Linkov, I., Ferguson, E., Magar, V. (in press). Real Time and Deliberative Decision Making: Application to Risk Assessment for Non-chemical Stressors. Springer, Amsterdam.

Modeling Stakeholder Preferences with Probabilistic Inversion: Application to Prioritizing Marine ecosystem Vulnerabilities

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Abstract

Keywords: Discrete Choice, random utility, probabilistic inversion, iterative proportional fitting, marine ecosystem,

1. Introduction

This article presents an analysis of the 64 experts' rankings of 30 scenario's involving threats to coastal ecosystems. The elicitation protocols were designed and executed by NCEAS. Experts are asked to rank the five scenarios posing the greatest threats and the five scenarios posing the least threats. The goal of this study was to find weights for criteria which adequately model these stakeholders' preferences and which can be used to predict the scores of other scenarios. Probabilistic inversion (PI) techniques to quantify a model of ecosystem vulnerability based on five criteria.

Other 'multi-criteria' weighting methods (Linkov et al 2005) require stakeholders to evaluate the criteria directly. Of course, the weights assigned to a criterion cannot be assessed independently of the scale on which the *all* criteria scores are measured – a fact which is sometimes overlooked. The present approach asks the stakeholders to rank scenarios, instead of evaluating criteria. Criteria weights are then derived so as to fit as well as possible the stakeholder preference rankings. This has the significant advantage of allowing us to assess the validity of our fitted model of stakeholder preference.

Probabilistic inversion denotes the operation of inverting a function at a probability distribution, rather than at a point. Such problems arise in quantifying uncertainty in physical models (Kurowicka and Cooke, 2006, Du et al2006 , Kraan and Bedford 2005). One has uncertainty distributions on observable phenomena, either from data or

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from expert judgment, and one wishes to find a distribution over the parameters of a predictive model, such that one recovers the observed distributions when the parameter distributions are 'pushed through' the model. PI algorithms used in the past were computationally intensive, involving sophisticated interior point optimization techniques, duality theory as well as ad hoc steering (Kraan 2002). Recent computational advances (Volmel 1999, Matus 2007) clarify the mathematical foundations for PI and yield simple algorithms with proven convergence behavior, suitable for use by non-specialists. The results depend on a variant of the classical Iterative Proportional Fitting algorithm (Kruithoff 1937, Deming and Stephan 1944, Csiszar 1975, Du et al 2006, Matus2007)

In stakeholder preference modeling, the data is discrete choice preference data elicited from a set of stakeholders. The distributions to be inverted are those of indicator variables such as

Alternative *i* is better than alternative *j*;
Alternative *i* is ranked 3rd in the given set of alternatives.

We are interested in the probability of such variables taking the values "yes" or "no" in a set of stakeholders. We can measure these probabilities by querying a large representative set of stakeholders. Existing discrete choice , or random utility, techniques construct a value or utility function from discrete choice data (Andersen et al 1996, Train 2003, Siikimaki and Layton 2007, McFadden 1974, Torgersen 1958, Thurston 1927, Bradley and Terry 1952) , and they strongly restrict the form of the utility functions. Using PI, this form can be inferred from choice data.

We first discuss the data, then address model adequacy, and model fit. Summary statistics for the 30 scenarios are then given. The conclusion of this analysis is that the data are broadly consistent with a linear model of stakeholder preferences.

2. Data

The 30 threat scenarios were scored on 5 criteria,
C1 Spatial scale,
C2 Frequency,
C3 Trophic (functional) impact,
C4 Recovery time. and
C5 Resistance

The stakeholders' preference data is modeled with a linear model:

1) $\textit{Score for scenario } S = \Sigma_{i=1\ldots5} \textit{ score of } S \textit{ on } C_i \times \textit{weight for } C_i.$

The weights are random variables which are non-negative and sum to 1. The (joint) distribution for the weights is modeled to represent the distribution of weights in a population of stakeholders, of which the 64 elicited experts are a random sample. Since the weights are normalized, the scores are transformed so that the product *score* $\times$ *weight* are positive and fall within the same range. Spatial scale is given in km^2, and the values for spatial scale range from *0.1*km^2 to *50,000*km^2. These values are transformed to $ln(100m^2)$, whose values thus range from 2.3 to 15.4. Frequency was scored as *ln(360*# /year)*. Resistance is scored as the percent of species affected per trophic layer. These transformations are chosen for mathematical convenience.

A salient feature of these data is dominance. Scenario A dominates scenario B from above if A's scores on all five criteria are greater or equal to the scores of B. A dominates B from below if A's scores on all five criteria are less than or equal to those of B. If A dominates B from above, then B can never be ranked above A in any model which computes the scenario score as a monotonic function of the 5 criteria scores. The presence of dominated scenarios enables us to analyze whether the experts' rankings are broadly consistent with a monotonic model of criteria scores.

3. Model adequacy

Of the 30 scenarios, only 7 were non-dominated. This means that none of the 23 scenarios dominated from above could be ranked 1 by a stakeholder whose preferences were consistent with the model. In fact, 22.4% of the top rankings were inconsistent in this sense. 77.6% of the top rankings went to 4 of the 7 non-dominated scenarios. A scenario dominated from above by two or more scenarios could not consistently be ranked second; in fact 23.7% of the second rankings were inconsistent in this sense. Dominance from below was much less prevalent than dominance from above.

In view of the large number of dominated scenarios, we view the percentages of inconsistent rankings as indicating that the stakeholders' preferences were broadly, though not wholly, consistent with a monotonic model[1]. We therefore proceeded to fit the linear model (1).

The 30 scenarios and their criteria scores are shown in Table 1. The non-dominated scenarios are highlighted.

Table 1: scenarios and criteria scores

Nr	Code	scenario	scale ln((km*10)^2)	Frequency ln(360*#/yr)	func TL	recov yrs	Resistance
1	am	Aquaculture: marine plant	5.298317	11.77221	1	1	0.2
2	as	Aquaculture: shellfish	6.214608	11.77221	1	0.1	0.05
3	cl	Climate change: sea level rise	13.81551	5.192957	2	5	0.2
4	ct	Climate change: sea temp	15.42495	5.886104	3	50	0.25
5	cu	Climate change: UV	13.81551	3.583519	1	1	0.05
6	ca	Coastal engineering: habitat alteration	4.60517	5.886104	4	25	0.75

[1] If the 64 experts had chosen their top ranked scenatio at random, the probability that 14 or fewer would chose one of the 23 dominated scenarios is in the order 10^{-20}.

7	dh	Direct human impact: trampling	9.615805	11.77221	2	25	0.35
8	fd	Fishing: demersal destructive	6.684612	2.890372	4	0.5	0.1
9	fn	Fishing: demersal non-destructive low bycatch	2.302585	2.890372	1	0.5	0.1
10	fa	Fishing: non-destructive artisanal	4.60517	2.890372	1	1	0.5
11	fp	Fishing: pelagic high bycatch	6.214608	1.280934	1	0.5	0.05
12	fr	Fishing: recreational	6.684612	9.837348	2	5	0.2
13	fu	Freshwater input: increase	6.907755	4.276666	2	1	0.1
14	is	Invasive species	14.50866	11.77221	1	20	0.25
15	ma	Military activity	6.907755	8.371011	1	5	0.1
16	nh	Nutrient input: causing HAMs	9.21034	4.276666	2	1	0.1
17	nz	Nutrient input: causing hypoxic zones	6.684612	4.276666	3	1	0.05
18	no	Nutrient input: into oligotrophic waters	8.29405	4.969813	1	0.5	0.3
19	og	Ocean dumping: lost fishing gear	2.302585	5.886104	3	3	0.15
20	os	Ocean dumping: ship wrecks	3.912023	2.890372	4	10	0.5
21	ox	Ocean dumping: toxic materials	6.907755	2.890372	1	1	0.1
22	po	Ocean pollution	6.907755	6.579251	1	3	0.2
23	pa	Pollution input: atmospheric	9.615805	3.583519	1	0.5	0.2
24	pi	Pollution input: inorganic	8.29405	4.276666	2	3	0.2
25	pr	Pollution input: organic	8.517193	5.192957	2	5	0.2
26	ps	Power, desalination plants	4.60517	11.77221	3	10	0.5
27	sr	Scientific research: collecting	2.302585	8.371011	1	2	0.15
28	sd	Sediment input: decrease	3.912023	1.280934	1	0.5	0.05
29	si	Sediment input: increase	10.81978	5.192957	2	10	0.3
30	ts	Tourism: surfing	2.302585	10.49127	1	1	0.05

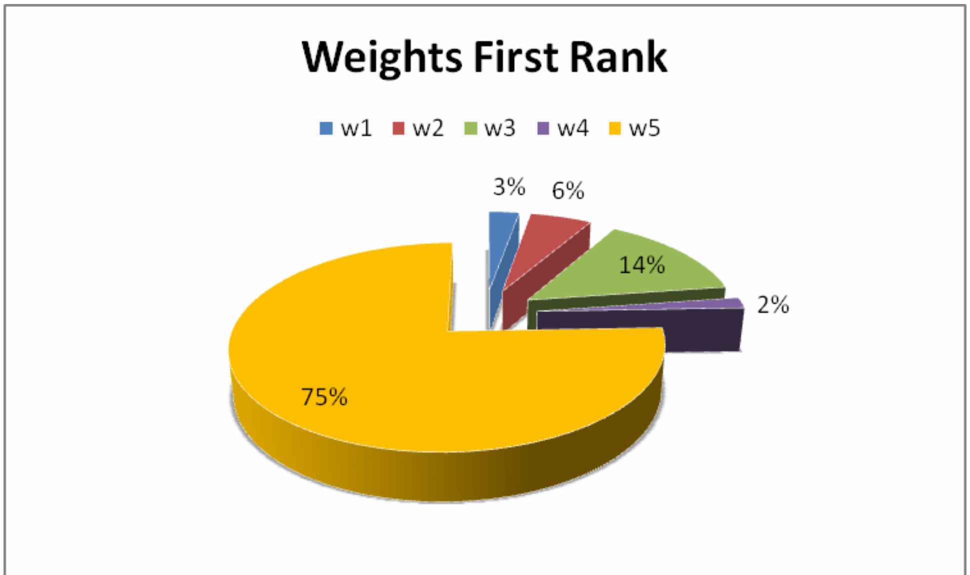
4. Model fitting: Criteria weights

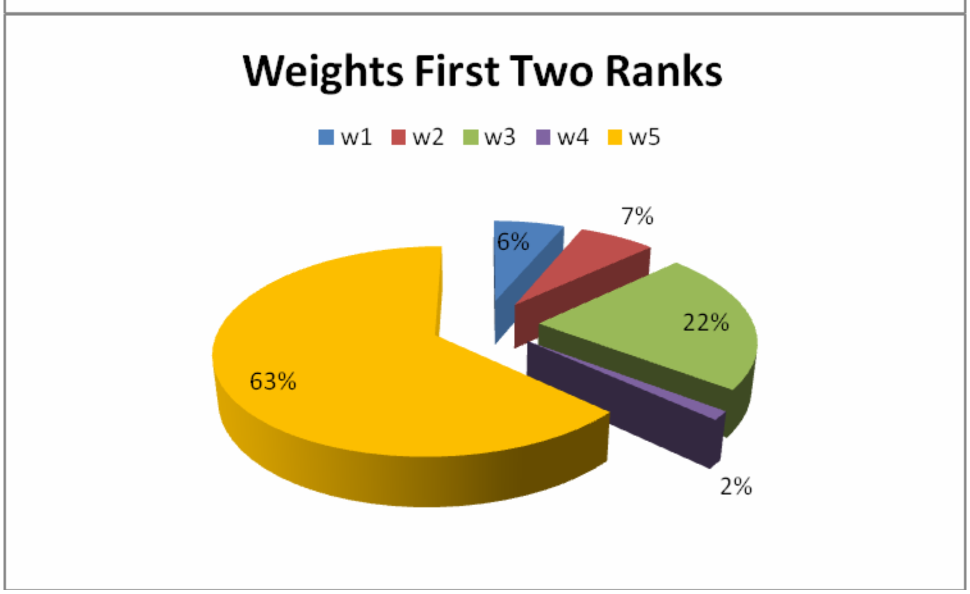
We fit the linear model by finding a distribution over criteria weights which fit as well as possible the probabilities of rankings given by the stakeholders. The fitting is done by probabilistic inversion. We start with a non-informative distribution over criteria weights (which however are constrained to add to 1). We then adapt this distribution so as to optimally recover the stakeholders' rankings. That is, if we sample randomly from the adapted distribution, the probability of drawing a set weights with which scenario A is ranked first equals, to the extent possible, the percentage of experts who ranked A first, and so on. The fitting based on first ranks fits only the percentages for the scenarios which were ranked first. The fitting based on the first two ranks fits only the percentages for the scenarios ranked 1 or 2, etc.

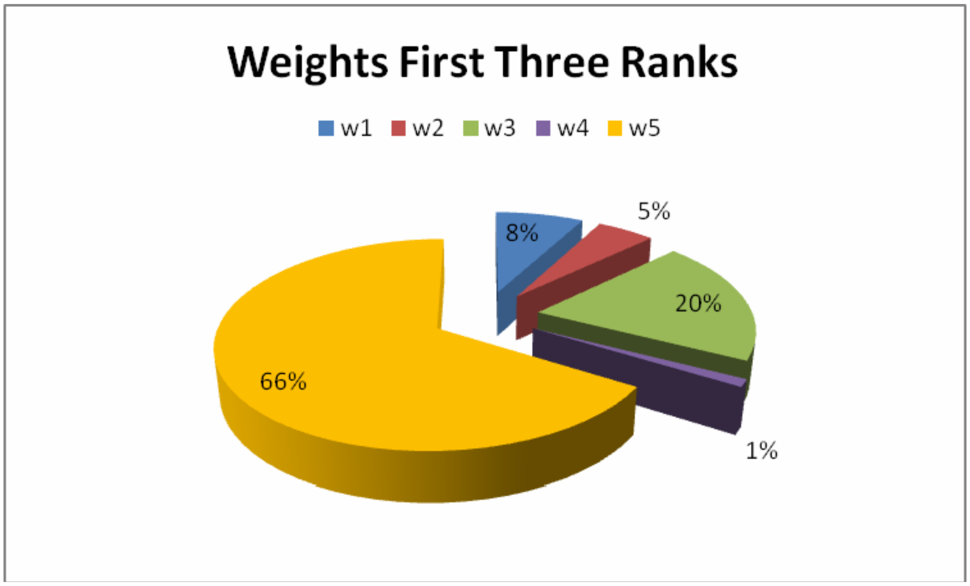
Because we are fitting a linear model, the expected score of any scenario may be computed by using the expected values of the criteria weights in the adapted distribution. A new scenario, not among the original 30, can be scored by multiplying its (transformed) criteria scores with the expected weight of each criterion. This of course is the great advantage of a linear model, and explains the preference for this model above more complex models, even though the latter might yield a better fit. Figure 1 shows the expected criteria weights based on fitting only the first ranks, the first two ranks, the first 3 ranks and the first 4 ranks, and finally, based on all ranks. We observe that these expected weights do not change significantly between the 2,3 and 4 ranks options. Using all ranks causes changes, and also causes greater variance in the criteria scores (see Table 4)

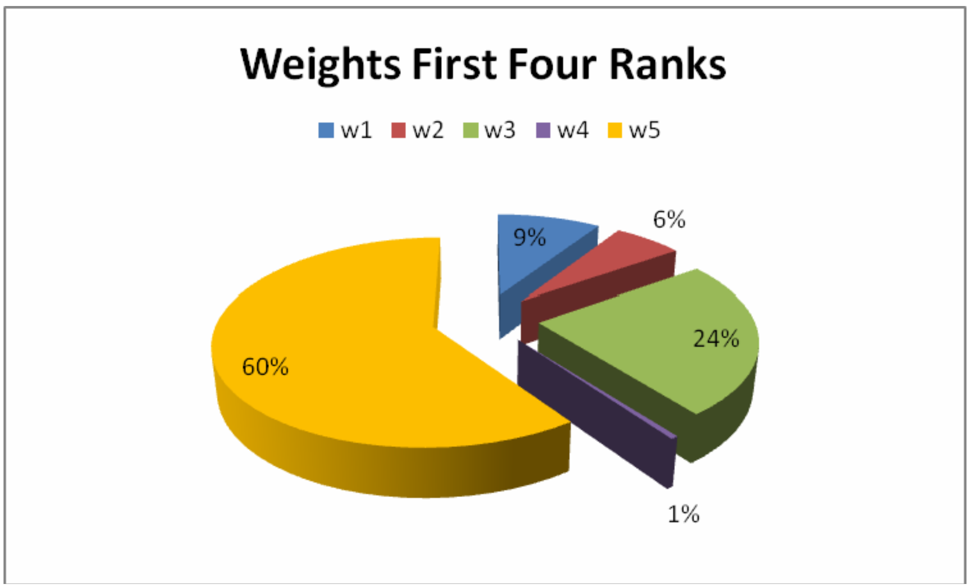
Although the expected weights are most important in using the model, it is also of interest to examine the distributions of weights. Figure 2 shows the cumulative distribution functions of the five weights in the four cases mentioned shown in Figure 1. The joint distributions for one rank, four ranks and all ranks are shown in Figure 3.

Figure 1: Expected criteria weights based on ranks, 1, 1&2, 1&2&3, 1&2&3&4, and All ranks






Weights First Three Ranks
w1
w2
w3
w4
w5
8%
5%
20%
1%
66%


Weights First Four Ranks
w1
w2
w3
w4
w5
9%
6%
24%
1%
60%

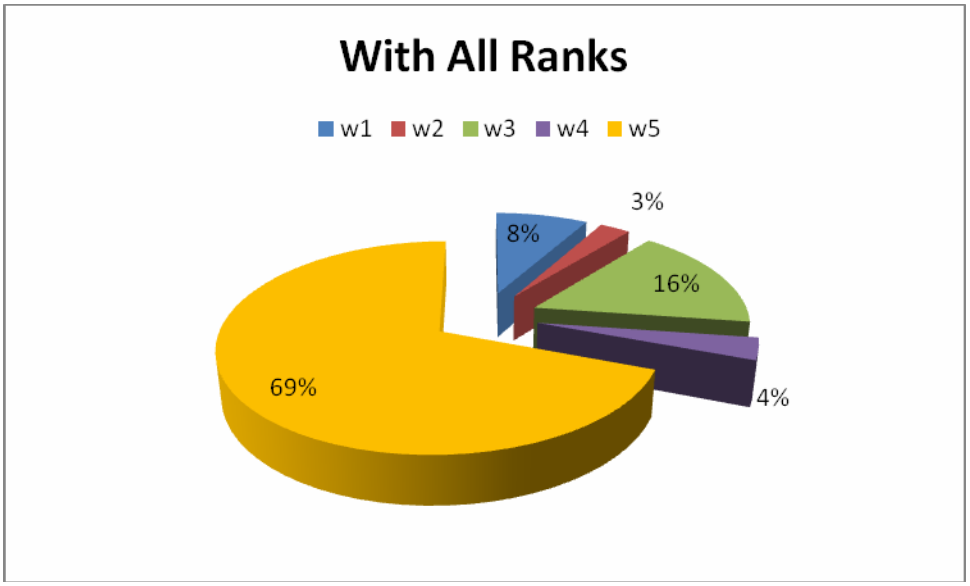
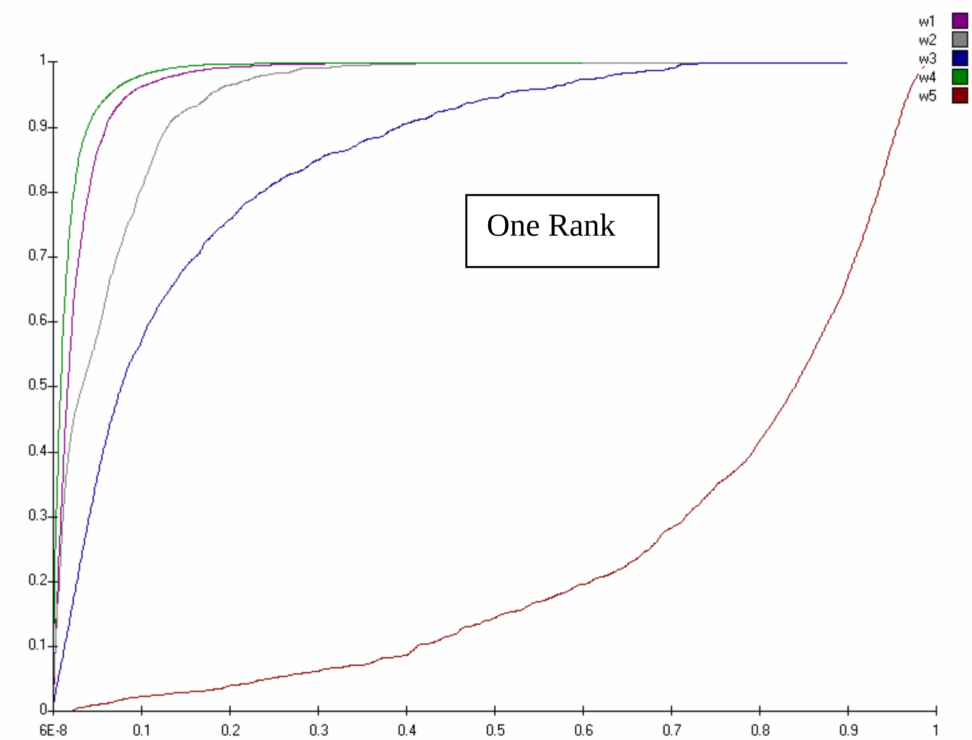


Figure 2: Cumulative weight distributions based on rank 1, 1&2, 1&2&3, 1&2&3&4, and all ranks



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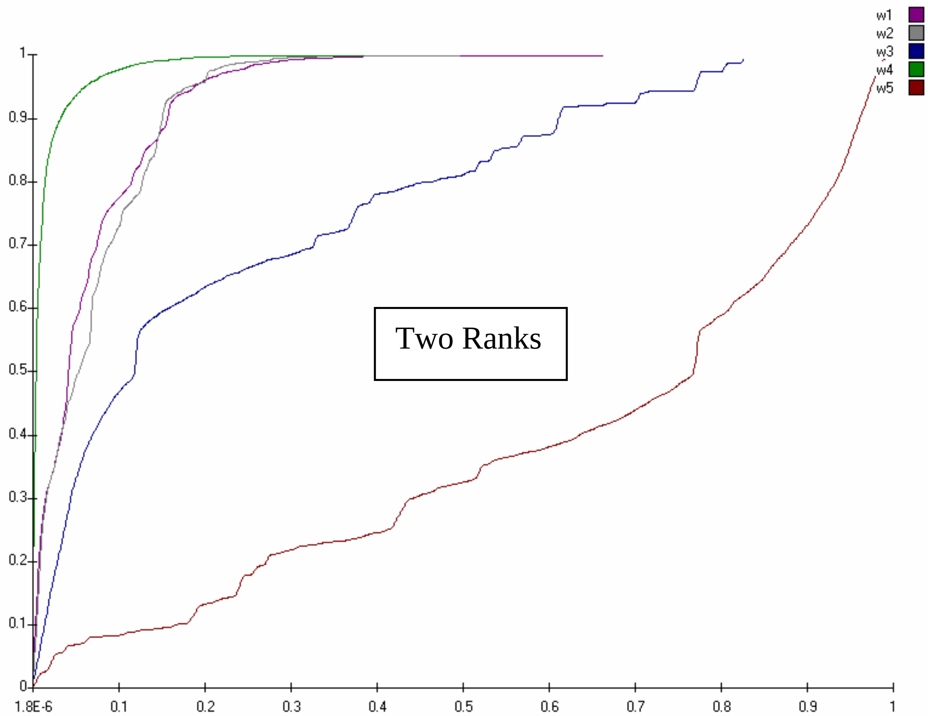
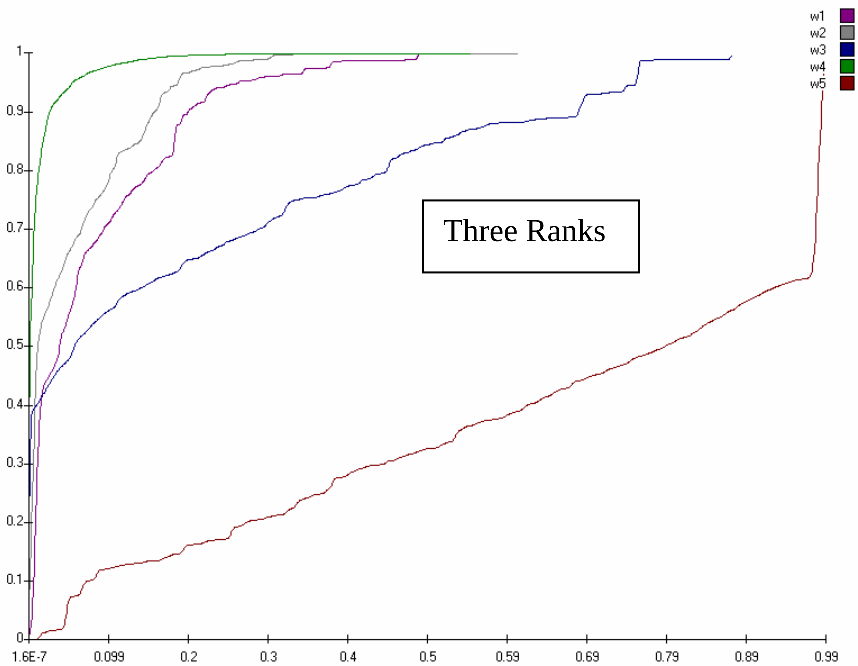
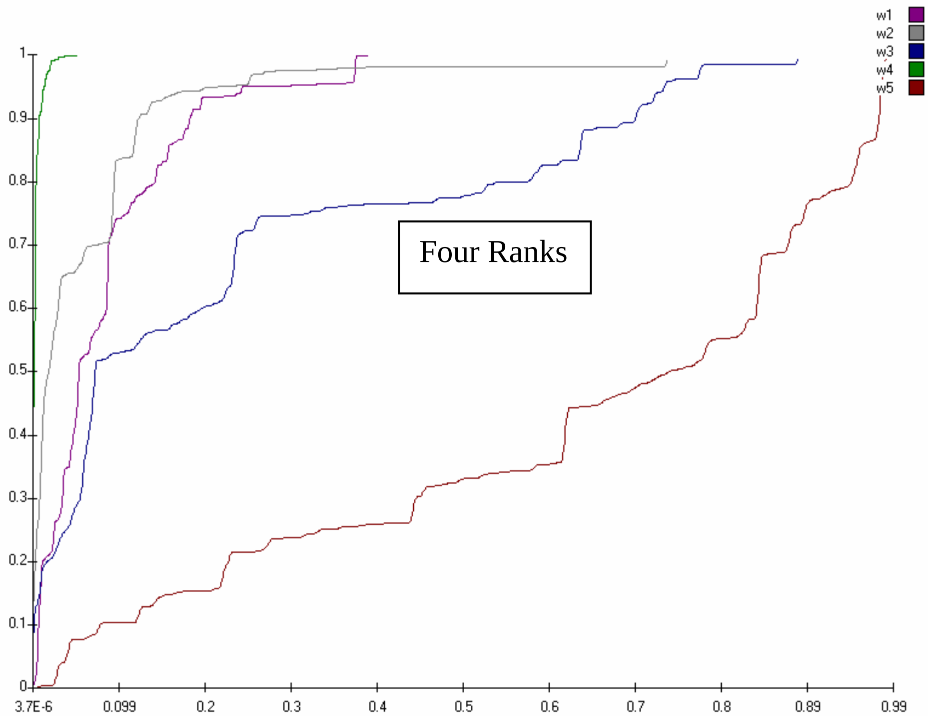
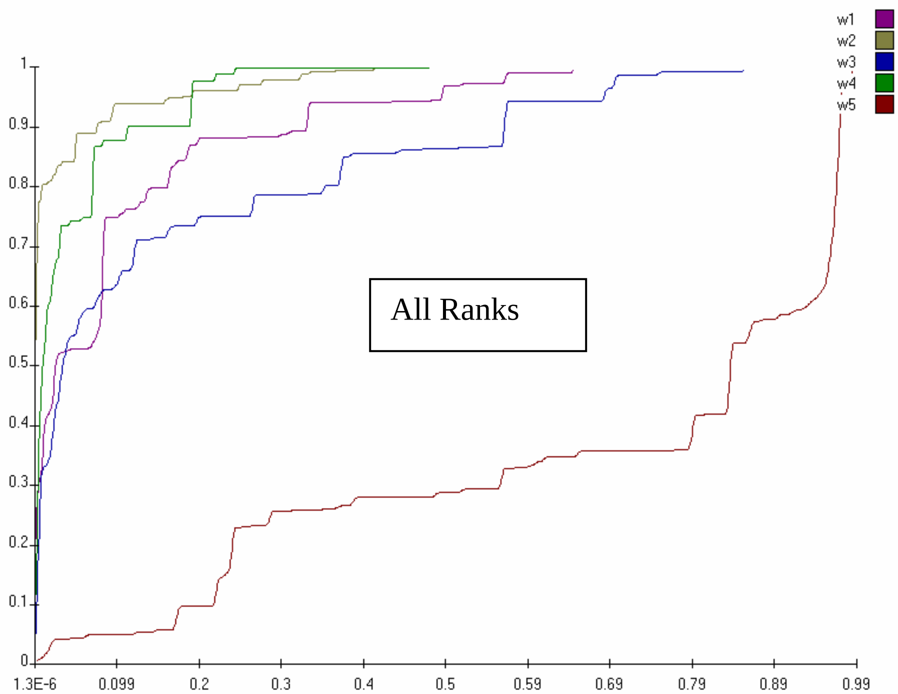




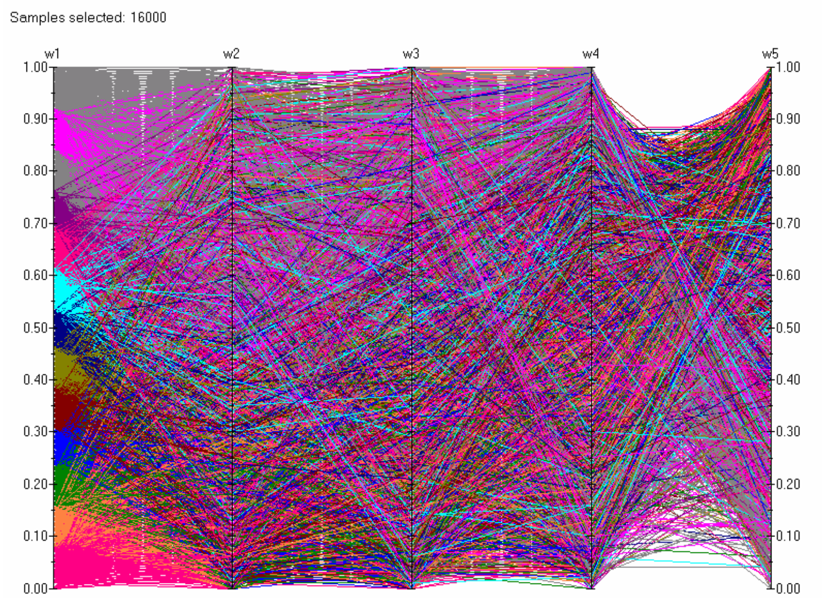




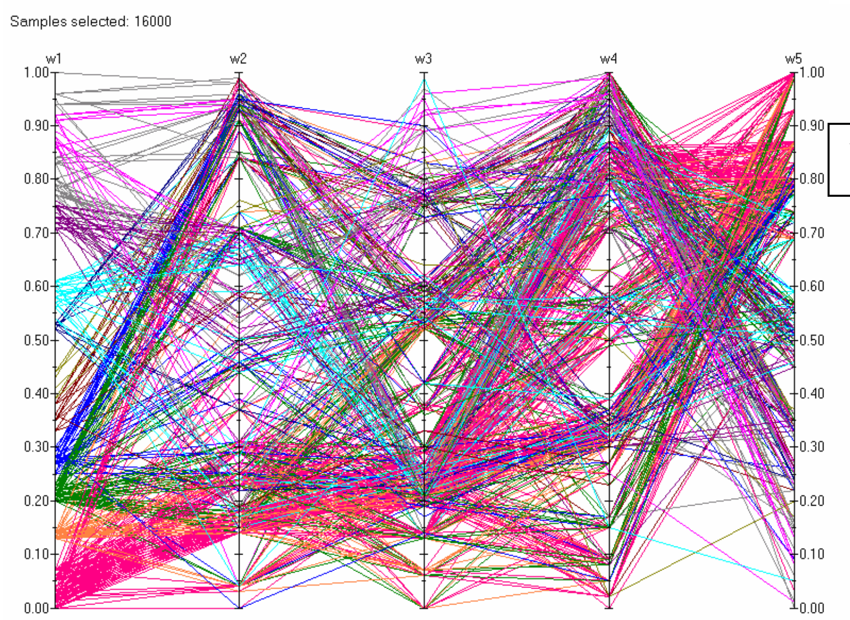
The "right most" cumulative distributions indicate greater importance. The picture from Figure 2 echoes that in Figure 1, for the first two ranks; Resistance is most important, followed by trophic impact. Of course, we must bear in mind that these results are relative to the scaling chosen to represent the criteria scores.

Figures 1 and 2 show that the means values and marginal distributions are somewhat similar in all fitting situations. The joint distributions, however, are quite different. One sample of weights represents one virtual stakeholder. If we plot these 5 weights on 5 vertical lines, we get a jagged line representing one virtual stakeholder. If we plot 16,000 such lines we get a picture of the population of stakeholders. We say that the stakeholder weights have *interactions* if eg knowledge that a stakeholder assigns high weight to the criteria “frequency” gives significant information regarding weights for other information. A quick visual impression of the joint distributions is given by “percentile cobweb plots” shown in Figure 3. Instead of the weights themselves, Figure 3 plots the weights’ percentiles, as this makes the dependence structure more visible. Evidently, the joint distributions are complex, and are different for the different fitting situations. A detailed analysis of interactions is not undertaken here. It is worth noting that the probabilistic inversion *infers* the dependence structure from the stakeholder data, it does *not* assume or impose any structure.

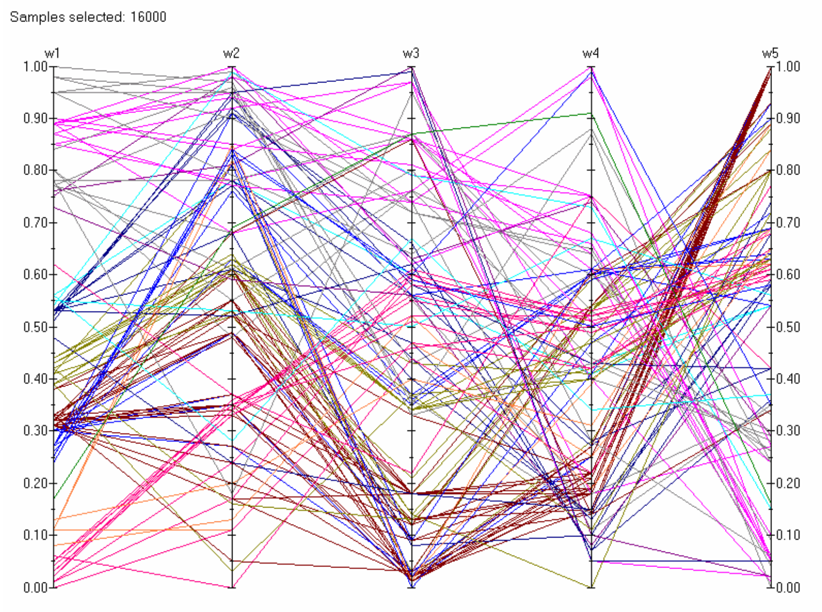
Figure 3 Percentile cobweb plots for criteria weights, fitting 1 rank, 4 ranks and all ranks



One rank



Four Ranks



All ranks

Table 2 shows the predicted probabilities of rankings based on the fitting in the four cases discussed above. Thus “prediction I” indicates the prediction based on fitting only the first ranked scenarios, etc. The first column gives the constraints. “#S4=1” denotes the constraint that scenario 4 was ranked 1. The last column shows that 34.3284% of the stakeholders ranked scenario 4 as first. Using the fitting based only on the first ranks predicts that 34.2395% of the population of stakeholders would rank scenario 4 as first. Similarly, 43.5945% of the population would rank S4 first, using the fitting based on the first four ranks. Of course, owing to the presence of inconsistent rankings, the fitting can never be perfect. Indeed, 22.4% of the first ranks were inconsistent with the model; as we fit the 77.6% of the consistent rankings, the remaining probability mass must be distributed over the other feasible rankings. Some of the discrepancies are sizeable, as in the case of #S20=5 for the prediction based in the top 4 ranks. On the whole, however, the predictions do capture the drift in the stakeholder preferences. Fitting all ranks is numerically quite burdensome and conflates issues that determine the most serious and least serious threats. The ranking based on the top four rankings presents the best compromise. The complete list of predictions, including the bottom ranked scenarios, is given in the spreadsheet.

Table 2: Model predictions and stakeholder probabilities for top 5 rankings

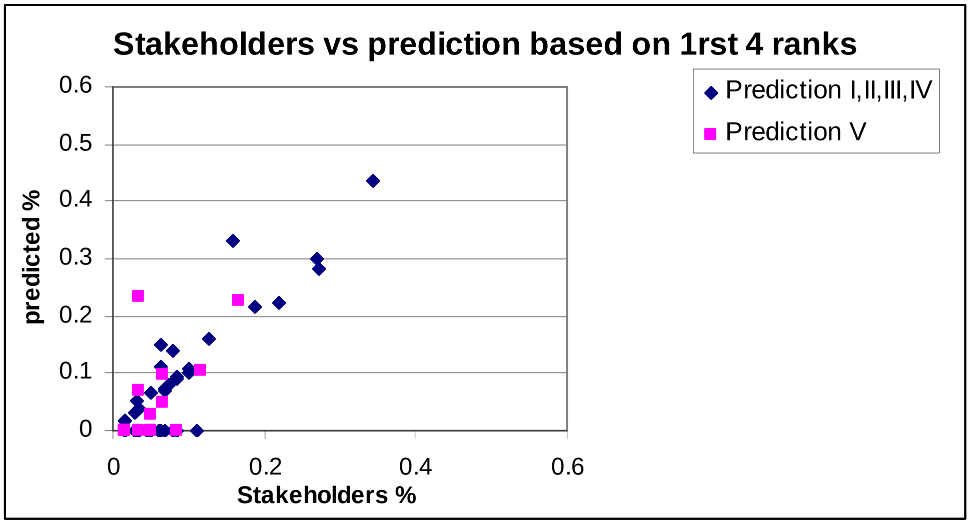
Constraint	**Prediction I**	**Prediction I,II**	**Prediction I,II,III**	**Prediction I,II,III,IV**	**Stakeholders**
#S3=1	0	0	0	0	0.059701
#S4=1	0.342395	0.342755	0.341965	0.435945	0.343284
#S6=1	0.26949	0.26866	0.41639	0.30077	0.268657
#S7=1	0.029575	0.029895	0.045345	0.032945	0.029851
#S8=1	0	0	0.0114	0	0.014925

#S9=1	0	0	0	0	0.014925
#S11=1	0	0	0	0	0.014925
#S12=1	0	0	0	0	0.014925
#S14=1	0.07436	0.07483	0.058015	0.07997	0.074627
#S16=1	0	0	0	0	0.029851
#S19=1	0	0	0	0	0.014925
#S22=1	0	0	0	0	0.044776
#S25=1	0	0	0	0	0.014925
#S28=1	0	0	0	0	0.029851
#S29=1	0	0	0	0	0.029851
#S2=2	0	0	0	0	0.033898
#S3=2	9.00E-05	0.03394	0.044245	0.03922	0.033898
#S4=2	0.22947	0.221345	0.17127	0.221765	0.220339
#S5=2	0	0	0	0	0.016949
#S6=2	0.475345	0.05114	0.06627	0.066135	0.050847
#S7=2	0.15567	0.067585	0.08245	0.068115	0.067797
#S8=2	4.50E-05	0.06791	0.043195	0.07251	0.067797
#S9=2	0	0	0	0	0.016949
#S11=2	0	0	0	0	0.016949
#S14=2	0.027475	0.269985	0.185495	0.28291	0.271186
#S16=2	0	0	0	0	0.050847
#S18=2	0	0	0	0	0.016949
#S20=2	0.023845	0.017015	0.021355	0.01741	0.016949
#S22=2	0	0	0	0	0.050847
#S23=2	0	0	0	0	0.016949
#S24=2	0	0	0	0	0.016949
#S29=2	0	0	0	0	0.050847
#S2=3	0	0	0	0	0.031746
#S3=3	0.001495	0.00841	0.192355	0.330465	0.15873
#S4=3	0.07984	0.065605	0.076875	0.14857	0.063492
#S6=3	0.070665	0.206345	0.07133	0.11314	0.063492
#S7=3	0.573185	0.46151	0.081615	0.140115	0.079365
#S8=3	0.000535	0.00528	0.03277	0.05139	0.031746
#S9=3	0	0	0	0	0.015873
#S12=3	0	0	0	0	0.063492
#S14=3	0.07303	0.064935	0.127605	0.16163	0.126984
#S16=3	0	0	0	0	0.031746
#S17=3	0	0	0	0	0.063492
#S18=3	0	0	0	0	0.031746
#S20=3	0.096785	0.158235	0.01575	0.018925	0.015873
#S21=3	0	0	0	0	0.015873
#S22=3	0	0	0	0	0.015873
#S24=3	0	0	0	0	0.015873
#S25=3	0	0	0	0	0.111111
#S26=3	0.104365	0.029315	0.01598	0.01813	0.015873

#S29=3	0	0	0	0	0.079365
#S3=4	0.013715	0.06871	0.013675	0.217435	0.186441
#S4=4	0.150775	0.112465	0.01503	0.039245	0.033898
#S5=4	0.00012	0.0036	0.037195	0.03923	0.033898
#S6=4	0.088925	0.341735	0.20989	0.095785	0.084746
#S7=4	0.119585	0.19476	0.258015	0.10914	0.101695
#S8=4	0.00309	0.007405	0.00279	0.073695	0.067797
#S11=4	0	0	0	0	0.033898
#S12=4	0.00171	0	0	0	0.084746
#S14=4	0.28505	0.07844	0.023485	0.090985	0.084746
#S16=4	0	0	0	0	0.033898
#S17=4	0	0	0	0	0.016949
#S18=4	0	0	0	0	0.016949
#S20=4	0.09903	0.079465	0.363005	0.017635	0.016949
#S22=4	0	0	0	0	0.067797
#S24=4	0	0	0	0	0.016949
#S25=4	0	0	0	0	0.050847
#S29=4	5.00E-06	8.50E-05	0	0.10243	0.101695
#S2=5	0.012245	0.00236	0.003495	0.070515	0.033333
#S3=5	0.013985	0.08453	0.02758	0	0.05
#S4=5	0.118865	0.167065	0.018225	0.02704	0.05
#S6=5	0.05437	0.07894	0.097435	0.096855	0.066667
#S7=5	0.11205	0.14602	0.415105	0.22786	0.166667
#S8=5	0.003385	0.009125	0.005715	5.00E-06	0.016667
#S12=5	0.00935	0.000515	2.00E-05	0.00039	0.016667
#S13=5	0	0	0	0	0.016667
#S14=5	0.1848	0.16412	0.11366	0.103565	0.116667
#S16=5	0	0	0	0	0.083333
#S17=5	0	0	0.0114	0	0.016667
#S19=5	0	0	0	0	0.016667
#S20=5	0.171445	0.086395	0.0883	0.233055	0.033333
#S22=5	0	0	0	0	0.033333
#S24=5	0	0	0	0	0.016667
#S25=5	0	0	0	0	0.05
#S26=5	0.25515	0.16456	0.10033	0.04926	0.066667

Figure 4 shows the information in 2 graphically. On the horizontal axis are stakeholders' percentages for rankings of scenarios; on the vertical axis are the predicted percentages based on the fitted model. The diamonds are scenarios which were ranked 1st, 2nd, 3rd or 4th. These percentages were used to fit the model. The squares are scenarios that were ranked 5th. We see that these percentages are reasonably well predicted with the model. Scenarios which plot on the horizontal axis correspond to rankings which are inconsistent with the model.

Figure 4: Predictions based on ranks 1 to 4, of stakeholder percentages for first four ranks(diamonds), and for 5th ranks (squares)



5. Scenario Scores

Table 3 shows the mean, variance and standard deviation of the 5 criteria weights and the 30 scenarios, based on the first four ranks. Table 4 gives the same information based on all ranks. Note that the variances in table 4 tend to be larger, sometimes much larger. The top ranked scenario S4 has a variance of 3.7 based on four ranks, and 17.2 based on all ranks. This suggests that trying to fit the top *and* bottom ranks just muddies the water – it does not give sharper insight into the factors determining high threat scenarios.

Table 3: Scenario scores

	With First four Rank		
Variable	Mean	Variance	SD
S1	1.571627	1.361583	1.166869
S2	1.561265	1.523243	1.234197
S3	2.21414	2.102801	1.450104
S4	2.901093	3.702064	1.924075
S5	1.762755	1.649106	1.284175
S6	2.328421	1.463672	1.209823
S7	2.419887	2.441138	1.562414
S8	1.82512	1.731593	1.3159
S9	0.693118	0.192708	0.438985
S10	1.146136	0.213732	0.462312
S11	0.923444	0.412429	0.642206
S12	1.844039	1.547017	1.243791
S13	1.445583	0.919627	0.958972
S14	2.539913	2.953413	1.71855
S15	1.471662	1.07466	1.036658
S16	1.656548	1.225883	1.107196

S17	1.638698	1.351518	1.162548
S18	1.446388	0.687576	0.829202
S19	1.405202	1.034591	1.017149
S20	1.8579	1.164557	1.079147
S21	1.117534	0.532685	0.729853
S22	1.412338	0.793808	0.890959
S23	1.465325	0.856217	0.92532
S24	1.642434	1.050101	1.024744
S25	1.728704	1.177705	1.085221
S26	2.21956	1.944399	1.394417
S27	1.06476	0.677409	0.823049
S28	0.712479	0.227958	0.47745
S29	2.024424	1.513184	1.230115
S30	1.1292	1.023464	1.011664

Table 4: Scenario scores using all ranks

	With All Ranks		
Variable	**Mean**	**Variance**	**SD**
S1	1.117964	1.724503	1.313203
S2	1.06512	2.114992	1.454301
S3	2.066086	4.996609	2.23531
S4	4.195709	17.18708	4.145731
S5	1.613338	4.405373	2.098898
S6	2.688947	4.172604	2.042695
S7	2.725864	6.269598	2.503917
S8	1.395671	2.080742	1.442478
S9	0.52834	0.254782	0.504759
S10	1.040633	0.481748	0.694081
S11	0.822006	0.931956	0.965379
S12	1.510104	2.293545	1.514445
S13	1.16709	1.585074	1.258997
S14	2.768019	7.953683	2.820227
S15	1.274146	1.974905	1.405313
S16	1.383625	2.432888	1.559772
S17	1.263349	1.874394	1.369085
S18	1.264006	1.687574	1.299067
S19	1.038872	0.961196	0.980406
S20	1.785677	1.71396	1.309183
S21	0.981132	1.232438	1.110152
S22	1.220056	1.596429	1.263499
S23	1.30219	2.184506	1.478007
S24	1.445351	2.066399	1.437497
S25	1.567832	2.362426	1.537019

S26	1.918288	2.360835	1.536501
S27	0.757252	0.650984	0.806836
S28	0.605471	0.430738	0.656306
S29	2.050589	3.662534	1.913775
S30	0.701122	0.902079	0.949778

6. Conclusion

By design, this study involved many dominated scenarios. This enabled us to test the extent to which the stakeholder preferences were consistent with a model for scenario scores based on a monotonic function of the 5 criteria scores. A stakeholder who prefers a dominated to a non-dominated scenario is not consistent with any such model. Of course, this does not mean that such a stakeholder is inconsistent, it simply means that his/her preferences are not consistent with this type of model. In view of the large number of dominated scenarios, we may conclude that these stakeholders are broadly, though not wholly consistent with such a monotonic model. A more complex model, possibly involving other criteria or interactions of criteria might produce a better fit, but such models would be much more cumbersome in practice.

The linear model (1) is one type of monotonic model. Owing to the above noted inconsistencies, it can never yield a perfect fit, but it does seem to capture the main drift of the stakeholder preferences. This means that the expected weights (Figure 1) can be used to score coastal ecosystem threat scenarios, provided their scores on the 5 criteria are given and scaled appropriately.

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