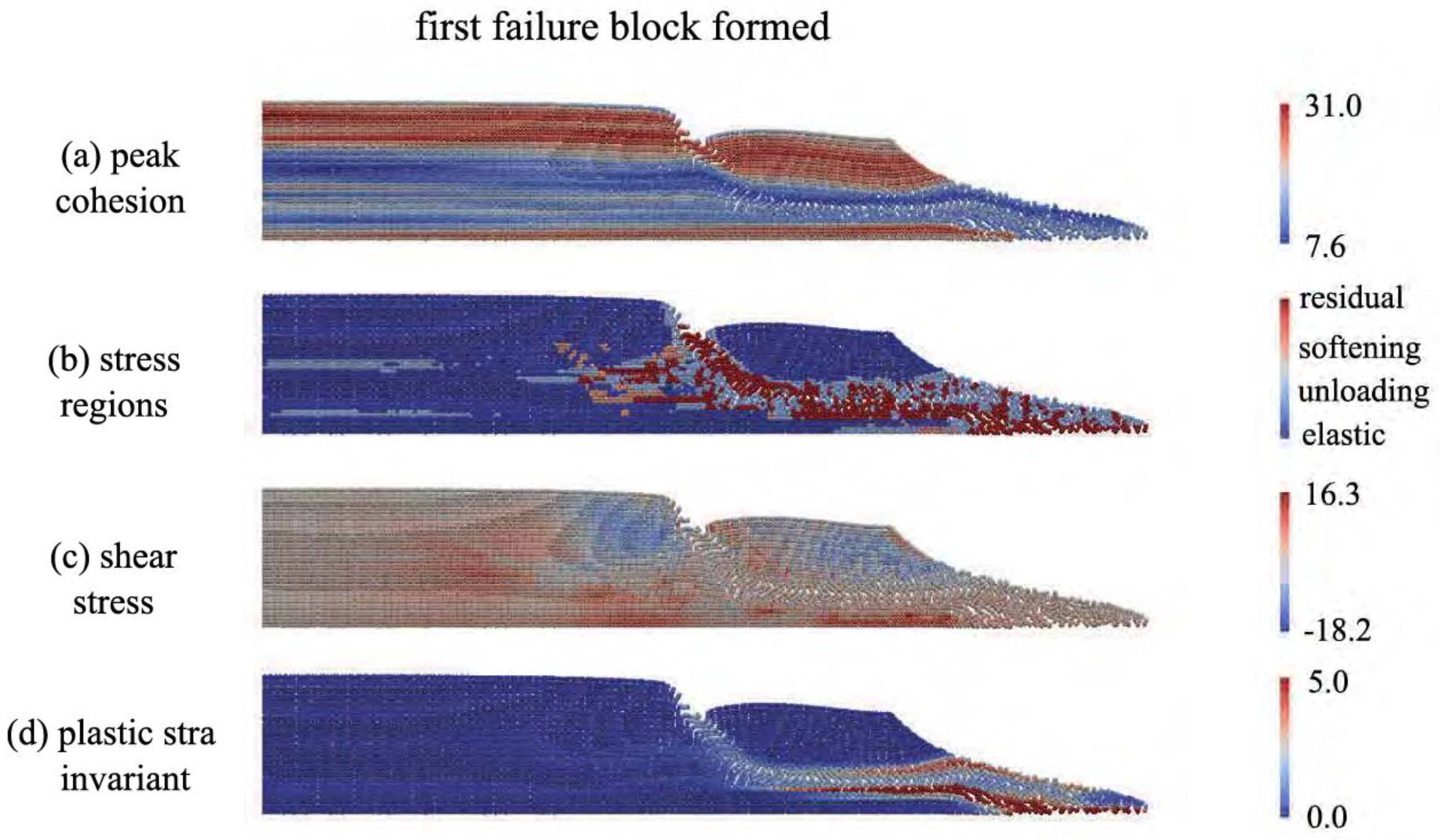
Slope failure – spatially variable materials



Wang, Hicks, Vardon, 2016, Géotechnique Letters

Examples

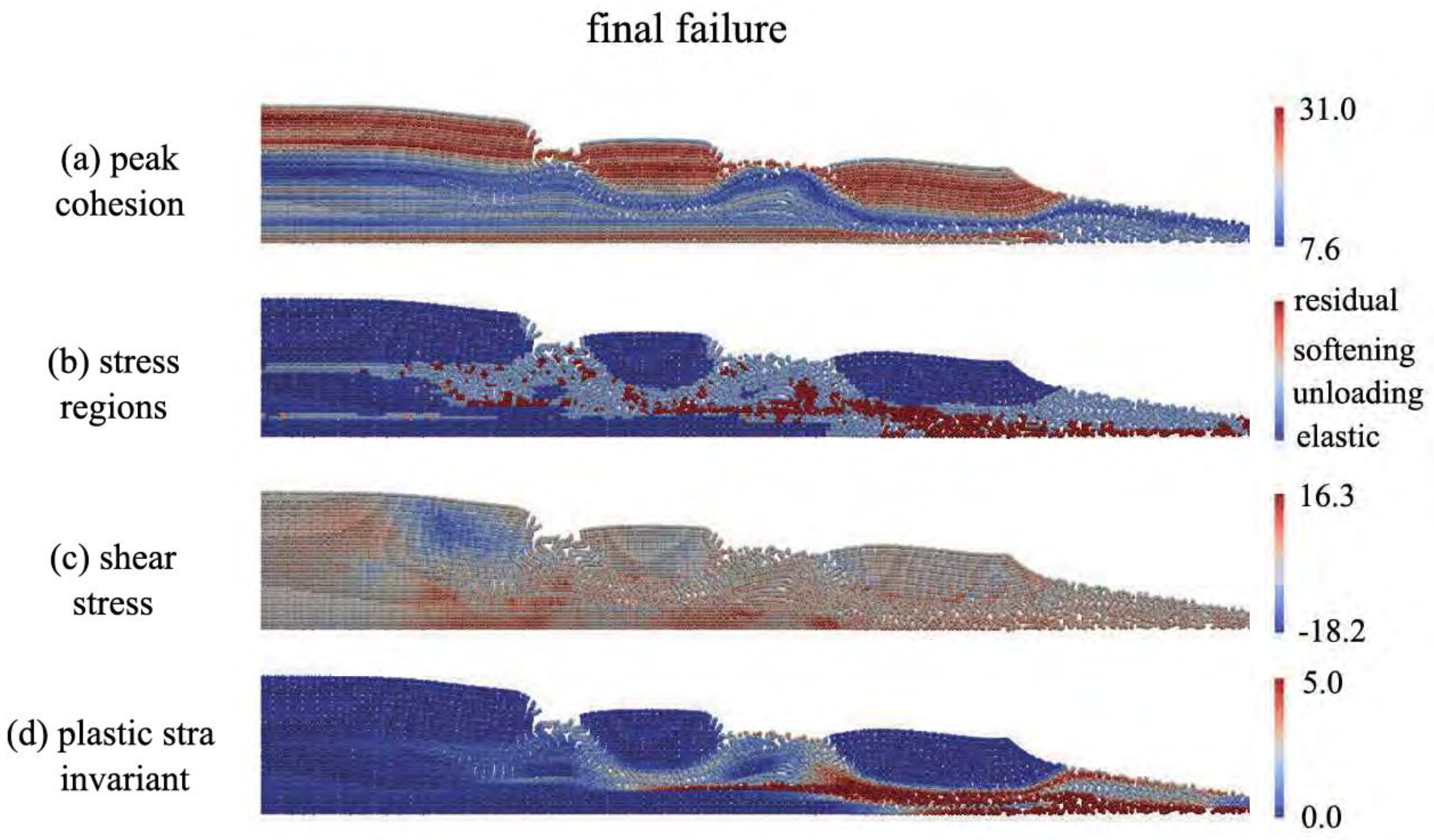
Slope failure – spatially variable materials



Wang, Hicks, Vardon, 2016, Géotechnique Letters

Examples

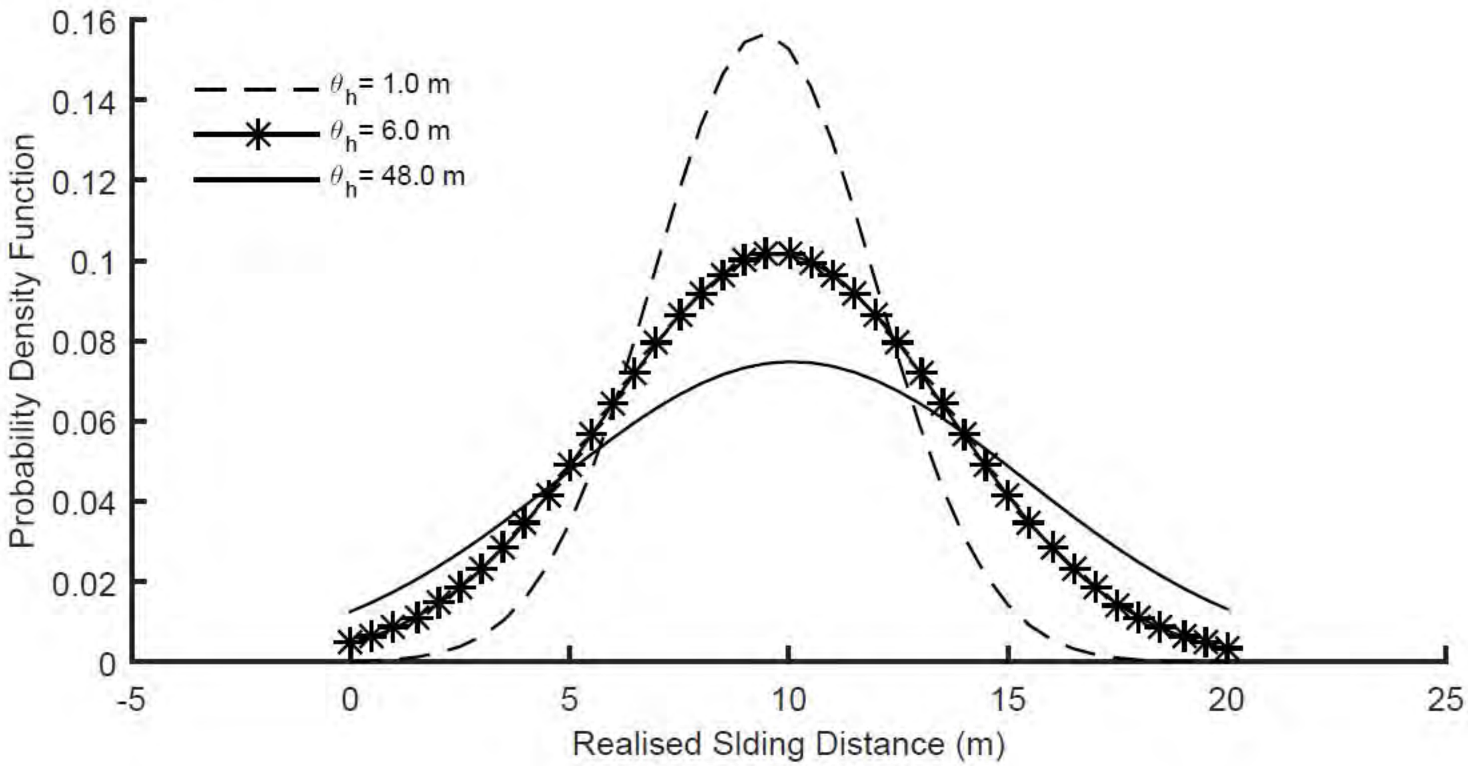
Slope failure – spatially variable materials



Wang, Hicks, Vardon, 2016, Géotechnique Letters

Examples

Slope failure – spatially variable materials

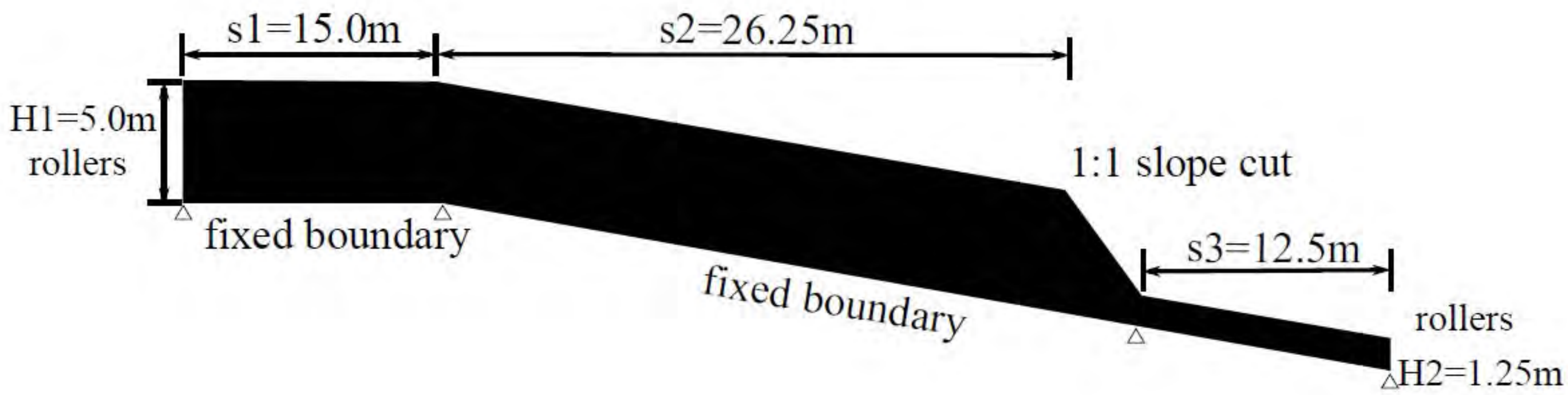


Wang, Hicks, Vardon, 2016, Géotechnique Letters

Examples

Slope failure – progressive and retrogressive

- Long slope – landslides
- ~15,000 material points

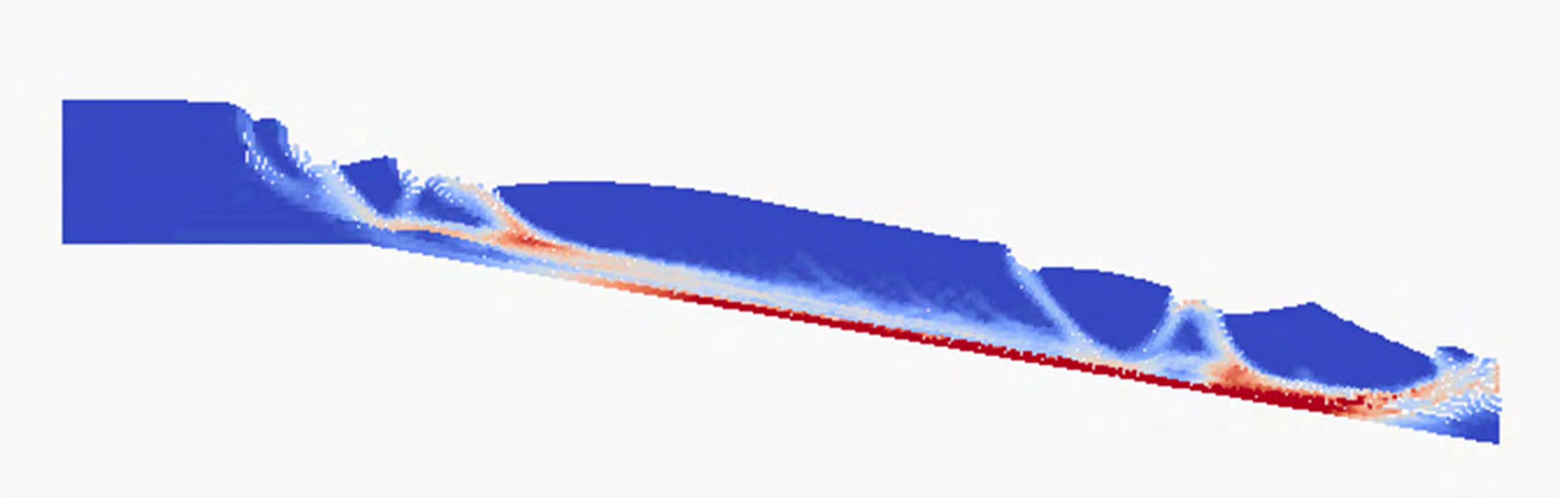


Wang, Vardon, Hicks, 2016b, Computers and Geotechnics

Examples

Slope failure – progressive and retrogressive

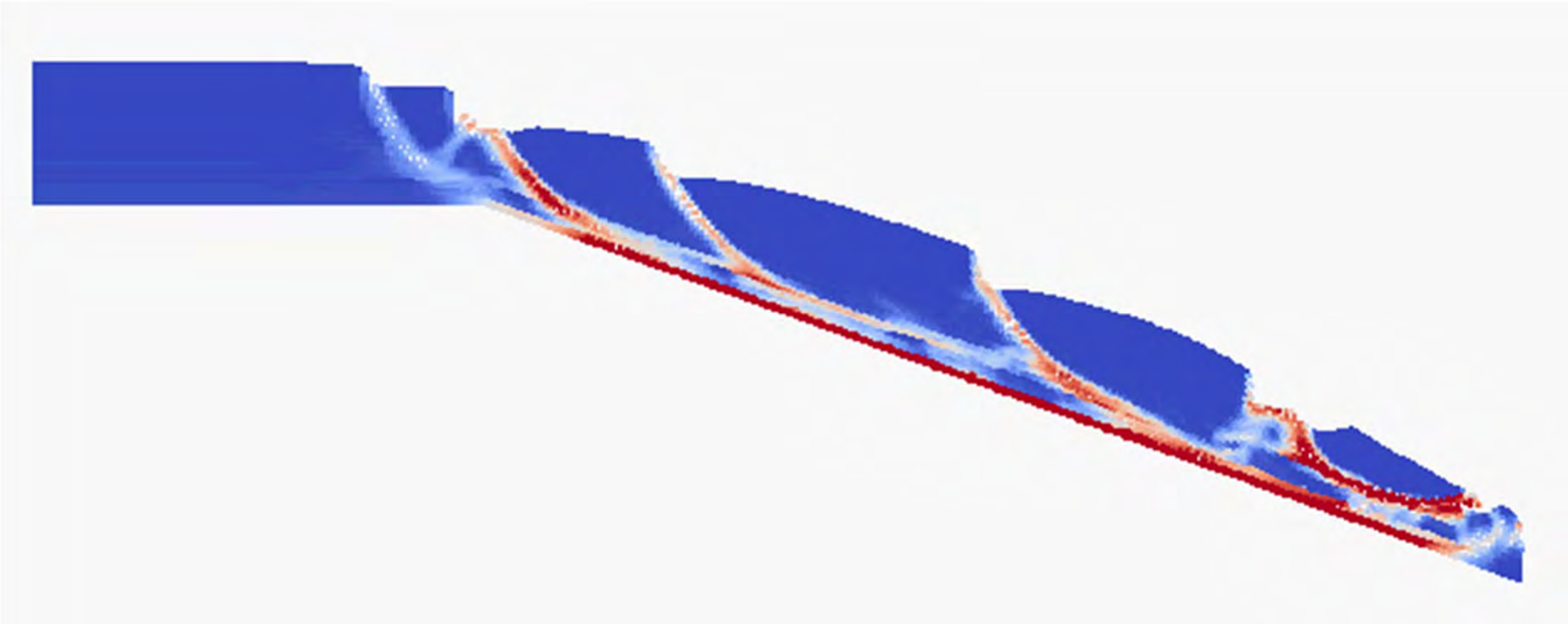
- 10° slope



Examples

Slope failure – progressive and retrogressive

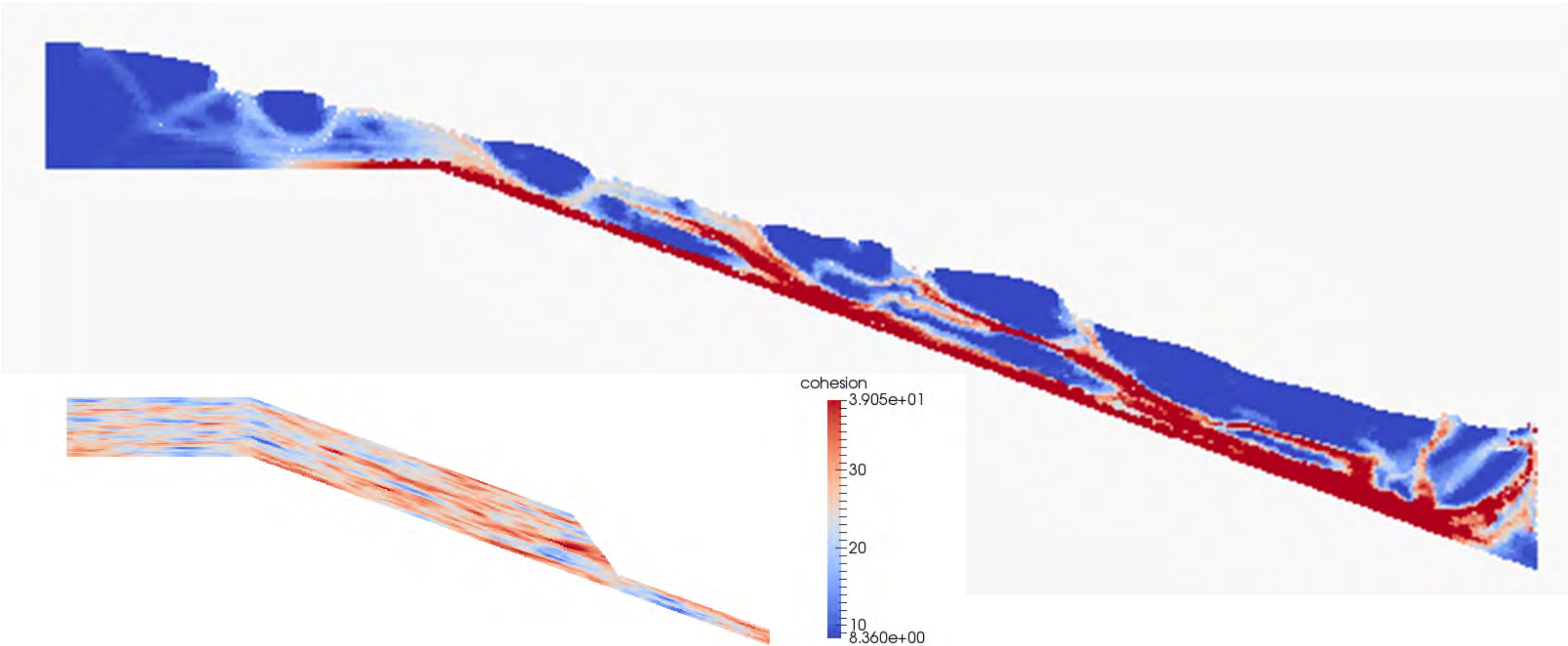
- 20° slope



Examples

Slope failure – progressive and retrogressive

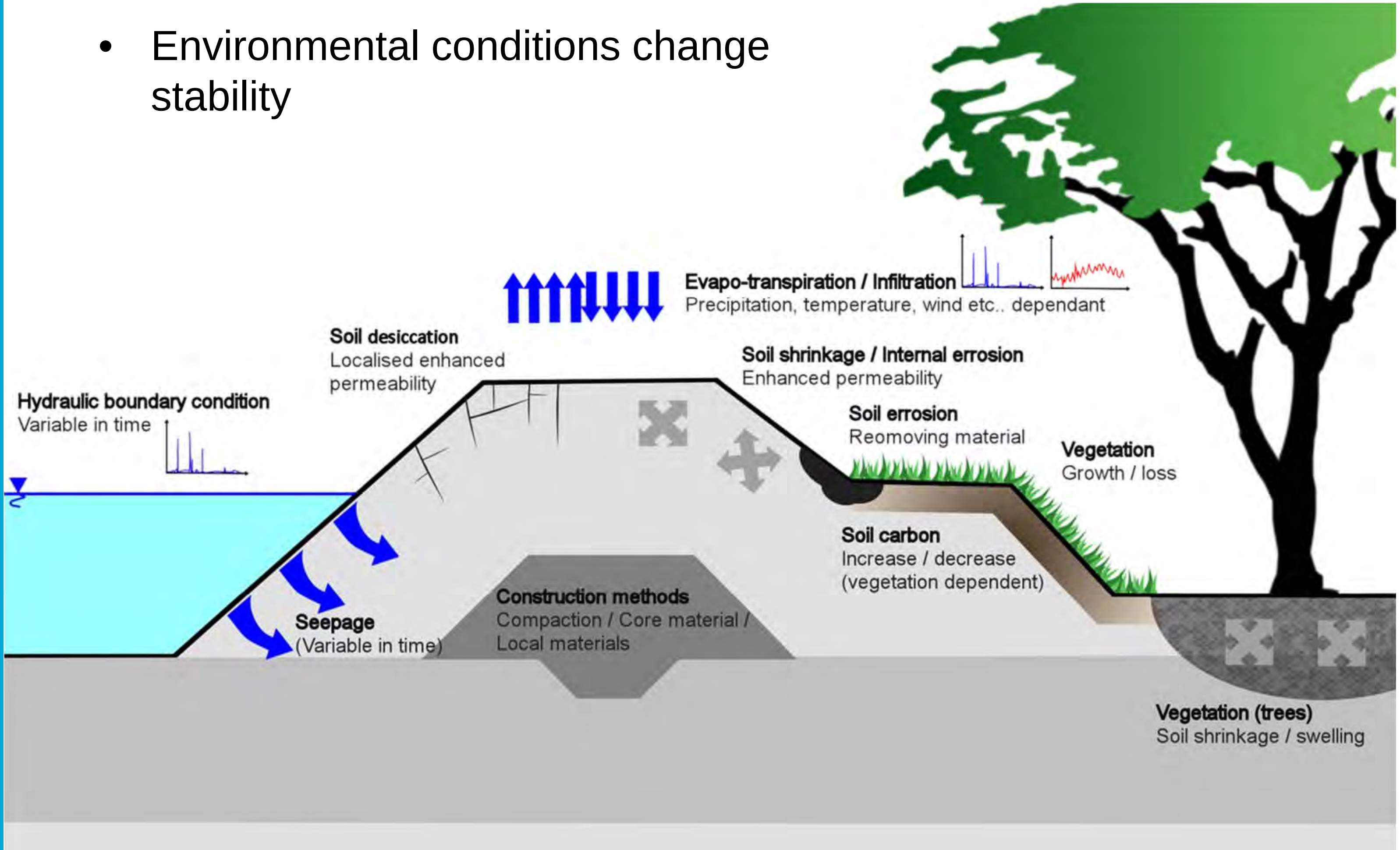
- 20° slope – heterogeneous material properties



Wang, Hicks, Vardon, 2016, Géotechnique Letters

Environmental slope processes

- Initially stable slopes
- Environmental conditions change stability



Vardon, 2015, Environmental Geotechnics

Examples

Rainfall induced slope failure

- Coupled HM behaviour is needed.

- Use momentum conservation for the dynamic problem:

- Conservation of water momentum

$$\rho_w \mathbf{a}_w = \nabla p_w + \rho_w \mathbf{b} - \frac{n S_w \mu_w}{k} \cdot (\mathbf{v}_w - \mathbf{v}_s)$$

- Conservation of mixture momentum

$$(1-n)\rho_s \mathbf{a}_s + n S_w \rho_w \mathbf{a}_w = \nabla \cdot \boldsymbol{\sigma} + (1-n)\rho_s \mathbf{b} + n S_w \rho_w \mathbf{b}$$

- Bishop’s stress as the effective stress measure

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}' + \mathbf{m}(\chi p_w)$$

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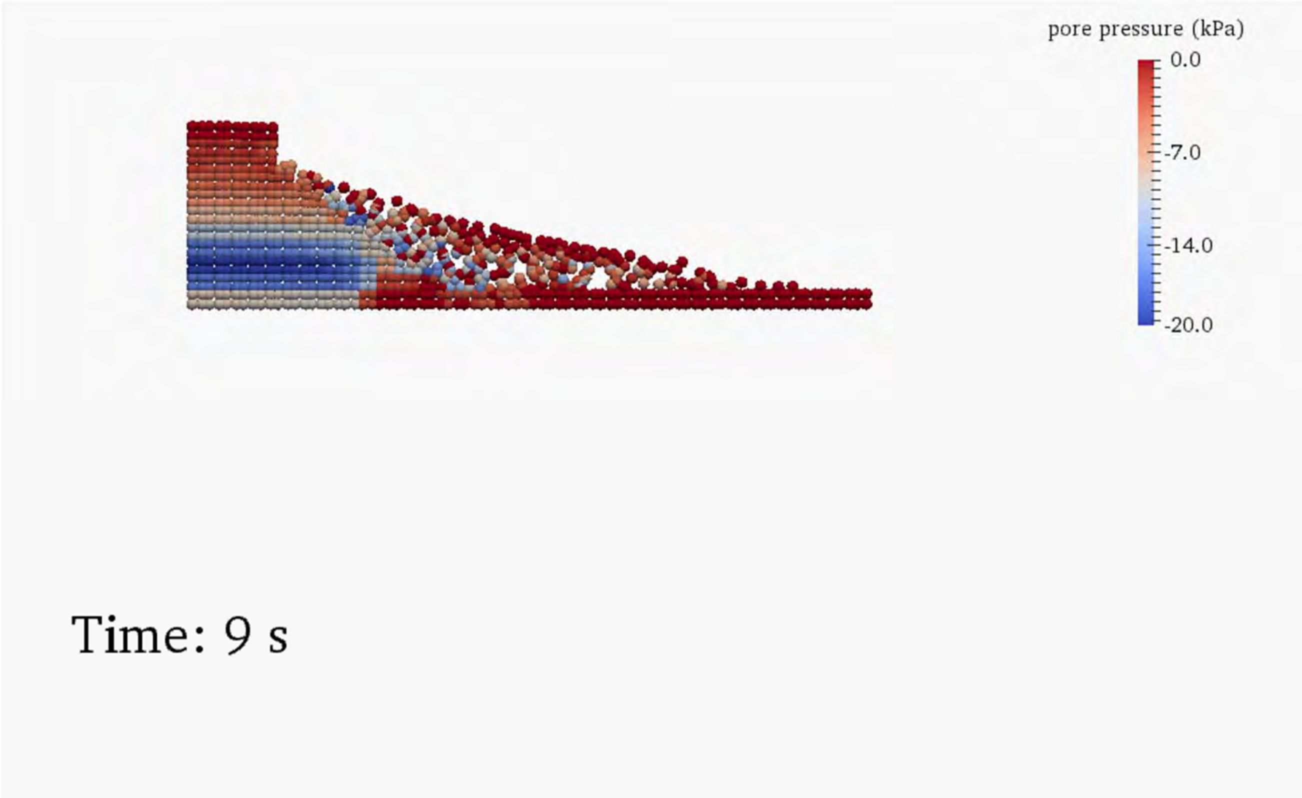
Examples

Rainfall induced slope failure

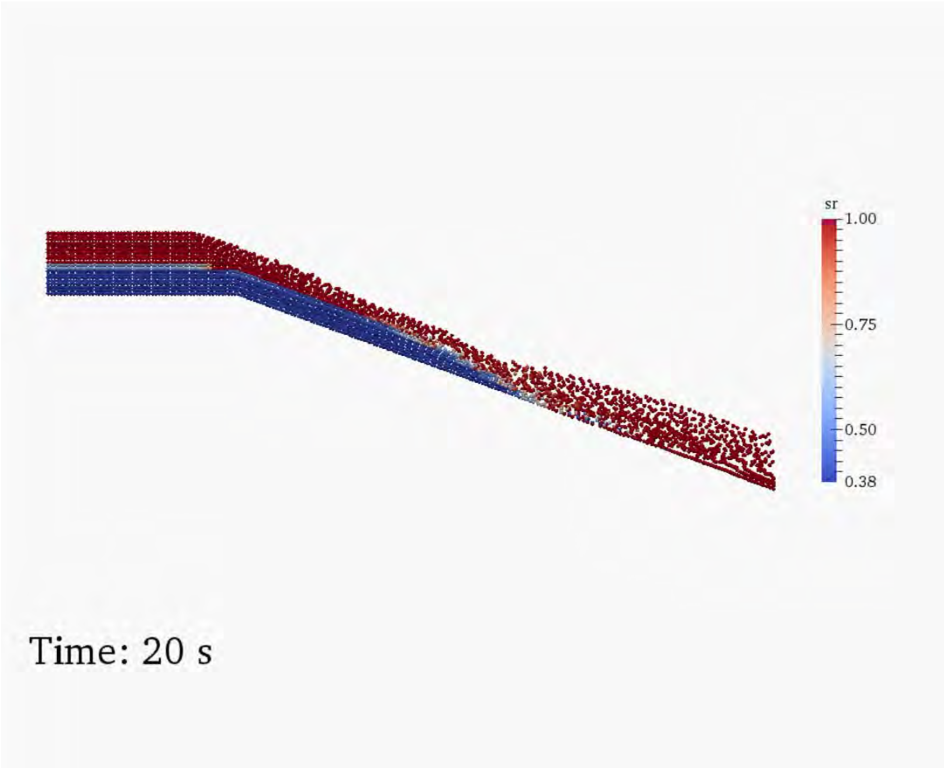


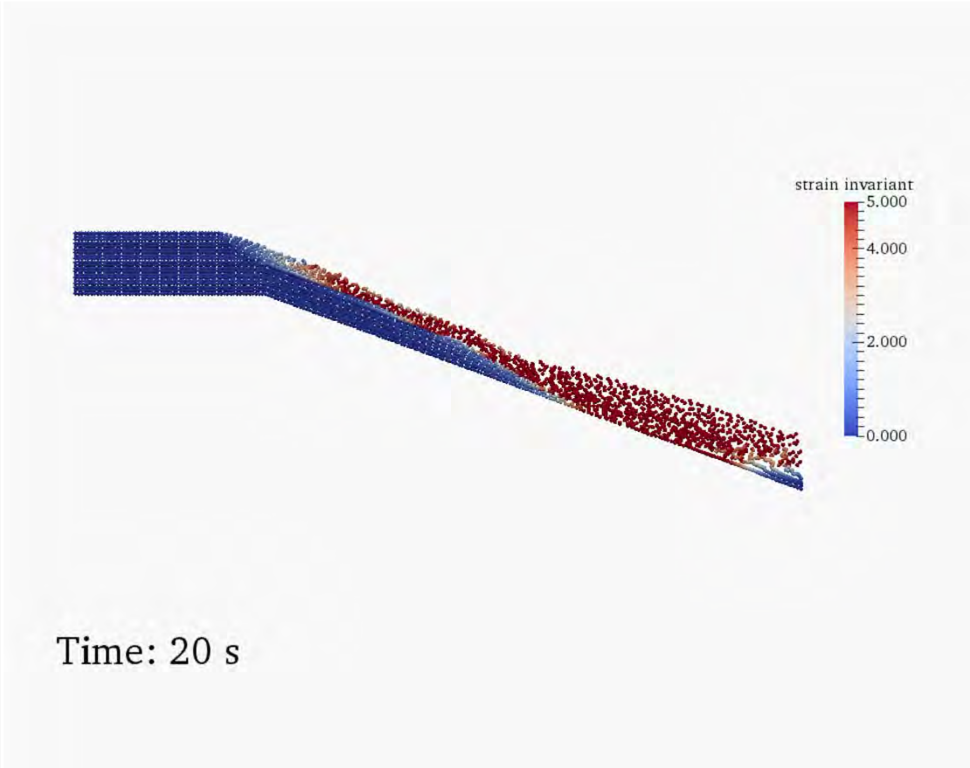
Examples

Rainfall induced slope failure - 20°

Degree of saturation

Plastic strain invariant (shear)





Challenges and future

- Stress oscillations and complex constitutive modelling.
- Interaction with other materials – contact.
- Setting boundary conditions well, where material boundaries do not coincide with elements.

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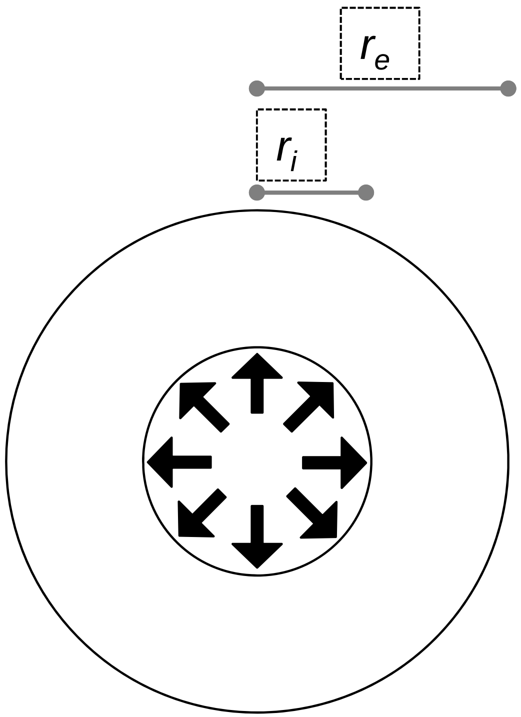
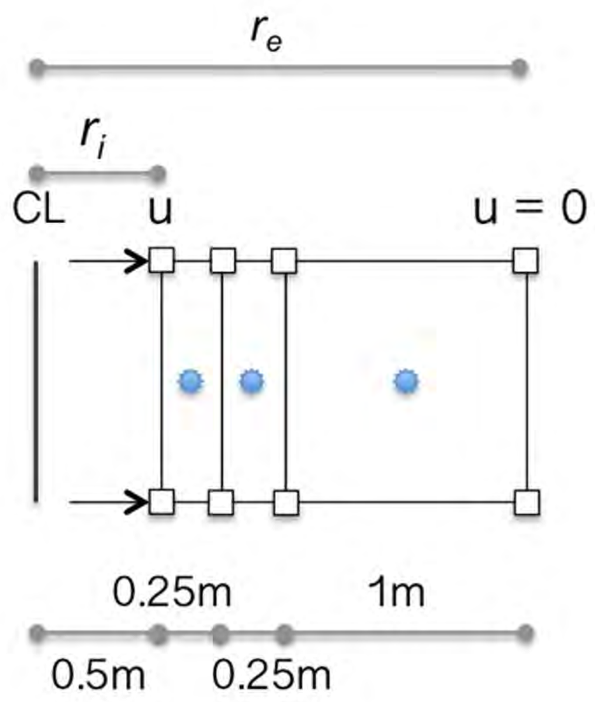
Challenges and future

Stress oscillations - example



Top view



Side view

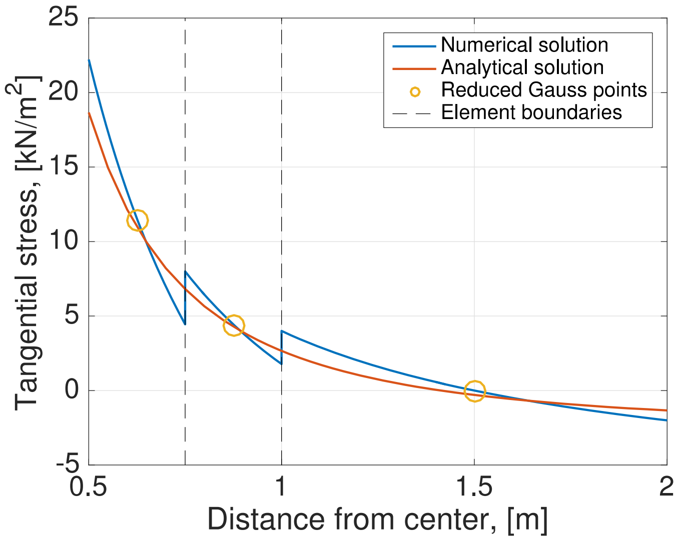
Pruijn, 2016, MSc thesis

Challenges and future

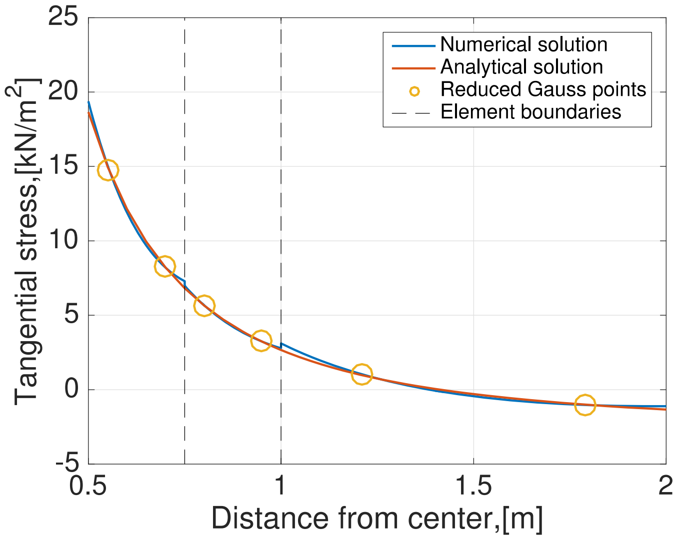
Stress oscillations - example

Tangential stresses

Linear element



Quadratic element



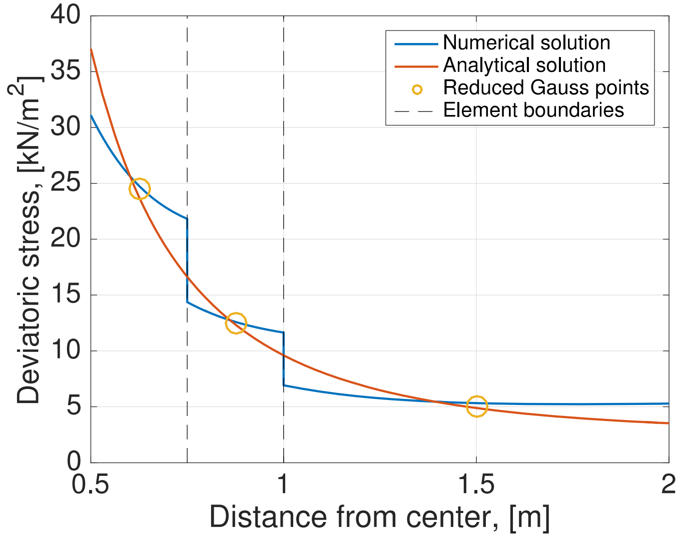
Pruijn, 2016, MSc thesis

Challenges and future

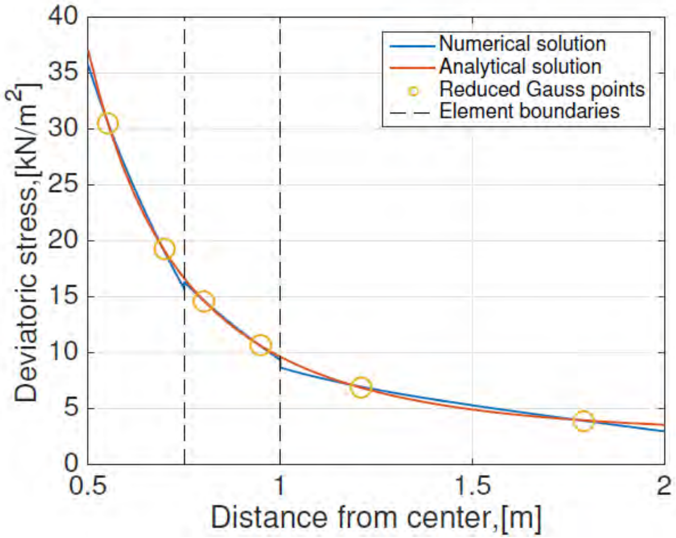
Stress oscillations - example

Deviatoric stresses

Linear element



Quadratic element



Pruijn, 2016, MSc thesis

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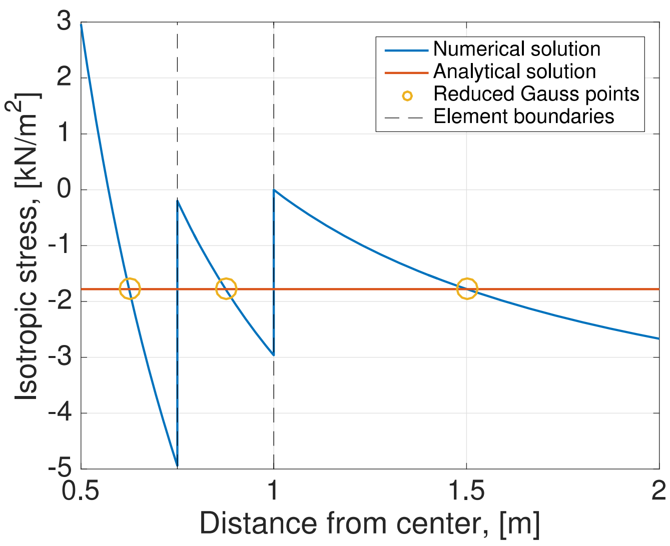
Challenges and future

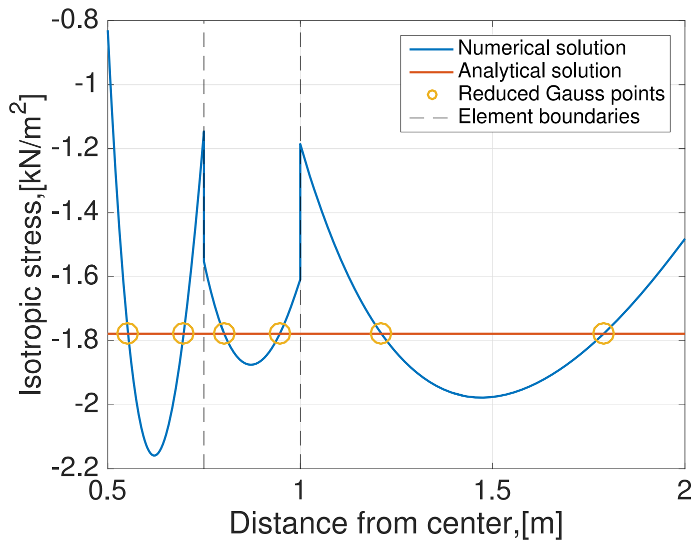
Stress oscillations - example

Isotropic stresses

Linear element

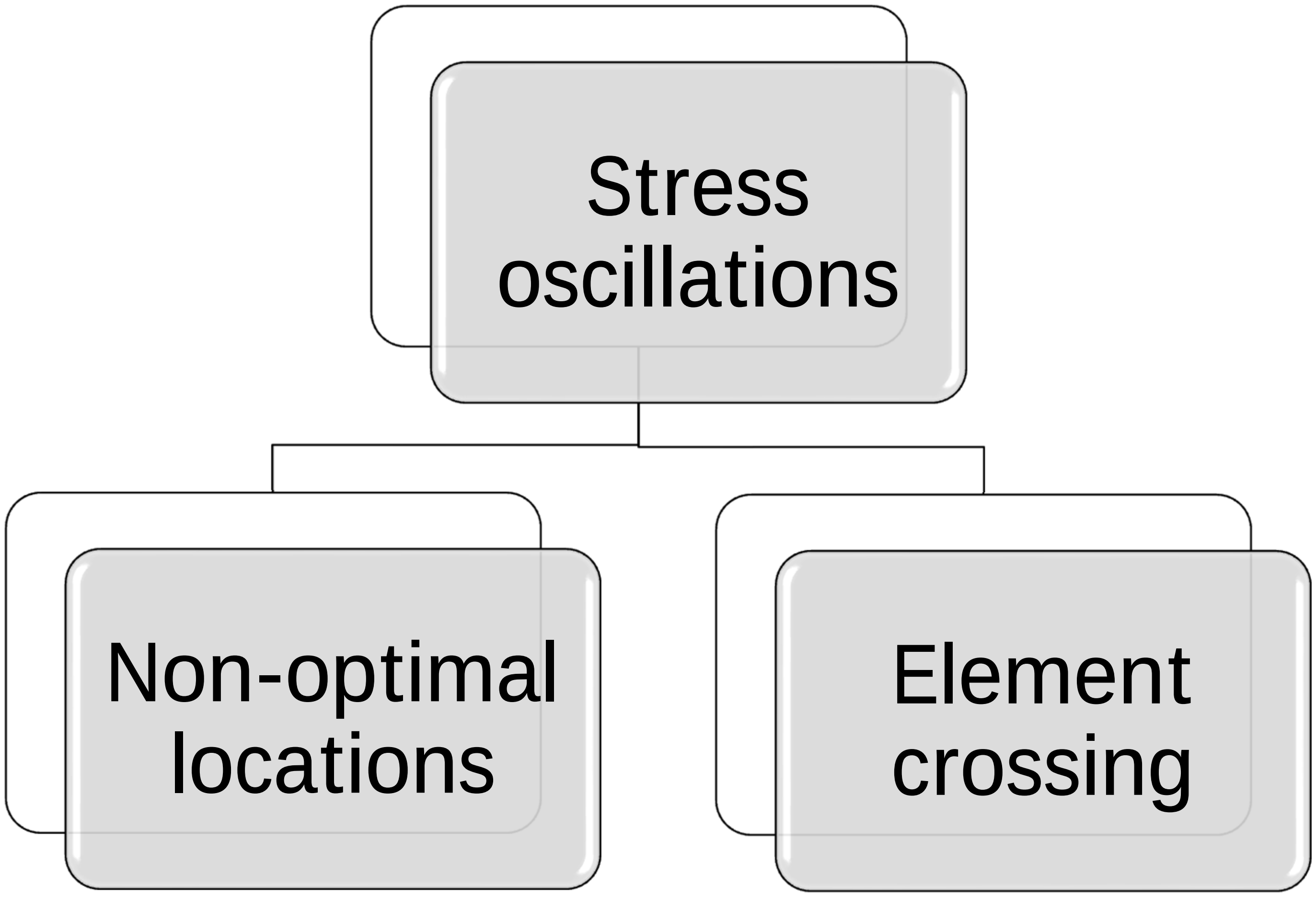


Quadratic element



This means that stress-dependent constitutive models will have problems

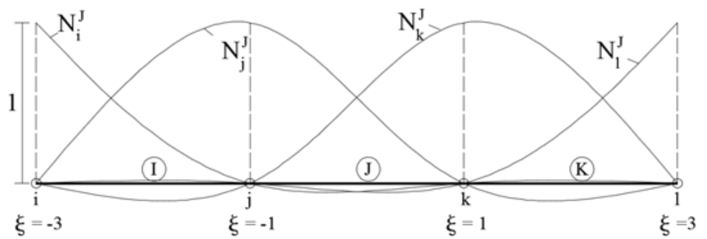
Pruijn, 2016, MSc thesis

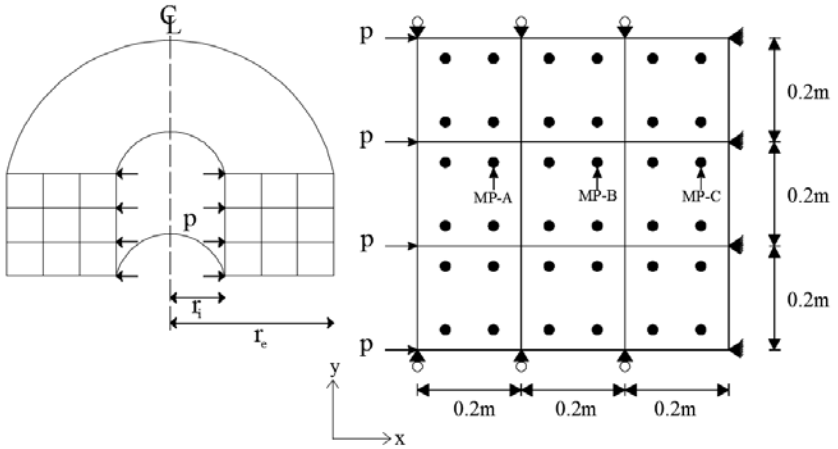

Stress oscillations
Non-optimal locations
Element crossing

Challenges and future

Stress oscillations - example

- CMPM - composite material point method
- Displacement with FEM – low order
- Stress recovery using more nodes – high order

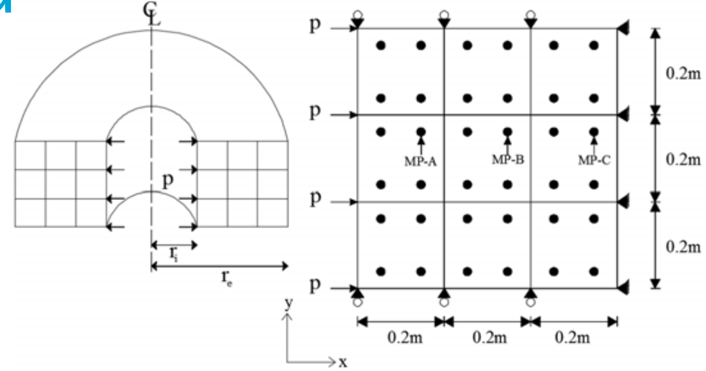


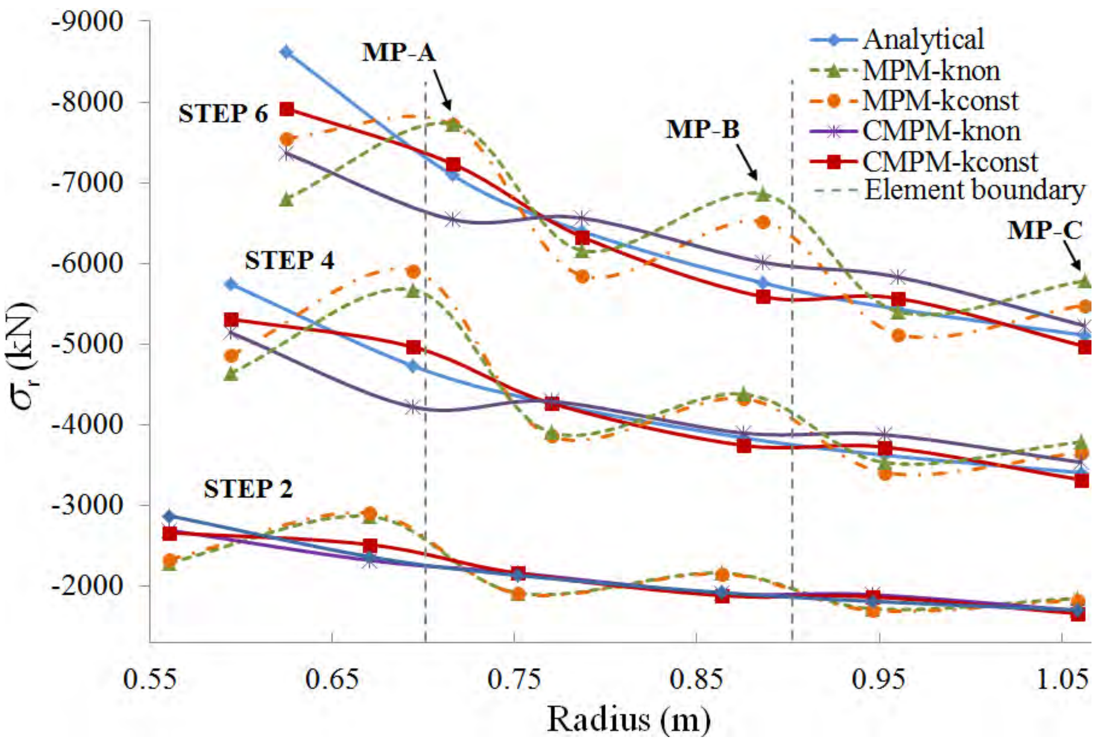


Acosta Gonzalez, Vardon, Hicks, 2017, MPM 2017

Challenges and future

Stress oscillations - example

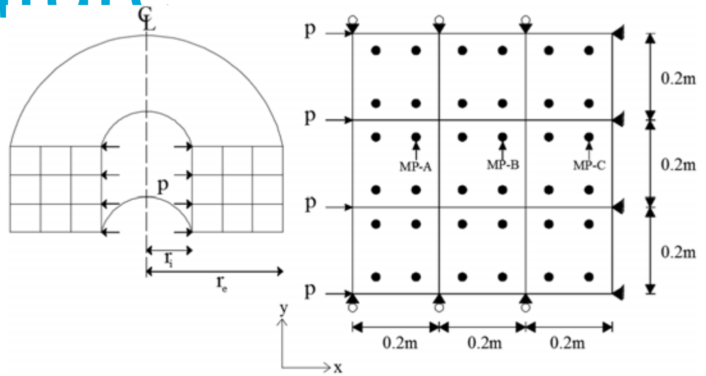


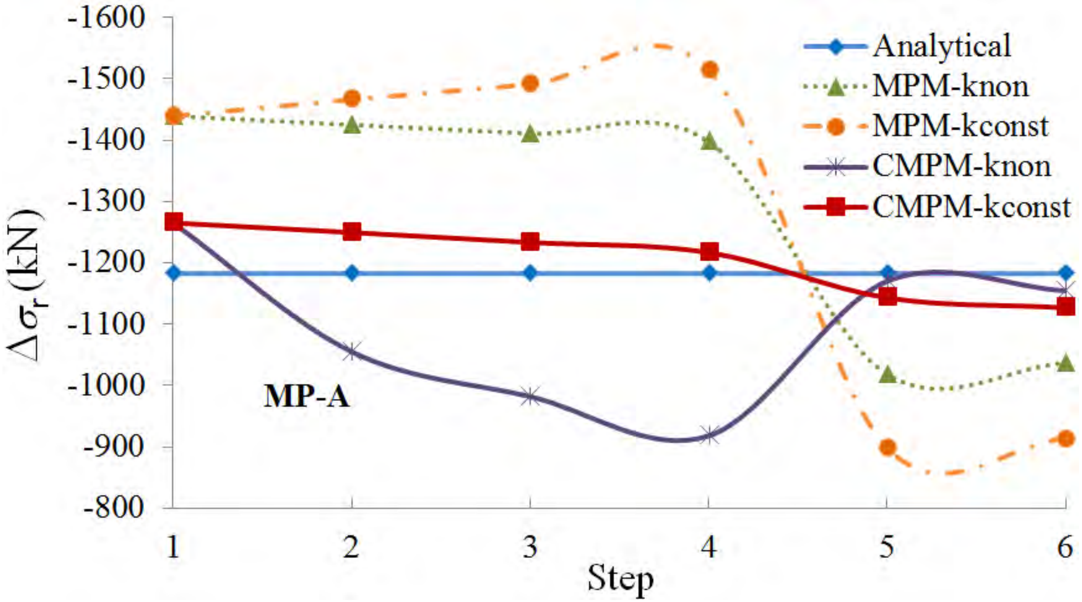


Acosta Gonzalez, Vardon, Hicks, 2017, MPM 2017

Challenges and future

Stress oscillations - example

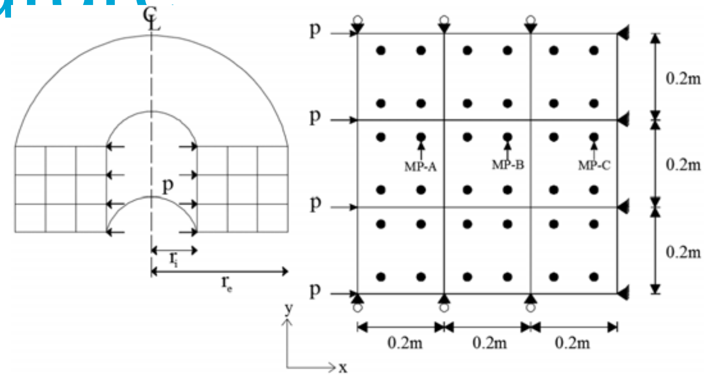


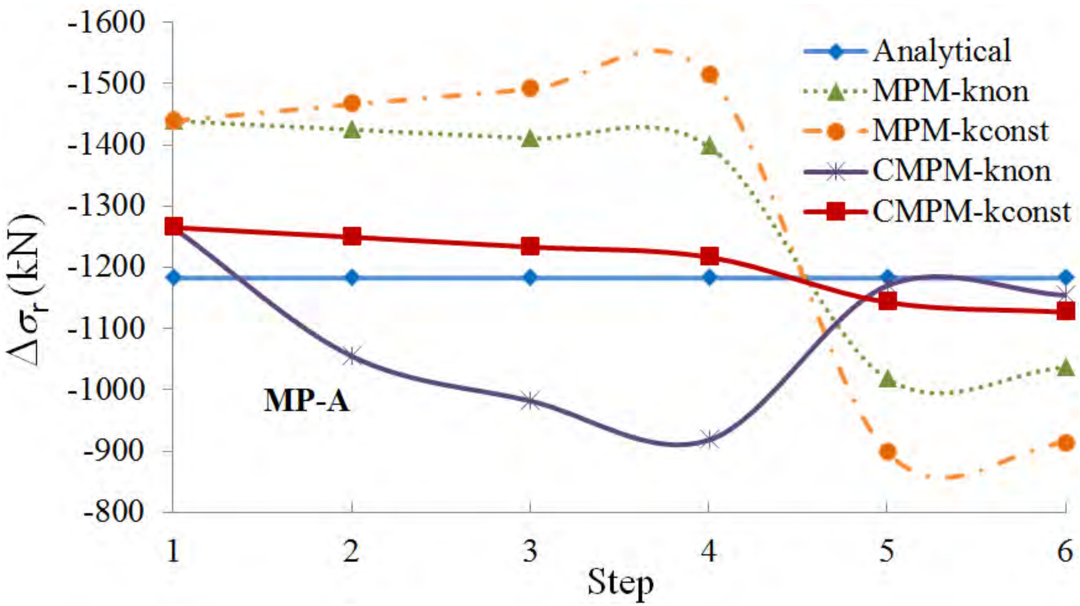


Acosta Gonzalez, Vardon, Hicks, 2017, MPM 2017

Challenges and future

Stress oscillations - example

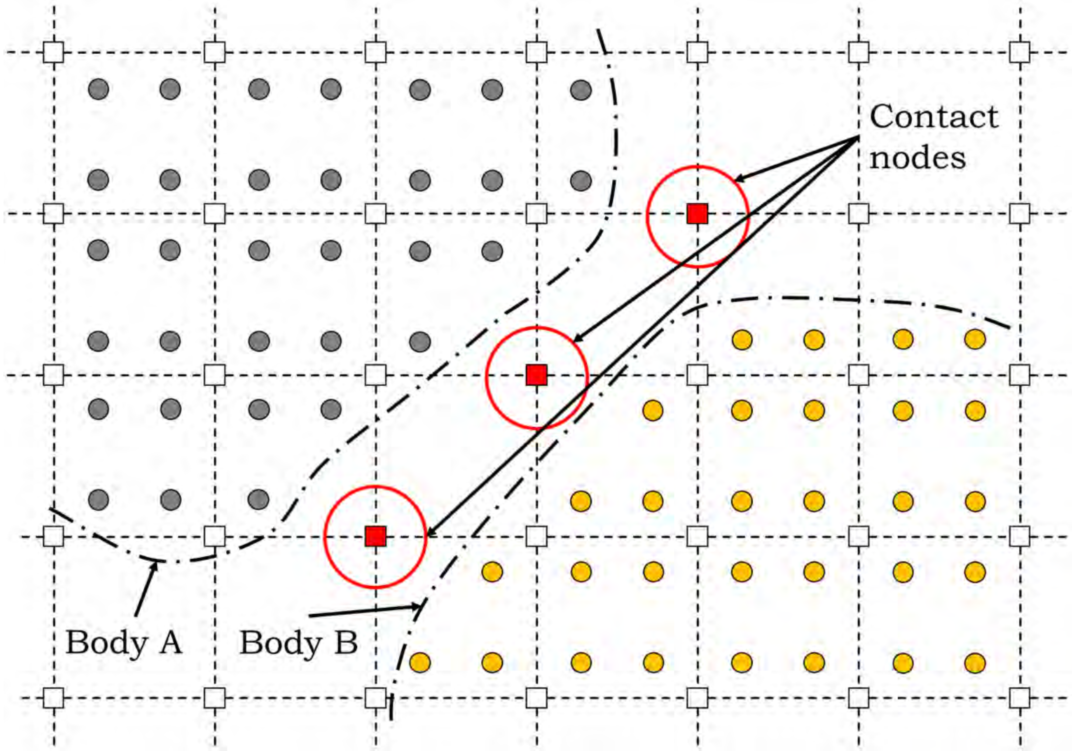




Acosta Gonzalez, Vardon, Hicks, 2017, MPM 2017

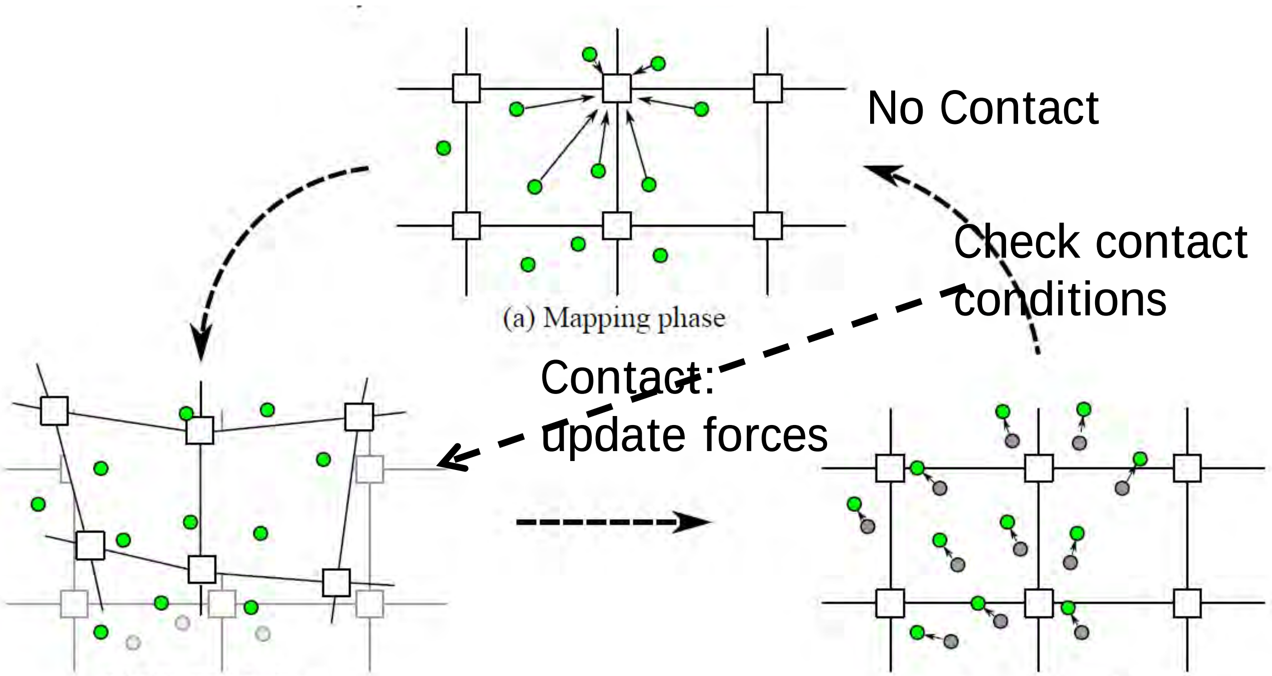
Challenges and future

Contact



Challenges and future

Contact



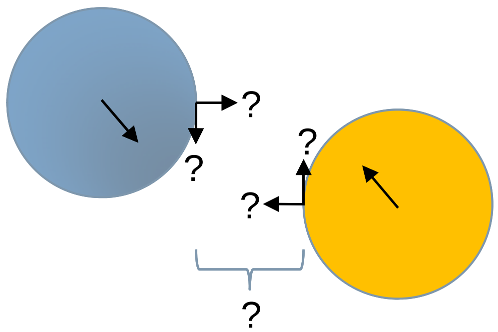
(a) Mapping phase

(b) UL-FEM phase

(c) Convection phase

Challenges and future

Contact



Impenetrability

$$g \geq 0$$
$$\sigma_n \leq 0$$
$$g\sigma_n = 0$$

(Hertz-Signorini-Moreau conditions)

Stick-slip conditions

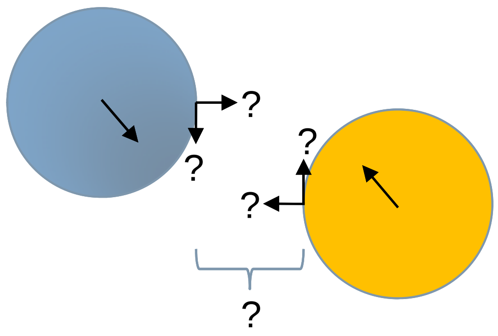
$$|v_t| \geq 0$$
$$\mu|\sigma_n| - |\sigma_t| \geq 0$$
$$|v_t|(\mu|\sigma_n| - |\sigma_t|) = 0$$

(Coulomb friction)

Pantev, 2016, MSc thesis

Challenges and future

Contact



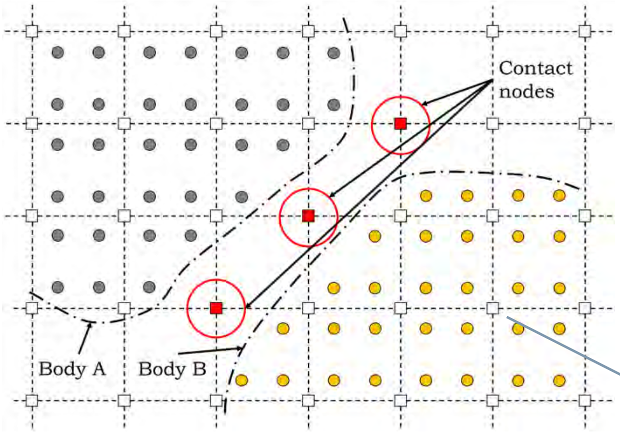
We need to find:

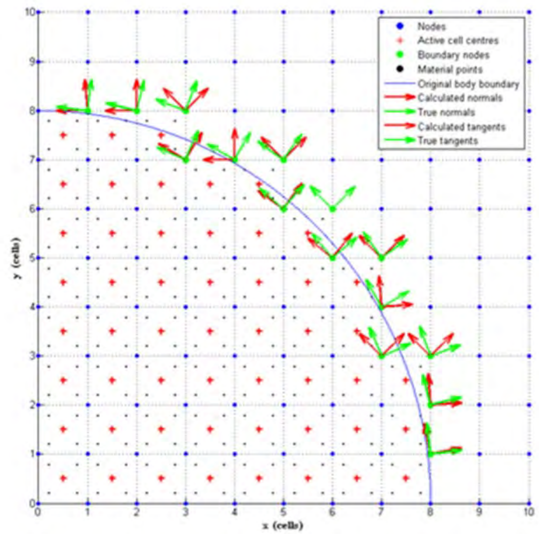
- The contact point
- The normal and tangential direction at the contact point
- Local velocity (momentum) at contact point

Challenges and future

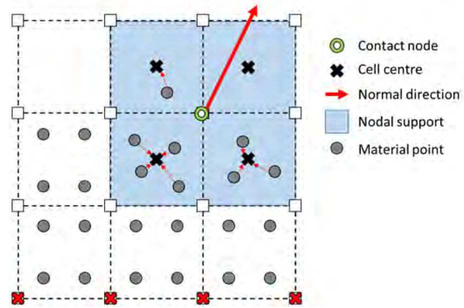
Contact

Resolve contact at the background grid nodes





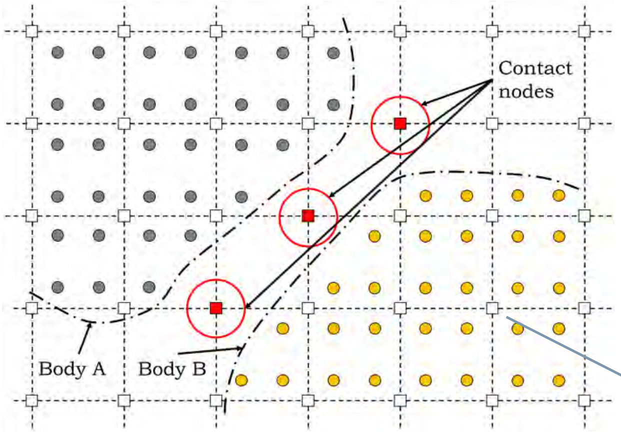
Determine normal from mass/density gradient at grid nodes

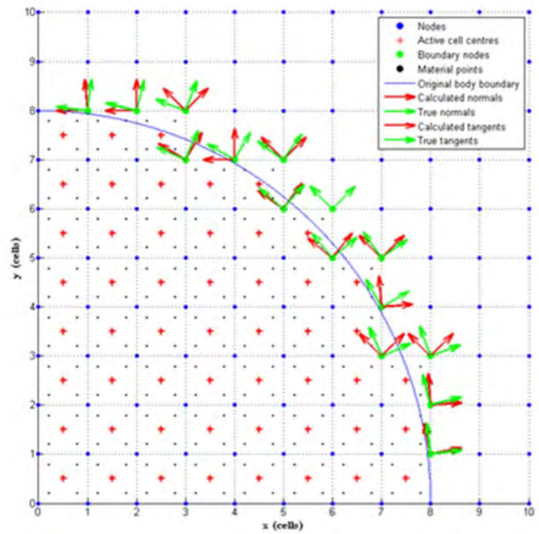


Challenges and future

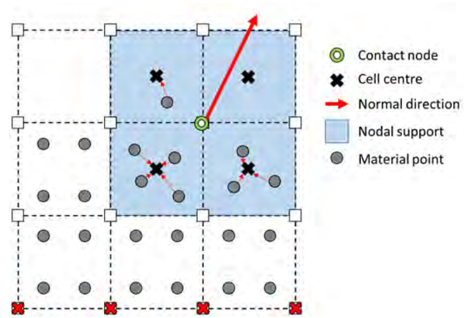
Contact

Resolve contact at the background grid nodes





Determine normal from mass/density gradient at grid nodes



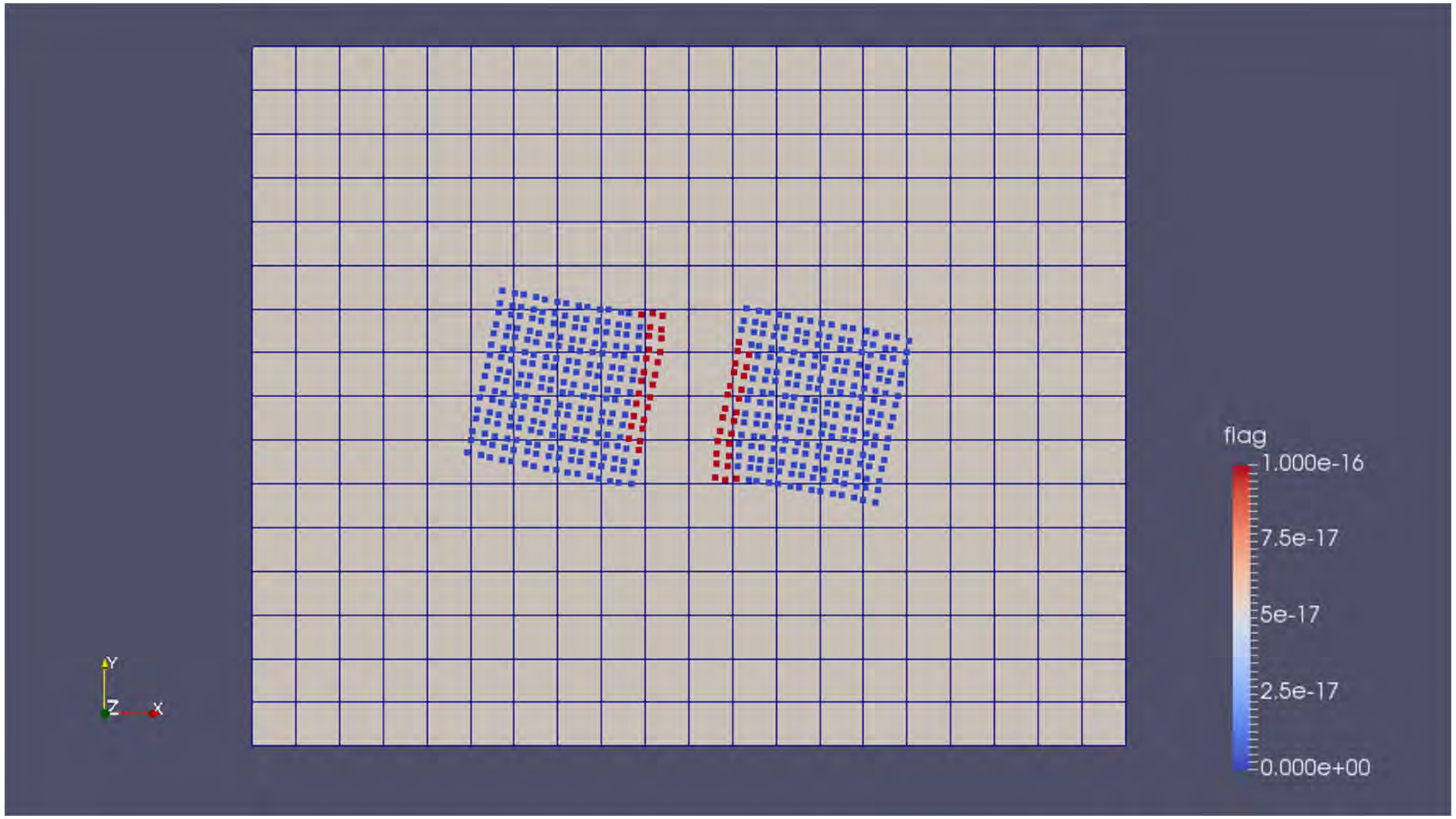
Multiple ways:

- Here using multi velocity grid
- Can ‘control’ distance by use of density or distance from nodes (mesh independent).

Challenges and future

Contact

- Diagonal collision
- No slip between blocks
- Like MPM w/o contact

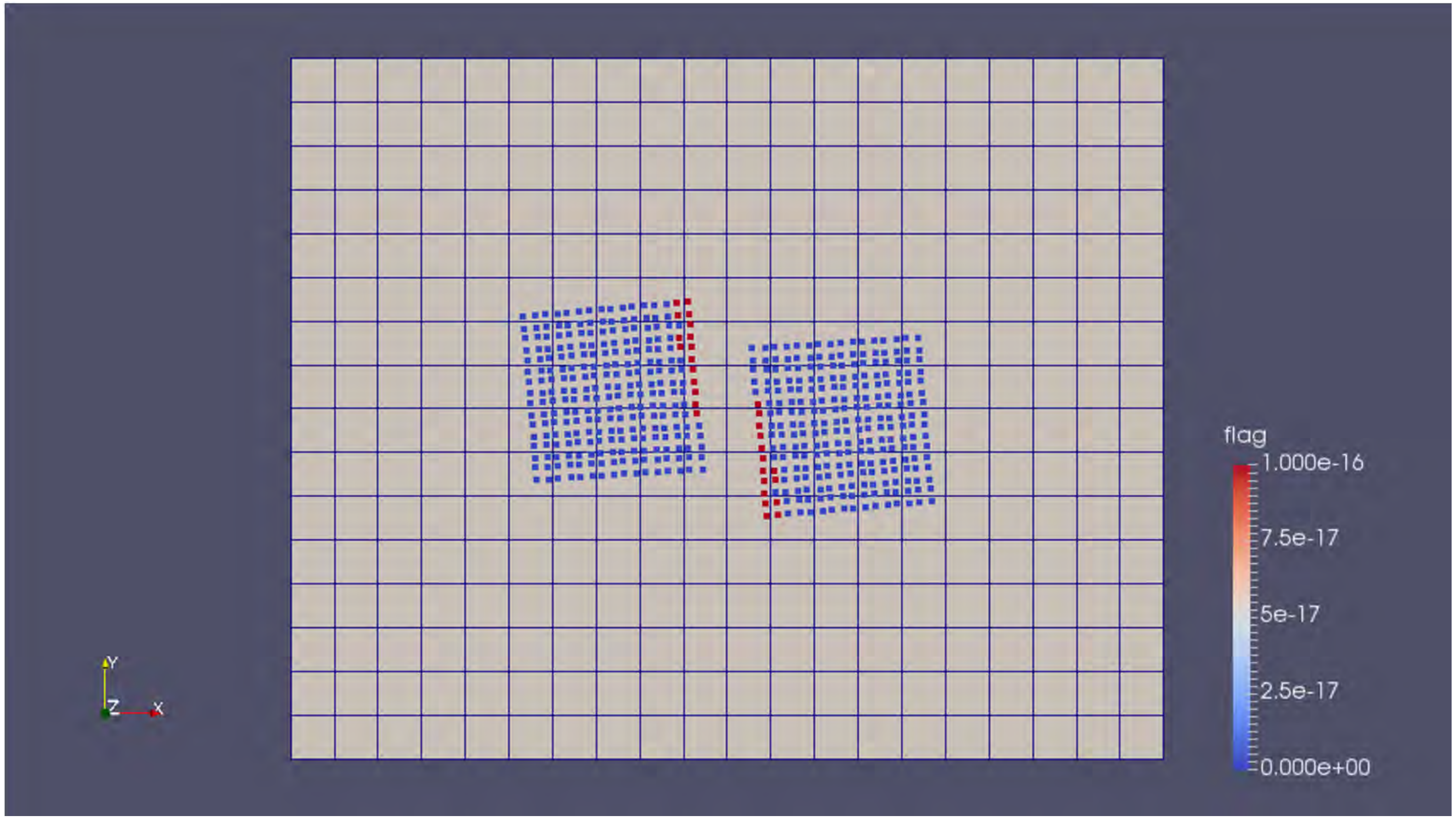


Pantev, 2016, MSc thesis

Challenges and future

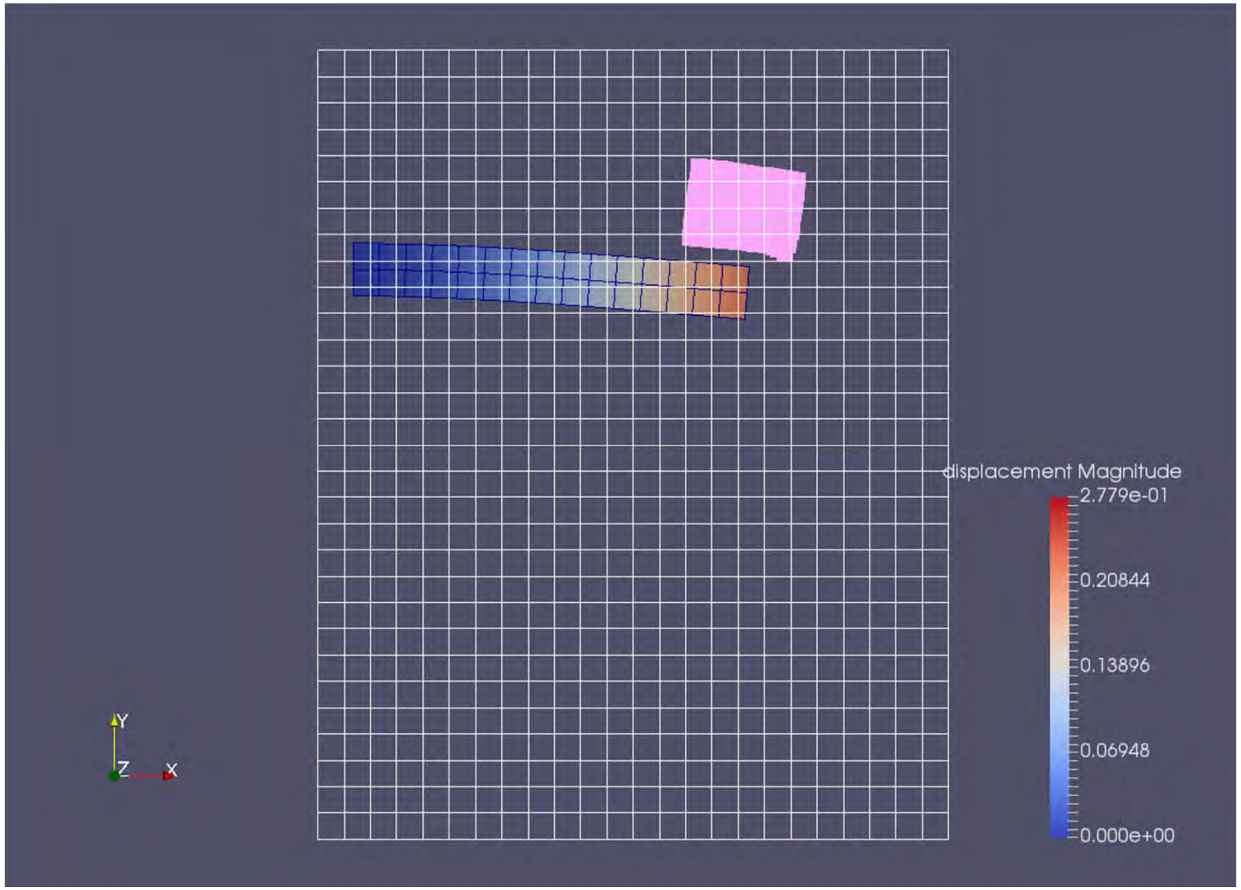
Contact

- Diagonal collision
- Frictionless
- Only impenetrability



Challenges and future

Contact



Cantilever beam solved with FEM
8-node quadrilateral elements

Block solved with MPM
16 MPs per element

Block slides off depending on friction

Conclusions

- Method for tackling large (arbitrary) displacements.
- Able to solve geotechnical behaviour.
- Need post-failure material behaviour.
- Still some improvements needed in the method to be commonly applied in practice.

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