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# Processing Expert Judgements in Accident Consequence Modelling

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## 1 Introduction

In doing uncertainty analysis on Accident Consequence Codes, the uncertainty analyst has to construct a distribution on the *target variables*, the uncertain code input parameters. To illustrate the construction of this distribution in this project, we make use of an example taken from the Foodchain module. We will start off with a brief introduction on the methodology on structured expert judgement elicitation used in this project. Due to the methodology a mathematical technique, termed Probabilistic inversion, had to be developed. Besides eliciting information on marginal distributions of elicitation variables (quantities for which the experts have to provide assessments), the experts provided information on dependencies among a selection of elicitation variables. In order to show the effect of correlation in doing uncertainty analysis, we concluded with a comparison between correlated versus uncorrelated propagation of the distribution on the target variables.

## 2 Structured Expert Judgement Elicitation

We will make use of an example taken from the Foodchain module to illustrate a vital part of the methodology on structured expert judgement elicitation adopted in this project. The example focuses on the movement of radioactive material in rootcrops, which is modeled in FARMLAND [2] using a compartmental model. Project staff regarded that it would be sufficient to do uncertainty analysis on a simplified version of the compartmental model for rootcrops as implemented in FARMLAND, see Figure 1. For this example, the target variables are the transfer coefficients [3] $(k_{21,n}, k_{24,n}, k_{45,n}, k_{51,n})$ for nuclide $n$. Project staff decided to determine a distribution on the target variables for the nuclides *Sr* and *Cs* only.

Based on Figure 1, a set of first order differential equations can be constructed which, with the appropriate initial conditions, can be solved and fully specifies the movement of the material between the compartments. A vital part of the methodology on structured expert judgement adopted in this project says that experts should on provide information on quantities which are measurable, in principle and with which the experts are familiar. To summarize, the quantities for which the experts have to provide information should be such that the experts can envision an experiment which measures these quantities. The transfer coefficients were regarded as non-measurable and therefore it was decided that they could not serve as elicitation variables. Instead, elicitation variables were formulated on the concentrations of *Sr* and *Cs* in the edible portion (Tuber) of rootcrops at different times before harvest:

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[2] FARMLAND is a computer code developed at the National Radiological Protection Board (NRPB) which calculates dose coefficients in a variety of plants and animals.

[3] Transfer coefficients $k_{lj,n}$ represent the proportion of radioactive material for nuclide $n$ moved from box $l$ to box $j$.

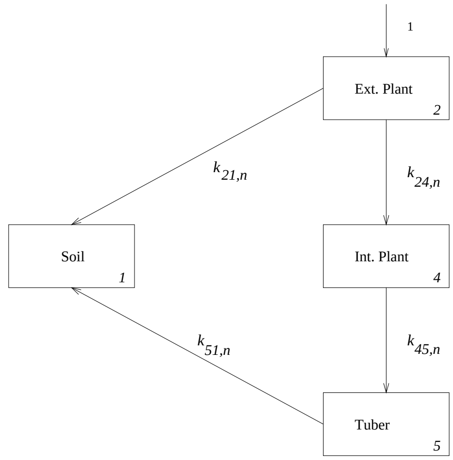


Figure 1: *Simplified compartmental model for rootcrops, for nuclide n*

*What is the concentration (Bq kg$^{-1}$ wet weight) of Sr and Cs in the edible portion of root crops at harvest, for a single deposition intercepted by the plant of 1 Bq m$^{-2}$ occuring 15, 30, 60, 90 days before harvest?*

The experts provided 5%, 50% and 95% quantiles of their distributions for the question above. The assessments of the experts were aggregated using equal weights to obtain 5%,50% and 95% quantiles of the distribution of the Decision Maker (DM), see Table 1.

| | Concentration of *Cs* in Tuber at time *t* | | | Concentration of *Sr* in Tuber at time *t* | | |
|---|---|---|---|---|---|---|
| | 5% | 50% | 95% | 5% | 50% | 95% |
| 15 days | 1.72e-6 | 8.08e-3 | 1.06e-1 | 9.61e-5 | 6.18e-6 | 1.59e-3 |
| 30 days | 1.46e-5 | 1.02e-2 | 1.14e-1 | 1.42e-6 | 9.63e-6 | 1.61e-3 |
| 60 days | 1.34e-4 | 1.96e-2 | 1.37e-1 | 3.48e-6 | 3.48e-5 | 1.96e-3 |
| 90 days | 1.42e-4 | 2.83e-2 | 1.80e-1 | 9.73e-6 | 1.15e-4 | 2.06e-3 |

Table 1: *Quantile information of the marginal distributions of the Decision Maker for rootcrops.*

Note, the expert/DM data give information on the concentration at different times for 'compartment 5', the Tuber, in Figure 1. The solution of the set of differential equations for compartment 5, $m_{5,n}(t)$ for nuclide $n$ is:

$$m_{5,n}(t) = \frac{k_{45,n}\,k_{24,n}\,e^{-k_{51,n}t}}{(k_{24,n}+k_{21,n}-k_{51,n})(k_{45,n}-k_{51,n})} - \frac{k_{45,n}\,k_{24,n}\,e^{-k_{45,n}t}}{(k_{24,n}+k_{21,n}-k_{45,n})(k_{45,n}-k_{51,n})} + \\ + \frac{k_{45,n}\,k_{24,n}\,e^{-(k_{21,n}+k_{24,n})\,t}}{(k_{24,n}+k_{21,n}-k_{51,n})(k_{24,n}+k_{21,n}-k_{45,n})} \tag{1}$$

# 3 Probabilistic Inversion

Suppose we had a distribution over the target variables for nuclide $n$. We could then push this distribution through the compartmental model and obtain a distribution over, for example the retention of nuclide $n$ at various times in the Tuber, using Equation 1. The problem at hand involves reversing this procedure: we have quantiles of distributions over the retention of nuclide $n$ in the Tuber at certain times, as given in Table 2; thus we seek a distribution over the target variables of nuclide $n$ which, when pushed through the compartmental model, yields quantiles over retentions of nuclide $n$ in the Tuber agreeing with those from the DM. Hence our problem is one of probabilistic inversion.

Mathematically, let $H$ represent a distribution of the DM over retention in given compartments at given times. Let $F$ represent a distribution over the target variables in the rootcrop model, and let $G(F)$ represent the 'push-through' distribution over retention in the Tuber at the given times, obtained by pushing the distribution $F$ through the model $G$. Then our problem may be represented as: Find $F$ such that $G(F) \sim H$, where '$\sim$' means 'has the same distribution as', or equivalently, $F \sim G^{-1}(H)$. Note that a probabilistic inverse $G^{-1}(H)$ may not exist, and if it exists it will be in general not unique. Therefore we must have a method of selecting a preferred distribution in case of non-uniqueness and a method of choosing a best fitting distribution in case of non existence, for details see [2],[3]. Throughout the whole project, information on 33 models has been probabilistically inverted:

**Dispersion & deposition** : Gaussian model for 4 stability classes and Wet deposition for Methyl-Iodide, Elemental Iodine and Aerosol particles.

**Foodchain** : Soil migration model for 2 nuclides, Grain model for 2 nuclides and Rootcrop model for 2 nuclides.

**Early Health Effects** : Mortality for GI-tract, Lung and Whole body and morbidity for Skin and Lung.

**Internal Dosimetry** : Lung model, absorption to the blood for 7 different nuclides, Systemic retention for 7 different nuclides.

Note also that the technique of probabilistic inversion is not restricted to expert judgement only. Distributions obtained from a series of experiments under similar conditions can also be used as input.

Based on the elicited information and Equation 1, a distribution on $(k_{21,n}, k_{24,n}, k_{45,n}, k_{51,n})$ for each nuclide $n$ was determined, see Table 2. Table 3 compares quantile information between the DM and the 'push-through' obtained from the probabilistic inversion.

| Target variable | 5% | 50% | 95% | Target variable | 5% | 50% | 95% |
|---|---|---|---|---|---|---|---|
| $k_{21,Cs}$ | 2.43e-4 | 2.22e-3 | 4.09e-1 | $k_{21,Sr}$ | 2.54e-4 | 9.00e-3 | 9.53e-2 |
| $k_{24,Cs}$ | 6.58e-6 | 5.42e-3 | 4.25e-2 | $k_{24,Sr}$ | 4.27e-7 | 3.11e-6 | 2.31e-4 |
| $k_{45,Cs}$ | 6.29e-4 | 2.58e-2 | 2.94e-1 | $k_{45,Sr}$ | 8.36e-3 | 4.27e-2 | 2.28e-1 |
| $k_{51,Cs}$ | 2.81e-5 | 3.10e-2 | 1.47e-1 | $k_{51,Sr}$ | 5.39e-6 | 1.98e-3 | 2.96e-2 |

Table 2: *Quantile information of the marginal distributions on the target variables.*

| | | $m_{5,Cs}(t)$ | | $m_{5,Sr}(t)$ | |
|---|---|---|---|---|---|
| | Percentiles | DM | Prob. Inv. | DM | Prob. Inv. |
| 15 days | 5% | 1.72e-6 | 1.72e-6 | 9.61e-7 | 3.61e-7 |
| | 50% | 8.08e-3 | 8.08e-3 | 6.18e-5 | 6.18e-5 |
| | 95% | 1.06e-1 | 1.06e-1 | 1.59e-3 | 1.59e-3 |
| 30 days | 5% | 1.46e-5 | 5.80e-6 | 1.42e-6 | 1.41e-6 |
| | 50% | 1.02e-2 | 1.02e-2 | 9.63e-6 | 2.04e-5 |
| | 95% | 1.14e-1 | 1.13e-1 | 1.61e-3 | 2.01e-3 |
| 60 days | 5% | 1.34e-4 | 1.76e-5 | 3.48e-6 | 5.22e-6 |
| | 50% | 1.96e-2 | 1.97e-2 | 3.48e-5 | 3.49e-5 |
| | 95% | 1.37e-1 | 1.37e-1 | 1.96e-3 | 1.79e-3 |
| 90 days | 5% | 1.42e-4 | 2.04e-5 | 9.73e-6 | 9.66e-6 |
| | 50% | 2.83e-2 | 6.60e-3 | 1.15e-4 | 7.15e-5 |
| | 95% | 1.80e-1 | 1.85e-1 | 2.06e-3 | 1.99e-3 |

Table 3: *Comparison of quantile information between Decision Maker (DM) and 'push-through' obtained from Probabilistic Inversion (Prob. Inv.)*

To summarize, probabilistic inversion supplied the uncertainty analyst with 2 distribution; a distribution on the target variables for *Cs* and a distribution on the target variables for *Sr*. The distributions were represented in the uncertainty analysis by marginal distributions with rank correlation matrices. Project staff recognized that there may be a potential dependence between the retention of *Cs* and *Sr* in the Tuber of the rootcrop. The next section focuses on how this dependence was elicited from the experts and how it was used to construct *one* distribution on the target variables of both *Cs* and *Sr*.

# 4 Elicitation of Dependence

In the course of the project, a new method for eliciting dependencies from experts has been developed, for a detailed description see [2]. The method developed is quick and easily understandable to the experts. The dependence is specified by a conditional probability among the elicitation variables of interest. For rootcrops the experts provided their conditional probability[4] for the following question:

> *Consider an experiment in which a unit Cs and a unit Sr is deposited on a rootcrop. Given that the concentration of strontium in the Tuber in this experiment 15 days before harvesting is above its median value, what is the probability that the concentration of caesium in the Tuber in the same experiment 15 days before harvesting is above its median value as well?*

The conditional probabilities of the experts were combined and translated into *one* rank correlation for the DM, $\tau_{DM}(m_{5,Cs}(15d.), m_{5,Sr}(15d.))$. Next, the distributions on the target variables for $Cs$ and $Sr$ were linked using the simulation package UNICORN [1]. UNICORN allows the user to specify a dependence structure among target variables via an acyclic graph. Modelling dependence via an acyclic graph it is ensured that the resulting rank correlation matrix is positive definite, see [4]. An acyclic dependency graph has been constructed among the target variables for both *Cs* and *Sr*, based on the rank correlation matrices obtained from the probabilistic inversion. Next, the dependency graphs were linked in such a way that the rank correlation between 'push-through' distributions of $m_{5,Cs}(15d.)$ and $m_{5,Sr}(15d.)$ equaled $\tau_{DM}(m_{5,Cs}(15d.), m_{5,Sr}(15d.))$. In this way *one* distribution on the target variables of both *Cs* and *Sr* is constructed. The resulting dependency graph is shown in Figure 2

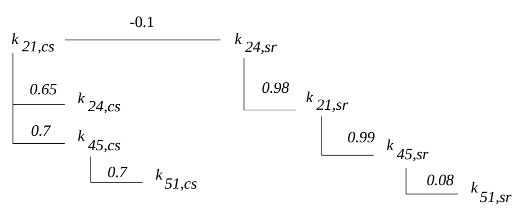


Figure 2: *Dependency graph with rank correlations among target variables for rootcrops.*

# 5 Influence of Correlations

The purpose of this section is to illustrate the effect of taking account of correlations among the target variables in doing uncertainty analysis. In performing the uncertainty analysis, the distribution on the target variables will be represented by marginal distributions together with a rank correlation matrix. The simulation package UNICORN can deal with this kind of representation of a distribution. The simulation results (correlated and uncorrelated) for both *Cs* and *Sr* are given in Table 4. For the uncorrelated propagation the range factors do not vary much in time, whereas the range factors in the correlated propagation decrease with time. The 5%-tiles of the correlated and uncorrelated propagation at all time-points are roughly the same, but the 95%-tiles

[4] Briefly, a conditional probability of 0 indicates that 2 elicitation variables are rank correlated by -1, a conditional probability of 0.5 indicates that 2 elicitation variables are uncorrelated and a conditional probability of 1 indicates that 2 elicitation variables are rank correlated by 1.

| | | *Cs* | | *Sr* | |
|---|---|---|---|---|---|
| | Percentiles | Correlated | Uncorrelated | Correlated | Uncorrelated |
| 15 days | 5% | 1.42e-6 | 1.58e-6 | 3.79e-7 | 4.55e-7 |
| | 50% | 7.38e-3 | 5.28e-3 | 1.10e-5 | 9.09e-6 |
| | 95% | 9.1e-2 | 1.2e-1 | 1.13e-3 | 6.84e-4 |
| Range factor | | 6.4e4 | 7.6e4 | 3000 | 1503 |
| 30 days | 5% | 4.94e-6 | 3.63e-6 | 1.42e-6 | 1.45e-6 |
| | 50% | 1.35e-2 | 1.168e-2 | 3.32e-5 | 2.55e-5 |
| | 95% | 1.27e-1 | 2.34e-1 | 1.83e-3 | 1.97e-3 |
| Range factor | | 2.6e4 | 6.45e4 | 1288 | 1359 |
| 60 days | 5% | 1.24e-5 | 5.84e-6 | 4.69e-6 | 3.16e-6 |
| | 50% | 1.53e-2 | 1.84e-2 | 7.39e-5 | 5.53e-5 |
| | 95% | 1.38e-1 | 3.55e-1 | 1.88e-3 | 4.36e-3 |
| Range factor | | 1.1e4 | 6.1e4 | 401 | 1380 |
| 90 days | 5% | 9.11e-6 | 5.75e-6 | 8.08e-6 | 3.45e-6 |
| | 50% | 1.33e-2 | 1.89e-2 | 1.01e-4 | 7.7e-5 |
| | 95% | 0.15 | 4.07e-1 | 1.8e-3 | 5.85e-3 |
| Range factor | | 1.6e4 | 7.1e4 | 223 | 1696 |

Table 4: *Simulation results (correlated and uncorrelated) for rootcrops for Cs and Sr (5000 samples)*

start off as being almost the same but in time the difference between correlated versus uncorrelated propagation is observed. Clearly the effect of the correlations in performing the uncertainty analysis cannot be ignored.

## 6 Conclusion

By making use of the rootcrop model from the Foodchain module, this paper illustrated the construction of the joint distribution on the target variables. Due to the methodology on structured expert judgement elicitation adopted in this project, two mathematical techniques (Probabilistic Inversion and the elicitation of dependencies) had to be developed, which provide the uncertainty analysis with information of the required distribution. Based on all information the uncertainty analyst constructed the distribution on the target variables. Finally, the effect of correlations in performing uncertainty analysis has been studied. The results clearly showed the effect of the correlations, which let to the conclusions that for this case the correlations also have to be taken into account. Although the rank correlations in the rootcrop example were large, cases have been studied for which the rank correlations among the target variables were small, but the difference between correlated propagation versus uncorrelated propagation was large, citeesrel97.

## References


[1] R.M. Cooke. *UNICORN; Methods and Code for Uncertainty Analysis.* AEA Technology, Warrington, Cheshire (UK), 1995.

[2] J.A. Jones, J. Ehrhardt, L.H.J. Goossens, J. Brown, R.M. Cooke, F. Fischer, I. Hasemann, and B.C.P. Kraan. Probabilistic accident consequence uncertainty assessment using cosyma: Methodology and processing techniques. Technical Report EUR 18827, National Radiological Protection Board and Forschungszentrum Karlsruhe GmbH and Delft University of Technology, 1999.

[3] B.C.P. Kraan. *Joint CEC/USNRC Uncertainty analysis on Accident Consequence Codes using Expert Judgment.* PhD thesis, Delft University of Technology, 2000.

[4] A.M.H. Meeuwissen and R.M. Cooke. Tree dependent random variables (revised version). Technical Report TUD-REPORT 94-28 and ISSN 0922-5641, Delft University of Technology, 1994.