

## STAKEHOLDER PREFERENCE ELICITATION

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April 5, 2005-04-05

**Abstract:** I review arguments why stakeholder preferences cannot be modeled as utilities, multicriteria or otherwise. An approach to stakeholder preferences based on well known models for consumer preference in market research is proposed. Simple paired comparisons is used to represent group preferences on an affine unique scale, and regression is used to “explain” these preferences in terms of scores on a number of criteria. Using the rich body of standard regression techniques, we can analyse degree of fit, and we can deal with dependence in the “criteria”. The tasks in stakeholder preference modeling can be apportioned between analysts, experts and stakeholders.

### 1. Introduction

Multi criteria methods are emerging in the area of risk analysis and risk management (Linkov et al, 2005). It is appropriate to recall the classical arguments why stakeholder preferences cannot be modeled as utilities, multicriteria or otherwise, in the sense of rational decision theory. An approach to stakeholder preferences based on well known models for consumer preference in market research is proposed (for a review see Cooke 1991). Simple paired comparisons is used to represent group preferences on an affine unique scale, and regression is used to “explain” these preferences in terms of scores on a number of criteria. Similar approaches to modeling valuations can be found in (McFadden 1974, Koop et al. 1994, Kind 1996, Saloman 2004, McCabe et al 2004). These approaches often use logit regression to model valuation of health states which can then be compared with "Standard Gamble" trade-off elicitation common in multi attribute utility theory (McCabe et al 2004, Torrance et al. 1996) Using the rich body of standard regression techniques, we can analyse degree of fit, and we can deal with dependence in the “criteria”. The tasks in stakeholder preference modeling can be apportioned between analysts, experts and stakeholders.

### 2. Classical Utility Theory

'Classical utility theory' designates a body of techniques for evaluating outcomes that derive from the individual rational agent paradigm. The most well know of these are

*Multi-attribute utility theory (MAUT)*

*Multi-criteria decision Making (MCDM)*

*Analytic Hierarchy Process (AHP)*

Multi-attribute utility theory tries to describe utility functions when the outcomes are multi dimensional (or multi attribute). *If* the preferences satisfy certain stringent independence axioms, *then* the utility on outcomes can be represented as a weighted sum of the utilities on each dimension, where the weights assigned to a dimension reflect the value of that dimension. Under these special circumstances, we can think of preferences of outcomes as

$$\text{Preference (outcome)} = \sum_i \text{Preference for attribute}_i \times \text{score of outcome on attribute}_i.$$

The MAUT axioms state, roughly, that the preference for attribute<sub>i</sub> does not depend on the scores on the attributes.

MCDM is a simplification of MAUT where the independence axioms are not rigorously checked, but shorthand techniques are used to develop 'preferences for criteria'. AHP involves cascaded sets of criteria. It is not consistent with the decision theory of a rational agent *unless* a common baseline utility (corresponding to the utility value zero) is agreed beforehand (see below).

Extending classical utility theory so as to be useful in social decision making has proven difficult, (French, 1988, Savage 1972, Arrow, 1963, Arrow and Raynaud, 1986), and well-meaning practitioners re-commit the same mistakes year after year, to wit:

- 1) ASSUMING that a group can be treated as a rational individual. The following (Condorcet voting paradox) shows why this is NOT true. Consider a population with preferences ( $X > Y$  means X is preferred to Y):

1/3 of population: *Beethoven* > *Bach* > *Mozart*

1/3 of population: *Mozart* > *Beethoven* > *Bach*

1/3 of population: *Bach* > *Mozart* > *Beethoven*

Then there is a 2/3 majority for each of the following pairwise preferences:

*Beethoven* > *Bach*

*Bach* > *Mozart*

*Mozart* > *Beethoven*

This is evidently intransitive. An individual with intransitive preferences could never choose, but would continually cycle through the alternatives. It has long been recognized that groups choosing by a majority rule will not in general have transitive preferences (May, 1952). Arrow and Raynaud (1986) summarize attempts to introduce axioms ensuring the transitivity of majority preference.

- 2) ASSUMING without verification that preference can be expressed in terms of preferences for criteria (also known as coordinates, attributes, dimensions) and

scores on the criteria, as in MAUT. The following preference pattern is eminently reasonable, yet inconsistent with the MAUT axioms:

- a. If unemployment is low and pollution is high, Prefer: *Close a dirty factory*, to *Keep dirty factory open*
- b. If unemployment is high and pollution is low, Prefer : *Keep the dirty factory open* to *Close the dirty factory*.

MAUT assumes that the trade-off between criteria is constant across the decision space. Preferences for criteria are measured by observing choice behavior as criteria are traded-off. Thus, subjects are asked, *If a policy A raised pollution by X but decreased unemployment by Y relative to policy B, would you choose A or B?* The above example shows that the rate at which a subject trades off may depend on values of pollution and unemployment for policy B.

- 3) ASSUMING without verification that the criteria scores are independent in the population of alternatives being compared. If two criteria, say CO<sub>2</sub> emissions and SO<sub>2</sub> emissions tend to favor and disfavor the same alternatives, then MCDM and MAUT would tend to introduce double counting. The extreme case of this arises if we have two cost criteria, cost in US dollars and cost in EURO's. Both are important but using both would clearly be double counting. A general strategy for gaming an MCDM exercise is to inject many criteria that favor your preferred alternatives (and which are therefore correlated). MAUT and MCDM have no prophylactic against correlated criteria and double counting.
- 4) ASSUMING that preference ratio's are meaningful at an individual or group level, without giving an operational definition. The presence or absence of operational definitions is illustrated as follows:
  - a. John prefers *Close dirty factory* to *Keep dirty factory open*  $\Rightarrow$   
OBSERVABLE BEHAVIOR: Vote to close
  - b. John' preference ratio *Close dirty factory* / *NOT close dirty factory* = 9  $\Rightarrow$   
OBSERVABLE BEHAVIOR?????

Asking for preference ratio's is just like asking for 'temperature ratio's'. Ask a friend "how much hotter is boiling water than freezing water: (i) about the same (ii) hotter (iii) much hotter, or (iv) very much hotter?" Now interpret the answers as temperature ratio's. Does that make sense? On the Fahrenheit scale the ratio is  $212/32 = 6.625$ . On the Kelvin scale it is  $373.15 / 273.15 = 1.366$ . On the centigrade scale this ratio is  $100 / 0 = \infty!$  Yet these three scales all measure the same thing, namely temperature. Utilities, like temperatures, are measurable on an affine or interval scale. If we fix a baseline zero value, then we can compare ratio's of utility *differences* (of course this ratio could also be negative!). Applications of the Analytic Hierarchy Process (AHP) require participants to state their 'degrees of preference' without establishing a baseline of zero utility. These degrees of preference are interpreted as 'preference ratio's' that have no operational meaning. Given the total lack of operational meaning for "preference degrees" the wide currency of the AHP method is cause for sober reflection, and underscores the difficulty of bringing rational methods to bear on real problems.

### 3. Stakeholder preferences

Group preferences do not satisfy the axioms that enable utility theory. On the other hand, there are techniques from consumer research which yield consumer preference functions. We shall speak of 'stakeholder preferences' to designate the values of a group or collective body charged with taking and/or implementing decisions. We will model these preferences with techniques from consumer preference theory.

Classical approaches (Thurstone, 1927, Bradley, Terry, 1952, Mosteller, 1952, Bradley 1953, David 1957, Torgerson 1958, McFadden 1974) elicit pairwise preferences from experimental subjects, and "scale" the data so as to yield a preference scale which can be related to market share. Of course, "market share" is just a convenient metaphor; "group preference" or "stakeholder preference" is a better designation for present purposes. Similar approaches have been used to evaluate different health outcomes for decisions regarding treatment selection in pre-paid medical plans, and recently, in valuing health outcomes (Kind 1996, Torrance et al 1996, Saloman 2004, McCabe et al 2004).

Using standard techniques, we can analyse the degree of agreement among the stakeholders, and the degree to which the modeling hypotheses for deriving scale values fit the data. Once the scaled values have passed these statistical tests, we can use standard regression modeling to "explain" the preferences by regressing the scores on explanatory variables. A rich body of standard techniques is at our disposal to

- analyse how well a regression model explains the scaled values,
- derive confidence bounds for the weights of the explanatory variables, and
- analyse the correlation in the weights' sampling distributions.

For the current problems, the Thurstone pair wise comparison methods are most promising of the standard approaches. The simplest version, which will be illustrated here, makes the following assumption: after choosing the same zero and unit values on their respective utility scales, each stakeholder has a value for each alternative, and the values are independently normally distributed in the population of stakeholders with constant fixed variance. The placement of the means is determined only up to a choice of zero and choice of unit; that is these values are determined up to a positive affine transformation. This is desirable for two reasons:

- This is the same invariance structure of individual utility functions
- This is very convenient if the scale values are to be explained by some other mode, for example a regression model (see below).

Other types of paired comparison models are discussed in section 7.

#### **4. Implementation**

The implementation involves three groups of 'players'; an analysis team, an expert team and a set of stakeholders. Their roles are described briefly below.

##### *Analysis team*

The analysis team defines a set of alternatives (eg policies for coastal defense) and a set of criteria (eg cost, expected number of fatalities, breach frequency, expected environmental impact, etc). A set of experts is also defined. The analysis team monitors and manages the entire process, and performs the mathematical analysis.

##### *Expert team*

The expert team scores each alternative with respect to each of the criteria. The experts may also provide feedback on the sets of alternatives and criteria, possibly iterating the definitions.

##### *Stakeholders*

The set of consumers are given the alternatives with their scores on the criteria. Each consumer expresses his/her pair wise preference for each pair of alternatives.

#### **5. Analysis**

The analysis team analyses the consumer preferences for consistency and significance, according to standard methods. This results in an affine unique consumer preference function that assigns a preference to each alternative. The preference values are regressed on the set of criteria. This yields a coefficient  $B_i$  for criteria  $C_i$ , which optimally express the preference for each alternative as a linear function of the criteria scores. For alternative  $a$  and criteria  $C_1 \dots C_n$ , we thus have:

$$\text{PREF}(a) = B_1 \times C_1(a) + \dots B_n \times C_n(a) + \text{error}.$$

$C_i(a)$  is the score of alternative  $a$  on criteria  $C_i$ . Unlike multicriteria analysis, the  $B$ 's need not be positive and need not add to one; thus, they cannot be interpreted as "weights" for the criteria. However, they best explain the preference values in a least squared sense. Standard tools are available to analyse the error and assess the degree to which the criteria scores explain the consumer preferences.

If the preferences are adequately explained by the model, the results are communicated to the problem owner. New alternatives can be evaluated using the regression model without iterating the consumer preference elicitation.

If the fit is not satisfactory, new criteria can be formulated and the regression step can be iterated. The regression model can also be extended to include interaction terms. This requires iterating the expert scoring and regression analysis, but does not require iterating the consumer elicitation. More details are illustrated in the following toy example.

## 6. Toy example

We consider a toy example for modeling group preferences for auto's. We want to model stakeholder preferences for auto's and use this model to predict future preference behavior and drive design improvements.

The analysis team selects 5 autos (policies) which cover the field parsimoniously, namely FOCUS, ASTRA, ROLLS, BMW, KA, XSRA. An expert team scores each autos on the criteria: PRICE, MONTHLY PAYMENTS, MILAGE, PASSENGER ROOM. Notice that a criteria like ROOM cannot be monotonic in value, whatever that may mean. A stakeholder will have an ideal size, and deviations above or below will be less desirable. Notice also that the criteria scores will evidently be strongly correlated.

These scores are passed to ten stakeholders, who evaluate the policies pairwise. Suppose the following preference matrix emerges ( the first cell entry 6.0/10 means that 6 of the 10 stakeholders preferred the FOCUS to the ASTRA).

### PREFERENCE MATRIX

Item	ASTRA	ROLLS	BMW	KA	XSRA
1 FOCUS	6.0/10	8.0/10	8.0/10	6.0/10	7.0/10
2 ASTRA		8.0/10	6.0/10	6.0/10	7.0/10
3 ROLLS			2.0/10	2.0/10	2.0/10
4 BMW				1.0/10	4.0/10
5 KA					6.0/10

The stakeholder preference summary matrix shows the number of times that each stakeholder preferred each auto to some other auto. The rightmost column shows the number of circular triads in each stakeholder's paired comparisons. With 3 or more circular triads, the null hypothesis that the stakeholder in question has given his preferences at random CANNOT be rejected. The analysis team might decide to re-elicite stakeholders Rock, Cliff, Ridge and Ruby.

### STAKEHOLDER PREFERENCE SUMMARY MATRIX

Stakeholder	FOCUS	ASTRA	ROLLS	BMW	KA	XSRA	Triads
Rock	4.0	4.0	1.0	1.0	2.0	3.0	4
Cliff	3.0	2.0	1.0	4.0	4.0	1.0	4
Ridge	4.0	2.0	0.0	2.0	3.0	4.0	3
Crystal	4.0	5.0	1.0	1.0	2.0	2.0	2
Jade	4.0	1.0	1.0	2.0	5.0	2.0	2
Pebble	2.0	3.0	5.0	4.0	1.0	0.0	0
Shale	3.0	2.0	0.0	1.0	5.0	4.0	0
Flint	4.0	4.0	0.0	1.0	2.0	4.0	1
Opal	4.0	4.0	0.0	1.0	4.0	2.0	1
Ruby	3.0	4.0	1.0	2.0	3.0	2.0	6
Total	35.0	31.0	10.0	19.0	31.0	24.0	

Preference values are shown below. Three common models for analysing paired comparison data are shown. Thurstone C is the model used here. (For a discussion see

next section.) The others are shown for the sake of comparison. For both Thurstone models a Chi square statistic tests the hypothesis that the model assumptions hold. The value 7.0349 is far from significant, thus the data do not lead to rejecting the model (the NEL (Bradley Terry) model would also be unrejected at 4.9667).

PREFERENCE VALUES

Item name	NEL(Bradley-Terry)	Thurstone C	Thurstone B
1. FOCUS	0.2998	0.4525	0.8546
2. ASTRA	0.2193	0.2699	0.5098
3. ROLLS	0.0416	-0.7015	-1.3273
4. BMW	0.0896	-0.2981	-0.5278
5. KA	0.2193	0.3117	0.5570
6. XSRA	0.1304	-0.0345	-0.0663
Goodness of fit :	4.9667	7.0349	
(Chi-square, DF=	10	10	

It is convenient to put the preferences from the Thurstone C model and the criteria scores in one matrix. We also add a column of "1's". The effect of adding this column is to enable the criteria scores to reflect deviations from the mean, when the criteria scores are standardized.

CRITERIA SCORES

PREF	CONST	PRICE	MONTHLY	MILAGE	ROOM
0.4525	1.0000	20.0000	0.2000	14.0000	4.0000
0.2699	1.0000	25.0000	0.1800	12.0000	6.0000
-0.7015	1.0000	40.0000	0.5000	9.0000	8.0000
-0.2981	1.0000	45.0000	0.4800	8.0000	7.0000
0.3117	1.0000	12.0000	0.1200	20.0000	3.0000
-0.0345	1.0000	15.0000	0.1500	16.0000	4.0000

The regression analysis finds criteria scores which yield the best linear fit to the preferences, in the sense of least squares. These scores and the resulting error are shown below:

$$PREF = CRITERIA SCORES \times REG.COEFF'S + ERROR$$

$$= CRITERIA SCORES \times \begin{matrix} 1.6908 \\ 0.0409 \\ -4.0369 \\ -0.0403 \\ -0.2126 \end{matrix} + ERROR$$

$$ERROR = \begin{matrix} 0.1654 \\ 0.0423 \\ 0.0539 \\ -0.0808 \\ 0.0577 \\ -0.2385 \end{matrix}$$

We see that the fit is not spectacularly good. Of course the number of alternatives is small for the number of regression coefficients to be estimated. The point is to illustrate the analytical tools which can be applied. We can analyse the covariance matrix of the regression coefficients and derive confidence bounds for the coefficient estimates. The diagonal terms are the variances of the criteria, the off-diagonal terms the covariances. We see that the standard deviation of the criterion PRICE is  $\sqrt{(0.0009)} = 0.03$ . This large deviation reflects of course the small number of alternatives evaluated in this toy example. In this case, the problem owner would be told that the model of stakeholder preferences was not very good; we could not claim that the coefficient for price was significantly different from zero.

REG COEFF'S COVARIANCE MATRIX

	CONST	PRICE	MONTHLY	MILEAGE	ROOM
C	1.8396	-0.0270	1.1116	-0.0758	-0.0812
Pr	-0.0270	0.0009	-0.0389	0.0012	-0.0001
Mnth	1.1116	-0.0389	2.4437	-0.0435	-0.0347
Mi	-0.0758	0.0012	-0.0435	0.0032	0.0029
RM	-0.0812	-0.0001	-0.0347	0.0029	0.0103

The off diagonal terms indicate that the fluctuations in the criteria values are not independent.

REGRESSION COEFFICIENT PRODUCT MOMENT CORRELATION

	CONST	PRICE	MONTHLY	MILEAGE	ROOM
	1.0000	-0.6749	0.5243	-0.9873	-0.5894
	-0.6749	1.0000	-0.8426	0.6907	-0.0257
	0.5243	-0.8426	1.0000	-0.4918	-0.2185
	-0.9873	0.6907	-0.4918	1.0000	0.4967
	-0.5894	-0.0257	-0.2185	0.4967	1.0000

Suppose that, given scores on the criteria, the preferences were sampled from a distribution in accord with the assumptions of the regression model. Then on repeated samples, the criteria coefficients found by the least squares algorithm would fluctuate with variances on the diagonal of the covariance matrix, and the scores would be correlated as in the correlation matrix. We could obtain uncorrelated estimates of the coefficients by carefully choosing our options (cars) in such a way that the criteria scores are uncorrelated. This would be a so-called orthogonal design. Having an orthogonal design is convenient but by no means necessary.

On the Thurstone C model, each consumer's utility for the 6 cars is modeled as 6 samples from independent normal variables  $X_1 \dots X_6$  with unit standard deviation and with means given by the preference scores. The probability that  $X_j$  is most preferred is modeled as the probability that  $X_j = \max(X_1 \dots X_6)$ , and this probability is the predicted market share. It can be computed by simulation, and we find:

CAR	PREDICTED MARKET SHARE
1. FOCUS	0.28
2. ASTRA	0.22

3. ROLLS	0.04
4. BMW	0.09
5. KA	0.23
6. XSRA	0.14

Note the similarity to the Bradley-Terry scale values, which solves for market share directly from the paired comparison data (see below). We cannot extract a preference ordering from the stakeholders in this toy example, owing to intransitivities. If we could extract such an ordering, we could compare it to the predicted market shares and derive an additional check on the modeling assumptions. Pairwise comparisons are intended to deal with the volatility of consumer preference by allowing each alternative to be judged several times, in combination with all other alternatives.

## 7. Discussion

We briefly summarize the assumptions underlying the models Thurstone C, Thurstone B and (NEL) Bradley Terry (there is a Thurstone A model, but it is not tractable). These are compared briefly with the logit regression approach. For a fuller discussion see (Cooke 1991).

*Thurstone C:* We assume that each stakeholder has an affine unique utility value for each alternative. We can arbitrarily choose 2 alternatives and scale them equal to 0 and 1. We do this for all stakeholders, using the same alternatives. The utility values for all stakeholders are now on a common scale. Each expert's values for the remaining alternatives are modeled as a sample from a random vector. We assume that the components of this vector are independent normals with unit variance and means which are solved from the paired comparison data.

*Thurstone B:* This is identical to Thurstone C, except that the components of the random vector of stakeholder values are not independent, but the correlation of values is constant. The solution algorithm for either model yields values determined on an *interval scale*; that is, the values are uniquely determined when a "zero" and "unit" are fixed.

The Thurstone models make assumptions which are compatible with the theory of rational preference at the individual level, and yield preference values which are affine unique.

*NEL / Bradley Terry:* NEL denotes Negative Exponential Life model. In the NEL model each stakeholder performs a thought experiment to determine which of two independent components with exponential life distributions outlives the other. The Bradley-Terry model assumes that values of alternatives are determined on a ratio scale, and that the probability that a stakeholder prefers alternative  $i$  over  $j$  is  $V_i/(V_i+V_j)$ , where  $V_i$  is the value of alternative  $i$ . Note that this ratio is invariant under linear transformations, but not under affine transformations. The computational algorithms of these two models are identical, and yield estimates of values  $V_i$  determined on a ratio scale. The Bradley Terry solution algorithm assumes that each choice event is modeled as an independent coin-

tossing experiment. The estimates of the values  $V_i$  are obtained by maximal likelihood. The Bradley-Terry assumptions would hold if each stakeholder were sampled *once* for each pairwise comparison. If each stakeholder assesses all pairs, then obviously is preferences for alternatives  $i$  and  $k$  cannot be independent of the preferences for  $(i, j)$ , and  $(j, k)$ .

The logit models, like the Bradley Terry model, assume that  $P(i > j) / P(j > i) = V_i / V_j$ , and find the scale values by log linear regression. The probability of choosing alternative  $i$  over  $j$  is  $\exp(u_i) / (\exp(u_i) + \exp(u_j))$  where  $u_i$  is a "population utility" about which the individual utility values are distributed according to an extreme value distribution. Note the similarity with the Bradley Terry model by putting  $u_i = \ln(V_i)$ .

In conclusion we remark that the stakeholder preference method sketched above has no problem dealing with intransitive group preferences. With reference to the Bach, Beethoven, Mozart example, these alternatives would all receive the same scale value. Perhaps the main virtue of the stakeholder preference approach is that it enables standard checks on model fit and model adequacy.

Finally, there are other approaches currently under development, of which one is worth mentioning here. Suppose each stakeholder in the population has a preference ranking. We could then ask, which distribution over preference rankings would produce the paired comparison data, if we sampled stakeholders randomly from the population and asked them to express their preference for each pair of alternatives? New mathematical techniques of probabilistic inversion can be used to determine the best fitting distribution over rankings.

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