Risk Management and Model Uncertainty in Climate Change

Roger M. Cooke[1]
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Abstract: This paper explores the issues of risk management and model uncertainty in climate change. Examples are drawn from the Integrated Assessment Model DICE, but apply more generally. Techniques for dealing with model uncertainty include structured expert judgment with independent expert panels, stress testing, canonical variations and probabilistic inversion.

Keywords: Climate change, integrated assessment models, risk management, uncertainty analysis, structured expert judgment

1. Introduction

The standard economic approach to analyzing the climate change problem has been to search for efficient abatement policies. Many Integrated Assessment Models (IAMs) achieve this objective by maximizing the present value of consumption, equating the marginal benefits of abatement in terms of reduced climate damages with the marginal costs of reducing emissions. Every trader, banker, and investor knows that maximizing expected gain entails a trade-off with risk. In finance, there are many of ways of addressing the risk, such as hedging, insuring, maintaining capital reserves, short selling, throwing a financial Hail Mary pass, or doing nothing and rolling the dice. The standard economic approach does not consider the risks associated with a climate policy that maximizes expected gain. According to the theory of rational decision (Savage 1954), preferences can always be represented as expected utility, hence from this viewpoint, any aversion to risk could be folded into the rational agent's utility function. This theory, recall, applies to rational *individuals*; groups of rational individuals do not comply with the axioms of rational decision theory.

Weitzman (2009) has recently called attention to the risks of climate change[2], arguing that current approaches court probabilities on the order of 0.05~0.01 of consequences that would render life as we know it on the planet impossible. One may quibble with Weitzman's numbers, but they are not outside the realm of mainstream climate science, and no scientist argues that consumption as we know it would persist in a 10°C warmer world. What is the plan to manage this tail risk, risk from extreme events in the tail of a distribution[3]?

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[1] Resources for the Future and Department of Mathematics, Delft University of Technology

[2] Weitzman (2009) estimates a probability on the order of 0.05 of 10°C or more warming in 200 years, as a result of doubling pre-industrial greenhouse gases, and a probability of 0.01 of 20°C or more warming. High warming could trigger positive feedback effects, leading to even higher temperature increases. For example, the Paleocene-Eocene thermal maximum is speculated to have released one sixth of the carbon content of methane hydrate estimated to underlie world oceans today, and raised mid latitude mean annual terrestrial temperatures by 4-5° C, having profound effects of evolution of the world's fauna (Gingerich 2006).

[3] Ackerman et al. (2009) characterize the impact of this shift as follows: "How would this perspective change our approach to climate economics and policy choices? Economics would find itself in a humbler role, no longer charged with determining the optimal policy. Instead,…there remains the extremely complex and intellectually challenging tasks—for which the tools of economics are both appropriate and powerful—of determining the least-

The fact is that 'professional risk taking organizations' do manage risk, and not by bending the utility function of a representative consumer. Rather, they employ techniques like probabilistic design and quantitative risk analysis to quantify risk and optimize expected gain under a risk constraint. Nuclear power plants are currently designed not to exceed a yearly probability of serious core damage of 10^{-4} per reactor per year. Banks and Insurance companies under the Basel II and Solvency II protocols sequester capital reserves to cover a one-in-200 year loss event. The Dutch sea dykes employ probabilistic design to hold the yearly inundation probability under 10^{-4}. Ultimately, managing societal risk is a problem of group decision, not of maximizing expected utility of an individual rational agent.

Risk management shifts the research question from 'how does the optimal abatement level change for different parameter values?' to 'how does our policy choice fare under the distribution of future conditions we may face?' As such, it places the quantification of uncertainty in the foreground. Uncertainty quantification is more than putting distributions on model's parameters. The antecedent question is 'is it the right model?' This is the question of model uncertainty.

It is a truism that all models are false. We speak of model uncertainty if multiple models exist for describing the phenomena which cannot be ruled out. In that case uncertainty might not be well captured by putting distributions on the parameters of *one* model. Moreover, if uncertainty quantification is done by independent experts, we cannot presume that they all subscribe to one and the same model. Techniques for coping with model uncertainty discussed here are:

-stress testing
-canonical variations
-structured expert judgment

Stress testing means checking that the models behave reasonably when parameters are given extreme, though possible, values. Failing a stress test indicates model uncertainty. Canonical variations are used to explore the model space for alternative models. Gone are the days when quantification of the uncertainties was left to the modelers themselves. At the state of the art, quantification is done by domain experts in a rigorous and transparent manner. That entails that experts quantify their uncertainty over model independent phenomena that are observable in principle if not in fact. This uncertainty must be pulled back onto the parameter space of any given model, and the degree to which the observable uncertainty can be captured by a model's parameter uncertainty forms an important aspect of model evaluation. If there were no uncertainty, there would be no recourse to expert judgment. In as much as structured expert judgment involves quantifying uncertainty, probabilistic inversion may be seen as fitting models to expert judgment data.

These features were deployed in the uncertainty analyses of accident consequence models for nuclear reactors conducted jointly by the European Union and the U.S. Nuclear Regulatory Commission. With 2,036 elicitation variables assessed by 69 experts spread over 9 panels and a budget of $7 million 2010 dollars (including expert remuneration of $15,000 per expert), this suite of studies is representative of the state of the art. It established a benchmark for expert

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cost global strategy for achieving those [risk management] targets…". Stern (2008) writes "As economists, our task is to take the science, particularly its analysis of risks, and think about its implications for policy."

elicitation, performance assessment with calibration variables, expert combination, dependence elicitation and dependent uncertainty propagation —features largely absent from the IAM discussion. The techniques discussed here have been extensively applied and exhaustively reviewed. For a summary see a special (Radiation Protection and Dosimetry, 2000), complete references may be found in (Cooke and Kelly 2010). For good order, there are also Bayesian approaches to model uncertainty, anchored in the Bayesian inference paradigm.

Shifting to a risk management focus will also suggest corresponding changes in policy approaches. Instead of seeing the policy problem as one of pricing the world's greatest externality, the policy question is how much are we willing to spend to buy down the risk of catastrophic impacts and what is the most cost-effective way to do so? After all, we can't short the planet.

This paper is not an application of uncertainty analysis, nor is it an exhaustive catalogue of techniques for coping with model uncertainty. It argues that model uncertainty should be taken seriously in the IAM community, and presents techniques successfully deployed in the past for doing so. That said, it is likely that the climate change problem will pose new challenges which can only be met by marshalling the best efforts from diverse disciplines.

A concluding section summarily compares the approaches to risk in the climate change debate and in the banking and insurance sectors.

2. Stress Testing: Pollyanna and Chicken Little

Uncertainty analysis often takes models outside their comfort zone where they are – hopefully – empirically grounded. Kann and Weyant (2000) note that many models are not calibrated for extreme values of their inputs since they are structured to provide close estimates for small perturbations of parameter values. Stress testing is preformed to check that the models remain realistic and capture the relevant possibilities as their parameters are fed values at the "limits of the possible". Pollyanna and Chicken Little both sit at the table. If either of these conditions fail, then uncertainty cannot be represented via distributions over the models parameters.

Stress testing is illustrated with the economic growth dynamics used in many IAMs. IAMs specify economic damages as a function of temperature change, and model use growth dynamics to model their impact on output and utility. For example, damages $\Omega(t)$ at time t induced by temperature change T(t) relative to pre-industrial mean temperature are represented in DICE as factor that reduces economic output:

(1) $$\Omega(t) = 1/[1 + 0.0028T(t)^2].$$

The standard Cobb Douglas production function expresses output Q(t) as a function of total factor productivity, A(t) (a parameter evolving over capturing technological change), capital stock, K(t), and labor (assumed to be the population), N(t). Temperature induced damages $\Omega(t)$ and abatement efforts $\Lambda(t) \in (0,1)$ reduce output:

(2) $$Q(t) = \Omega(t)[1-\Lambda(t)]A(t)K(t)^{\gamma}N(t)^{1-\gamma}.$$

Capital in the next time period is the depreciated capital of the previous time period (at rate δ), plus investment (output minus consumption):

$$(3) \qquad K(t+1) = (1-\delta)K(t) + Q(t) - C(t)$$

Substituting (2) into (3) and replacing the difference equation with a differential equation, this growth model reduces to a Bernoulli equation solved by Jacob Bernoulli in 1695. If we put T(t) = Λ(t) = 0, then (3) reduces to a common growth model whose empirical basis is a subject of a debate (see eg D. Romer 2006, chapter 3) which the IAM community has generally skirted. As standard macro economics texts (eg Romer 2006, Barro and Sala-i-Martin 1999) do not reference this fact, a derivation is given in the appendix[4].

It is important to notice that, according to (3), the rate of change of capital depends *only* on current values of the quantities in (2). There is no other "stock variable" the accumulation of which could influence capital growth. To give a prosaic example, letting K(t) denote cyclists' altitude on a mountain, does the rate of change of K depend only on the current K and the current added energy of peddling, regardless whether the cyclist is going up or down hill? Authors who suggest that growth might be path dependent draw attention to this assumption. (Stern 2008, 2009). We use the term "Bernoulli dynamics" to describe any growth model based on the Bernoulli equation.

An illustrative stress test of Bernoulli dynamics focuses on the growth model (3) without climate damage. Assume there is no temperature change, no abatement and take constant savings rate constant at 20%. Total factor productivity and population are held constant at their initial values in DICE, and we use DICE values for other parameters[5]. Figure 1 shows two capital trajectories. The first trajectory starts with an initial capital of 1\$, that is, $\$1.5\times10^{-10}$ for each of the earth's 6.437×10^{6} people. The second trajectory starts with an initial capital equal to ten times the DICE2007 initial value. The limiting capital value is independent of the starting values – with a vengeance: the two trajectories are effectively identical after 60 years. The same will hold for any intermediate starting capital. If this were true we would have needed no Marshall plan, and after 5000 years of civilization we shouldn't expect differences in per capita output. Such obviously unrealistic consequences underscore the need for circumscribing the empirical domain of application of the Bernoulli dynamics. Regardless whether the model adequately describes small departures from an equilibrium state, its use for long term projections inevitably entails this sort of behavior[6]. The IAM DICE computes out to 2595, and the IAM FUND (Anthoff and Tol, 2008) goes out to 3000. Putting uncertainty distributions on the model's parameters will not change that.

[4] There is a good Wikipedia page: http://en.wikipedia.org/wiki/Bernoulli_differential_equation

[5] DICE uses: δ=0.1, A=0.02722, N = 6514 [10^{6}], K(0) = 137[10^{12}\$], savings rate = 0.2, and γ=0.3

[6] Although DICE uses a capital depreciation rate of 10%, D. Romer suggests 3 ~ 4% (Romer, 2006, p.25); in this case convergence would occur in 150 years, after 60 years, capital with \$1 starting value would be slightly more than half that of the \$1800 trillion trajectory.

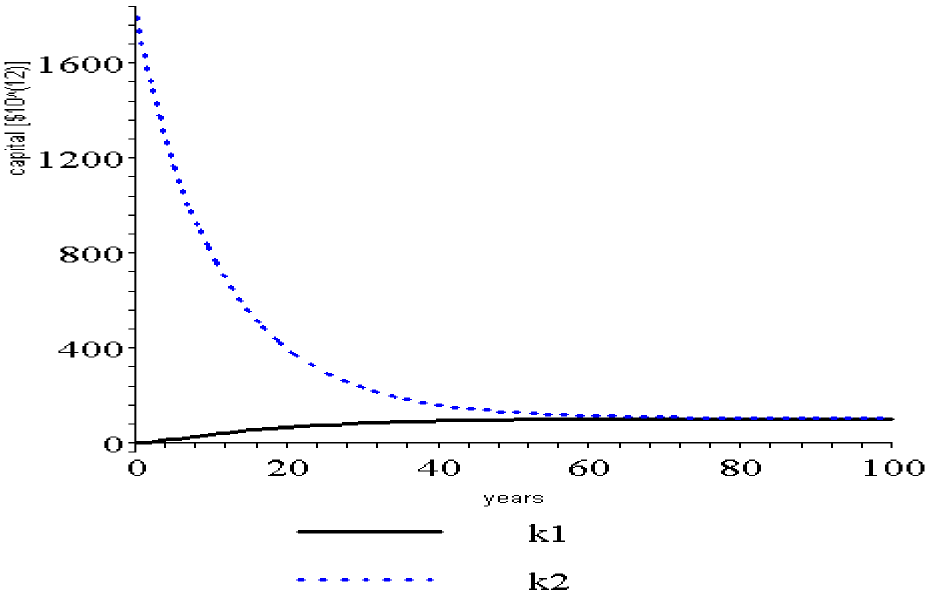

Figure 1: *Two capital trajectories with DICE with default values, no temperature rise, no abatement K1(0) = 1$ and K2(0) = 1800 trillion $*

Output is determined by (2) and shows the same behavior. To dispel the suggestion that Figure 1 is an idle theoretical exercise, Figure 2 computes the output trajectories for the paths in Figure 1, this time using the DICE2009XL software[7]. The behavior at zero is influenced by the granularity of the 10 year time steps in DICE, but the rate of convergence is similar.

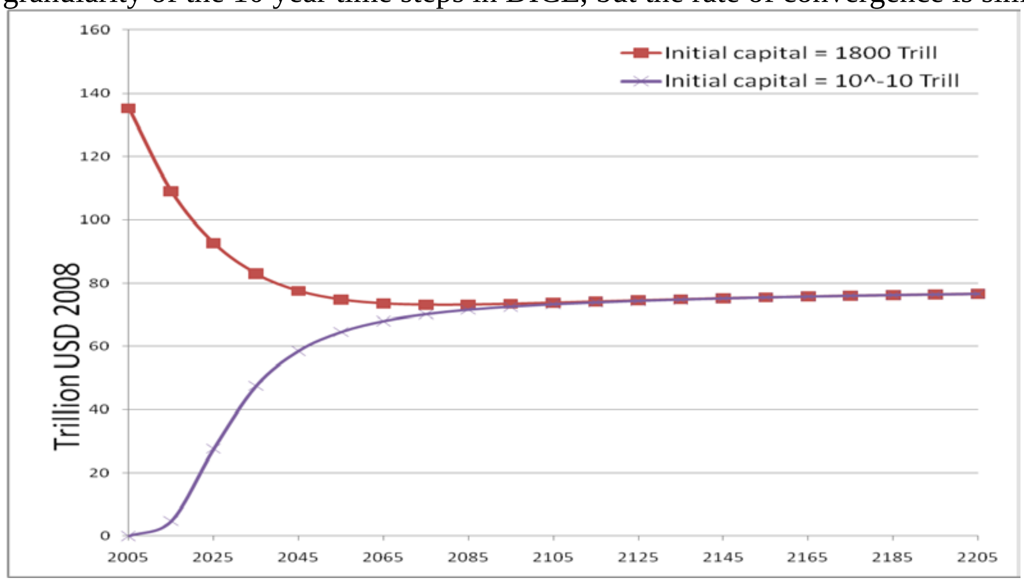

Figure 2. *Output gross of abatement cost and climate damage ($trill 2000 USD) Base case, no temperature damage, no abatement, constant population, constant total factor productivity (0.0307951), initial output from production function and DICE defaults for other parameters (DICE 2009 EXCEL version).*

[7] Dice2009_EXCEL_051109aa, accessed Monday, July 19, 2010

There has been much discussion about the form of the damage functions in IAMS, and several modelers (eg Pizer, 1999, Nordhaus 2008) have put distributions on model parameters and presented the Monte Carlo results as uncertainty analysis. In light of the evident lack of realism in the fundamental growth dynamics, even without temperature damage, one must question the wisdom of using such models to advise and inform climate policy, without considering model uncertainty.

3. Canonical Model Variation

It is often noted that simple models like the above cannot explain large differences across time and geography between different economies, pointing to the fact that economic output depends on many factors not present in such simple models. To "save the phenomena" researchers have proposed enhancing the basic model with inter alia social infrastructure, government spending, human capital, knowledge accretion, predation and protection, rent seeking, extortion and expropriation (see Romer 2006, chapter 3, Barro and Sala-i-Martin 1999, ch. 12). Interest in geographical covariates has recently been rekindled (Dell et al 2009, Nordhaus et al 2006, Nordhaus, 2006) Before adding epicycles to the simple Cobb-Douglas model, it is well to consider whether other growth dynamics with comparable prime facie plausibility can be formulated within this restricted modeling vocabulary.

We illustrate with one variation based on the following simple idea: Gross World Production (GWP [trill USD 2008]) produces pollution in the form of greenhouse gases. Pollution, if unchecked, will eventually destroy production allowing pollution to recede, where after production can resume. This simple observation suggests that Lotka-Volterra dynamics might provide a perspective which an uncertainty analysis ought not rule out.

Greenhouse gases are modeled with the carbon cycle in DICE. It is often said that emissions [GTC] are a fixed fraction of GWP (Kelly and Kohlstad 2001). To see where this leads, we take the emission fraction fixed at 0.1, taken from the period 2015 – 2025 in DICE. Greenhouse gases, converted to ppmC, determine the equilibrium temperature rise above pre-industrial levels according to:

$$(4) \qquad T(GHG(t)) = cs \times \ln(GHG(t)/280)/\ln(2).$$

where *cs* is the climate sensitivity parameter (the use of equilibrium as opposed to transient temperature is a simplification that could be easily removed). Real GWP has grown at an annual rate of β = 3% over the last 48 years (this includes population growth) [8]. Dell, Jones and Olken (2009) argue that rising temperature decreases the growth rate of GWP. Using country panel data, within-country cross-sectional data and cross country data they derive a temperature effect which accounts for adaptation. On their analysis, yearly growth, after adaptation, is lowered by δ =0.005 per degree centigrade warming. This gives the following system, where ε is the ratio of emissions [GTC] to output [Trill USD 2008]

$$(5) \quad GHG(t+1) = 0.988 \times GHG(t) + 0.0047 \times Biosphere(t) + \varepsilon \times GWP(t)$$
$$(6) \quad Biospere(t+1) = 0.9948 \times Biosphere(t) + 0.012 \times GHG(t) - 0.0005 \times DeepOceans(t)$$
$$(7) \quad DeepOceans(t+1) = 0.999 \times DeepOceans(t) - 0.0001 \times Biosphere(t)$$

$$(8) \quad GWP(t+1) = [1+ \beta - \delta \times (T(GHG(t)))]GWP(t).$$

The first three equations reflect the carbon cycle in DICE, while the last equation incorporates warming-induced damages on economic growth. Initial values for the atmospheric GHG stock, terrestrial and shallow ocean biosphere C stock and deep ocean C stock are taken from DICE .[9]. If T were linear in GHG, this would be a simple Lotka-Volterra type non-linear dynamical system. To appreciate what this means, write the change in GWP as $\beta\, GWP(t) - \delta.$ $(T(GHG(t)))GWP(t)$. The increment $\beta GWP(t)$ is reduced by a damage term. For fixed asset level the damage in GWP would be proportional to GHG, and for fixed GHG the decrement is proportional to the asset level. Of course T(GHG) is not linear, but the morphology of a simple non –linear system is still at work. As T gets large, GWP declines at an increasing rate; that is, GWP collapses. This conclusion will not surprise readers of Jared Diamond (2005), but to the best of the author's knowledge there is no macro-economic model for collapse.

Several authors[10] have suggested that climate damages might hit capital stocks. If capital has a positive real rate of return, say β, and capital growth is negatively impacted by a term linear in T(GHG(t), then a similar system would result. Capital replaces GWP in equation (5) and GWP in equation (2) is replaced by a production function giving output as a function of capital, labor and productivity. This approach is not used here, as the empirical literature supporting a climate effect on growth is concerned with GWP and not capital, but its behavior is very similar to that described below.

Figure 3 shows GWP and GHG as functions of time out to 500 yrs, with all variables at their nominal values. GWP collapses. Greenhouse gases also recede, but not to their initial level; as the carbon stock in the biosphere and the deep ocean has gone up, and these reservoirs serve as a source to the atmosphere long after industrial emissions have declined. Evidently, different ways of modeling the impact of climate change damages give qualitatively different predictions, and steady state values may not be relevant for current policy choices. Neither theoretical nor empirical evidence excludes the Lotka-Volterra type of interaction between damages and production presented here. A credible uncertainty analysis should fold in this and other possibilities, which brings us to the next point of examining a distribution of future conditions for a given policy choice.

[9] The transfer coefficients in DICE are converted to yearly rates. Of course, the transfer coefficients in the carbon cycle would not remain fixed over long time scales owing to features like die off of the ocean's phytoplankton, deforestation, and release of methane from the oceans.

[10] See Frankhauser and Tol (2005) and Ackerman et al (2009). Nordhaus (1999) discussed the possibility that climate induced damages hit capital stocks: loss of coastal assets due to sea-level rise, destruction of crops, devaluation of cold-weather capital like ski resorts, and accelerated obsolescence of indoor climatization systems. More serious might be shifting rainfall patterns necessitating new water infrastructure, deteriorating social infrastructure due to migration, war, predation etc, and destruction of natural capital in the form of fertility of soils and fecundity of oceans (Boyce et al 2010). Rapid temperature changes over annual or decadal scales, so-called "climate flickering," would likely make impacts on capital worse (Hall and Behl 2006). In addition Sterner and Persson (2008) show that climate damages could also alter relative prices; if this is the case it can justify more restrictive emissions controls. Note that the discount rate plays no role in our discussion.

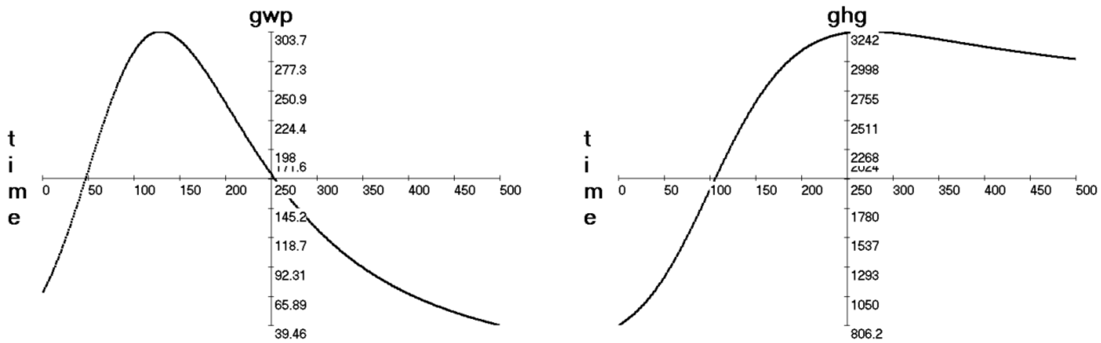

Figure 3: *The impact of climate damages on GWP (left) and greenhouse gases [GTC](right)*

3.1 Les Jeux Sont Faits

At Monte Carlo casinos, you place your bets before the roulette wheel is spun. Similarly, the policy questions we face under the large uncertainties and tail risks of climate change are not addressed by re-optimizing IAMs at parameter values sampled from a distribution. We are not afforded a peek at the true values of uncertain parameters before choosing a consumption-abatement path. Instead, the risk manager seeking to safeguard our future must assess the distribution of possible outcomes once a policy path is chosen. Policies can then be adapted to keep the risk within tolerable bounds.

Much realism can be added to the Lotka Volterra model. It is easy to see that decreasing the constant emission rate ε will postpone but not prevent the GWP collapse; the humps are merely shifted to the right in Figure 3. Similarly, decreasing the damage rate δ will allow us to get richer before GWP collapses; the humps get higher. A different fate within this simple model can be achieved only if, sooner or later, the emission rate effectively goes to zero. (Averting collapse could also be achieved if the damage rate went to zero, but with constant emission rate, this would lead to temperatures at which life is unsustainable). In DICE's "no policy" base case, the emission fraction [GTC/GWP] goes from 0.13 to 0.011 in 200 years. The required reductions would depend on new technologies whose existence is uncertain. To capture this uncertainty with a simple model, we replace ε with the time dependent emission factor $0.1\times\exp(-t \times \alpha)$, where t is time in years, and α is log uniformly distributed on $[10^{-6}, 10^{-4}]$. Thirty samples from the distribution of this emissions factor are shown in Figure 4.

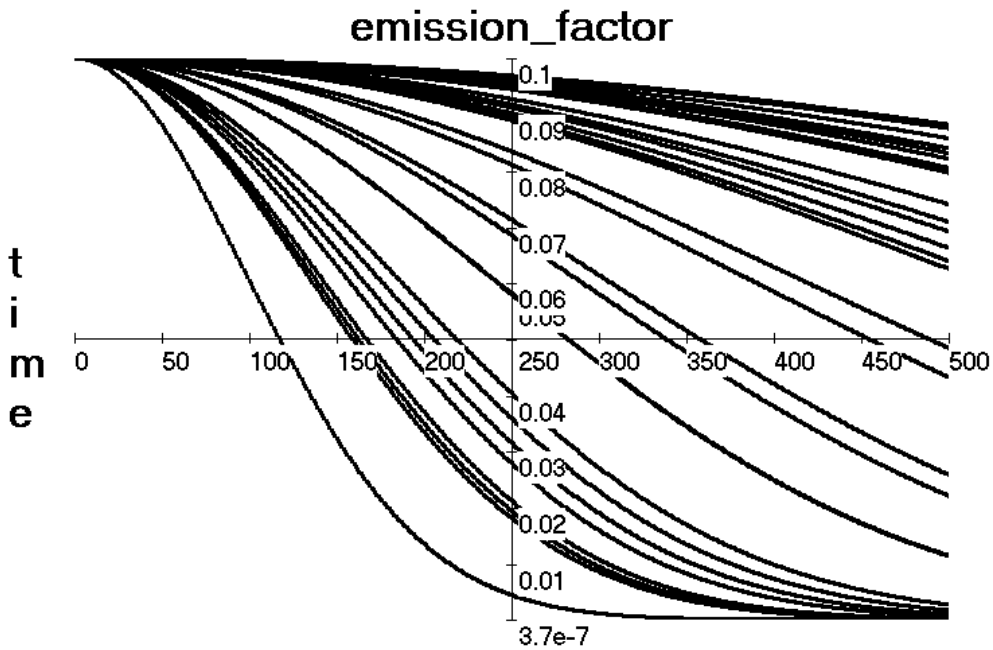

Figure 4; Thirty Emission factor paths [GTC/ $Trill USD2008]

All paths start at 0.1, and the emission factor after 250 years ranges from 0.097 to 0.005. Of course, how these different emission paths effect GWP will depend on all the other uncertainties. Choosing 'ball park' distributions for these parameters[11], Figure 5 shows thirty paths for GWP and temperature

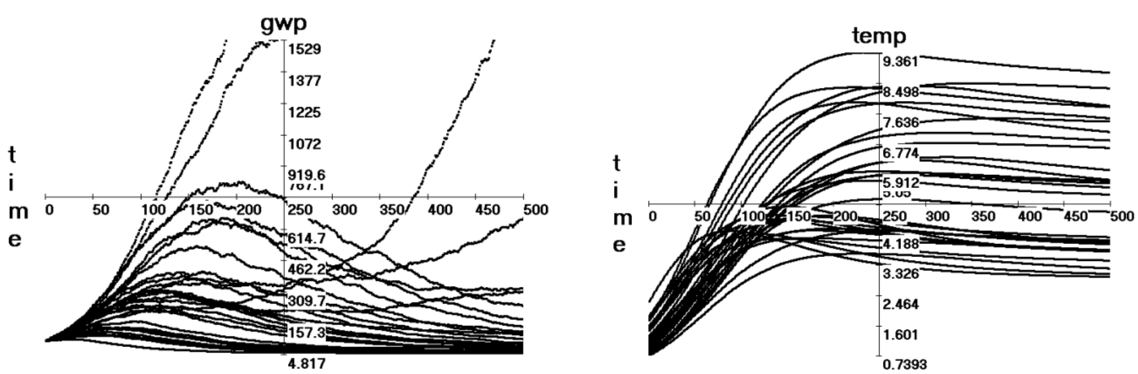

Figure 5: Thirty GWP (left) and Temperature (right) paths

[11] Climate sensitivity is epistemic (uncertain, but constant through each run, t = 0…500) and Beta distributed on [1, 15] with parameters (4, 24), mean 3. The damage parameter δ, log uniform distributed on [0.004, 0.01], and the emission rate parameter α, log uniform on [10^{-6},10^{-4}] are also epistemic. Lower values of δ lead to temperatures at which life is unsustainable, pointing to a weakness of linear damage models in this context.. Only the intrinsic growth rate of production is aleatoric (independently sampled for each time t) and Beta distributed on [0.01, 0.06] with parameters (5, 7) and mean 0.03. In Figures 4 and 5, all these distributions are independent.

For the most aggressive reduction paths in Figure 4, GWP enjoys uninterrupted growth, on other paths GWP collapses; the timing and height at collapse depend on all uncertain parameters. Maximum temperatures range from 9.4C to 3.7C. The happy growth rates in Figure 5 arise if the emission reduction rate is very aggressive, the climate sensitivity is very low, and the damage rate δ is very low; and none of these factors by itself is sufficient. Note that the costs of the different emission reduction policies are not reflected in the GWP paths.

The issue of dependence has been largely absent from discussions of uncertainty in the IAM community. These uncertainties are subjective and dependencies may be caused by putative physical coupling (e.g., high climate sensitivity might be caused by feedbacks that also exacerbate damage rates, high damage rates may drive aggressive emissions reductions). Alternatively, dependence may result from pooling experts' assessments (a mixture of independent distributions is not generally independent) or from post-processing (see next section). The graphs in Figure 6 use the same marginal distributions as Figure 5 but now with a dependence structure reflecting a positive dependence between *cs* and δ and a negative dependence between δ and α; the exact manner in which this is done are not of interest here.

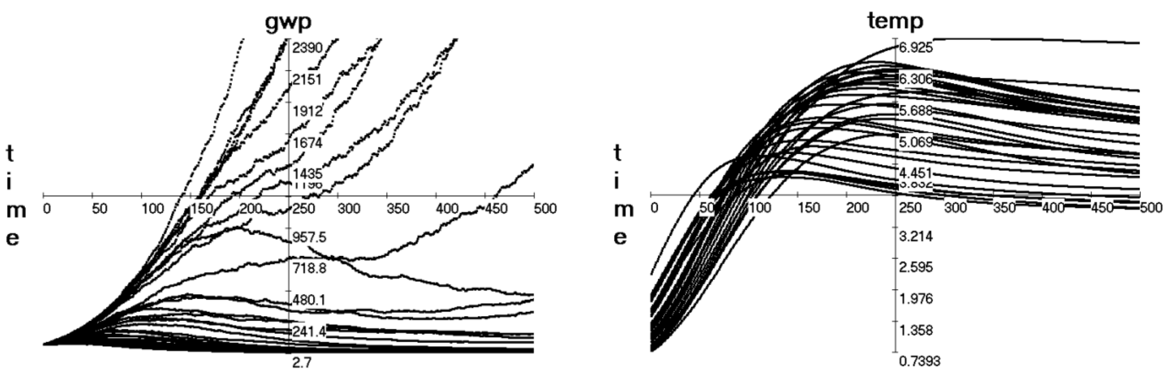

Figure 6: *Thirty GWP (left) and Temperature (right) paths with dependence*

The dependence has significantly lowered our uncertainty in temperature, maximum temperatures now range from 4.4C to 6.9C, instead of 3.7C to 9.4C. About one third of our growth paths are happy. Another way to appreciate the impact of dependence is to consider two emission fraction reduction policies shown in Figure 7. The lower policy is the most aggressive policy in Figure 4. It reduces the emission fraction from 0.1 to 0.05 in 117 years. The second milder policy reaches 0.05 in 185 years.

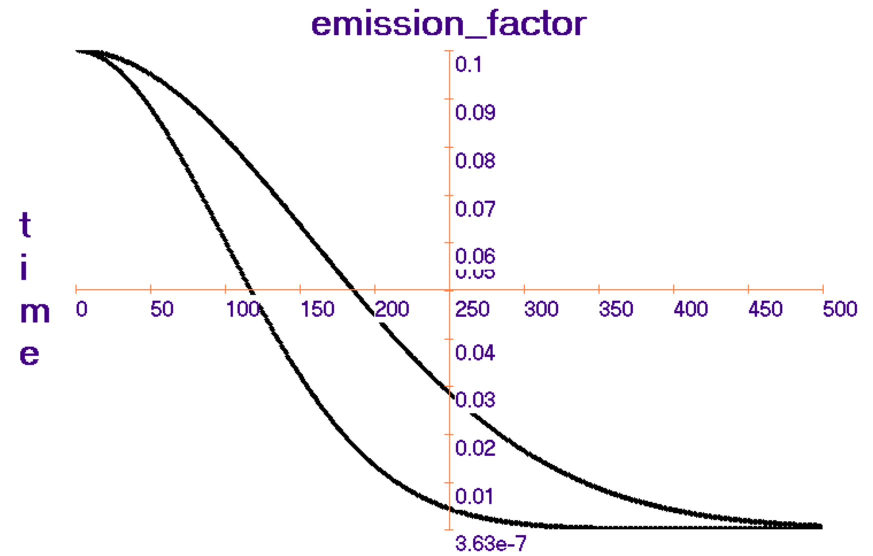

Figure 7: *Two emission factor reduction policies.*

Figure 8 shows that the aggressive policy under the independence assumption is roughly comparable to the milder policy under dependence, with regard to maximum temperature.

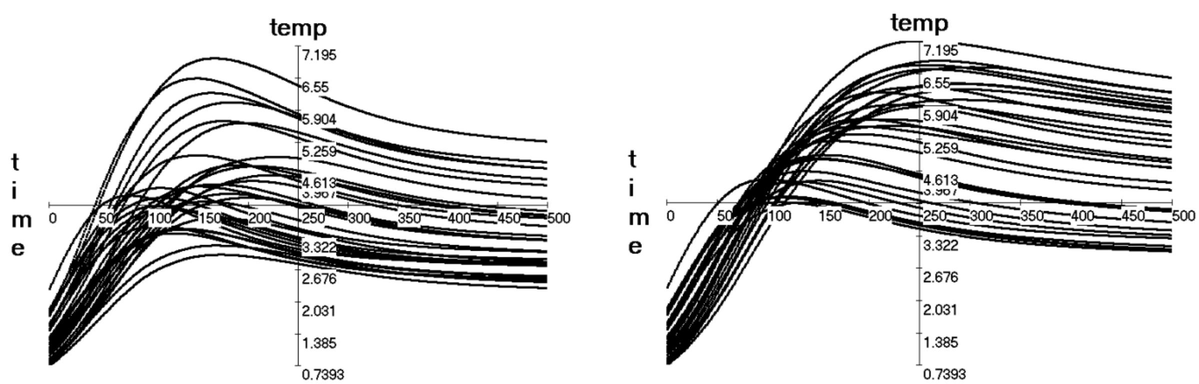

Figure 8: *Temperature scenarios for aggressive reduction policy with independence (left) and for milder policy with dependence (right).*

Of course, other dependence structures could lead to very different pictures, and the independence assumption is not generally conservative. The figures merely serve to illustrate that dependence cannot be ignored. For good order, these sketches do not constitute an analysis of uncertainty; they are arguments for doing a proper analysis of uncertainty.

4. Structured Expert Judgment for Quantifying Uncertainties

Uncertainty analysis with climate models must be informed by the broad community of climate experts - not simply the intuitions or proclivities of modelers - through a process of structured expert judgment.[12] Experience teaches that independent experts will not necessarily buy into the models whose parameter uncertainties they are asked to quantify. Hence, experts must be queried about observable phenomena, results of thought-experiments if you will, and their uncertainty over these phenomena must be 'pulled back' onto the parameters of the model in question. This process is analogous to the process by which model parameters would be estimated from data, if there were data. The new wrinkle is that data are replaced by experts' uncertainty distributions on the results of possible, but not actual, measurements. The 'pull back' process is called probabilistic inversion, and has been developed and applied extensively over the last two decades (see Kraan and Bedford, 2005, Kurowicka and Cooke, 2006, Cooke and Kelly 2010). In general, an exact probabilistic inverse does not exist, and the degree to which a model enables a good approximation to the original distributions on observables forms an important aspect of model evaluation. The details of the expert judgment process are outside the scope of this paper, but three features deserve mention.

(i) Experts are regarded as statistical hypotheses, and their statistical likelihood and informativeness are assessed by their performance on calibration questions from their field whose true values are known post hoc. Experts' ability to give statistically accurate and informative assessments is found to vary considerably.
(ii) Experts' uncertainty assessments can be combined using performance based weights.
(iii) Dependence, either assessed directly by experts or induced by the probabilistic inversion operation, is a significant feature of an uncertainty analysis.

An application of uncertainty analysis in the climate change arena will doubtless pose new and unforeseen challenges and will require solutions beyond the current state of the art. However, the problems are sufficiently serious to warrant an expenditure of effort at least comparable to best efforts made in the past.

The importance of a structured expert judgment approach to uncertainty quantification was illustrated in the uncertainty analysis of the consequence of accidents with nuclear reactors, mentioned in the introduction (Harper et al 1995, Cooke and Kelly 2010). Throughout the 1980s, complex models were built to predict the dispersal of a radioactive source term, the resulting health and economic damage, as well as effects of countermeasures. Initially, the modelers, like those at the Kernforschungszentrum Karlshue (KfK), quantified the uncertainties in their models themselves. Table 1 shows the ratio of the 95^{th} to the 5^{th} percentiles for the centerline (peak) concentration and lateral spread of airborne radioactive material 10 km downwind, under neutral atmospheric conditions, and also dry deposition velocity. The KfK values are the result of "in house" uncertainty quantification by the modelers themselves. The EU-USNRC values resulted from a structured expert elicitation using 8 international experts vetted by an independent steering committee. The expert elicitation produced a much greater range of uncertainty than the modelers did themselves.

Ratio: 95 %-tile / 5%-tiles of uncertainty distributions		
	KfK	**EU-USNRC**
Peak centerline concentration per unit released, 10km downwind, neutral stability	3	174
Crosswind dispersion coefficient, 10 km downwind, neutral stability	1.46	11.7

[12] Nordhaus' (1994) expert survey on climate change may be compared with the protocols followed in the EU-USNRC studies (Cooke and Goossens 2000). Notably absent in Nordhaus' study is validation of expert performance.

Dry deposition velocity 1 μm aerosol, wind speed 2 m/s	30.25	300

Table 1: *Ratio of 95- and 5 percentiles of uncertainty distributions computed by National Radiological Protection Board (NRPB) and Kernforschungszentrum Karlsruhe (KfK), and by the European Union, US Nuclear Regulatory Commission (EU-USNRC).*

5. Implications for risk management

A graph as shown in Figure 3 raises the question 'What risk of collapse are we willing to run? How much are we willing to pay to buy down the probability of collapse?' These questions represent a fundamentally different set of concerns than those of a social planner optimizing expected utilities for a representative consumer. Several policy approaches to this range of potential outcomes have been discussed in the literature, including hedging strategies (Manne and Richels 1995) and adaptive strategies (Lempert, Schlesinger and Bankes 1996).

Regulations for Banks and Insurance companies, such as the Basel II and Solvency 2 Protocols in the EU, instruct companies to manage the risk of extreme events using a value-at-risk (VAR) framework. A target insolvency probability is chosen or set through regulation, typically 1-in-200 per year, and the firm must maintain capital reserves to cover losses from a 1-in-200 year loss event (McNiel et al 2006). If Weitzman is right, society is currently much more cavalier with the survivability of the planet than banks and insurance companies with their own solvency.

The closest to a solvency constraint in the current climate change arena may be "dangerous anthropogenic interference" (DAI),[13] and debates over how to define it are ongoing. Definitions of DAI run the gamut from crossing some physical threshold, such as the melting of the Greenland ice sheet (e.g., Hansen 2005), to reducing GHG to some level, to individual perceptions of danger (Dessai, Adger, Hulme et al. 2004). Defining DAI is not a solely scientific issue, but involves the public and their governments as well (Oppenheimer 2005). Studies have suggested that business-as-usual emissions could result in probabilities of DAI far exceeding 1-in-200 (Mastrandrea and Schneider 2004; McInerney and Keller 2008).

For use as a risk constraint in risk management, the probability of the undesired event should be calculated as a function of our policy choices. For illustration, suppose our DAI is defined as mean temperature greater than 3°C. By doing simulations similar to those described above, we can search for an emission reduction policy which keeps the probability of exceeding 3°C warming below a specified threshold, say 0.02. For example, with the model used in Figures 4 and 5, emission policy shown in Figure 9 is found to satisfy this constraint. Of course much is left out of this picture. The cost of this policy is not computed, and there may be more efficient ways of satisfying this risk constraint. We will also learn about the effects of greenhouse gas emissions as we go forward. These models merely serve to illustrate the calculations that would implement a risk management policy.

[13] Signatories to the United National Framework Convention on Climate Change (UNFCCC) pledged to stabilize greenhouse gas concentrations "in the atmosphere at a level that would prevent dangerous anthropogenic interference with the climate system. Such a level should be achieved within a time frame sufficient to allow ecosystems to adapt naturally to climate change, to ensure that food production is not threatened and to enable economic development to proceed in a sustainable manner." 193 countries are parties to the UNFCCC, suggesting that DAI is as close to a global consensus on a climate change "solvency constraint" the world is likely to achieve.

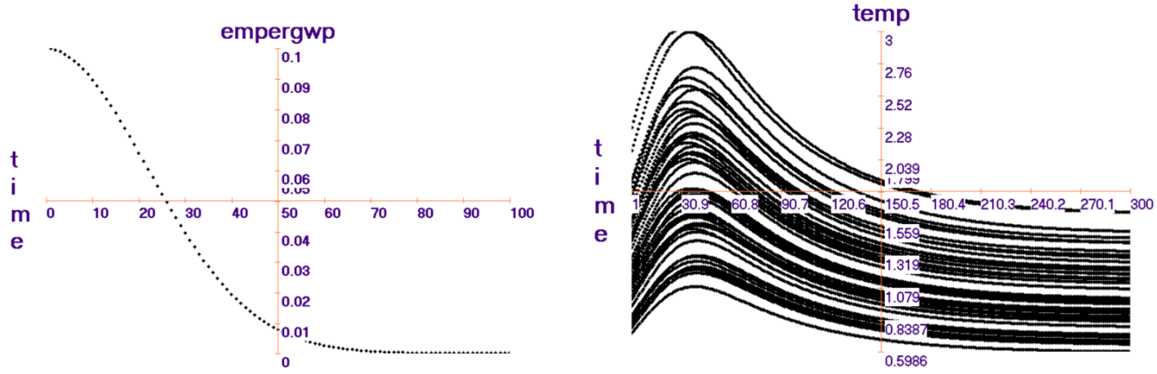

Figure 9: Emission reduction path and 50 temperature paths, all with maximum below 3°C.

These reflections challenge us to deploy risk management strategies on a global scale. We suggest this begin with (i) stress testing models, (ii) exploring alternative models, and (iii) quantifying uncertainty in such models via structured expert judgment. We are condemned to choose a climate policy without knowing all the relevant parameters, but we are not condemned to ignore the downside risks of our choices.

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Appendix

The model (3) reduces to a differential equation solved by Jakob Bernoulli in 1695. Putting $C(t) = \kappa Q(t)$, $(d/dt)K(t) = K'(t)$ (3) :

A.1) $K'(t) + \delta K(t) = B(t)K(t)^{\gamma}$; $K(t) > 0$; $B = \kappa\Lambda(t)\, A(t)N(t)^{1-\gamma}/(1 + \psi T^{p}(t))$.

Set $w = K^{1-\gamma}$. Then $w' = (1-\gamma)K^{-\gamma}K'$. Dividing by $K(t)^{\gamma}$, the equation (A.1) becomes:

$w'/(1-\gamma) + \delta w = B.$

Multiply both sides by $(1-\gamma)e^{(1-\gamma)\delta t}$ to get

$e^{(1-\gamma)\delta t}\, w' + (1-\gamma)e^{(1-\gamma)\delta t}\, \delta w = (d/dt)\,(e^{(1-\gamma)\delta t}\, w) = (1-\gamma)e^{(1-\gamma)\delta t}\, B,$

If B is constant, the solution is

$$e^{(1-\gamma)\delta t}\, w(t) = (1-\gamma)B \int_{u=o..t} e^{(1-\gamma)\delta u}\, du + w(0).$$

Make the change of variable x = t – u, and write this as

$$w(t) = (1-\gamma)B \int_{x=o..t} e^{-(1-\gamma)\delta x}\, dx \ + e^{-(1-\gamma)\delta t} w(0);$$

A.2) $K(t) = [(1-\gamma)B \int_{x=o..t} e^{-(1-\gamma)\delta x}\, dx \ + e^{-(1-\gamma)\delta t} K(0)^{(1-\gamma)}]^{1/(1-\gamma)}.$

If we set $K'(t) = 0$ in A.1, or let $t \to \infty$ in A.2, we find that the steady state value of capital is given by $K^* = (B/\delta)^{1/(1-\gamma)}$; this value is approached as $t \to \infty$ regardless of the initial value of capital. If there is a constant temperature rise of T°C above pre-industrial levels, then the steady state value becomes $K^* = (\Omega B/\delta)^{1/(1-\gamma)}$, where $\Omega(T) = 1/(1+0.0028388T^2)$. Steady state output is then $Q^*(t) = \Omega K^{*\gamma} AN^{1-\gamma}$. For linear temperature increase up to T in t = 200 years, we multiply the integrand in (A.2) by $\Omega(Tt/200)$.

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