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Conceptual Fallacies in Subjective Probability

Roger M. Cooke

ABSTRACT. Subjective probability considered as a logic of partial belief succumbs to three fundamental fallacies. These concern the representation of preference via expectation, the measurability of partial belief, and the normalization of belief.

Introduction

Subjective probability encompasses a spread of theories sharing the following two principles:

(I) Degree of partial belief is represented by a subjective probability measure.

(II) Preference behavior is represented by expected utility.

The founding fathers of subjective probability, Ramsey [1], DeFinetti [2] and Savage [3], give a very strong reading to the first principle. In Ramsey's words, the axioms of probability, interpreted subjectively, constitute the "logic of partial belief". Since any conceivable event can be invested with a degree of partial belief, this entails that subjective probability must be defined for every conceivable event.[1] By a conceivable event I mean an event which can be thought of by the agent, the degree of partial belief need not be positive.

The goal of subjective probability theories may be put in the following highly schematic form: Define subjective probability and utility in such a way that X is preferred to Y if and only if the subjective expected utility of X is greater than that of Y. The representation of preference in terms of expected utility should exist for all rational preference behavior, and it should be unique up to some reasonable equivalence.

The subjectivists' achievements are quite impressive, and have contributed greatly to our understanding of probability and rational choice. Nonetheless, the above principles involve fundamental conceptual fallacies; and the concept of subjective probability underlying them is in my opinion ultimately unsuitable for representing partial belief and rational decision.

Three fallacies are discussed in this paper, the expectation fallacy, the measurability fallacy, and the normalization fallacy.

The expectation fallacy

There are classical puzzles involving expectation, such as the St. Petersburgh paradox, which can be satisfactorially resolved in subjective probability. The principles of subjective probability, however, generate new problems of their own. Most theories of subjective probability discuss preference as a relation between acts. A minority [4, 5] sees preference as a relation between events. We discuss the expectation fallacy first in terms of acts, and then briefly in terms of events.

Acts are conceived as consequence-valued functions on possible worlds. Subjective probability requires that act f is preferred to act g, written $f > g$, if and only if the expected utility of f, written $E(f)$ is greater than that of g. Let A be an event, that is, a set of possible words. Let $E(f \mid A)$ denote the conditional expected utility of f given A (that is, the expectation of f restricted to A, divided by the probability of A). If subjective probability is defined for every conceivable event, then conditional expectation must also be defined for every conceivable event (by convention, we assign the value zero to $E(f \mid A)$ if $p(A) = 0$).

In this situation it is simply a fallacy to maintain that expectation can represent preference. I give the name *first person* events to the class of events whose defining conditions involve the agent's own actions. Indeed, with any act f, we may associate an event f, namely the event that the agent performs act f. It is clear that first person events may be the objects of partial belief and should therefore obey the 'logic of partial belief'. We shall see shortly that the measurement of subjective probability for first person events causes some difficulty, but for the present we assume that they can be assigned subjective probabilities in some meaningful way. If this is so, then we must be able to decompose the expectation of f in the following way

$$E(f) = E(f \mid f)p(f) + E(f \mid f')p(f');$$

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where f' denotes the complement of f, i.e. the event that act f is not performed.

Two questions arise at this point. First, what meaning can be given to the expectation of f conditional on the event that f is not performed? Second, why should this latter quantity be relevant in evaluating f?

These questions receive no satisfactory answer in the classical theories of subjective probability where preference is unconditional. However, both questions can be circumvented in theories of conditional preference or theories involving act-dependent probabilities (see references [5]–[10]). It is essentially a matter of eliminating the irrelevant term $E(f \mid f')$ and stipulating that $f > g$ if and only if $E(f \mid f) > E(g \mid g)$.

Nevertheless, the term $E(f \mid f)$ is also very problematic. Suppose I have decided not to do f; then clearly my degree of partial belief in the event f is zero, and the expression $E(f \mid f)$ is either undefined or is defined to be zero. In either case, distinctions in preference between acts which I have decided not to perform cannot be represented. Moreover, the decision not to perform these acts is presumably made on the basis of expectations. The very words 'suppose I were to do f...' often have the unmistakably subjective connotation of introducing a supposition contrary to fact. Summarizing, if subjective probability is defined for every event in which the agent can entertain partial belief, then a preference between acts which the agent is certain not to perform cannot be represented by (conditional) expectation.

Krantz and Luce [8] have given perhaps the most sophisticated and the most successful theory of conditional preference. In this theory it is stipulated that $p(f) > 0$ for every available act f. Of course this is mathematically convenient, but philosophically it is quite unsatisfactory. In a well-defined decision problem the agent is certain to perform the act with the highest (conditional) expected utility. All the other acts are certain not to be performed. The theory is clearly unable to represent the agent's preferences in this case. If we constrain the agent to invest a little partial belief in these rejected acts so that they acquire a positive probability, then we effectively forbid the agent from *doing* anything.

Preference is sometimes conceived as a relation between events, represented by expected utility. It is easily shown that the problems encountered above arise in this approach also. Let $U(.)$ be a utility function defined on possible words. The expected utility of an event A is defined to be

$$E(A) := \int_{s \in A} U(s)\, \mathrm{d}p(s)/p(A).$$

If we substitute for A a first person event of probability zero, then the expected utility of A is evidently not defined.

The measurability fallacy

As noted above, principle I entails that subjective probability is attributed to every conceivable event. Most subjectivists believe that the subjective probability of every event can also be measured. This belief is fallacious. I shall show that there are at least two classes of events which frustrate all measurement schemes proposed to date. One class, the first person events, we have met already. The second class I call the extreme events. Roughly speaking, an extreme event is one in which 'nothing matters anymore'. A precise definition will be given presently. Subjectivist measurement schemes may be divided into three general categories; those which use personal betting rates, those which use quadratic loss functions, and those which use a qualitative probability relation. Among the personal betting rate schemes we may distinguish the following three approaches: the vulgar Ramsey approach, the real Ramsey approach, and the DeFinetti approach. DeFinetti's approach is wholely inadequate on grounds that have nothing to do with first person or extreme events. However, since this approach has never been criticised in the literature, so far as I know, I shall discuss it briefly, after discussing the first two betting rate schemes. The vulgar Ramsey approach is by far the most common and underlies most applications in decision theory.

Personal betting rates: vulgar Ramsey

According to this scheme one proceeds by establishing an equivalence in preference between a lottery on an event A with payoffs in money, and an amount of money. Putting the lottery in curly brackets, suppose we observe

$$\{\$x \text{ if } A;\ \ \$y \text{ if } A'\} \cong \$z\,,$$

where '$\cong$' denotes equivalence in preference. Letting $U(.)$ denote an (independently determined) utility function on money, the subjective probability of A is then defined to be

$$p(A) := [U(z) - U(y)]/[U(x) - U(y)]\,;$$

which equation is derived by applying principle II. Note that this definition is meaningful only if all preference equations involving A, perhaps with different cash payoffs, yield the same value for $p(A)$.

In general, the probability of first person events cannot be defined in this way. Let f denote the act of destroying \$5 of one's own money. It is reasonable to suppose that the subject is indifferent about performing f, if he wins five dollars by doing so. It follows that the subject will endorse the following two preference equations

$$\{\$5 \text{ if } f;\quad \$0 \text{ if } f'\} \cong \$0$$

$$\{\$10 \text{ if } f;\quad \$0 \text{ if } f'\} \cong \$5\,.$$

From the first equation it follows that $p(f) = 0$, and from the second we find

$$p(f) = [U(5) - U(0)]/[U(10) - U(0)] > 0.$$

In plain English, the problem is that the 'probability' of a first person event may be profoundly disturbed by making it the subject of a lottery.

A different sort of problem arises with the following event: W_x := 'total thermonuclear war breaks out before year x'. Since nothing matters after total thermonuclear war, we may well observe the following betting behavior. For a fixed positive amount a, and for any negative amount Y

$$\{\$a \text{ if } W'_x;\quad \$Y \text{ if } W_x\} > \$0\,.$$

In this case the vulgar Ramseyian would conclude:

$$p(W'_x) > [U(0) - U(Y)]/[U(a) - U(Y)]\,.$$

Letting $a \to 0$ and or $Y \to -\infty$, he would conclude that $p(W'_x) = 1$. This subjective probability is well defined, but its value is absurd. We could re-run the above argument for any value of x and thereby conclude that the subject believes total thermonuclear war to be impossible. A similar argument could be given for the event D_x = 'the subject dies before year x'. With similar reasoning one could conclude that the subject considers himself immortal.

These are examples of extreme events, and we shall return to these examples in discussing the normalization fallacy. We conclude that first person events and extreme events cannot in general be assigned subjective probabilities in the vulgar Ramsey approach.

Personal betting rates: Ramsey

Unlike most of his followers, Ramsey restricts the class of events for which subjective probability is defined. An event is called 'ethically neutral' for a subject if the subject is indifferent whether the event or its complement is realized. Ramsey says that subjective probability is defined for an event A if A is ethically neutral, or if A satisfies the following property: for every possible world s, there is a possible world q equivalent in preference to s such that A holds in q.[2] This property can be stated quite simply with the help of Ramsey's utility function $U(.)$. $U(.)$ is defined over the set S of possible worlds and takes values in the set of real numbers $\mathbb{R}$. The above property simply comes down to this: $U(A) = U(S)$. In other words, every utility value is realized by some element in A, where A is now conceived as a subset of S.

Ramsey's idea seems to be the following. Preference equations are written in terms of values. Letting r and s denote utility values, the lottery

$$\{r \text{ if } A;\quad s \text{ if } A'\}$$

is understood to mean this: If A occurs, then the agent gets a possible world in A whose utility value is r, and if A' occurs, the agent gets a possible world in A' whose utility value is s. The point is that A need not be ethically neutral. The 'intrinsic value' of A is not able to distort the payoff in the lottery, since we select a particular realization of A which realizes the utility value r.

Suppose now that A is a first person event. If $r > s$ the agent will clearly choose to realize A, otherwise he will realize A'. In general, we have:

$$\{r \text{ if } f;\quad s \text{ if } f'\} \cong \max(r, s)\,.$$

This entails that:

$$\{r \text{ if } f;\quad s \text{ if } f'\} \cong \{s \text{ if } f;\quad r \text{ if } f'\}\,.$$

Suppose $r > s$. From the first lottery above we conclude that $p(f) = 1$, and from the second we conclude $p(f') = 1$.

Ramsey's approach is also beset with formal difficulties. It is reasonable to suppose that the event W'_x (total thermonuclear war does not break out before year x) has the property $U(W'_x) = U(S)$. However, W_x does not have this property. W_x does not contain any 'nice' possible worlds, especially if x falls within the near future. In short, the set of events for which subjective probability is defined is not closed under complementation.

Another formal problem is the following. Let $B := (s \in S \mid U(s) \leqslant r)$ for some real number r. B is not ethically neutral since every element in $B' = (s \in S \mid U(s) > r)$ is more desirable than every element of B. Moreover, B does not possess the property $U(B) = U(S)$. Subjective probability cannot be defined for B (or for B'). To a mathematician this means that the utility function $U(.)$ is not measurable with respect to the probability. This means that the expected utility of an event cannot be defined unless the probability is degenerate.

Although the real Ramsey is more sophisticated than the vulgar Ramsey, they are equally unsuccessful. Although the real Ramsey attempted to characterize the class of events for which subjective probability is defined, this attempt failed since this class is not closed under complementation. For the vulgar Ramsey extreme events get a well-defined, although absurd, subjective probability. For the real Ramsey their subjective probability cannot be defined. First person events elude both the real and the vulgar Ramsey.

Personal betting rates; DeFinetti

DeFinetti [2] has proposed a betting scheme which, in the context of principles I and II, falls completely wide of the mark. DeFinetti always assumes that moderate amounts of money are proportional to utility, and we shall grant this assumption for the sake of discussion. Suppose we are interested in someone's subjective probability that heads will turn up on the next toss of a given coin. DeFinetti's idea is as follows. We tell the subject: "We determine the stake on heads to be \$10. You may choose the stake on tails to be whatever you like, say Y. We then choose which side of the wager we take, and you have to play against us. I.e. we choose one of the two wagers:

(1) you win \$10 from us if heads, you pay us \$ Y if tails,
(2) you pay us \$10 if heads, you win \$ Y from us if tails.

DeFinetti reasons that the subject must choose Y such that his expectation on both wagers is equal to zero. Applying principle II, he concludes that the subjective probability of heads is $Y/(Y + 10)$.

If the subject really tries to maximize his expectation in this game, and if he utilizes all his partial beliefs, then he will reason in a very different manner. Suppose for example that the subject believes that the probability of heads is 1/2, and that he believes that we believe that the probability of heads is 4/5. If he sets $Y = 10$ he is certain that we will choose the second wager. Indeed, he believes that we believe that the two wagers are equivalent if $Y = 40$. He may therefore set $Y = 36$ in the knowledge that we will still go for the second wager, and he can look forward to an expected gain of \$13. In this simple case the ratio $Y/(Y + 10)$ approximates not the subject's partial belief in heads, but his estimation of our partial belief in heads. This measurement scheme simply measures the wrong partial belief.

Quadratic loss

The quadratic loss measurement scheme was introduced by Grayson [11] and exploited by DeFinetti. Its shortcomings are similar to those encountered in the Ramsey schemes, so we can be brief. For a given event A, we constrain the subject to play the following 'game'. He must give a number x_A between zero and one. If A occurs, he loses an amount proportional to $(1 - x_A)^2$; if A does not occur, he loses an amount proportional to $x_A{}^2$. Writing out his subjective expectation in this game and setting its derivative with respect to x_A equal to zero, it is easy to show that the subject's expected loss is minimized when $x_A = p(A)$.

If A is a first person event and if the stake is large, the subject will clearly put $x_A = 0$ or 1 and act accordingly. In other words, the partial belief in first person events is profoundly disturbed by playing the game. If we substitute for A the event W_x, then the subject will clearly put $x_A = 1$ in order to incur his loss after total thermonuclear war breaks out, in which case the loss does not matter anyway.

Qualitative probability

The most celebrated example of this approach is that of Savage. However, since Savage's definition of the qualitative probability relation relies on the so-called constant acts, and since they have been roundly criticised in the literature, it seems more appropriate to concentrate on the more recent attempts to define qualitative probability with the help of conditional preference. We simply note one conceptual problem with constant acts in the context of principles I and II, which has so far escaped notice. Since constant acts take the same consequences in every possible world, then we must have, for any constant act f

$$\text{for } s \in f;\ \hat{s} \in f';\quad f(s) = f(\hat{s});$$

that is, the consequence of doing any constant act is the same as the consequence of not doing the act.

I shall give a simplified version of the definition of qualitative probability in Krantz and Luce [8]. The conclusions reached here hit their theory; namely to discern a difference in (unconditional) qualitative probability, we must observe a preference between acts, both of which are certain to be performed. Moreover, they require $p(f) > 0$ for all available acts f. However, the method of derivation given here applies only to this simplified version of their theory.

We assume that preference is defined between acts restricted to subsets of possible worlds. The expression

$$f \mid A > g \mid B$$

is read as saying that performing act f in the event A is preferred to performing act g in the event B. Preference is represented by conditional expected utility. Consider two disjoint events, A and B, and suppose there are two acts, f and g, such that

$$f \mid A \cong g \mid B > g \mid A \cong f \mid B .$$

By definition, we say that A is qualitatively more probable than B if $f \mid A \cup B > g \mid A \cup B$.[3] To see the intention behind this definition, suppose A is qualitatively more probable than B. This is equivalent to:

$$E(f \mid A \cup B) > E(g \mid A \cup B) .$$

Using decompositions of the form:

$$E(f \mid A \cup B) = E(f \mid A) p(A) / p(A \cup B) +$$
$$+ E(f \mid B) p(B) / P(A \cup B)$$

and using the assumptions regarding f and g, this is equivalent to

$$[E(f \mid A) - E(f \mid B)]\ [p(A) - p(B)] > 0.$$

Since the first factor is positive, this means that

$$p(A) > p(B) .$$

This argument depends on the representation:

$$f \mid A > g \mid B \text{ if and only if } E(f \mid A) > E(g \mid B) .$$

However, in discussing the expectation fallacy we observed that the correct representation would be

$$f \mid A > g \mid B \text{ if and only if } E(f \mid A \cap f) > E(g \mid B \cap g).$$

It is easy to check that under this representation the above equivalence argument does not go through.

Some one wishing to salvage this definition of qualitative probability may proceed in one of two ways. First, one might require (as Krantz and Luce do, in effect) that $f \supset A \cup B$ and $g \supset A \cup B$. But now let $B = A'$; it follows that $p(f) = p(g) = 1$. The qualitative probability relation between A and A' gets revealed by a preference between acts, both of which are certain to be performed. Preference *behavior* has dropped out of sight. We might solicit verbal preference reports in this case, but then we might just as well ask the fellow if he thought A was more probable than A'.

The second possibility is to arrange things such that the events A, B are independent of the events f, g. In this case $p(A \cap f) = p(A)p(f)$, etc. Independence can be defined in terms of qualitative probability (see Krantz *et al.* [9]); but here we need a concept of independence which is more primitive than qualitative probability. To my knowledge, such a concept has not yet been proposed.

The problems which arose in measuring extreme events via betting rates also plague the qualitative probability approach. The machinery of conditional preference allows us to characterize these problems more precisely. We call an event A *extreme* if for all acts f and g, and for all $C \subset A$

$$f \mid A \cong g \mid C .$$

It is immediately apparent that the requirement

$$f \mid A \cong g \mid B > g \mid A \cong f \mid B$$

cannot be fulfilled if either A or B is extreme. (Extreme events are excluded in the Krantz Luce theory; they violate their axiom 9.i.) Extreme events cannot be put in a relation of qualitative probability to any other event.

To summarize the discussion of the measurement fallacy, subjectivists are not able to measure subjective probability for every event in which the subject may have partial belief. Nor have they succeeded in defining a field of events which are measurable.

The normalization fallacy

Subjectivists unanimously believe that belief is normalizable. In fact, it has never even been noticed that this position is not obvious. Belief can be regarded as a feeling like pleasure and pain. It would never occur to us that pleasure could be represented by a measure with values between zero and one. Why should this hold for belief? Most people would surely answer that belief has a natural supremum. No belief can be stronger than our belief in $A \cup A'$. In saying this, however, we distance ourselves from the notion that belief is simply a feeling whose intensity is to be measured. Tautologies are not all believed with equal intensity, since some of them have to be proved.

The belief in normalization is surely reinforced by the fact that subjectivists' measurement schemes seem to yield a normalized belief function. Referring to the (vulgar) Ramsey definition of subjective probability, it is easy to check that $p(A) = 1 - p(A')$. I have shown elsewhere that the interpretation of these measurement schemes which leads to a normalized measure is fallacious [12]. I shall recapitulate the non-technical aspects of that argument, concentrating on the betting rate approach.

Recall that options or lotteries can be nested in subjective probability. Instead of offering the subject a utility value r if event A occurs, I could just as well offer him another lottery with expected utility equal to r if event A occurs. When we reflect on the matter, all lotteries are really of the latter form. For example, to receive $1000 is to engage in an option whose outcomes may be different in different possible worlds. Letting r, q and t denote utility values, we assume that a preference equation of the form

$$\{r \text{ if } A;\ \ q \text{ if } A'\} \cong t$$

is in fact established by observing $h \cong k$, where h and k are acts such that $E(k) = t$ and $h := f \mid A \cup g \mid A'$ with $E(f \mid A) = r$ and $E(g \mid A') = q$. Letting S denote the set of all possible worlds, we define:

$$S_{fgk} := \{s \in S \mid f(s) = g(s) = k(s)\}.$$

Suppose we now ask the subject whether for him

$$f \mid A \cup g \mid A' \cong k?$$

Since these three acts all agree on S_{fgk}, it is for him a matter of determining whether

$$f \mid A \cap S'_{fgk} \cup g \mid A' \cap S'_{fgk} \cong k \mid S'_{fgk}.$$

Notice that the degree of partial belief in S_{fgk} plays no role in this evaluation. It follows that preference behavior with respect to lotteries involving f, g and k can at most reveal a *conditional* partial belief, where conditionalization is taken with respect to S'_{fgk}. Now let $\mathbb{F}_t$ denote the set of acts (or lotteries) available at time t, and define

$$S_t := \{s \in S \mid f(s) = g(s) \text{ for all } f, g \in \mathbb{F}_t\}.$$

Clearly, at time t we can at most measure partial belief conditional on the event S'_t.[4] Suppose we measure partial belief at instants t_i, $i = 1, 2, \ldots$. The question arises, can we fit all these conditional partial beliefs together so as to form a unique normalized probability over the whole $\bigcup_{i \in \mathbb{N}} S'_{t_i}$? As shown in [12], a necessary condition for knowing in advance that this is possible is that for some finite index I:

$$\bigcup_{i \in \mathbb{N}} S'_{t_i} = \bigcup_{i \in I} S'_{t_i}.$$

(Even if this holds, we need not have $\bigcup_{i \in I} S'_{t_i} = S$.) If we do not know whether this condition is fulfilled then we are not justified in assuming that the partial belief of the subject can be represented by a normalized measure.

We remark that $\cap S_t$ is an extreme event for the subject. We can now readily understand why events like W_x (total thermonuclear war breaks out before year x) have subjective probability zero on the vulgar Ramsey scheme. Such schemes measure at most partial belief conditional on $(\cap S_t)'$. Since $W_x \subset \cap S_t$, it follows that $W'_x \supset (\cap S_t)'$ so that

$$p(W'_x \mid (\cap S_t)') = 1.$$

The absurd conclusion that total thermonuclear war has subjective probability zero arises from mistaking a conditional probability for an absolute probability.

Conclusion

The expectation fallacy, the measurability fallacy and the normalization fallacy all arise when we try to interpret the laws of probability as the laws of partial belief, in the spirit of principles I and II. These fallacies could conceivably be avoided if we restrict the class of events for which subjective probability is defined to some subclass of the class of 'partially believable events'. This sort of approach may be attractive to mathematicians and decision theorists, but it should be very unappealing to philosophers. As philosophers we are obliged to look for models of rational decision with the strongest possible normative basis and the widest possible application. If we simply exclude important classes of events in order to derive a familiar mathematical representation, then we are unworthy of the title 'philosopher'.

A more exciting and philosophically more respectable alternative is to try improving our mathematical tools, so as to represent the full scale of partial belief and the full scale of rational decision. The fallacies discussed here indicate that the classical probability concept is not adequate to this task. The normalization fallacy suggests that we might better employ a conditional probability concept in the sense of Renyi [13]. The expectation fallacy suggests that we need to conditionalize on first person events of measure zero. Van Fraassen [14] has developed a probability concept, closely related to Renyi's with which this is possible. The measurement fallacy suggests that we must be able to retrieve the class of measurable events from the class of available acts. Instead of simply assuming that all events are measurable, or simply assuming that some field of measurable events is given a priori, we should attempt to start from a set of available acts, and derive the class of measurable events from the class of available acts.

Notes

[1] As is well known, under the usual set-theoretic assumptions, a countably additive probability measure cannot be defined for all subsets of the interval [0, 1] which agrees with the Lebesgue measure for intervals. Subjectivists typically require only finite additivity, and they point to the fact that a finitely additive probability measure which extends the Lebesgue measure for intervals can be constructed with the help of the (notoriously non-finitary) Hahn–Banach theorem. The strong reading of principle I is not unsound mathematically, but the appeal to the Hahn–Banach theorem in this context does not rhyme with subjectivists' finitary attitude in rejecting countable additivity.

[2] Ramsey defines a value as an equivalence class (under preference) of possible worlds. Preference is defined over possible worlds and lotteries in which the payoffs are values. Ramsey assumes that every real number is the utility of some possible world, i.e. $U(S) = \mathbb{R}$.

[3] The reader may verify that this definition makes sense when A or B are first person events.

[4] If we follow Savage and adopt the position that $\mathbb{F}_t$ should include all mathematically possible functions from possible worlds into consequences then clearly $S_t = \emptyset$. As indicated previous this position is no longer considered realistic. In any measurement situation we can offer at most a small finite number of acts.

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