Inspection and maintenance decisions based on imperfect inspections

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Presentation outline

- Introduction
- Model concept + gamma process
- Perfect & imperfect inspections
- Case study: inspecting a hydrogen dryer
- Conclusions

Introduction

Research objective

“To develop an easy-to-use model for making optimal inspection and maintenance decisions for pressurized vessels in the process industry, under the assumption that field measurement data is imperfect.”

Applying a stochastic process to the corrosion degradation mechanism

Corrosion state functions

The thinning due to corrosion state function is given by (American Petroleum Institute, 2000):

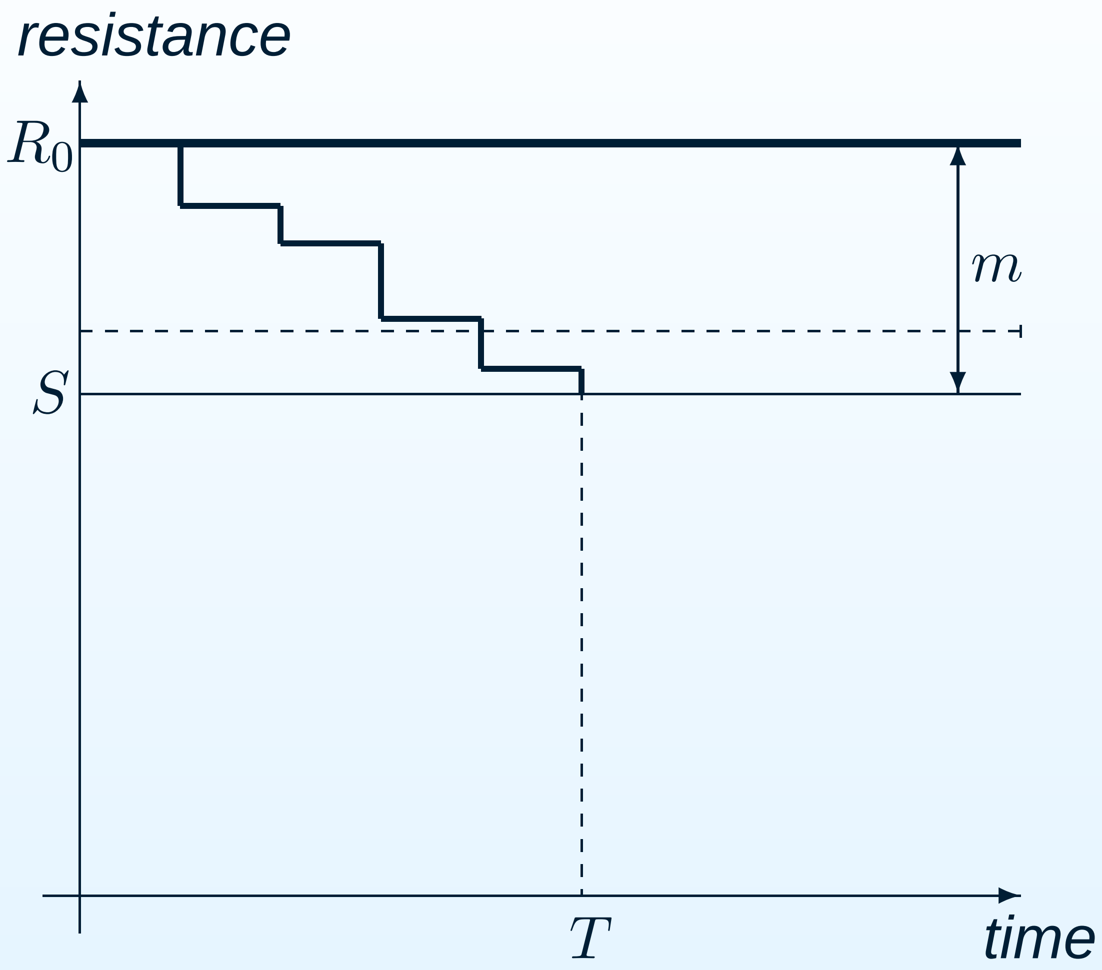
$$g(t) = \underbrace{S\left(1 - \frac{C \times t}{th}\right)}_{Resistance} - \underbrace{\left(\frac{P \times d}{2th}\right)}_{Stress},$$

$S =$	Material strength [MPa = 10bar]	$t =$	Time [yr]
$C =$	Corrosion rate [mm/yr]	$d =$	Diameter [mm]
$P =$	Operating pressure [bar]	$th =$	Thickness [mm]

The limit state is given by:

$$g(t) = 0 \Leftrightarrow th - C \times t = \frac{P \times d}{2S}$$

Stochastic deterioration



← Initial thickness

← Corrosion allowance

← Failure

The safety margin is given by:

$$m = th - \frac{P \times d}{2S}.$$

Bayesian gamma stochastic process decision model

Gamma process for corrosion

- The corrosion is modelled as a gamma process with independent increments: $X(t) \sim \text{Ga}(x|\alpha t, \beta)$ with $t \geq 0$.
- Assuming linear degradation we want

$$\mathbb{E}(X(t)) = \mu t \text{ and } \text{Var}(X(t)) = \sigma^2 t,$$

  where μ is the average amount of deterioration per time unit and σ is the standard deviation of the deterioration.
- For $X(t)$ it holds that $\mathbb{E}(X(t)) = \frac{\alpha}{\beta} t$ and $\text{Var}(X(t)) = \frac{\alpha}{\beta^2} t$, therefore the parameters of the process are given by

$$\mu = \frac{\alpha}{\beta}, \sigma^2 = \frac{\alpha}{\beta^2} \quad \Rightarrow \quad \alpha = \left(\frac{\mu}{\sigma}\right)^2, \beta = \frac{\mu}{\sigma^2}.$$

Fixing the standard deviation

- The mean and variance are uncertain, but by fixing the standard deviation relative to the mean, i.e.

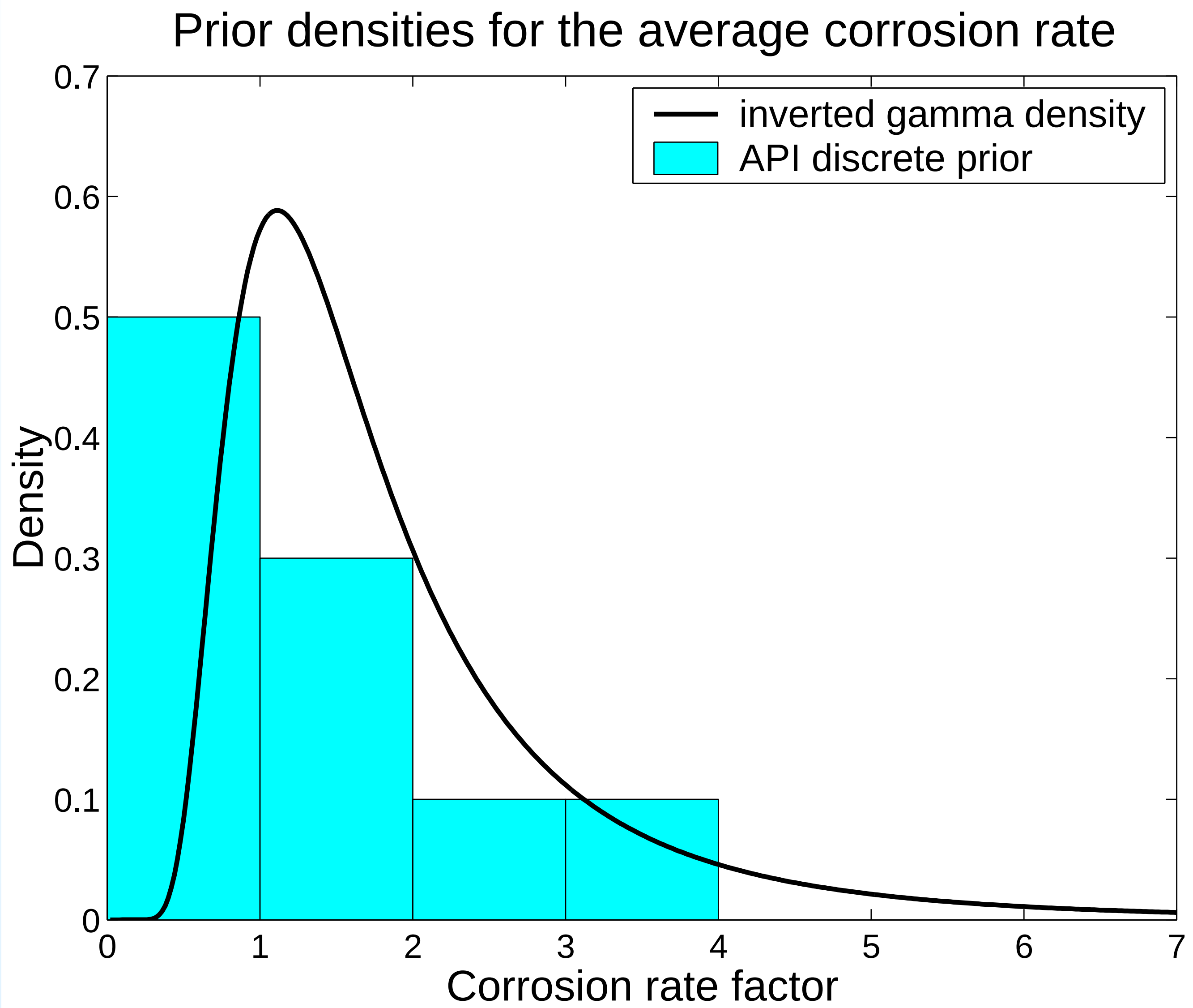
$$\sigma = \text{COV} \times \mu,$$

results in (ν = COV)

$$X(t) \sim \text{Ga}\left(x \,\middle|\, \frac{t}{\nu^2}, \frac{1}{\mu\nu^2}\right), \quad \nu > 0.$$

- This manipulation avoids the need for an assessment of the COV in the absence of sufficient data.

Inverted gamma prior



Perfect inspections (1)

- Prior density $\pi(\mu) = \text{Ig}(\mu|a,b)$, where the inverted gamma density with parameters $a > 0$ and $b > 0$ is defined as:

$$\text{Ig}\,(x|a,b) = \frac{b^a}{\Gamma(a)}\left(\frac{1}{x}\right)^{a+1}\text{exp}\left\{-\frac{b}{x}\right\},\;\; x \geq 0.$$

- Posterior density with one inspection:

$$\pi(\mu|x) = \text{Ig}\left(\mu\,\middle|\,\frac{t}{\nu^2}+a, \frac{x}{\nu^2}+b\right) \propto l(x|\mu)\pi(\mu),$$

where $l(x|\mu)$ is the likelihood of measurement x ($x > 0$) at time t and $\pi(\mu)$ is the prior for the average corrosion rate.

Perfect inspections (2)

- Multiple perfect inspections:

$$\pi(\mu|x_1,\ldots,x_n) =$$

$$= \text{Ig}\left(\mu \,\middle|\, \frac{\sum_{i=1}^{n} t_i - t_{i-1}}{\nu^2} + a \,,\, \frac{\sum_{i=1}^{n} x_i - x_{i-1}}{\nu^2} + b\right)$$

- due to our choice of $\nu = \sigma/\mu$, the posterior for multiple perfect inspections only depends on the last measurement x_n at time t_n. We assume that $t_0 = x_0 = 0$.

Imperfect inspections (1)

- Current non-destructive testing (NDT) techniques, like ultrasonic thickness measurements, are not capable of measuring the exact material thickness.
- To extend the existing Bayesian updating model such that it can cope with imperfect inspections, we use a stochastic process $Y(t)$ consisting of the actual deterioration process $X(t)$ plus a normally distributed measurement error:

$$Y(t) = X(t) + \epsilon \quad \text{with} \quad \epsilon \sim \mathcal{N}(0, \sigma_\epsilon).$$

Imperfect inspections (2)

- likelihood for 1 inspection:

$$l(y|\mu) = f_{Y(t)}(y) = f_{X(t)+\epsilon}(y) = \int_{-\infty}^{\infty} f_{X(T)}(y-\epsilon) f_{\epsilon}(\epsilon) d\epsilon$$

- likelihood for $K > 1$ inspections:

$$l(y_1, \ldots, y_K|\mu) = \prod_k l_{Y(k)-Y(k-1)}(y_k - y_{k-1}|\mu) =$$

$$= \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \prod_k f_{D_k}(y_k - y_{k-1} - \delta_k) f(\delta_1, \ldots, \delta_k) d\delta_1 \cdots d\delta_k$$

where $D_k = Y(k) - Y(k-1)$ and $\delta_k = \epsilon_k - \epsilon_{k-1}$.

Imperfect inspections (3)

- Using Monte Carlo integration, the likelihood is approximated by

$$l(y_1, \ldots, y_K|\mu) \approx$$

$$\frac{1}{N}\sum_{j=1}^{N}\left[\prod_k \text{Ga}\left(d_k - \delta_k^{(j)} \left| \frac{t_k - t_{k-1}}{\nu^2}, \frac{1}{\mu\nu^2}\right.\right) I_{[0,\infty)}(d_k - \delta_k^{(j)})\right],$$

as $N \longrightarrow \infty$ and where $d_k = y_k - y_{k-1}$ and $\delta_k^{(j)} = \epsilon_k^{(j)} - \epsilon_{k-1}^{(j)}$.

Expected average costs per unit time

- A useful cost based decision criterium is the expected average costs per time unit. For each inspection interval Δk we calculate the ratio of the expected (cumulative) cycle costs over the expected cycle length:

$$C(\rho, \Delta k) = \frac{\sum_{i=1}^{\infty} c_i(\rho, \Delta k) p_i(\rho, \Delta k)}{\sum_{i=1}^{\infty} i p_i(\rho, \Delta k)},$$

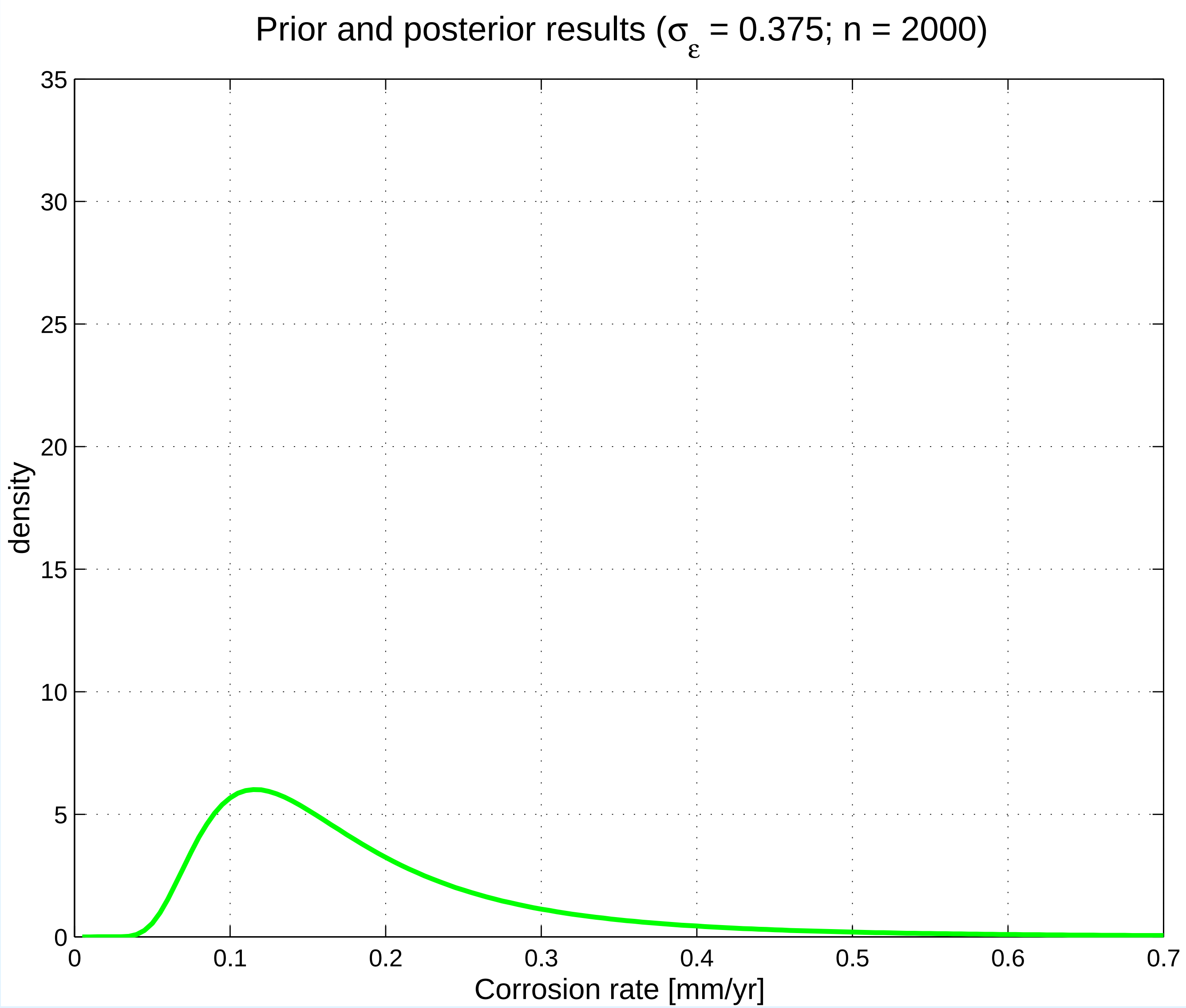
  where ρ defines the replacement level (i.e. the corrosion allowance)

Case study: inspecting a hydrogen dryer

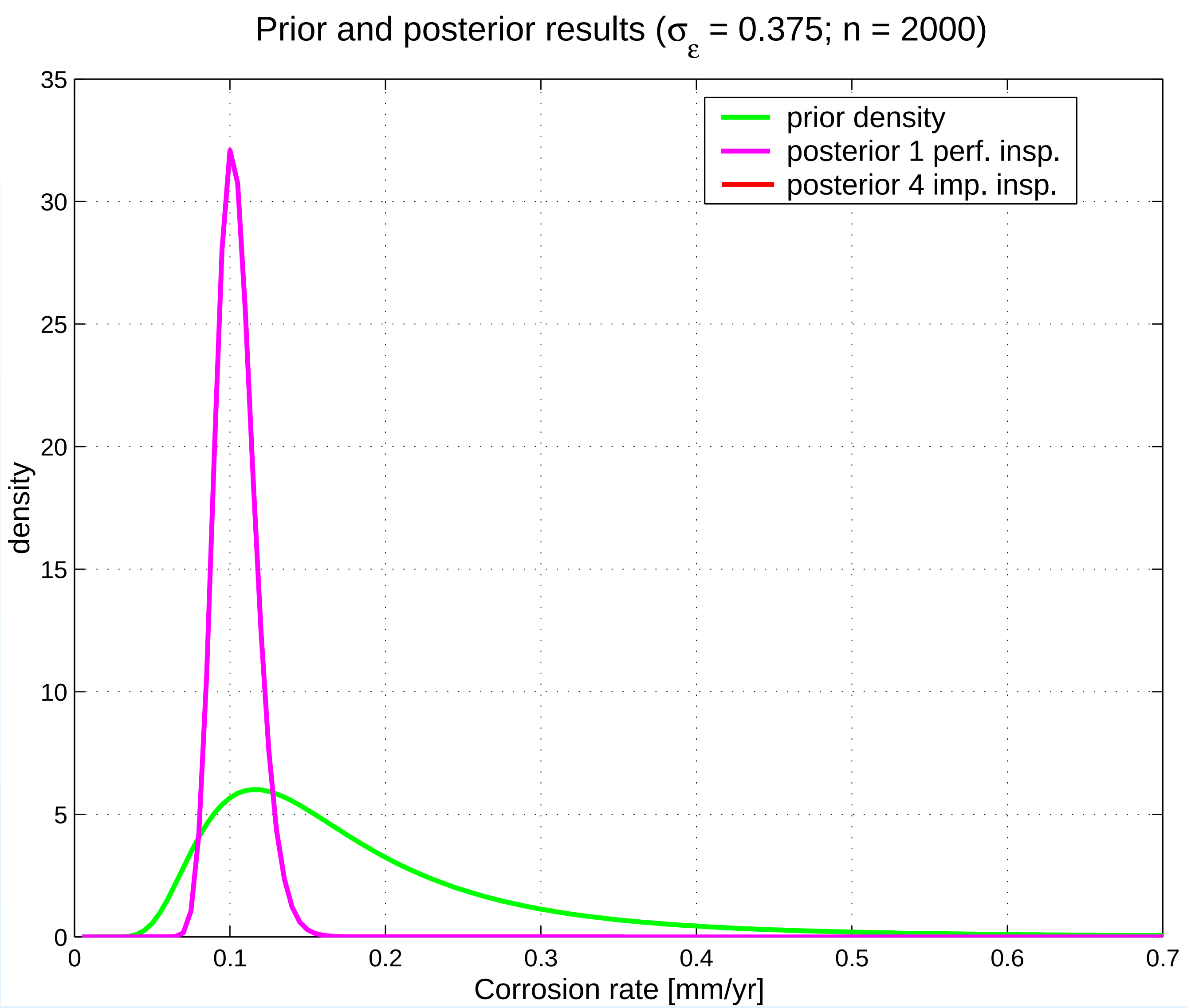
Vertical drum input data

Service start:	1977	
Operating pressure:	32	bar
Drum diameter:	1180	mm
Initial thickness:	15+12% = 16.8	mm
Corrosion allowance:	4.5	mm
Ultrasonic thickness measurements:		
1986:	15.6	mm
1990:	14.6	mm
1994:	14.2	mm
1998:	13.8	mm
Costs:		
Inspection:	10,000	$
Preventive replacement:	50,000	$
Corrective replacement:	1,000,000	$

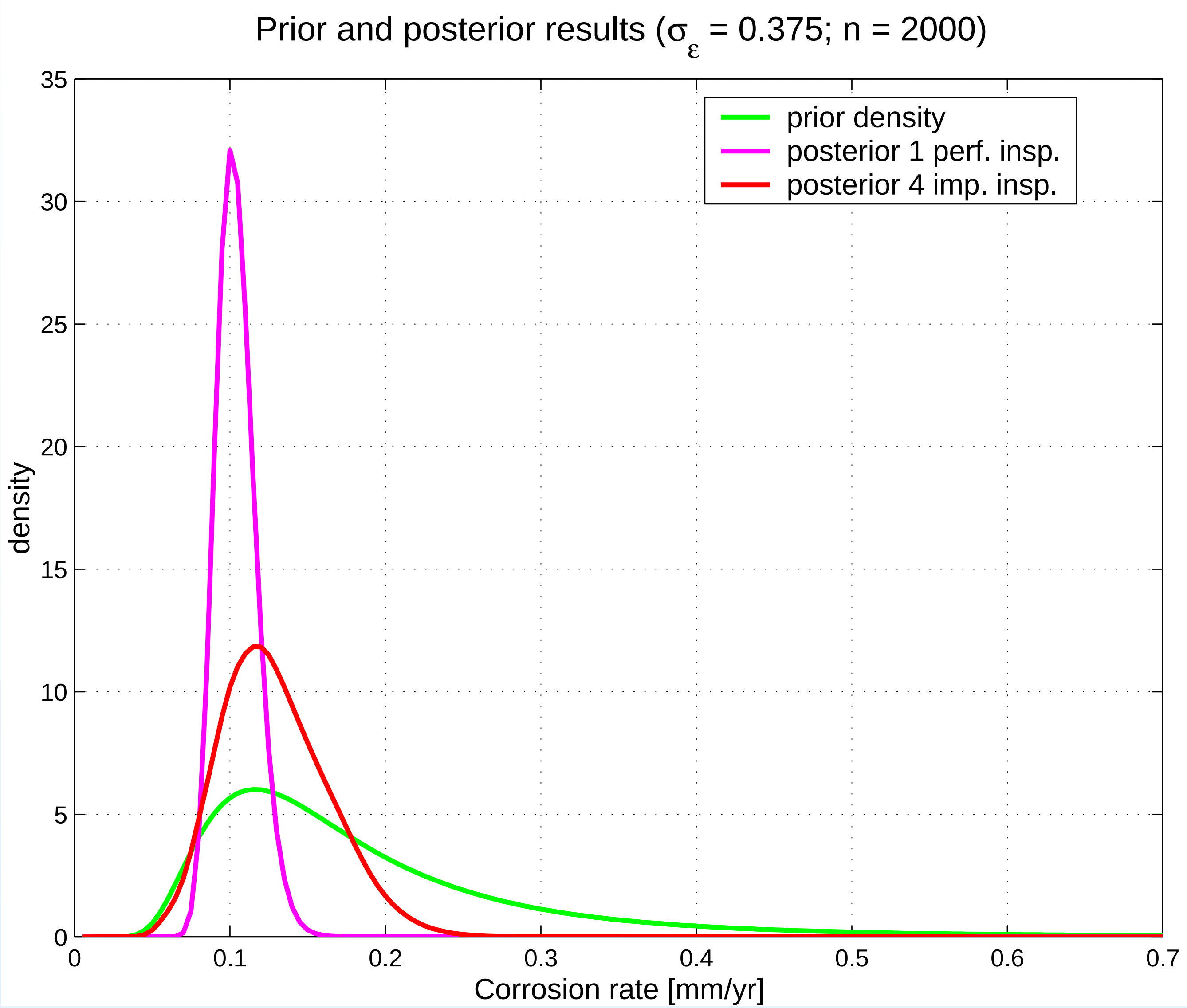
Continuous prior and posterior



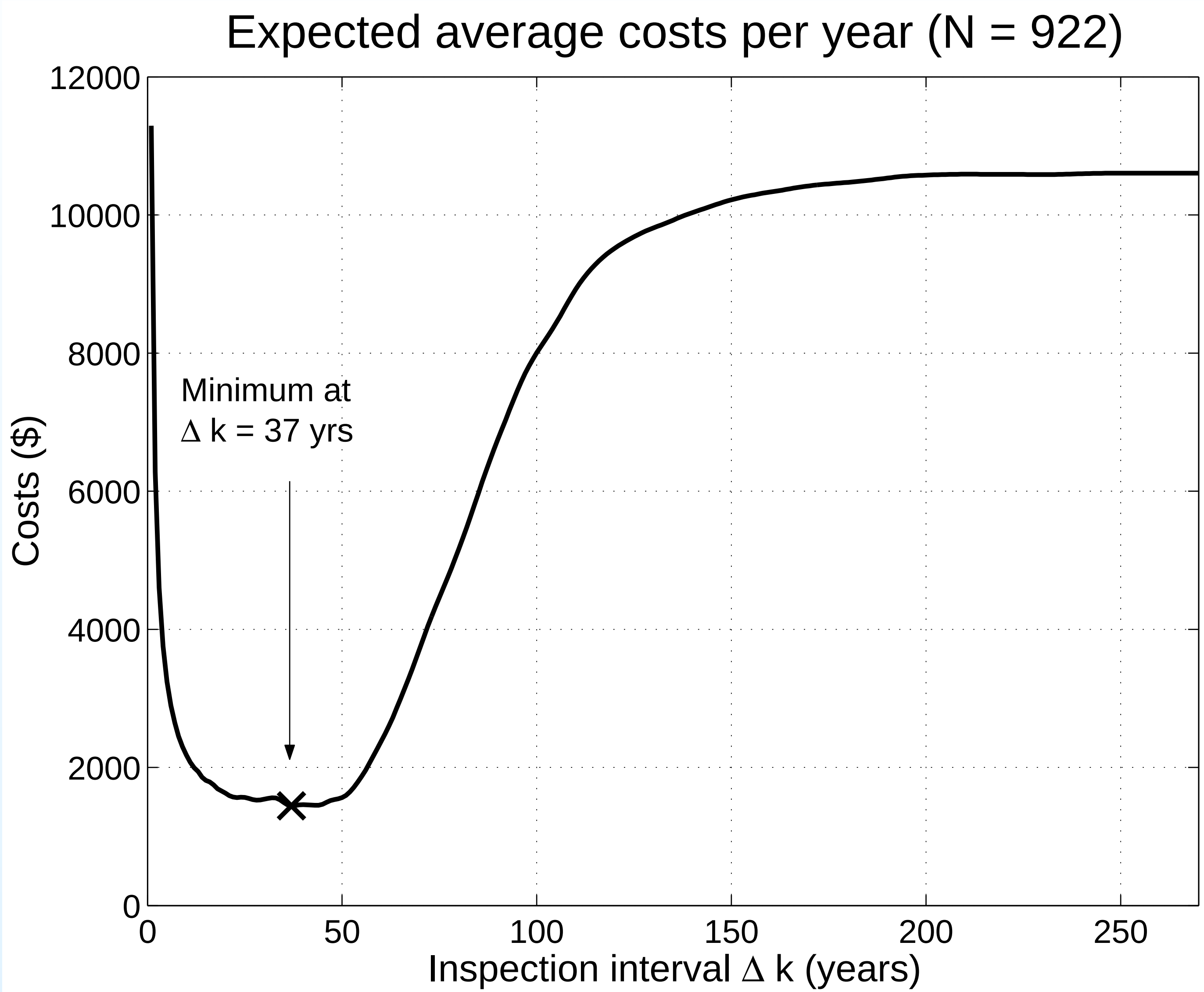
Continuous prior and posterior



Continuous prior and posterior



Expected average costs per unit time



Conclusions

- The Bayesian stochastic process with gamma distributed increments is a very good alternative to current methodologies. The expected average costs per year cost criterium is well suited for safe and economical decisions.
- Computationally this model is inefficient, but with the right assumptions and a proper implementation this can be greatly improved:
  - For example: we can choose an isotropic time grid (i.e. $t = 1/\alpha = \text{COV}^2$), such that the amounts of deterioration are exponentially distributed. This enables more analytic calculations, reducing the computational load.