Techniques for Modelling the Life-Cycle Cost of Civil Infrastructures

Jan van Noortwijk

Delft University of Technology, the Netherlands

HKV Consultants, Lelystad, the Netherlands

Leo Klatter

Ministry of Transport, Public Works and Water management

Civil Engineering Division, Utrecht, the Netherlands

Contents

Introduction

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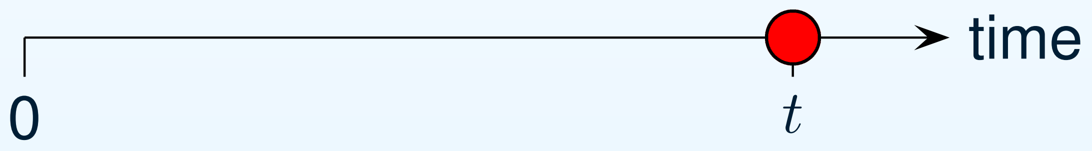
- Netherlands Directorate General for Public Works and Water Management is responsible for management of road bridges
- Management can be optimised by balancing maintenance and replacement of bridges using life-cycle costing:
  - information on time and cost of bridge replacement and maintenance is needed
- Paper has two objectives:
  - determining lifetime distributions for concrete bridges
  - computing the expected cost of replacement and maintenance of the current bridge stock as a function of time and age

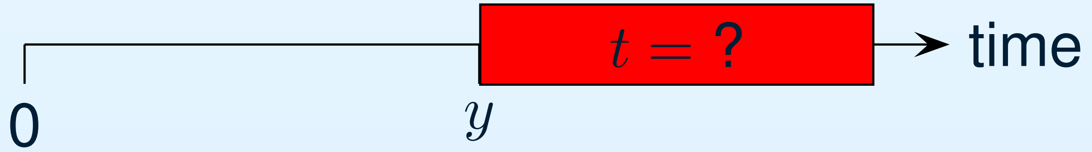
Lifetime distribution of bridges

Statistical analysis of bridge lifetimes and ages I

- Estimating lifetime solely on the basis of replacement times:
  - expected bridge lifetime is underestimated
- This problem can be resolved by statistical analysis on:
  - lifetimes of demolished bridges (complete observations):
    - lifetime is known to be t



  - current ages of existing bridges (right-censored observations):
    - unknown lifetime t > current age y



Statistical analysis of bridge lifetimes and ages II

- Fit Weibull distribution:
  - properly models ageing of bridges
- Use maximum-likelihood method:
  - complete and right-censored observations
- Obtain conditional probability distribution of residual lifetime given current age:
  - left-truncated Weibull distribution

Weibull distribution

A random variable X has a Weibull distribution with shape parameter $a > 0$ and scale parameter $b > 0$ if the probability density function of X is given by

$$\ell\left(x|a,b\right) = \mathrm{We}(x|a,b) = \frac{a}{b}\left[\frac{x}{b}\right]^{a-1}\exp\left\{-\left[\frac{x}{b}\right]^{a}\right\}I_{(0,\infty)}(x),$$

where $I_A(x) = 1$ if $x \in A$ and $I_A(x) = 0$ if $x \notin A$ for every set A.

The survival function is defined by

$$\bar{F}(x|a,b) = 1 - F(x|a,b) = \exp\left\{-\left[\frac{x}{b}\right]^{a}\right\}$$

with expected value $E(X) = b\Gamma(a^{-1}+1)$.

Left-truncated Weibull distribution

Condition on current life or age y and determine the conditional probability that lifetime X exceeds x given $X > y$:

$$\Pr\{X > x|X > y\} = \bar{F}(x|a,b,y) = \exp\left\{-\left[\frac{x}{b}\right]^a + \left[\frac{y}{b}\right]^a\right\}$$

for $x > y$.

Probability density function of left-truncated Weibull distribution:

$$\text{LTW}(x|a,b) = \frac{a}{b}\left[\frac{x}{b}\right]^{a-1}\exp\left\{-\left[\frac{x}{b}\right]^a + \left[\frac{y}{b}\right]^a\right\} I_{(y,\infty)}(x).$$

where $X - y$ is the residual (or excess) lifetime for a bridge having age y.

Likelihood of lifetimes and ages

- Let $\mathbf{x} = (x_1, \ldots, x_r)'$ denote a random sample of r complete lifetimes.
- Let $\mathbf{y} = (y_1, \ldots, y_m)'$ denote a random sample of m right-censored lifetimes (ages).
- Maximise the likelihood function

$$\ell\left(\mathbf{x}, \mathbf{y} \middle| a, b\right) = \prod_{i=1}^{r} \ell\left(x_i \middle| a, b\right) \prod_{j=1}^{m} \bar{F}\left(y_j \middle| a, b\right).$$

Dutch stock of concrete bridges: Lifetimes

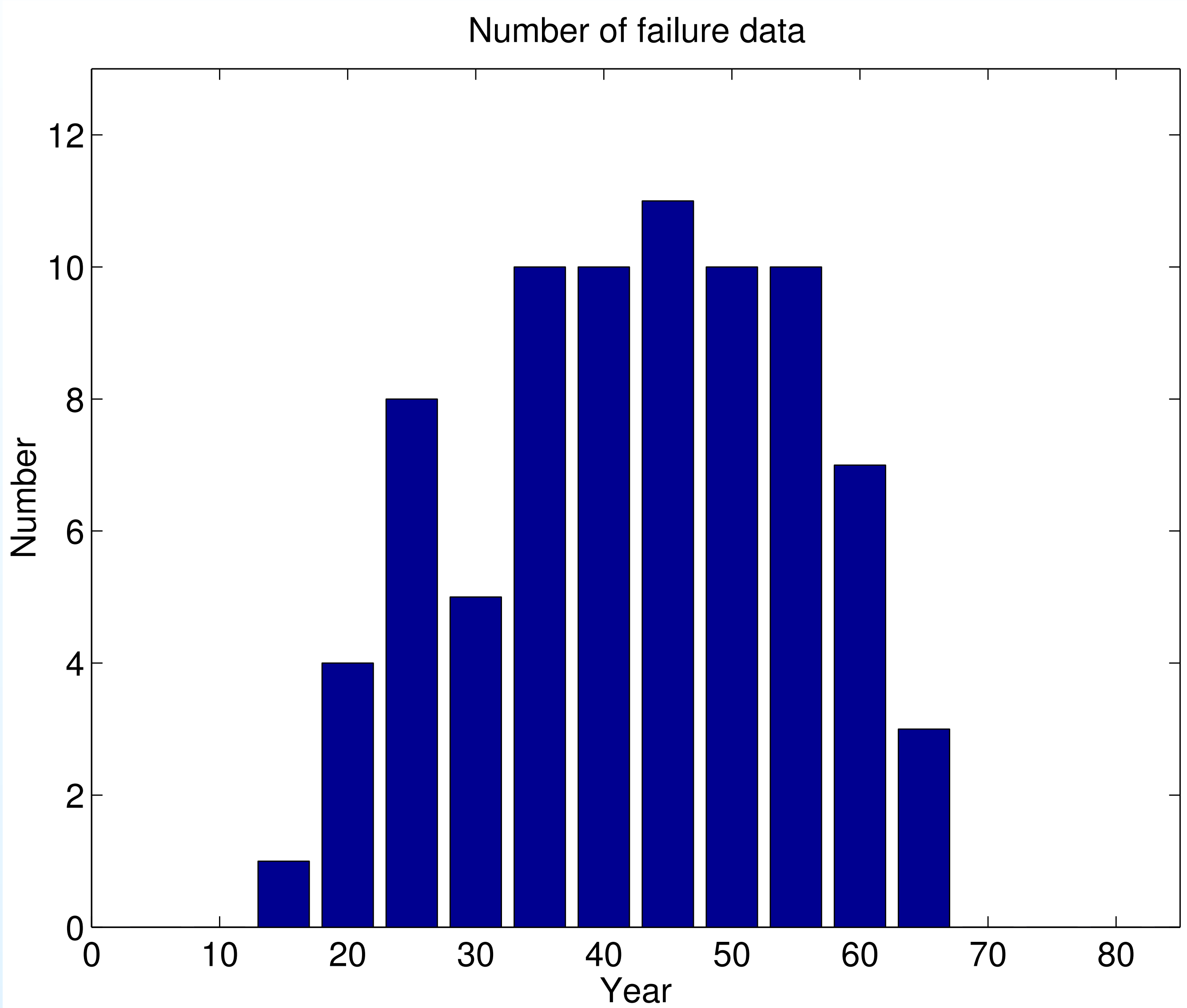
Stock of concrete bridges

- Dutch stock of concrete bridges and viaducts in and over the highway
- Observed lifetimes and ages of concrete bridges were aggregated:
  - $r = 79$ lifetimes of demolished bridges (all with length less than 200 m)
  - $m = 2974$ ages of existing bridges
  - for 94 concrete bridges, the year of construction was unknown
- Right-censored observations available in terms of units of time of 1 year
  - discrete-time stochastic process

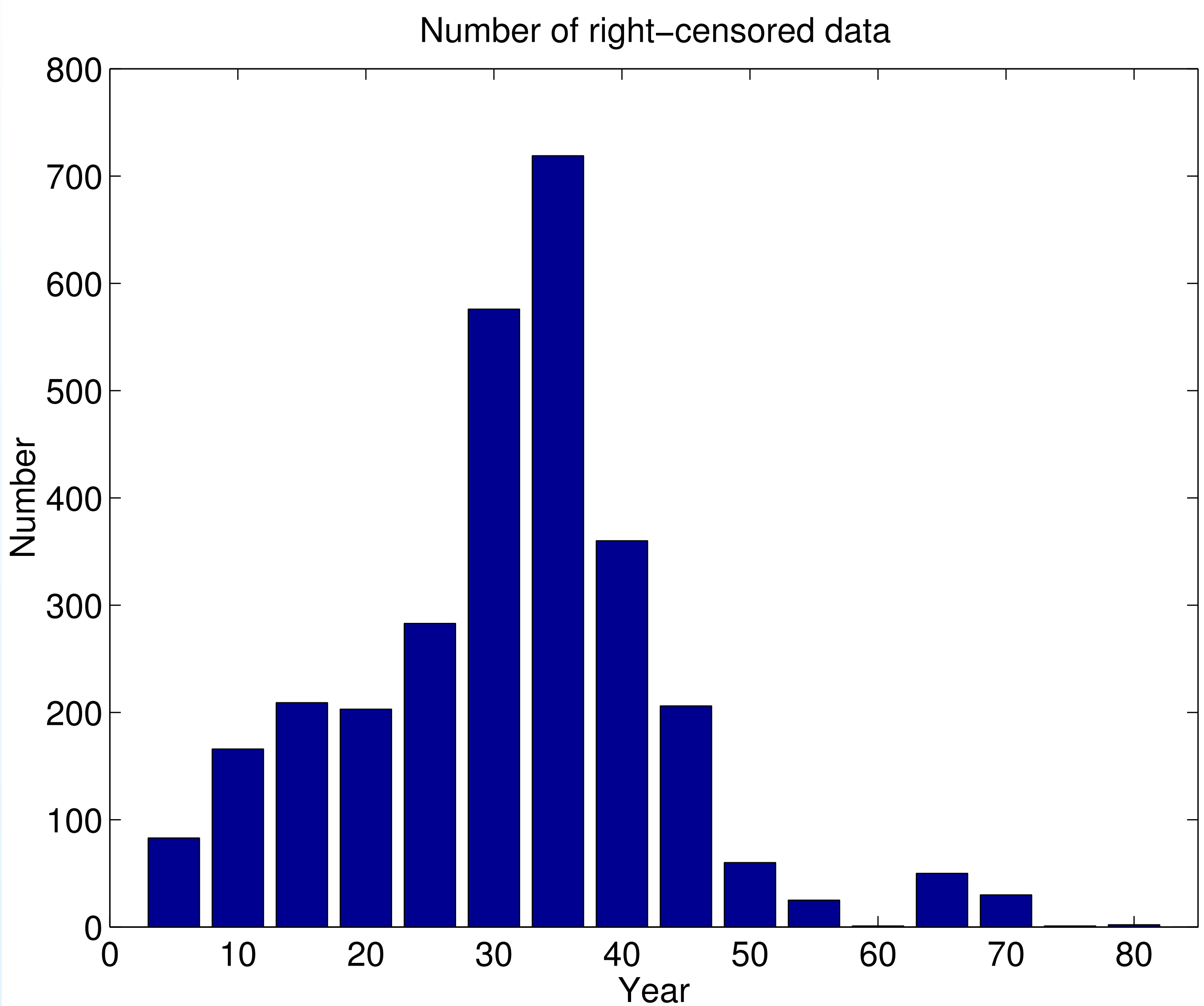
Types of failure and changes in design

- Most demolished bridges were *not* replaced due to technical failure, but due to a change in functional or economical requirements
- Not enough information for making a distinction between the technical, functional and economical lifetime
- Possible changes in bridge design over time could not be taking into account

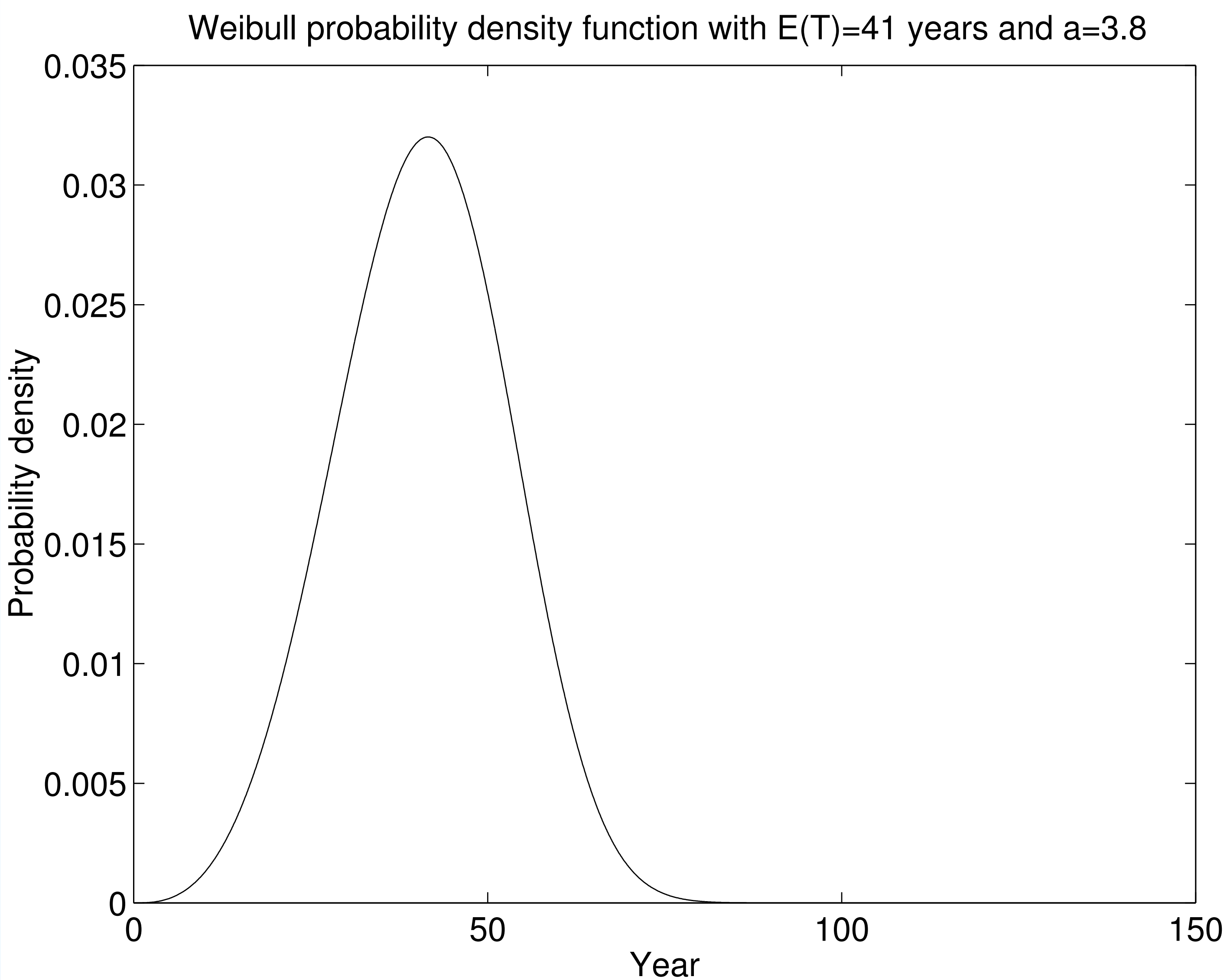
Number of demolished bridges



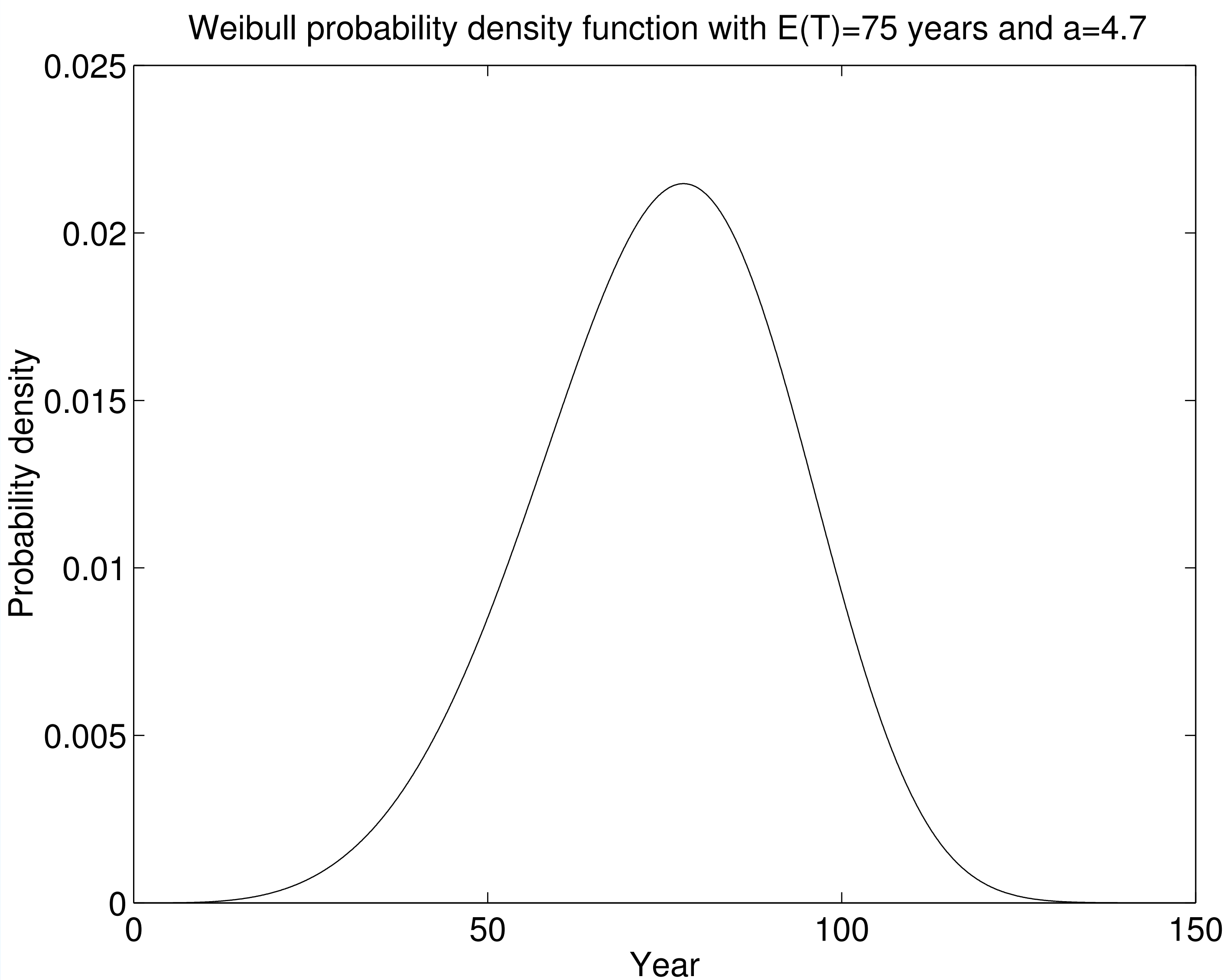
Number of existing bridges



Lifetime distribution: Demolished bridges



Lifetime distribution: Demolished and existing bridges



Expected cost of bridge replacement

Bridge replacement as a renewal process

- Estimate future expected cost of replacing the bridge stock
  - as a function of time
  - when the current ages are given
  - while taking account of the uncertainty in the lifetime

  by applying techniques from renewal theory
- Assume bridge replacement can be modelled as a discrete renewal process:
  - renewals are the replacements
  - after each renewal we start (in a statistical sense) all over again

Discrete renewal process

- A discrete renewal process $N(n)$, $n = 1, 2, 3, \ldots,$ is a non-negative integer-valued stochastic process that registers the renewals in time interval $(0, n]$
- Let the renewal times $T_1, T_2, T_3, \ldots,$ be non-negative, independent, identically distributed, random quantities having the discrete probability function

$$\Pr\{T_k = i\} = p_i = F(i|a, b, 0) - F(i-1|a, b, 0),$$

$i = 1, 2, \ldots,$ where p_i represents the probability of a renewal in unit time i

Expected number and cost of renewals

- Expected number of renewals solves recursive equation

$$E(N(n)) = \sum_{i=1}^{n} p_i \left[1 + E(N(n-i))\right]$$

  for $n = 1, 2, 3, \ldots$ and $N(0) \equiv 0$.

- When the cost of a renewal equals c, the expected cost over the bounded horizon $(0, n]$ is

$$E(K(n)) = cE(N(n)).$$

- Expected long-term average number of renewals per unit time is

$$\lim_{n\to\infty} \frac{E(N(n))}{n} = \frac{1}{\sum_{i=1}^{\infty} i p_i} = \frac{1}{\mu}$$

  being the reciprocal of the mean lifetime μ.

Delayed renewal process

- Expression for $E(N(n))$ can be extended to the situation in which the first bridge has age $y \geq 0$.
- Discretise the probability distribution of the residual lifetime:

$$\Pr\{\tilde{T} = i|y\} = q_i(y) = F(y+i|a,b,y) - F(y+i-1|a,b,y),$$

$i = 1, 2, \ldots$

- Expected number of renewals in time interval $(0, n]$ when the first bridge has age y can then be written as

$$E(\tilde{N}(n,y)) = \sum_{i=1}^{n} q_i(y) \left[1 + E(N(n-i))\right].$$

- Because the renewal process only starts from the second renewal on, the stochastic process $\{\tilde{N}(n,y), n = 1, 2, 3, \ldots\}$ is called a *delayed renewal process*.

Expected cost of bridge replacement

- Expected cost of replacement of a bridge stock can be obtained by summing the expected replacement cost over the current ages $y_1, \ldots, y_m$:

$$E(\tilde{K}(n)) = \sum_{j=1}^{m} c_j E(\tilde{N}(n, y_j)),$$

where c_j is the cost of replacing the jth concrete bridge.

- Expected cost in unit time i is

$$E(\tilde{K}(i)) - E(\tilde{K}(i-1)), \quad i = 1, \ldots, n.$$

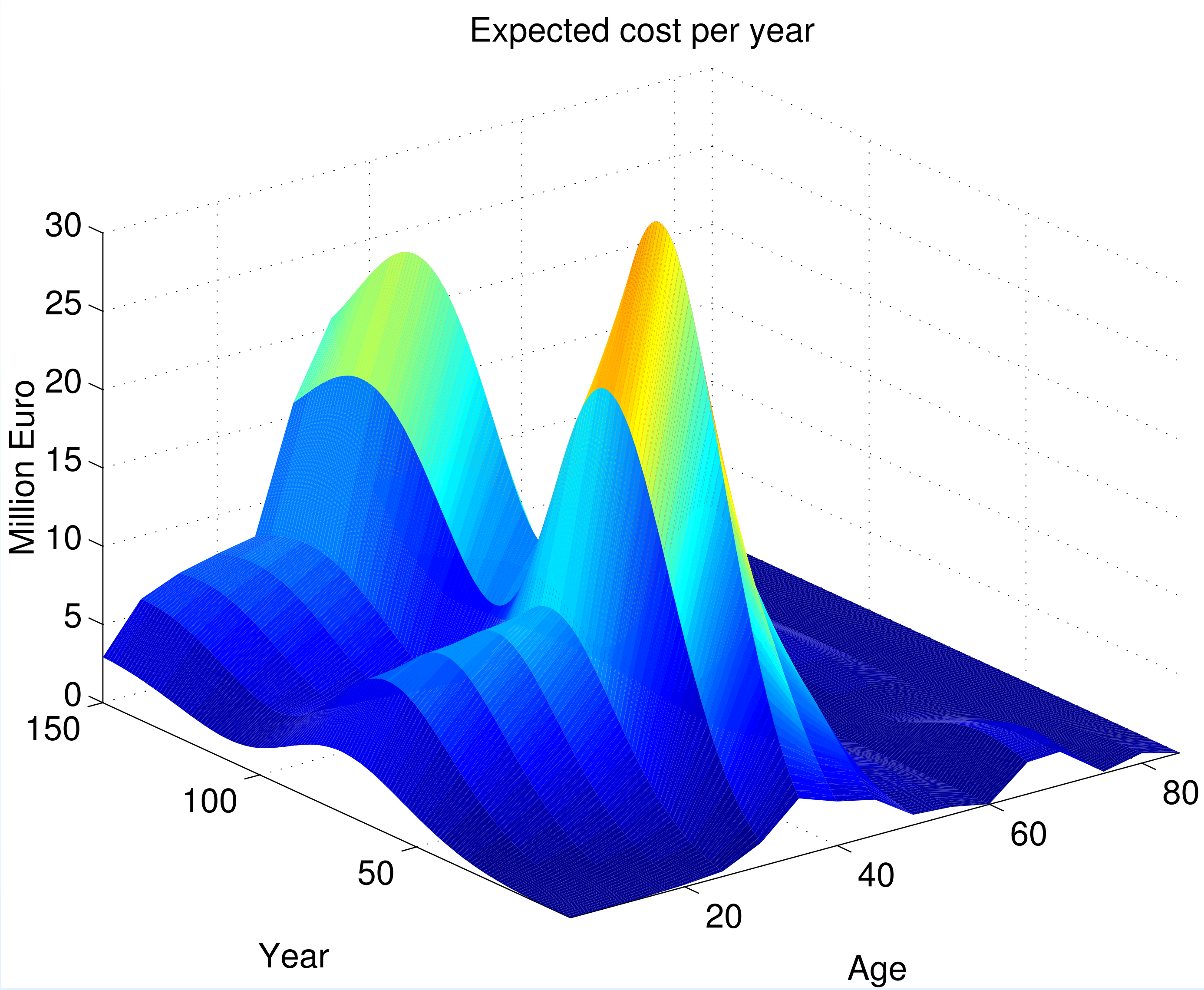
Dutch stock of concrete bridges: Cost

Replacement value of Dutch bridge stock

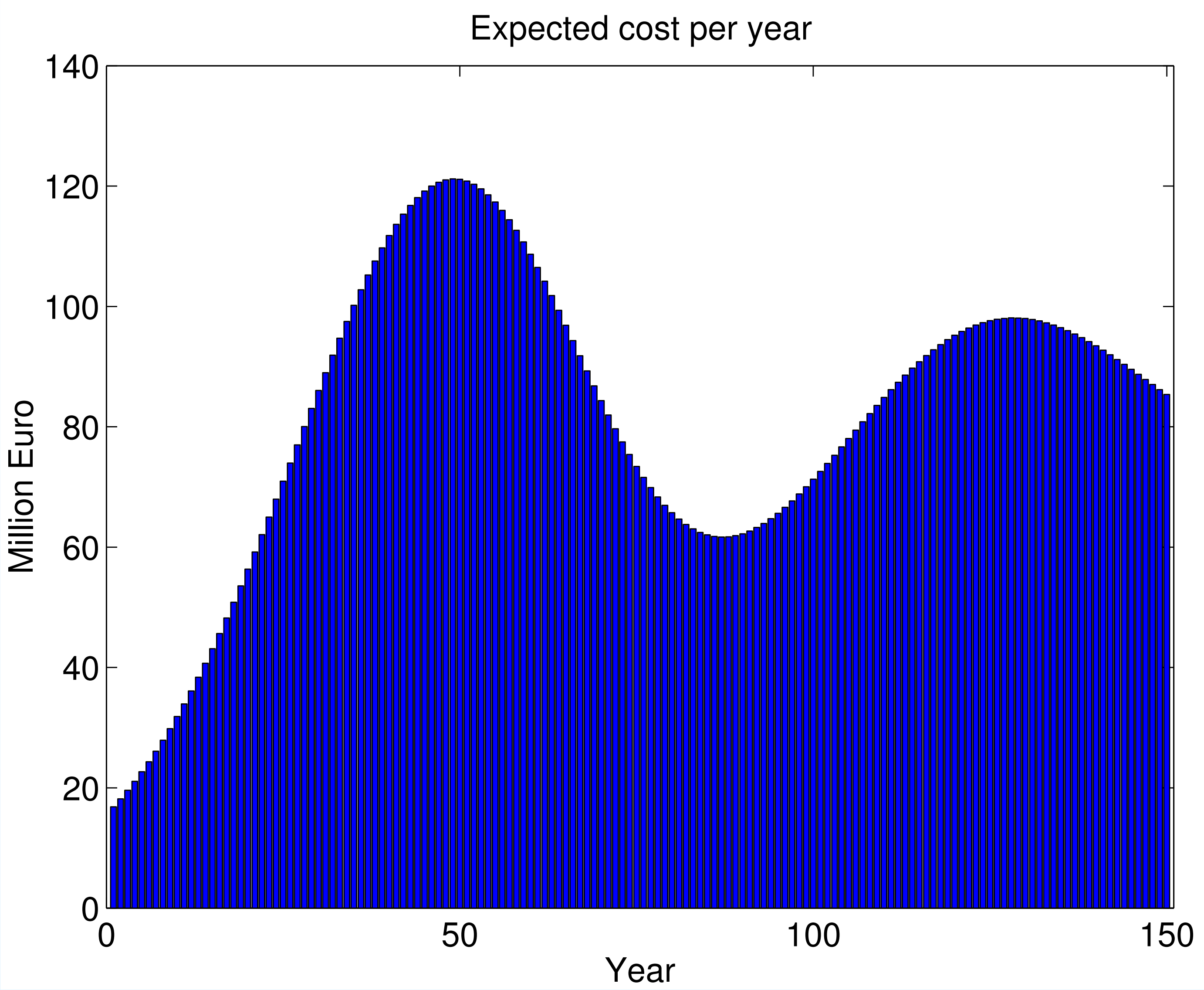
- Although an old bridge is seldom replaced by the same type of bridge, it is difficult to accurately assess the cost of such a new bridge
- Let cost of replacement therefore be independent of time:
  - replacement value of single bridge: $c = 2.15$ million Euro
- Replacement value of bridge stock
  - with known years of construction: 6.38×10^9 Euro
  - with unknown years of construction: 1.30×10^8 Euro
  - with correction factor for bridges with unknown years of construction:

$$\frac{6.38 \times 10^9 + 1.30 \times 10^8}{6.38 \times 10^9} = 1.02$$

Expected cost per unit time when ages are given



Expected cost per unit time summed over all ages

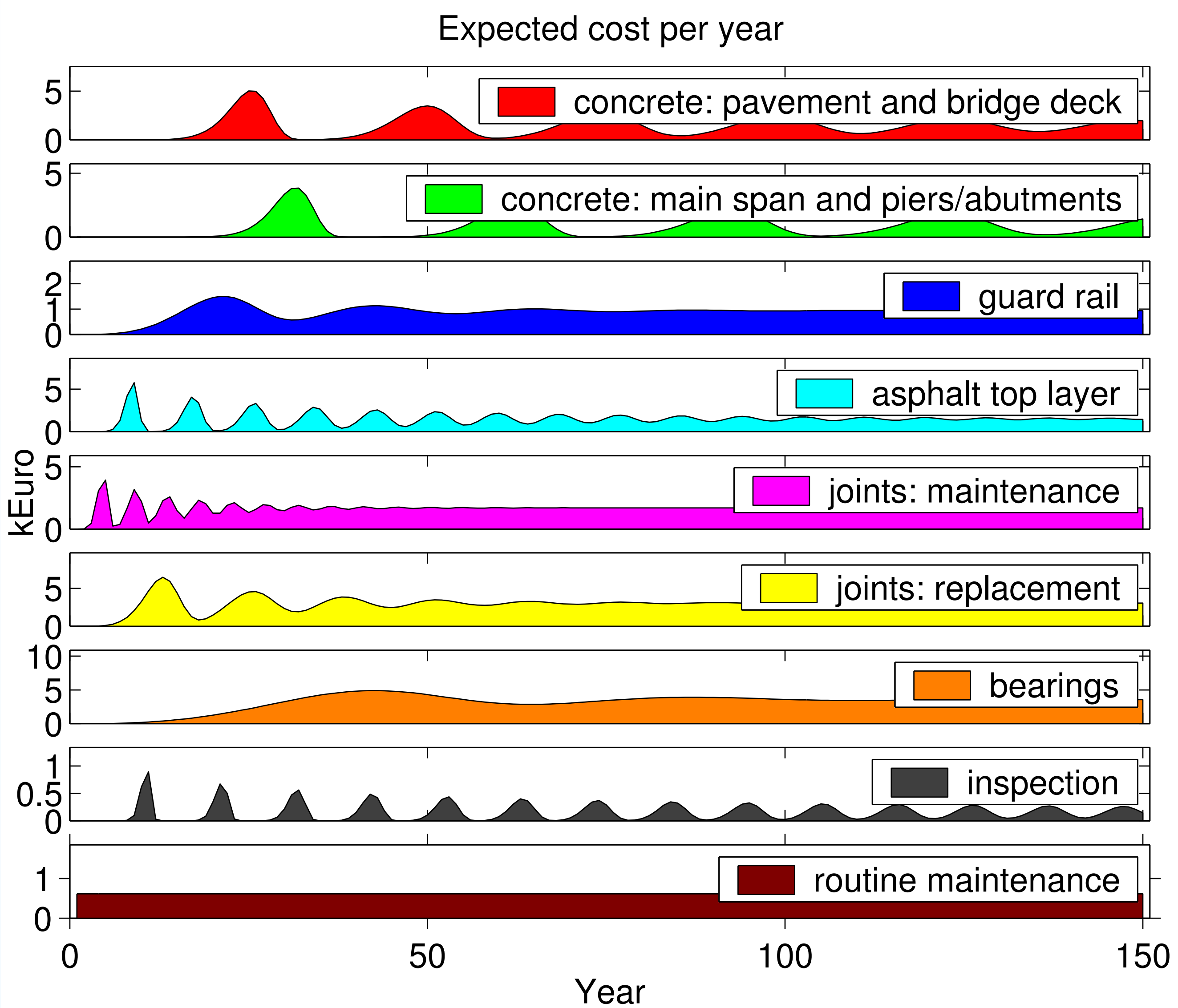


Maintenance of concrete bridge elements

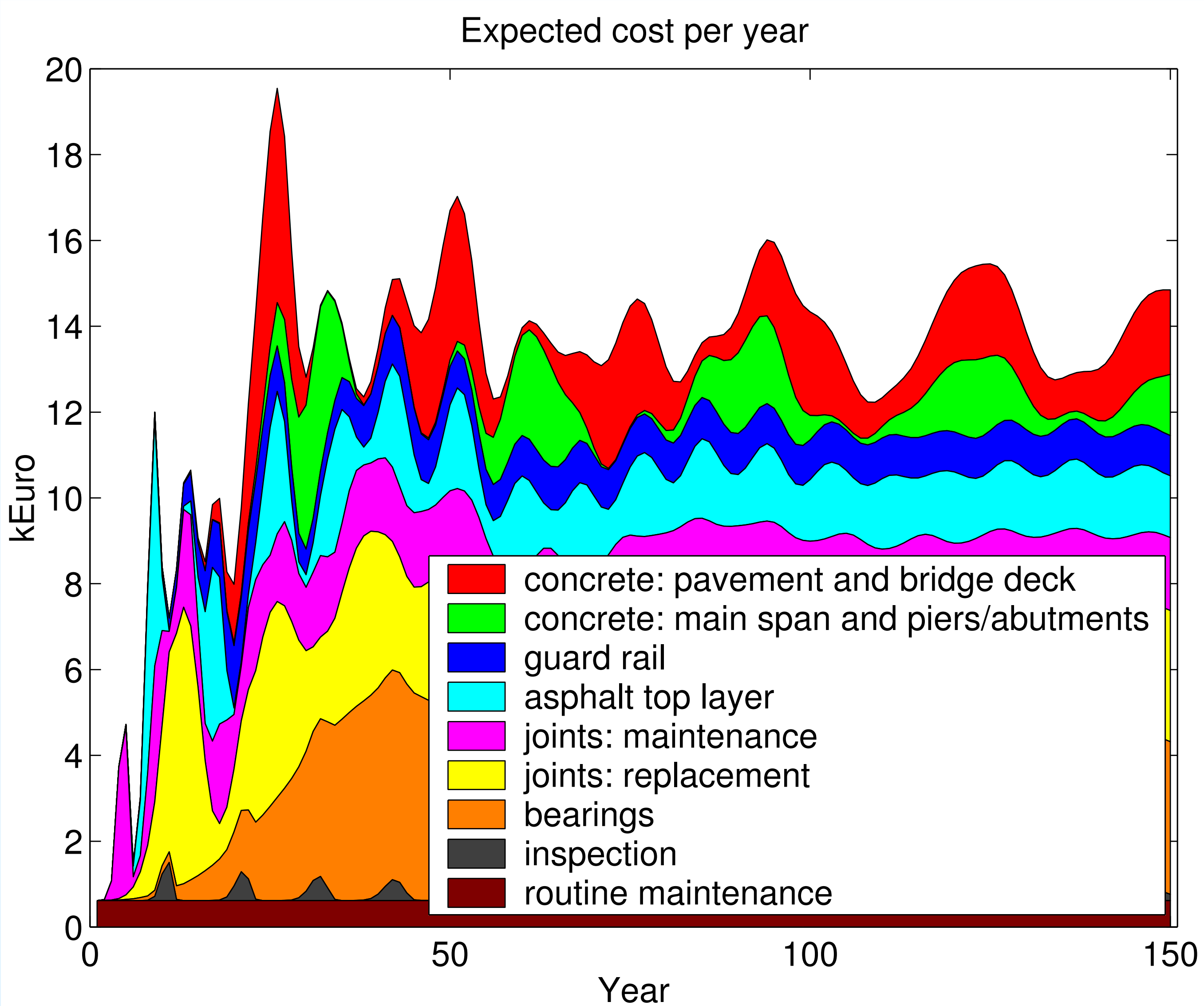
Bridge element	Cost [EUR]	Interval [year]	CV [-]
Concrete:			
- Pavement and bridge deck	36364	24	0.12
- Main span and piers/abutments	27360	30	0.10
Guard rail	19224	20	0.26
Asphalt top layer	12852	8	0.11
Joints: maintenance	7648	4	0.15
Joints: replacement	38240	12	0.20
Bearings	145000	40	0.30
Inspection	1650	10	0.06
Routine maintenance	617	1	0.00

CV = Coefficient of Variation

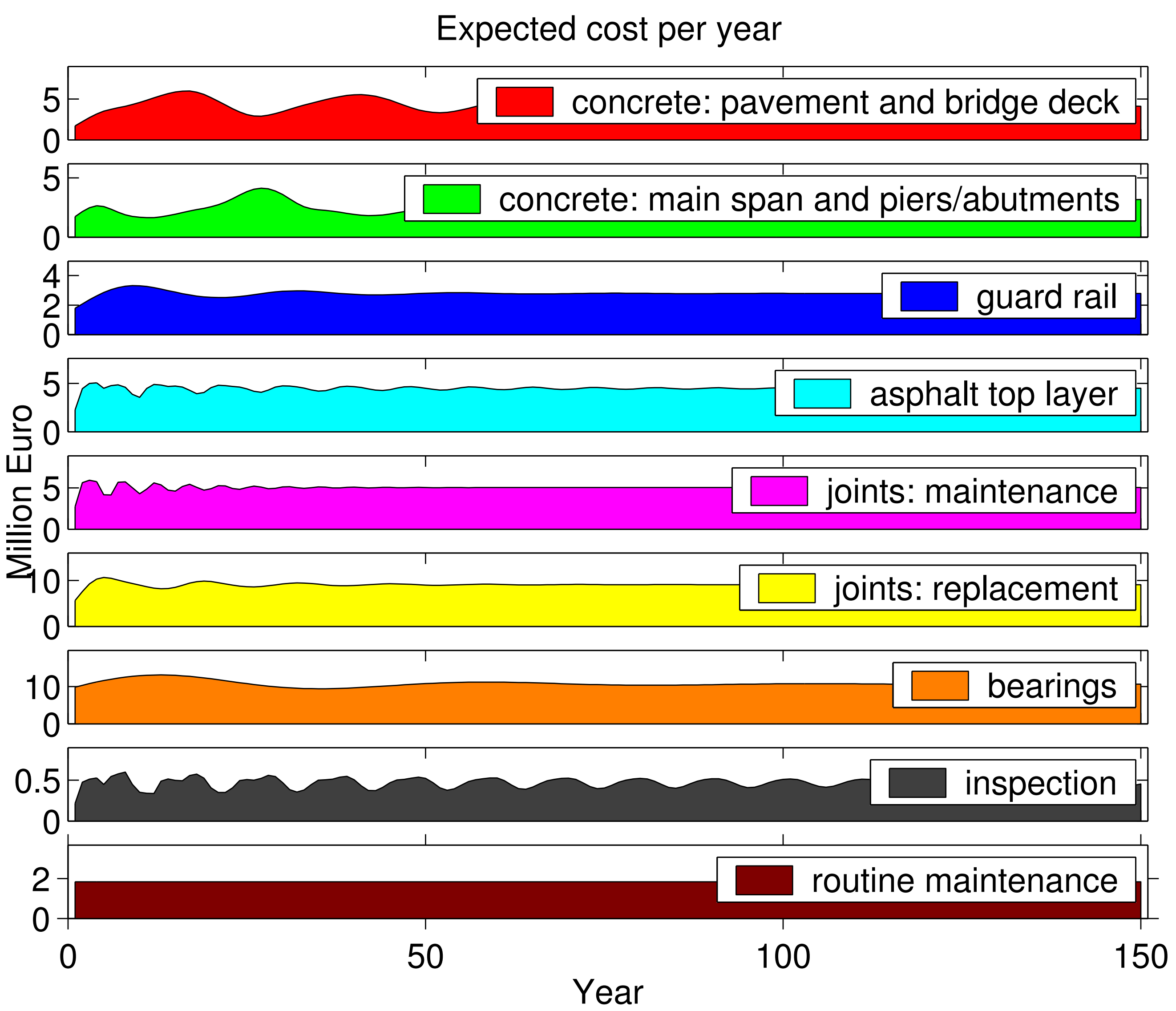
Maintenance cost of bridge elements for one bridge I



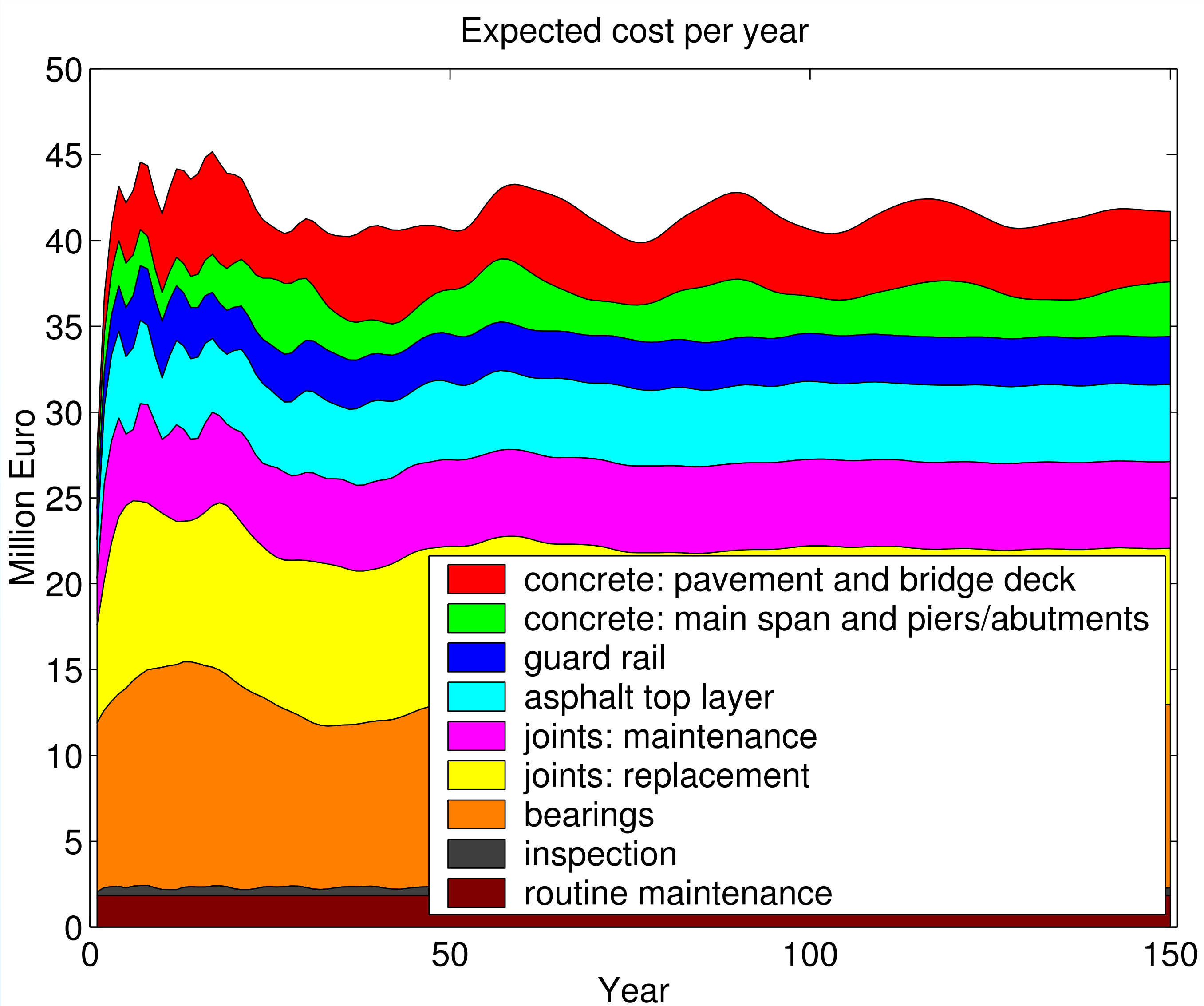
Maintenance cost of bridge elements for one bridge II



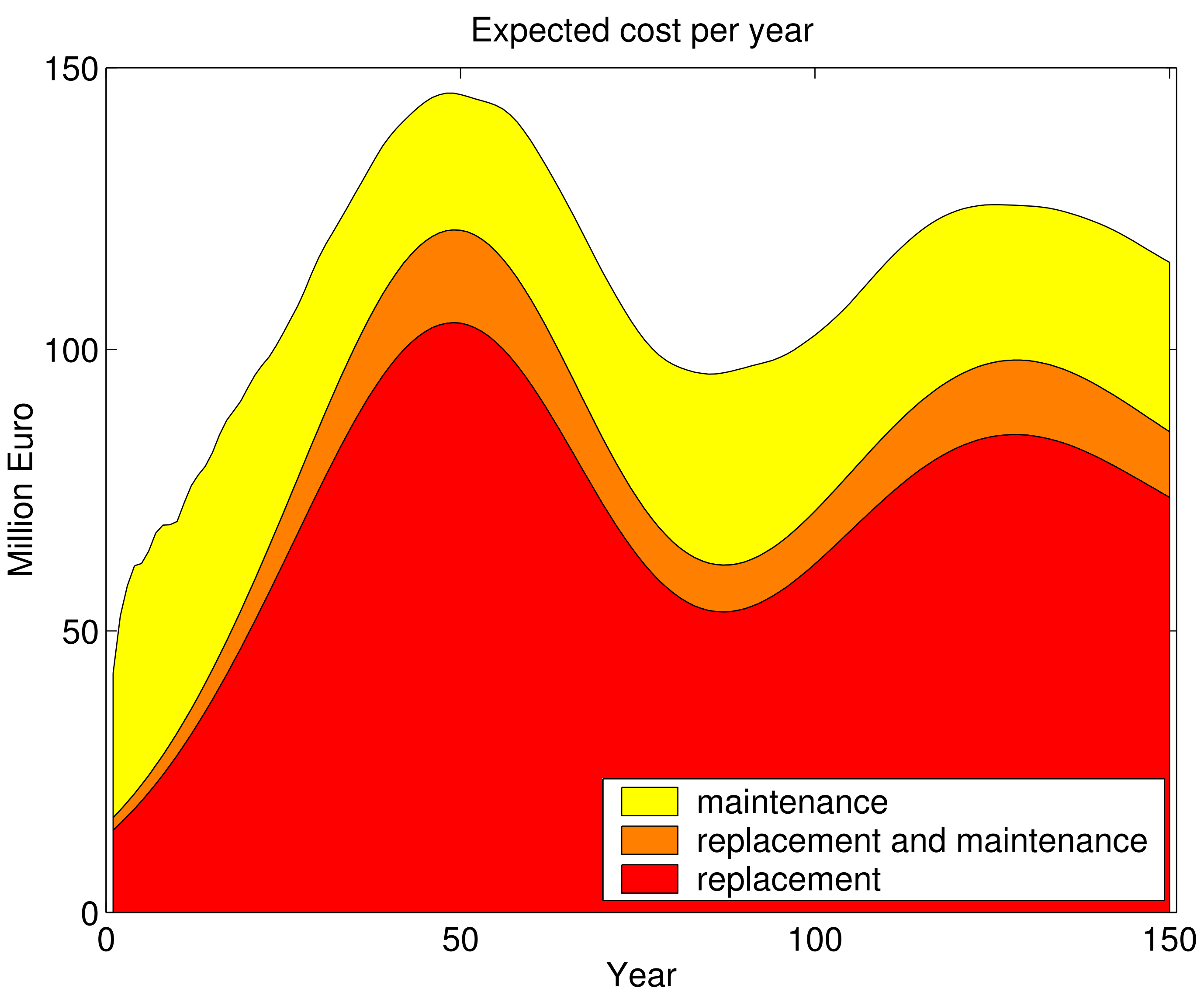
Maintenance cost of bridge elements for all bridges I



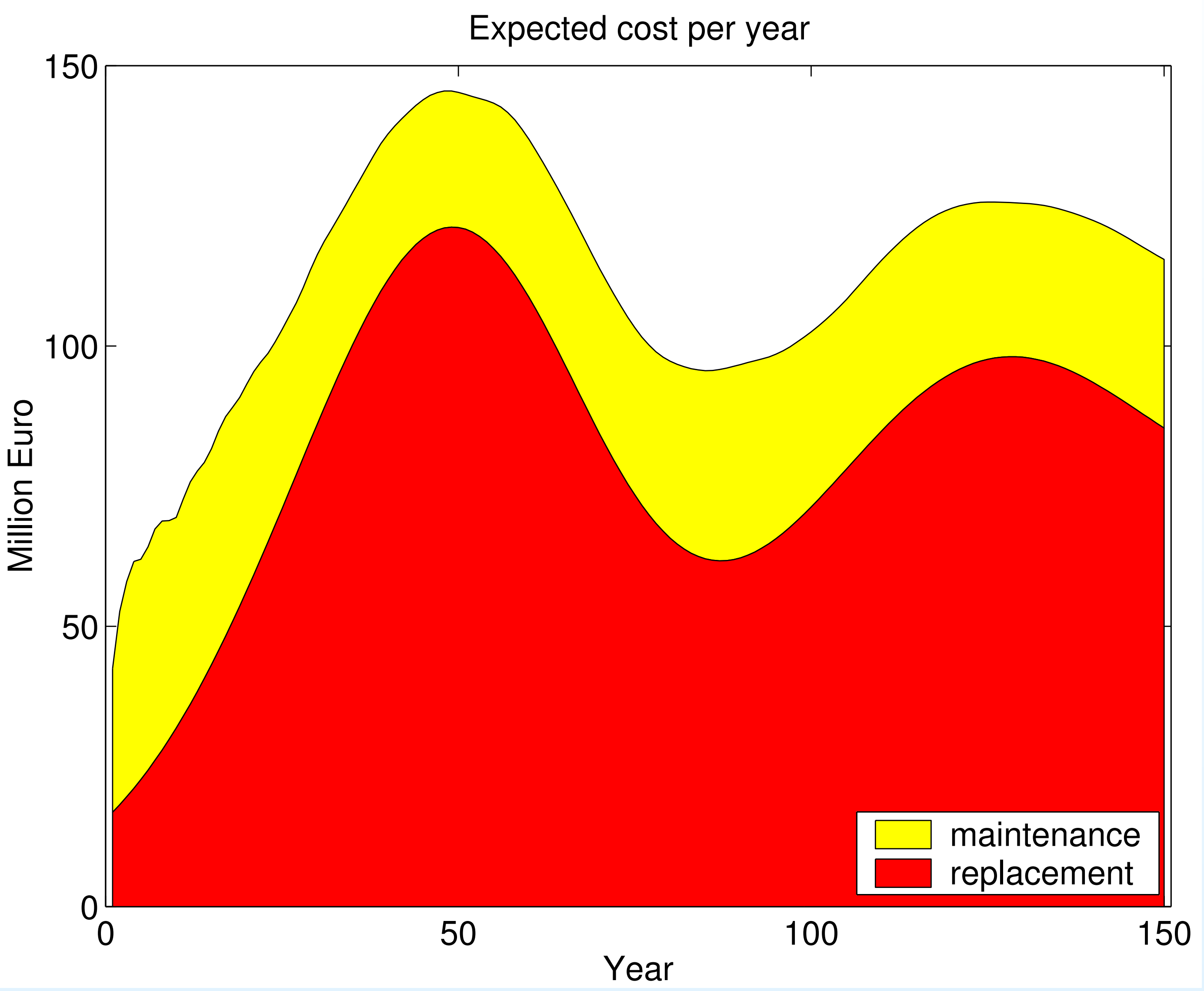
Maintenance cost of bridge elements for all bridges II



Cost of maintenance and replacement for all bridges



Cost of maintenance and replacement for all bridges



Long-term expected average cost per unit time

- Cost of replacing one concrete bridge: 2.15 Million Euro
- Long-term expected average cost per year for one bridge:
  - replacement cost: 28 kEuro per year
  - maintenance cost: 15 kEuro per year
- Long-term expected average cost per year for bridge stock:
  - replacement cost: 86 Million Euro per year
  - maintenance cost: 44 Million Euro per year
- For the purpose of life-cycle costing:
  - expected cost of replacement as a function of time
  - expected average cost of maintenance per year

Conclusions

Conclusions

- Lifetime distribution of Dutch concrete bridges has been determined
- Weibull distribution was fitted to
  - complete lifetimes of demolished bridges
  - current ages of existing bridges
- Expected value of the Weibull lifetime in agreement with usual design life:
  - 80 to 100 years
- Advantages of Weibull distribution:
  - possibility to properly model ageing
  - analytically derive the conditional probability density function of residual lifetime when current age is given

Conclusions (continued)

- Using renewal theory, the expected cost of replacing and maintaining the bridge stock has been determined:
  - take account of ages of individual bridges
- Uncertainties in replacement and maintenance times:
  - cost is more spread out over time
- For the purpose of life-cycle costing:
  - expected cost of replacement as a function of time
  - expected average cost of maintenance per year
- Methodology can be extended:
  - different types of lifetime (such as technical, functional and economical)
  - possible change of bridge design over time