Conditional Dependence in the Markowitz Model

Results of conditional sampling of stocks in the AEX-index

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Chapter 1

Introduction and Elementary Investment Theory

Portfolio managers invest in a variety of financial securities in a way that the risk/return profile of their portfolio matches the managers risk preference. All securities have a different risk/return profile based on historical data. This profile can be visualized by a dot in Figure 1.

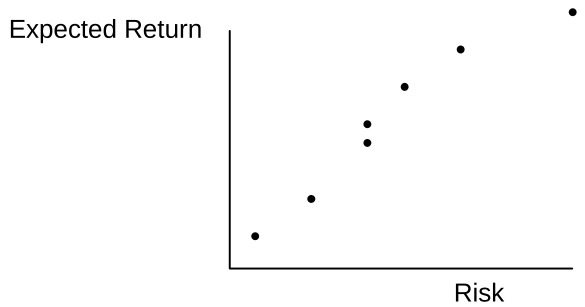


Figure 1 A typical Risk/Return graph. Risky assets generally yield higher returns. Treasury Bills can be found near the origin because of their low risk and low pay-off.

Markowitz (1959) showed that all the information needed to choose the appropriate securities is contained in three statistical measures: mean (Expected return), standard deviation (Risk) and the correlation with other securities return. The theory of Markowitz has been of great importance to the theory of portfolio selection, because simple statistical measures actually gave relevant insight in risk-bearing portfolios.
In the following, we give an example of a portfolio containing two securities.

We are going to construct a portfolio P consisting of a mixture of securities A and B. We denote the expected returns of A, B and P by μ_A, μ_B and μ_P. For $0\leq\alpha\leq1$ we choose P= αA + (1-α)B. Then μ_P equals $\alpha\mu_A + (1-\alpha)\mu_B$. The risk σ_P (standard deviation) equals

$$\sqrt{\alpha^2\sigma_A{}^2 + (1-\alpha)^2\sigma_B{}^2 + 2\rho(A,B)\alpha(1-\alpha)\sigma_A\sigma_B}\ ,$$

in which ρ(A,B) denotes the product moment correlation between A and B (by its definition always between –1 and 1, see Cooke (1999)).

We now vary α between 0 and 1, to gain portfolios P_α with other risk/return profiles than A and B (see Figure 2). The lines contain the risk/return profiles of all possible mixtures of securities A and B.
The following remarks can be made.

- The curve (a general example for the correlation between securities A and B) shows that there exists a portfolio consisting of a mixture of A and B, such that the risk of P is lower than the risk in A and B.
- If we have found two securities that are perfectly negatively correlated, this would in practice mean that we are able to construct a risk-free portfolio providing us μ_{-1} in a year. In general it is likely to find such portfolio that would be much more profitable than the risk-free interest rate.

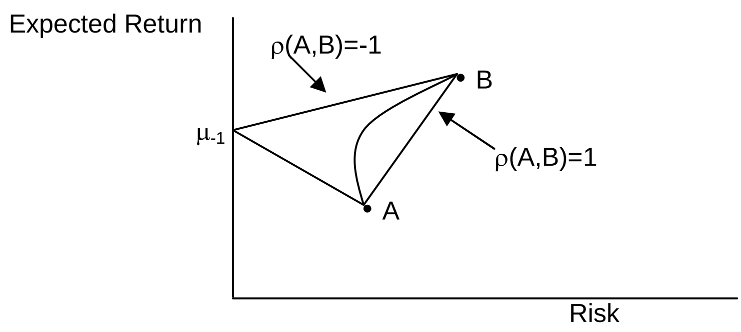


Figure 2 A Risk/Return graph of a portfolio containing all possible combinations of securities A and B, given their correlation ρ(A,B).

Unfortunately, in practice there are no securities with a product moment correlation of -1 (other than a short and a long position in the same security yielding an expected return of zero). This means that based on the original theory of Markowitz we are only able to construct portfolios within a certain area of Figure 2. This area is shown in Figure 3, which contains the convex cone, bounded by an efficient frontier that contains all possible portfolios in it. Much effort has been put in the work to stretch the efficient frontier to the northwest of the graph. Combining the efficient frontier with the risk-free interest μ_r that a financial institution would give for your money yields all possible portfolios.

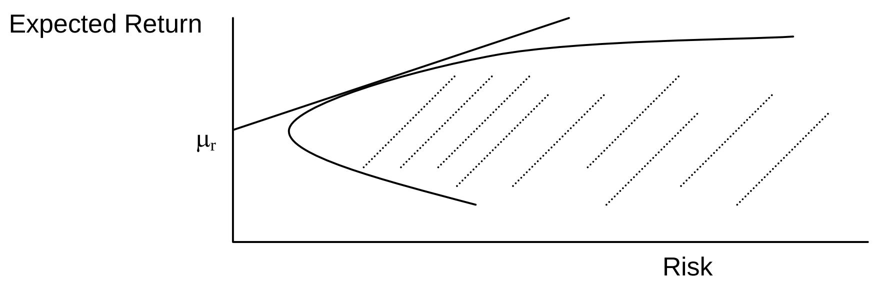


Figure 3 The Markowitz cloud with the risk-free asset.

In the following chapter the traditional method for statistical inference on the correlation between securities will be discussed. A more sophisticated method of conditional dependence modelling will be proposed. Chapter 3 discusses an example in which conditional dependence yields a lot of information about correlation that should change our belief in the risk involved in a certain portfolio. Chapter 4 contains a brief conclusion.

Chapter 2

Issues involved in product moment correlation

The simplicity of the model of Markowitz has made it enormously valuable to portfolio managers. We do not have to gather knowledge about every firm on aspects such as earnings, market share, strategy, dividend policy etc. Only three statistical measures are necessary to determine the risk/return profile of our portfolio. Since it is a model, Markowitz used several assumptions.

A basic assumption in the theory is that historical data provides useful information about the future. In this article we use that assumption too. This means that we assume historical values of returns, standard deviation, and correlation to be good estimates for future values of returns, standard deviation, and correlation.
Another important assumption is that a change in the dependency between securities can only be driven by new data. This article claims that simple correlation statistics does not reveal enough information about the correlation between securities and research on conditional dependence is proposed as a tool to quantify relevant dependence.

Consider for example the following situation. We own a portfolio consisting of several stocks on the AEX-index and these stocks are mixed in such a way that the risk/return profile of the portfolio matches our risk preference according to the Markowitz model.

Question: *Does any information about the height of a stock that's not in our portfolio change our belief in the risk/return profile of our portfolio?*
Answer: *No, since we invest according to the Markowitz model (because* μ, σ *and the statistically measured* ρ *between all stocks in our portfolio does not change).*

In the following chapter evidence is given that conditional dependence between securities cannot be neglected in portfolio selection. Some examples of conditional dependence between several stocks of the AEX-index will be visualized using the software package *Unicorn* (see Cooke (1999)). Cobweb plots show that conditional dependence between stocks A and B given the current height of a certain stock C, which does not necessarily have to be in our portfolio, should change our belief in the risk/return profile of our portfolio.

CHAPTER 3

Conditional Dependence Modeling of Stocks in the AEX-index

3.1 Historical data used

We have used daily closing prices of 12 stocks from the AEX-index in the years 1995 - Feb. 2000 (roughly 5 years). The data has been corrected for stock splits and has been adapted to the EURO currency. Data was taken from www.behr.nl.

3.2 Sampling

Figure 4 shows the historical data of the last 5 years projected in a Cobweb plot. Each vertical line represents one particular stock. Each broken line represents a sample, intersecting the vertical axes in the correct percentile. The scale used is percentiles which makes the dependence between different stocks visible. The 5 years of historical data used together yield 1255 samples.

Strongly positive correlated stocks have a cross density function that is uniformly distributed. Uncorrelated stocks have a cross density that is triangular. Highly negative correlated stocks are those, for which the lines cross in one point. This figure shows what we expected namely correlation values in the range of 0 to +1. Some of the cross densities are almost uniformly others are approximately triangular. Especially in the long term (10 years or more) stocks are highly positive correlated.

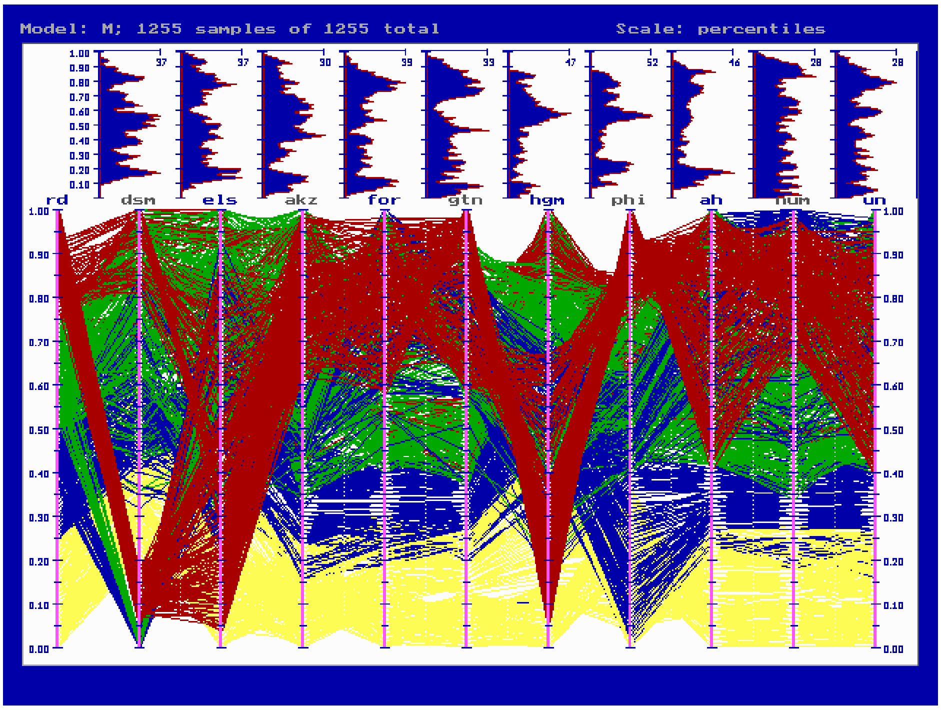


Figure 4 A cobweb plot containing 11 stocks and 1255 samples of daily closing prices. The cross density functions of Ahold (AH), Numico (NUM), and Unilever (UN) show highly positive correlation, whereas the cross density of Hagemeyer (HGM) and Philips (PHI) is more triangular shaped, indicating very low correlation between these two stocks over the last 5 years.

3.3 Conditionalization on the Royal Dutch stock

From Appendix A, report M.rep, we notice that the correlation between HGM and PHI is 0.03 over the last 5 years. However, we expect that it is possible to distinguish market situations for which this correlation is highly negative. To investigate the influence of the actual height of other stocks on the correlation between all stocks we select only the highest 50% stock values of Royal Dutch (RD). This procedure is repeated for the lowest 50% values of RD.

Figures 5a and 5b show the Cobweb plots after conditionalizing on the highest and lowest 50% values of RD over the last 5 years, respectively. The correlation matrix after conditionalizing on the RD stock value can be found in Appendix A, reports MRH.rep and MRL.rep.

According to the reports in Appendix A the following holds:

ρ(HGM,PHI)=0.03,
ρ(HGM,PHI | 50% ≤ RD ≤ 100%) = -0.61, and
ρ(HGM,PHI | 0% ≤ RD < 50%) = 0.30,

in which ' | 50% ≤ RD ≤ 100% ' denotes that conditionalization has been done on the highest 50% stock values of RD. This result yields the first indication that whenever portfolio selection is based on standard correlation calculations, it is likely to misjudge the actual risk/return in a portfolio.

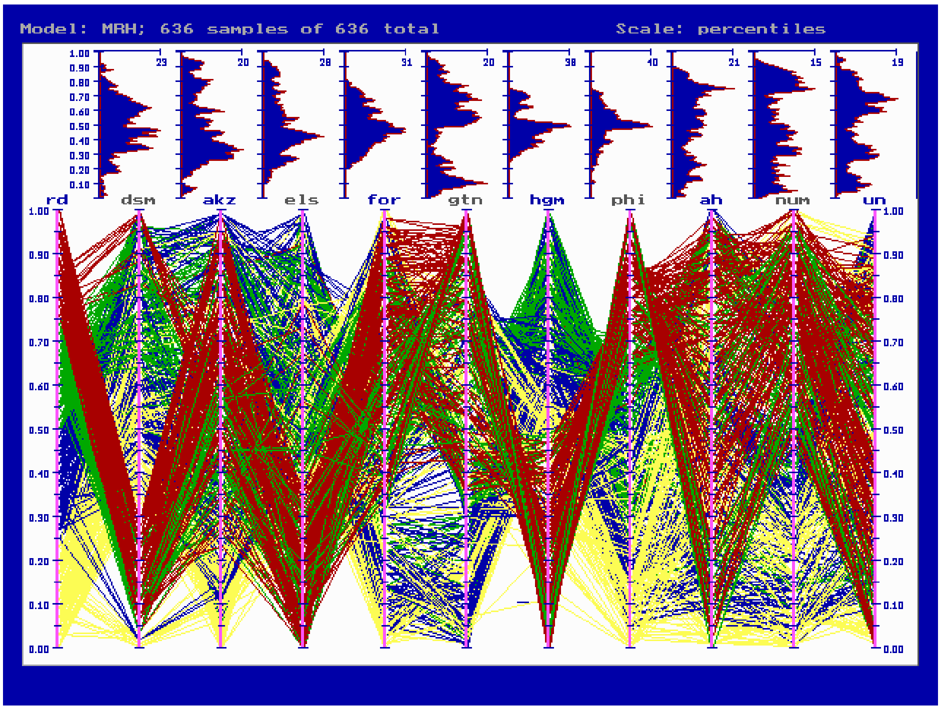
3.4 Conditionalization on the DSM stock

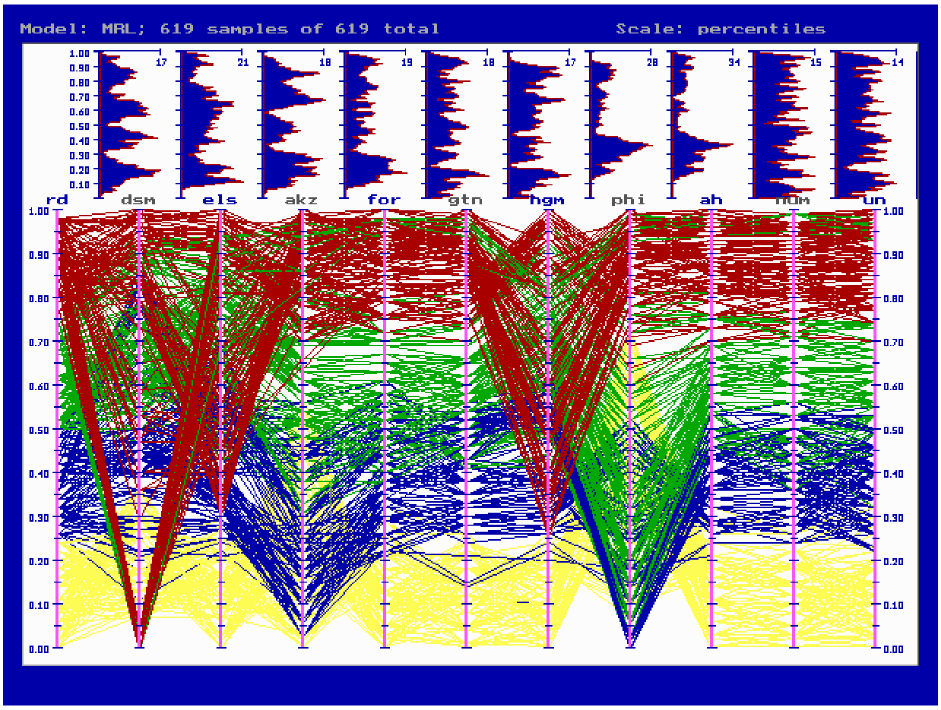
In this section we will perform a more sophisticated research on the correlation between the 11 stocks. After initial conditionalization on the RD stock, we will now distinguish the situation for the four quantiles of stock values of DSM. We get 8 correlation matrices shown in Appendix A, under the names MRHD1-MRHD4 and MRLD1-MRLD4. The Cobweb plots are shown in Figures 6a-6d and Figures 7a-7d, respectively.

The table below contains the most negative conditional dependencies between the different stocks over the last 5 years. The last column contains information on the conditional dependence between HGM and PHI.

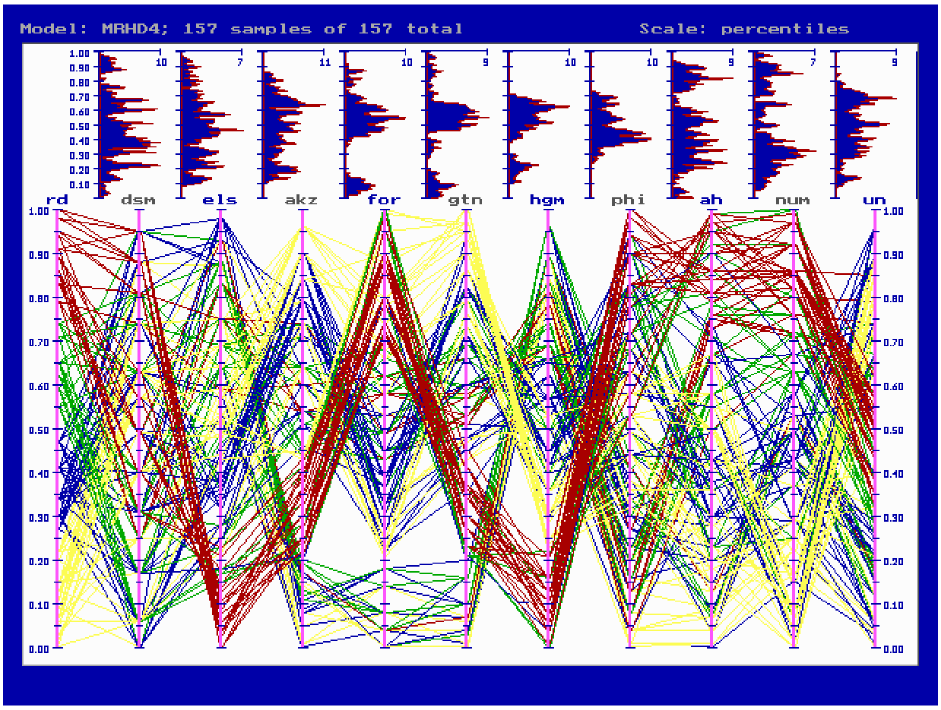
I	II	Given I and II the lowest correlations are:	Given I and II ρ(HGM,PHI) =
RD high	DSM highest 25%	ρ(FOR,HGM) = -0.83	-0.68
RD high	DSM 50%-75%	ρ(AGN,HGM) = -0.82	-0.31
RD high	DSM 25%-50%	ρ (ELS,GTN) = -0.81	-0.59
RD high	DSM lowest 25%	ρ(GTN,HGM) = -0.82	-0.67
RD low	DSM highest 25%	ρ(ELS,AGN) = +0.04	+0.49
RD low	DSM 50%-75%	ρ(DSM,ELS) = +0.22	+0.23
RD low	DSM 25%-50%	ρ(HGM,PHI) = -0.78	-0.78
RD low	DSM lowest 25%	ρ(DSM,NUM) = -0.82	+0.74

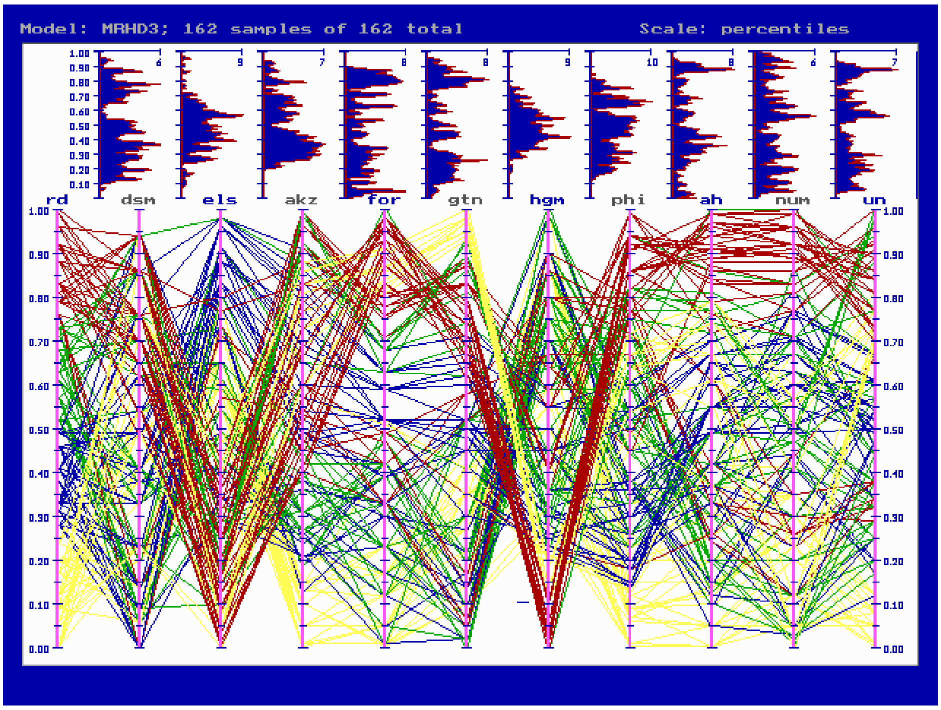
Although originally we expected a correlation of 0.03 between HGM and PHI, we shouldn't be so sure about that anymore. Especially when the RD stock is above average we measure a large negative correlation that must be considered in the process of portfolio selection. As stated in the first Chapter, we can reduce the risk involved in our HGM and PHI containing portfolio whenever that occurs. Also, when we are less risk-averse we can choose for a larger return.



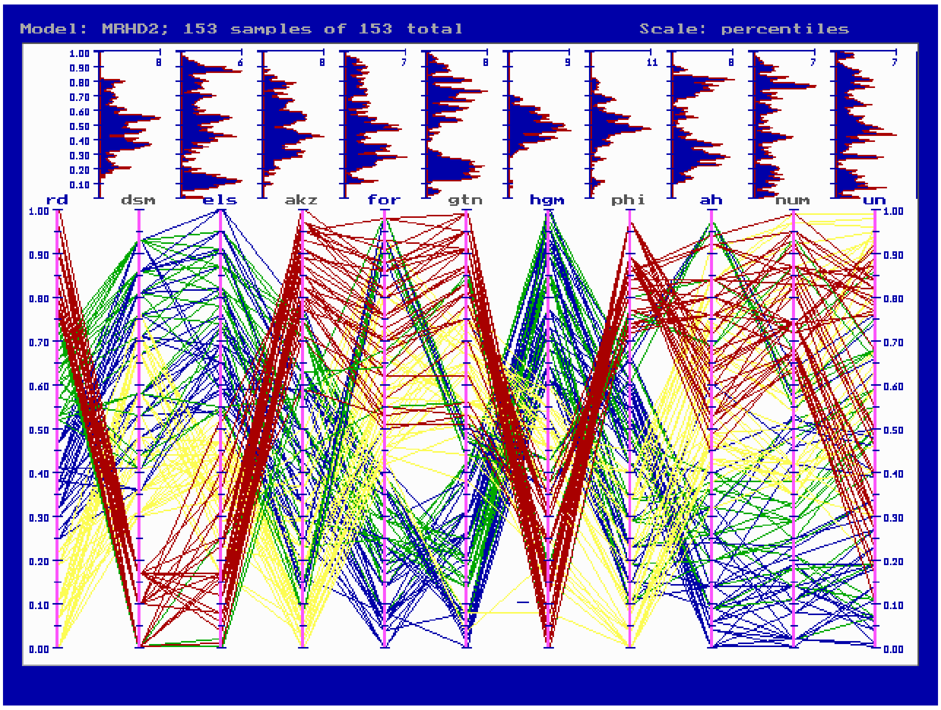


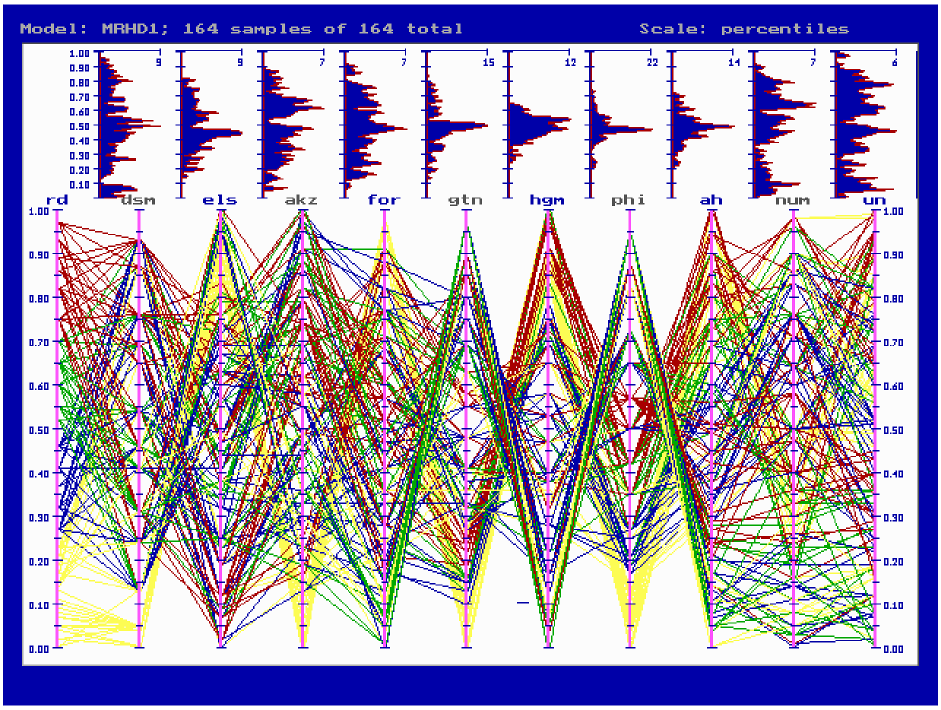
Figures 5a and 5b give indication of dependence between different stocks on the AEX-index given that the actual value of RD is within the 50% highest and lowest of all values in the last 5 years, respectively. Large negative conditional dependence between GTN and HGM as well as between HGM and PHI is indicated in Figure 5a. Most crossings are between the vertical lines are in the middle.



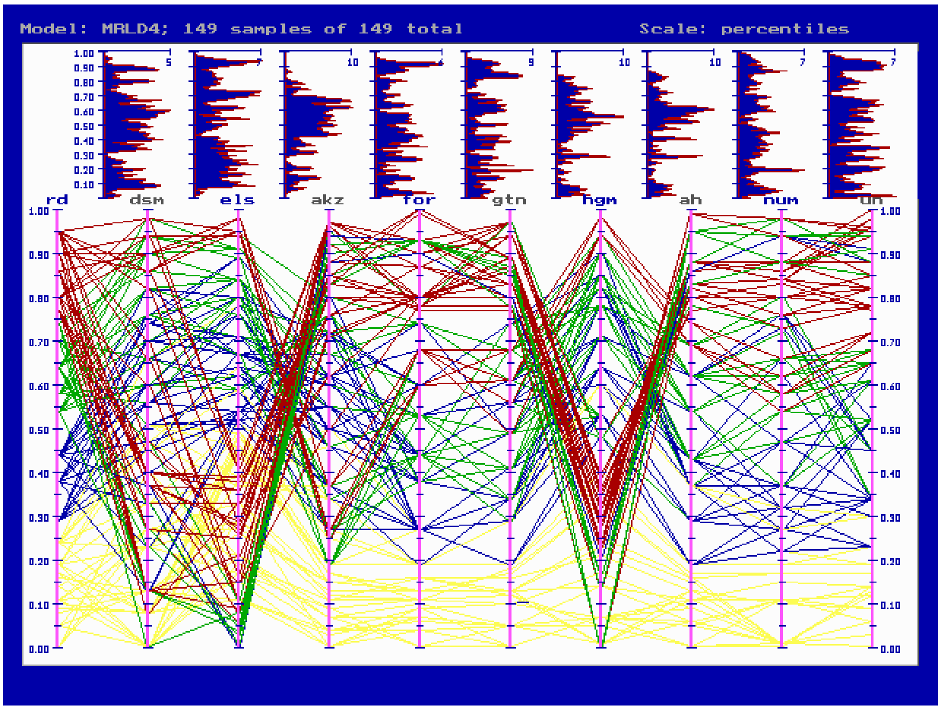


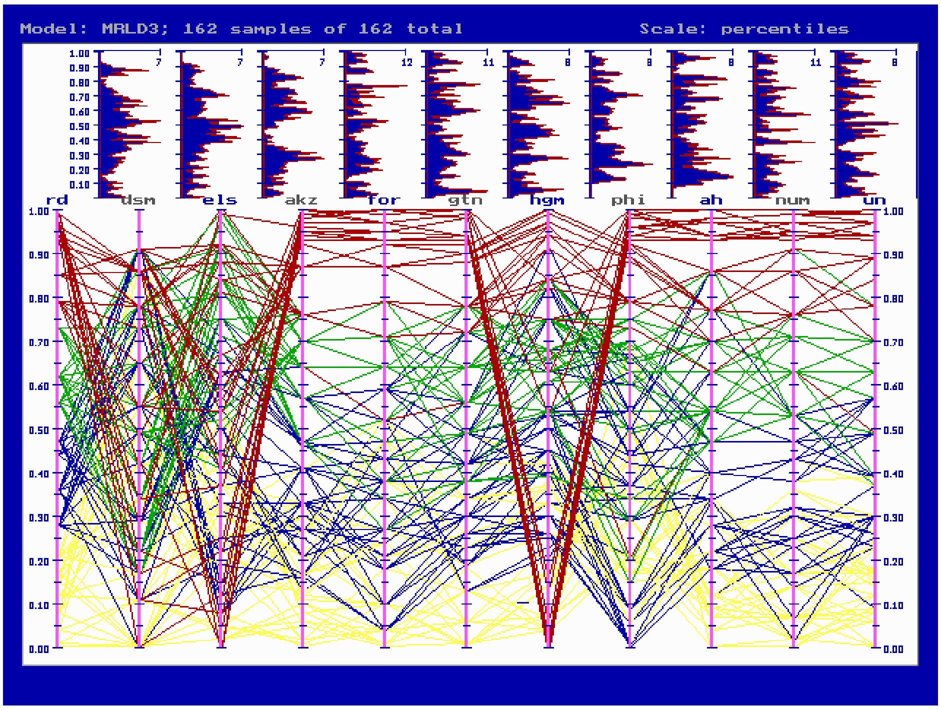
Figures 6a and 6b give an indication of the correlation between 11 stocks after conditionalizing initially on the highest 50% values of RD followed by conditionalization on the 4th and 3rd quantile of the DSM stock, respectively.



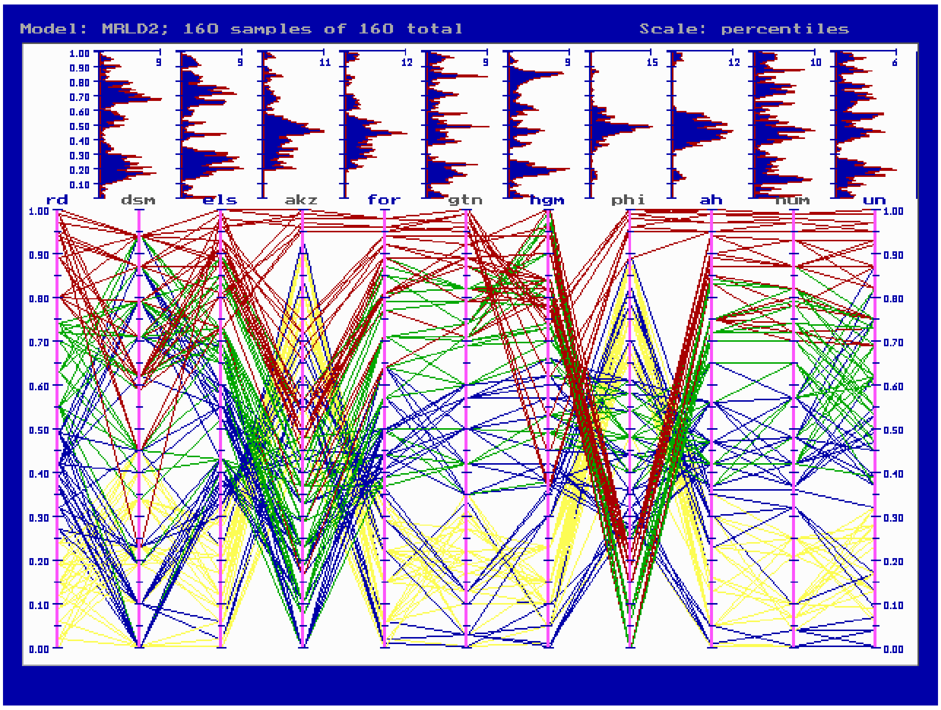


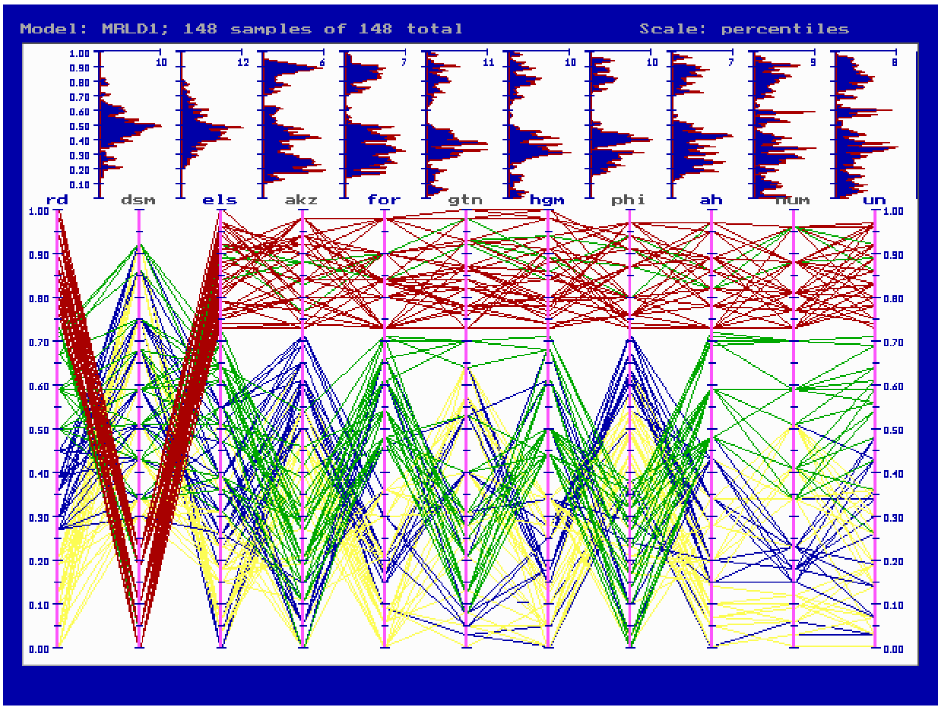
Figures 6c and 6d give an indication of the correlation between 11 stocks after conditionalizing initially on the highest 50% values of RD followed by conditionalization on the 2nd and 1st quantile of the DSM stock, respectively.





Figures 7a and 7b give an indication of the correlation between 11 stocks after conditionalizing initially on the lowest 50% values of RD followed by conditionalization on the 4th and 3rd quantile of the DSM stock, respectively.





Figures 7c and 7d give an indication of the correlation between 11 stocks after conditionalizing initially on the lowest 50% values of RD followed by conditionalization on the 2nd and 1st quantile of the DSM stock, respectively.

Chapter 4

Conclusion

The theory of Markowitz for portfolio selection is widely used by portfolio managers to give an indication of the risk in their portfolio. The model is based on the assumption that only three statistical measures are needed for a proper risk analysis, the mean return of a security, the standard deviation of the return (risk), and the correlation between the securities in our portfolio.

This article has shown that basic statistical correlation calculations do not reveal all the information necessary to judge about risk involved in a portfolio. Instead, a simple example has proven that although a correlation of 0.03 was measured between the Philips and Hagemeyer stock over the years 1995- Feb. 2000 (both stocks are part of the AEX-index) it is possible to define 8 high/low situations of two other stocks in such way that the correlation is always within the ranges between [-0.78,-0.31] and [0.23,0.74]. Therefore, an extensive investigation to the influence of other stocks on the correlation between stocks in a certain portfolio should not be omitted.

On the other hand, speculators may take advantage of conditional dependence research to portfolios. It is likely to find highly negative dependencies, given current market conditions. The method discussed is in practice easy to incorporate in optimization models based on the Markowitz portfolio selection theory.

Appendix A

Unicorn Reports

On the following three pages the reports are given containing a brief overview on the product moment correlation referred to in Chapter 3.

The reports M, MRH, MRL refer to the product moment correlation of the 11 stocks over the last 5 years unconditionalized, and conditionalized on the highest, and the lowest 50% stock values of RD, respectively.

The four reports named MRHD1 - MRHD4 give correlation details on 11 stocks after conditionalizing on the highest 50% values of the Royal Dutch stock during the last 5 years. Further conditionalizing on the lowest 25% of DSM stock values yields MRHD1. MRHD2 - MRHD4 are calculated by gradually conditionalizing on the next 25% stock values of DSM.

The same holds for the reports MRLD1 - MRLD4 although for these reports we have initially conditionalized on the 50% lowest stock values of the Royal Dutch stock.

------------- New Report: M.REP-----------------------------+

Product Moment Correlation Matrix
....

Random Variable	2	4	5	6	7	8	9	10	11	12	13	14
2. agn	1.00	0.90	0.78	-0.27	0.17	0.98	0.81	0.10	0.89	0.70	0.75	0.89
4. ah	0.90	1.00	0.84	-0.04	0.46	0.95	0.73	0.43	0.96	0.65	0.87	0.96
5. akz	0.78	0.84	1.00	0.08	0.47	0.83	0.81	0.45	0.80	0.76	0.90	0.87
6. dsm	-0.27	-0.04	0.08	1.00	0.55	-0.21	-0.27	0.67	-0.17	-0.33	-0.03	0.04
7. els	0.17	0.46	0.47	0.55	1.00	0.29	0.24	0.88	0.42	0.12	0.43	0.45
8. for	0.98	0.95	0.83	-0.21	0.29	1.00	0.82	0.23	0.94	0.73	0.83	0.92
9. gtn	0.81	0.73	0.81	-0.27	0.24	0.82	1.00	0.13	0.81	0.92	0.83	0.69
10. hgm	0.10	0.43	0.45	0.67	0.88	0.23	0.13	1.00	0.37	0.03	0.42	0.45
11. num	0.89	0.96	0.80	-0.17	0.42	0.94	0.81	0.37	1.00	0.70	0.87	0.90
12. phi	0.70	0.65	0.76	-0.33	0.12	0.73	0.92	0.03	0.70	1.00	0.83	0.59
13. rd	0.75	0.87	0.90	-0.03	0.43	0.83	0.83	0.42	0.87	0.83	1.00	0.83
14. un	0.89	0.96	0.87	0.04	0.45	0.92	0.69	0.45	0.90	0.59	0.83	1.00

------------ New Report: MRH.REP----------------------------+

Product Moment Correlation Matrix
....

Random Variable	2	4	5	6	7	8	9	10	11	12	13	14
2. agn	1.00	0.64	0.39	-0.46	-0.60	0.92	0.70	-0.65	0.70	0.49	0.35	0.50
4. ah	0.64	1.00	0.30	-0.11	-0.35	0.79	0.31	-0.26	0.73	0.37	0.41	0.75
5. akz	0.39	0.30	1.00	0.05	-0.10	0.36	0.59	-0.10	0.26	0.54	0.52	0.48
6. dsm	-0.46	-0.11	0.05	1.00	0.72	-0.43	-0.51	0.87	-0.58	-0.50	-0.45	0.14
7. els	-0.60	-0.35	-0.10	0.72	1.00	-0.57	-0.59	0.82	-0.69	-0.61	-0.57	-0.12
8. for	0.92	0.79	0.36	-0.43	-0.57	1.00	0.63	-0.61	0.81	0.54	0.46	0.51
9. gtn	0.70	0.31	0.59	-0.51	-0.59	0.63	1.00	-0.66	0.60	0.73	0.56	0.32
10. hgm	-0.65	-0.26	-0.10	0.87	0.82	-0.61	-0.66	1.00	-0.68	-0.61	-0.53	0.01
11. num	0.70	0.73	0.26	-0.58	-0.69	0.81	0.60	-0.68	1.00	0.62	0.65	0.41
12. phi	0.49	0.37	0.54	-0.50	-0.61	0.54	0.73	-0.61	0.62	1.00	0.85	0.14
13. rd	0.35	0.41	0.52	-0.45	-0.57	0.46	0.56	-0.53	0.65	0.85	1.00	0.15
14. un	0.50	0.75	0.48	0.14	-0.12	0.51	0.32	0.01	0.41	0.14	0.15	1.00

------------ New Report: MRL.REP----------------------------+

Product Moment Correlation Matrix
....

Random Variable	2	4	5	6	7	8	9	10	11	12	13	14
2. agn	1.00	0.99	0.83	0.05	0.57	0.99	0.97	0.58	0.98	0.71	0.83	0.98
4. ah	0.99	1.00	0.83	0.03	0.63	0.99	0.96	0.61	0.99	0.69	0.84	0.98
5. akz	0.83	0.83	1.00	0.34	0.64	0.84	0.87	0.66	0.81	0.76	0.84	0.84
6. dsm	0.05	0.03	0.34	1.00	0.47	0.03	0.18	0.57	0.01	0.02	0.48	0.08
7. els	0.57	0.63	0.64	0.47	1.00	0.62	0.64	0.92	0.63	0.33	0.83	0.60
8. for	0.99	0.99	0.84	0.03	0.62	1.00	0.96	0.61	0.99	0.71	0.83	0.98
9. gtn	0.97	0.96	0.87	0.18	0.64	0.96	1.00	0.68	0.94	0.70	0.88	0.97
10. hgm	0.58	0.61	0.66	0.57	0.92	0.61	0.68	1.00	0.61	0.30	0.85	0.61
11. num	0.98	0.99	0.81	0.01	0.63	0.99	0.94	0.61	1.00	0.69	0.83	0.97
12. phi	0.71	0.69	0.76	0.02	0.33	0.71	0.70	0.30	0.69	1.00	0.53	0.71
13. rd	0.83	0.84	0.84	0.48	0.83	0.83	0.88	0.85	0.83	0.53	1.00	0.86
14. un	0.98	0.98	0.84	0.08	0.60	0.98	0.97	0.61	0.97	0.71	0.86	1.00

----------- New Report: MRHD1.REP---------------------------+

Product Moment Correlation Matrix

Random Variable	2	4	5	6	7	8	9	10	11	12	13	14
2. agn	1.00	0.43	-0.18	-0.19	-0.09	0.87	-0.22	0.26	0.37	-0.52	-0.39	0.26
4. ah	0.43	1.00	-0.13	0.28	-0.09	0.71	-0.77	0.84	0.66	-0.60	-0.14	0.92
5. akz	-0.18	-0.13	1.00	0.48	-0.20	-0.11	0.47	-0.26	-0.02	0.67	0.75	-0.19
6. dsm	-0.19	0.28	0.48	1.00	-0.47	-0.09	0.10	0.08	0.54	0.29	0.60	0.36
7. els	-0.09	-0.09	-0.20	-0.47	1.00	-0.07	-0.29	0.18	-0.47	-0.27	-0.41	0.00
8. for	0.87	0.71	-0.11	-0.09	-0.07	1.00	-0.49	0.54	0.47	-0.59	-0.29	0.52
9. gtn	-0.22	-0.77	0.47	0.10	-0.29	-0.49	1.00	-0.82	-0.30	0.80	0.52	-0.78
10. hgm	0.26	0.84	-0.26	0.08	0.18	0.54	-0.82	1.00	0.42	-0.67	-0.30	0.85
11. num	0.37	0.66	-0.02	0.54	-0.47	0.47	-0.30	0.42	1.00	-0.36	0.02	0.68
12. phi	-0.52	-0.60	0.67	0.29	-0.27	-0.59	0.80	-0.67	-0.36	1.00	0.82	-0.65
13. rd	-0.39	-0.14	0.75	0.60	-0.41	-0.29	0.52	-0.30	0.02	0.82	1.00	-0.20
14. un	0.26	0.92	-0.19	0.36	0.00	0.52	-0.78	0.85	0.68	-0.65	-0.20	1.00

----------- New Report: MRHD2.REP---------------------------+

Product Moment Correlation Matrix

Random Varble	2	4	5	6	7	8	9	10	11	12	13	14
2. agn	1.00	0.80	0.43	-0.60	-0.74	0.97	0.84	-0.76	0.89	0.52	0.39	0.62
4. ah	0.80	1.00	0.16	-0.50	-0.63	0.83	0.56	-0.59	0.85	0.42	0.31	0.80
5. akz	0.43	0.16	1.00	-0.53	-0.40	0.43	0.65	-0.46	0.50	0.85	0.90	-0.06
6. dsm	-0.60	-0.50	-0.53	1.00	0.82	-0.60	-0.80	0.87	-0.65	-0.74	-0.64	-0.19
7. els	-0.74	-0.63	-0.40	0.82	1.00	-0.64	-0.81	0.79	-0.77	-0.58	-0.47	-0.49
8. for	0.97	0.83	0.43	-0.60	-0.64	1.00	0.79	-0.74	0.88	0.56	0.42	0.56
9. gtn	0.84	0.56	0.65	-0.80	-0.81	0.79	1.00	-0.80	0.77	0.71	0.60	0.31
10. hgm	-0.76	-0.59	-0.46	0.87	0.79	-0.74	-0.80	1.00	-0.72	-0.59	-0.52	-0.35
11. num	0.89	0.85	0.50	-0.65	-0.77	0.88	0.77	-0.72	1.00	0.63	0.56	0.72
12. phi	0.52	0.42	0.85	-0.74	-0.58	0.56	0.71	-0.59	0.63	1.00	0.91	0.02
13. rd	0.39	0.31	0.90	-0.64	-0.47	0.42	0.60	-0.52	0.56	0.91	1.00	0.06
14. un	0.62	0.80	-0.06	-0.19	-0.49	0.56	0.31	-0.35	0.72	0.02	0.06	1.00

----------- New Report: MRHD3.REP---------------------------+

Product Moment Correlation Matrix

Random Variable	2	4	5	6	7	8	9	10	11	12	13	14
2. agn	1.00	0.83	0.72	0.50	-0.46	0.96	0.80	-0.82	0.66	0.67	0.43	0.77
4. ah	0.83	1.00	0.60	0.56	-0.38	0.90	0.58	-0.66	0.91	0.72	0.69	0.75
5. akz	0.72	0.60	1.00	0.49	-0.12	0.65	0.79	-0.51	0.48	0.66	0.47	0.87
6. dsm	0.50	0.56	0.49	1.00	-0.35	0.56	0.62	-0.23	0.52	0.60	0.39	0.60
7. els	-0.46	-0.38	-0.12	-0.35	1.00	-0.43	-0.36	0.24	-0.20	-0.63	-0.35	-0.28
8. for	0.96	0.90	0.65	0.56	-0.43	1.00	0.75	-0.80	0.77	0.65	0.46	0.75
9. gtn	0.80	0.58	0.79	0.62	-0.36	0.75	1.00	-0.59	0.43	0.61	0.23	0.85
10. hgm	-0.82	-0.66	-0.51	-0.23	0.24	-0.80	-0.59	1.00	-0.55	-0.31	-0.18	-0.58
11. num	0.66	0.91	0.48	0.52	-0.20	0.77	0.43	-0.55	1.00	0.57	0.62	0.64
12. phi	0.67	0.72	0.66	0.60	-0.63	0.65	0.61	-0.31	0.57	1.00	0.78	0.71
13. rd	0.43	0.69	0.47	0.39	-0.35	0.46	0.23	-0.18	0.62	0.78	1.00	0.54
14. un	0.77	0.75	0.87	0.60	-0.28	0.75	0.85	-0.58	0.64	0.71	0.54	1.00

----------- New Report: MRHD4.REP---------------------------+

Product Moment Correlation Matrix

Random Variable	2	4	5	6	7	8	9	10	11	12	13	14
2. agn	1.00	0.52	0.57	-0.10	-0.45	0.83	0.60	-0.77	-0.06	0.51	-0.09	0.72
4. ah	0.52	1.00	0.20	0.07	-0.62	0.78	0.01	-0.71	0.68	0.50	0.67	0.38
5. akz	0.57	0.20	1.00	-0.25	-0.06	0.36	0.85	-0.28	-0.20	0.14	-0.15	0.82
6. dsm	-0.10	0.07	-0.25	1.00	0.06	-0.03	-0.27	0.06	0.16	-0.02	0.21	-0.29
7. els	-0.45	-0.62	-0.06	0.06	1.00	-0.57	-0.03	0.83	-0.39	-0.79	-0.50	-0.42
8. for	0.83	0.78	0.36	-0.03	-0.57	1.00	0.31	-0.83	0.36	0.48	0.27	0.53
9. gtn	0.60	0.01	0.85	-0.27	-0.03	0.31	1.00	-0.22	-0.48	0.21	-0.37	0.83
10. hgm	-0.77	-0.71	-0.28	0.06	0.83	-0.83	-0.22	1.00	-0.29	-0.68	-0.29	-0.54
11. num	-0.06	0.68	-0.20	0.16	-0.39	0.36	-0.48	-0.29	1.00	0.19	0.83	-0.09
12. phi	0.51	0.50	0.14	-0.02	-0.79	0.48	0.21	-0.68	0.19	1.00	0.34	0.49
13. rd	-0.09	0.67	-0.15	0.21	-0.50	0.27	-0.37	-0.29	0.83	0.34	1.00	0.05
14. un	0.72	0.38	0.82	-0.29	-0.42	0.53	0.83	-0.54	-0.09	0.49	0.05	1.00

----------- New Report: MRLD1.REP----------------------------+

Product Moment Correlation Matrix

Random Variable	2	4	5	6	7	8	9	10	11	12	13	14
2. agn	1.00	1.00	0.91	-0.81	0.96	1.00	0.97	0.97	1.00	0.78	0.98	0.99
4. ah	1.00	1.00	0.93	-0.81	0.97	1.00	0.98	0.97	1.00	0.80	0.98	0.99
5. akz	0.91	0.93	1.00	-0.64	0.90	0.91	0.94	0.87	0.93	0.92	0.93	0.93
6. dsm	-0.81	-0.81	-0.64	1.00	-0.78	-0.82	-0.79	-0.81	-0.82	-0.55	-0.75	-0.77
7. els	0.96	0.97	0.90	-0.78	1.00	0.97	0.93	0.91	0.96	0.77	0.95	0.96
8. for	1.00	1.00	0.91	-0.82	0.97	1.00	0.97	0.97	0.99	0.77	0.98	0.98
9. gtn	0.97	0.98	0.94	-0.79	0.93	0.97	1.00	0.95	0.98	0.85	0.94	0.97
10. hgm	0.97	0.97	0.87	-0.81	0.91	0.97	0.95	1.00	0.97	0.74	0.94	0.96
11. num	1.00	1.00	0.93	-0.82	0.96	0.99	0.98	0.97	1.00	0.81	0.97	0.99
12. phi	0.78	0.80	0.92	-0.55	0.77	0.77	0.85	0.74	0.81	1.00	0.78	0.81
13. rd	0.98	0.98	0.93	-0.75	0.95	0.98	0.94	0.94	0.97	0.78	1.00	0.98
14. un	0.99	0.99	0.93	-0.77	0.96	0.98	0.97	0.96	0.99	0.81	0.98	1.00

----------- New Report: MRLD2.REP----------------------------+

Product Moment Correlation Matrix

Random Variable	2	4	5	6	7	8	9	10	11	12	13	14
2. agn	1.00	0.99	0.18	0.58	0.87	0.99	0.95	0.70	0.98	-0.23	0.96	0.94
4. ah	0.99	1.00	0.15	0.62	0.89	0.98	0.96	0.73	0.99	-0.27	0.97	0.93
5. akz	0.18	0.15	1.00	-0.05	-0.05	0.18	0.18	-0.32	0.13	0.51	0.20	0.13
6. dsm	0.58	0.62	-0.05	1.00	0.71	0.60	0.67	0.70	0.63	-0.34	0.56	0.47
7. els	0.87	0.89	-0.05	0.71	1.00	0.88	0.87	0.89	0.90	-0.52	0.83	0.82
8. for	0.99	0.98	0.18	0.60	0.88	1.00	0.95	0.72	0.98	-0.27	0.95	0.92
9. gtn	0.95	0.96	0.18	0.67	0.87	0.95	1.00	0.75	0.96	-0.31	0.93	0.91
10. hgm	0.70	0.73	-0.32	0.70	0.89	0.72	0.75	1.00	0.76	-0.78	0.66	0.67
11. num	0.98	0.99	0.13	0.63	0.90	0.98	0.96	0.76	1.00	-0.31	0.96	0.93
12. phi	-0.23	-0.27	0.51	-0.34	-0.52	-0.27	-0.31	-0.78	-0.31	1.00	-0.23	-0.22
13. rd	0.96	0.97	0.20	0.56	0.83	0.95	0.93	0.66	0.96	-0.23	1.00	0.91
14. un	0.94	0.93	0.13	0.47	0.82	0.92	0.91	0.67	0.93	-0.22	0.91	1.00

----------- New Report: MRLD3.REP----------------------------+

Product Moment Correlation Matrix

Random Variable	2	4	5	6	7	8	9	10	11	12	13	14
2. agn	1.00	0.93	0.84	0.25	0.41	0.96	0.90	0.36	0.95	0.89	0.91	0.86
4. ah	0.93	1.00	0.91	0.24	0.42	0.93	0.94	0.42	0.95	0.81	0.95	0.95
5. akz	0.84	0.91	1.00	0.40	0.39	0.89	0.88	0.45	0.87	0.76	0.92	0.92
6. dsm	0.25	0.24	0.40	1.00	0.22	0.31	0.26	0.35	0.24	0.28	0.27	0.31
7. els	0.41	0.42	0.39	0.22	1.00	0.36	0.38	0.74	0.43	0.26	0.33	0.42
8. for	0.96	0.93	0.89	0.31	0.36	1.00	0.90	0.37	0.94	0.90	0.91	0.88
9. gtn	0.90	0.94	0.88	0.26	0.38	0.90	1.00	0.44	0.95	0.81	0.93	0.95
10. hgm	0.36	0.42	0.45	0.35	0.74	0.37	0.44	1.00	0.42	0.23	0.31	0.49
11. num	0.95	0.95	0.87	0.24	0.43	0.94	0.95	0.42	1.00	0.84	0.94	0.93
12. phi	0.89	0.81	0.76	0.28	0.26	0.90	0.81	0.23	0.84	1.00	0.80	0.75
13. rd	0.91	0.95	0.92	0.27	0.33	0.91	0.93	0.31	0.94	0.80	1.00	0.93
14. un	0.86	0.95	0.92	0.31	0.42	0.88	0.95	0.49	0.93	0.75	0.93	1.00

----------- New Report: MRLD4.REP----------------------------+

Product Moment Correlation Matrix

Random Variable	2	4	5	6	7	8	9	10	11	12	13	14
2. agn	1.00	0.97	0.87	0.34	0.04	0.96	0.94	0.29	0.97	0.95	0.92	0.96
4. ah	0.97	1.00	0.90	0.40	0.16	0.97	0.96	0.38	0.98	0.95	0.95	0.97
5. akz	0.87	0.90	1.00	0.34	0.07	0.92	0.94	0.35	0.90	0.88	0.86	0.89
6. dsm	0.34	0.40	0.34	1.00	0.72	0.39	0.42	0.75	0.42	0.54	0.50	0.38
7. els	0.04	0.16	0.07	0.72	1.00	0.09	0.15	0.87	0.16	0.24	0.27	0.16
8. for	0.96	0.97	0.92	0.39	0.09	1.00	0.96	0.32	0.96	0.93	0.94	0.95
9. gtn	0.94	0.96	0.94	0.42	0.15	0.96	1.00	0.42	0.95	0.95	0.94	0.96
10. hgm	0.29	0.38	0.35	0.75	0.87	0.32	0.42	1.00	0.38	0.49	0.48	0.39
11. num	0.97	0.98	0.90	0.42	0.16	0.96	0.95	0.38	1.00	0.96	0.93	0.98
12. phi	0.95	0.95	0.88	0.54	0.24	0.93	0.95	0.49	0.96	1.00	0.94	0.96
13. rd	0.92	0.95	0.86	0.50	0.27	0.94	0.94	0.48	0.93	0.94	1.00	0.95
14. un	0.96	0.97	0.89	0.38	0.16	0.95	0.96	0.39	0.98	0.96	0.95	1.00

Appendix B

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