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W.T. van Horssen, and M.A. Zarubinskaya

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Abstract

In this paper we will study an elastic dissipation model for a cantilevered beam. This problem for a cantilevered beam has been formulated by D.L. Russell as an open problem in [1, 2]. To determine the relationship between the damping rates and the frequencies we will use a recently developed, adapted form of the method of separation of variables. It will be shown that the dissipation model as proposed by D.L. Russell for the cantilevered beam will not always generate damping. Moreover, it will be shown that some solutions can become unbounded.

1 Introduction

It is possible to use different approaches to describe energy dissipation in oscillating, elastic bodies such as beams (see [1, 2]). Many approaches (such as molecular theories) are too complicate to use and to analyze in practice. So, as a result different phenomenological theories are used and applied in mechanics. Of course every theory has its pros and cons. In particular, Russell notes in [1, 2] that it is clear that "viscous" damping models such as

$$\rho\frac{\partial^2 u}{\partial t^2} + 2\gamma\frac{\partial u}{\partial t} + \frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 u}{\partial x^2}\right) = 0,$$

which produce uniform damping rates, are inadequate if experimentally observed damping properties are to be incorporated in the model. Kelvin and Voigt noted at the end of the nineteenth century that damping rates tend to increase with frequency. Incorporated into the Euler-Bernoulli beam model their approach yields an equation of the form

$$\rho\frac{\partial^2 u}{\partial t^2} + 2\gamma\rho\frac{\partial^3}{\partial t\partial x^2}\left(EI\frac{\partial^2 u}{\partial x^2}\right) + \frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 u}{\partial x^2}\right) = 0.$$

In [3] Chen and Russell study models of the form

$$\ddot{x} + B\dot{x} + Ax = 0, \tag{1}$$

where A is an elasticity operator, and B is related in various ways to the positive square root, $A^{1/2}$, of A. For beam equations this approach was generalized and developed further by Russell in [1, 2]. More recent results on the nonnegative square root of fourth order derivative operators are obtained by Yao in [4].

In [1, 2] Russell introduces a new phenomenological dissipation model for beams, where the damping is assumed to be proportional to the bending rate of the beam. In fact the following equation is considered

$$u_{tt} - \delta u_{txx} + u_{xxxx} = 0,$$

where $u = u(x,t)$ is the displacement of the beam in vertical direction, and δ is a positive damping constant. No derivation of the dissipation term u_{txx} is given in [1, 2]. However, it is noted in [1, 2] that this new model has good mathematical properties. For instance, for initial value problems for simply supported beams, such as

$$\begin{aligned} &u_{tt} - \delta u_{txx} + u_{xxxx} = 0, \qquad 0 < x < \pi, \qquad t > 0, \\ &u(0,t) = u(\pi,t) = u_{xx}(0,t) = u_{xx}(\pi,t) = 0, \qquad t \geq 0, \\ &u(x,0) = f(x), \quad u_t(x,0) = g(x), \quad 0 < x < \pi, \end{aligned}$$

the solution for $0 < \delta < 2$ is given by (for $\delta \geq 2$ similar formulas can be derived)

$$u(x,t) = \sum_{n=1}^{\infty} e^{-\frac{\delta n^2}{2}t} \Big(A_n \sin\big(\frac{n^2}{2}\sqrt{4-\delta^2}t\big) + B_n \cos\big(\frac{n^2}{2}\sqrt{4-\delta^2}t\big) \Big) \sin(nx), \qquad (2)$$

where

$$A_n = \frac{4}{n^2\pi\sqrt{4-\delta^2}} \int_0^{\pi} \Big(g(x) + \frac{n^2}{2}\delta f(x) \Big) \sin(nx)dx, \qquad B_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin(nx)dx.$$

Indeed it can easily be seen that the damping rate in this case increases with the frequency. Russell [1, 2] and MacCluer [5] observe that the damping operator B in (1) and the stiffness operator A in (1) often "commute", that is, B shares the eigenmodes of A, or equivalently the nonnegative square root of the fourth order derivative operator $\frac{\partial^4 u}{\partial x^4}$ is $-\frac{\partial^2 u}{\partial x^2}$. Unfortunately for a cantilevered beam it turns out that A and B do not commute. Russell notes that [2, p.375]: "The apparent necessity of discarding this model for this reason is a real disappointment ...", and MacCluer remarks that [5, p.114]: "It is certain that most remain to be discovered".

In this paper we will study the following initial value problem for a cantilevered beam

$$\begin{aligned} &u_{tt} - \delta u_{txx} + u_{xxxx} = 0, \quad 0 < x < \pi, \quad t > 0 \\ &u(0,t) = u_x(0,t) = u_{xx}(\pi,t) = u_{xxx}(\pi,t) = 0, \quad t \geq 0, \\ &u(x,0) = f(x), \quad \textit{and} \quad u_t(x,0) = g(x), \quad 0 < x < \pi, \end{aligned} \qquad (3)$$

where δ is a positive damping parameter. To find the relationship between the damping rates and the frequencies we will use the recently developed, adapted form of the method of separation of variables (see [6]). This paper is organized as follows. In section 2 of this paper we will discuss how this adapted version of the method of separation of variables can be applied to the initial value problem (3) for the cantilevered beam. It will turn out that we have to consider three different cases: $\delta = 2$, $\delta > 2$, and $0 < \delta < 2$. These three cases will be treated in sections 3, 4, and 5 respectively. Finally in section 6 of this paper some conclusions will be drawn and some remarks will be made.

2 On an adapted version of the method of separation of variables

The method of separation of variables is the oldest systematic method to find nontrivial solutions for (linear) partial differential equations. To study waves and vibrations Daniel Bernoulli, Euler, and D'Alembert used this method in the middle of eighteenth century. The method has been considerably refined and generalized during the last centuries, and remains a method of great importance and frequent use today. Recently it has been shown in [6] that the method can be applied to a much larger class of problems as is generally assumed. After substitution of a separated solution (that is, a solution of the form $X(x)T(t)$) into the partial differential equation, dividing by $X(x)T(t)$, and after differentiating the so-obtained equation sufficiently many times with respect to some of the independent variables, we can finally reduce the problem to ordinary differential equations. This adapted version of the method of separation of variables seems to be not (well-) known in the literature on partial differential equations. In this section we will show how the adapted method can be applied to the initial value problem (3) for the cantilevered beam.

First we are looking for a nontrivial solution in the form $X(x)T(t)$ which satisfies the partial differential equation (PDE) and the boundary conditions. Substituting this nontrivial solution into the PDE, and by dividing the so-obtained equation by $X(x)T(t)$, we find

$$\frac{\ddot{T}}{T} - \delta \frac{\dot{T}}{T}\frac{X''}{X} + \frac{X''''}{X} = 0, \tag{4}$$

where $' = \frac{\partial(...)}{\partial x}$ and $\dot{} = \frac{\partial(...)}{\partial t}$. Generally it is assumed that (4) can not be separated because of the mixed term $-\delta \frac{\dot{T}}{T}\frac{X''}{X}$. However, by simply differentiating (4) with respect to x or t (see also [6]) we can separate the variables in (4). For instance if we differentiate (4) with respect to t we obtain

$$\frac{d}{dt}\left(\frac{\ddot{T}}{T}\right) - \delta \frac{X''}{X}\frac{d}{dt}\left(\frac{\dot{T}}{T}\right) = 0,$$

which can easily be separated, yielding

$$\frac{X''}{X} = -\beta, \tag{5}$$

where β is a complex valued separation constant. From (5) it follows that $X'''' = -\beta X'' = \beta^2 X$, and then it can easily be deduced from (4) that *T(t)* has to satisfy

$$\ddot{T} + \delta\beta\dot{T} + \beta^2 T = 0. \tag{6}$$

And so, the problem has been reduced to ordinary differential equations. Finally by substituting the "separated" solution *X(x)T(t)* into the boundary conditions we obtain as usual a boundary value problem for *X(x)*

$$\begin{aligned} &X'' + \beta X = 0, \qquad 0 < x < \pi, \\ &X(0) = X'(0) = X''(\pi) = X'''(\pi) = 0, \end{aligned} \tag{7}$$

where β is a complex valued separation constant. It turns out that the boundary value problem (7) only has trivial solutions. We will omit these lengthy but elementary calculations. So, differentiation of (4) with respect to t leads for the cantilevered beams to trivial solutions. For a simply supported beam, however, it will lead to the following boundary value problem for *X(x)*:

$$\begin{aligned} &X'' + \beta X = 0, \qquad 0 < x < \pi, \\ &X(0) = X''(0) = X(\pi) = X''(\pi) = 0, \end{aligned}$$

which has nontrivial solutions $X(x) = sin(nx)$ for $n = 1, 2, 3...$. And these solutions will finally lead to the solution of the initial value problem for the simply supported beam as for instance given by (2) for $0 < \delta < 2$.

We can also differentiate (4) with respect to x to obtain

$$-\delta\frac{\dot{T}}{T}\frac{d}{dx}\Big(\frac{X''}{X}\Big) + \frac{d}{dx}\Big(\frac{X''''}{X}\Big) = 0,$$

which can also easily be separated yielding

$$\frac{\dot{T}}{T} = \lambda, \tag{8}$$

where λ is a complex valued separation constant. From (8) it follows that $\ddot{T} = \lambda\dot{T} = \lambda^2 T$, and then it can easily be deduced from (4) that *X(x)* has to satisfy the following boundary value problem

$$\begin{aligned} &X'''' - \delta\lambda X'' + \lambda^2 X = 0, \qquad 0 < x < \pi, \\ &X(0) = X'(0) = X''(\pi) = X'''(\pi) = 0, \end{aligned} \tag{9}$$

where $\lambda = \lambda_1 + i\lambda_2$ with λ_1 and $\lambda_2 \in \mathbb{R}$. By considering the characteristic equation

$$k^4 - \delta\lambda k^2 + \lambda^2 = 0 \quad \Longleftrightarrow \quad (k^2 - \frac{\delta\lambda}{2})^2 + \frac{\lambda^2}{4}(4 - \delta^2) = 0 \tag{10}$$

for the differential equation in (9) it is obvious that we have to consider three cases: $\delta = 2$, $\delta > 2$, and $0 < \delta < 2$. These three cases will be studied in the next three sections. It will be shown that nontrivial solutions for (9) can be found in all three cases. From (8) the time-dependent behaviour of a nontrivial solution *X(x)T(t)* for (3) can be determined. It is obvious from (8) that arbitrary vibrations of the cantilevered beam can only be damped out if all eigenvalues λ have a negative real part, that is, λ_1 should be negative for all vibration modes.

3 The case $\delta = 2$

In this section we will study the boundary value problem (9) with $\delta = 2$. The characteristic equation for the differential equation in (9) becomes in this case

$$(k^2 - \lambda)^2 = 0, \tag{11}$$

where $\lambda = \lambda_1 + i\lambda_2$ with λ_1 and $\lambda_2 \in \mathbb{R}$. It can be shown elementarily that for $\lambda_2 = 0$ the boundary value problem (9) has only trivial solutions. For $\lambda_2 \neq 0$ the characteristic equation (11) has as roots

$$\zeta_1 + i\zeta_2, \qquad \text{and} \qquad -\zeta_1 - i\zeta_2,$$

where

$$\zeta_1 = \sqrt{\frac{\sqrt{\lambda_1^2 + \lambda_2^2} + \lambda_1}{2}}, \quad \zeta_2 = \sqrt{\frac{\sqrt{\lambda_1^2 + \lambda_2^2} - \lambda_1}{2}}. \tag{12}$$

Each root has multiplicity two. Putting $k_1 = \zeta_1 + i\zeta_2$ the solution of the differential equation in (9) can now be written as

$$X(x) = C_1 \cosh(k_1 x) + C_2 \sinh(k_1 x) + C_3 x \cosh(k_1 x) + C_4 x \sinh(k_1 x), \tag{13}$$

where C_1, C_2, C_3, and C_4 are complex valued constants of integration. By substituting (13) into the boundary conditions in (9) we obtain a system of four linear, homogeneous equations for C_1, C_2, C_3, and C_4. To have a nontrivial solution the determinant of the coefficient matrix has to be zero, yielding

$$\sinh^2(k_1\pi) - k_1^2\pi^2 + 4 = 0. \tag{14}$$

Taking apart real and imaginary parts in (14) we get a system of two nonlinear equations for ζ_1 and ζ_2 (note that $k_1 = \zeta_1 + i\zeta_2$)

$$\begin{aligned} \cosh(2\pi\zeta_1)\cos(2\pi\zeta_2) &= \tfrac{1}{2}\big((2\pi\zeta_1)^2 - (2\pi\zeta_2)^2\big) - 7, \\ \sinh(2\pi\zeta_1)\sin(2\pi\zeta_2) &= 2\pi\zeta_1 2\pi\zeta_2. \end{aligned} \tag{15}$$

Using the formula manipulation package Maple numerical approximations of the solution of (15) can easily be obtained. Using these approximations and (12) the eigenvalues $\lambda = \lambda_1 + i\lambda_2$ can be approximated. The first six approximations of the eigenvalues λ of the boundary value problem (9) are listed in table 1.

	$\delta = 2$	
Nr.	λ_1	λ_2
1	0.072471	0.327553
2	-1.306096	2.005633
3	-4.930357	4.225684
4	-10.658699	6.772424
5	-18.442296	9.470416
6	-28.260886	12.309142

Table 1. Approximations of the first six eigenvalues $\lambda = \lambda_1 + i\lambda_2$ for the case $\delta = 2$.

The first eigenvalue has a positive real part. From (8) it can readily be seen that for this eigenvalue there exists a nontrivial solution *X(x)T(t)* of (3) which becomes unbounded for increasing times t. So for this first vibration mode there certainly is no energy dissipation.

4 The case $\delta > 2$

In this section we will study the boundary value problem (9) with $\delta > 2$. The characteristic equation for the differential equation in (9) is

$$k^4 - \lambda\delta k^2 + \lambda^2 = 0, \tag{16}$$

where $\lambda = \lambda_1 + i\lambda_2$ with λ_1 and $\lambda_2 \in \mathbb{R}$. It is easy to show that for $\lambda_2 = 0$ the boundary value problem (9) has only trivial solutions. For $\lambda_2 \neq 0$ it follows from the characteristic equation (16) that

$$k^2 = \lambda\Big(\frac{\delta}{2} + \frac{1}{2}\sqrt{\delta^2 - 4}\Big), \quad \text{or} \quad k^2 = \frac{\lambda}{\big(\frac{\delta}{2} + \frac{1}{2}\sqrt{\delta^2 - 4}\big)}. \tag{17}$$

Putting $a = \Big(\frac{\delta}{2} + \frac{1}{2}\sqrt{\delta^2 - 4}\Big)$ it follows from (17) that $k^2 = \lambda a$ or $k^2 = \frac{\lambda}{a}$. And so, the roots of the characteristic equation (16) are

$$ap_1, \quad -ap_1, \quad p_1, \quad \text{and} \quad p_1,$$

where $p_1 = \xi_1 + i\xi_2$ with

$$\xi_1 = \sqrt{\frac{\sqrt{\lambda_1^2 + \lambda_2^2} + \lambda_1}{2}} \qquad \text{and} \qquad \xi_2 = \sqrt{\frac{\sqrt{\lambda_1^2 + \lambda_2^2} - \lambda_1}{2}}. \tag{18}$$

For $\lambda_2 \neq 0$ the solution of the differential equation in (9) can now be written as

$$X(x) = C_1\cosh(p_1 x) + C_2\sinh(p_1 x) + C_3\cosh(ap_1 x) + C_4\sinh(ap_1 x), \tag{19}$$

where C_1, C_2, C_3, and C_4 are complex valued constants of integration. By substituting (19) into the boundary conditions in (9) we obtain a system of four linear, homogeneous equations for C_1, C_2, C_3, and C_4. To have a nontrivial solution the determinant of the coefficient matrix has to be zero, yielding

$$1 + a^4 + \frac{a}{2}(a-1)^2 \cosh\big((a+1)p_1\pi\big) - \frac{a}{2}(a+1)^2 \cosh\big((a-1)p_1\pi\big) = 0. \tag{20}$$

Taking apart the real and imaginary parts in (20) we finally obtain a system of two non-linear equations for ξ_1 and ξ_2 (note that $p_1 = \xi_1 + i\xi_2$).

$$\begin{aligned} &1+a^4+\frac{a}{2}(a-1)^2\cosh\big((a+1)\pi\xi_1\big)\cos\big((a+1)\pi\xi_2\big)-\frac{a}{2}(a+1)^2\cosh\big((a-1)\pi\xi_1\big)\cos\big((a-1)\pi\xi_2\big) = 0,\\ &\frac{a}{2}(a-1)^2 \sinh\big((a+1)\pi\xi_1\big) \sin\big((a+1)\pi\xi_2\big) - \frac{a}{2}(a+1)^2 \sinh\big((a-1)\pi\xi_1\big) \sin\big((a-1)\pi\xi_2\big) = 0. \end{aligned} \tag{21}$$

Numerical approximations of the solution of (21) can easily be obtained by using the formula manipulation package Maple. Using these approximations and (18) we can approximate the eigenvalues $\lambda = \lambda_1 + i\lambda_2$. The first six approximations of the eigenvalues λ are listed in table 2 for some specific values of the parameter $\delta > 2$.

	$\delta = 2.001$		$\delta = 3$		$\delta = 10$	
Nr.	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
1	0.071093	0.318035	0.035470	0.058214	0.012281	0.019740
2	-1.266093	1.943048	-0.675846	0.582716	-0.054805	0.055348
3	-4.779897	4.122121	-1.728249	0.623632	-0.197794	0.087744
4	-10.334167	6.553759	-3.707961	1.393278	-0.418701	0.130915
5	-18.384359	9.277380	-6.116124	1.063614	-0.705594	0.202380
6	-27.403149	11.889795	-9.027316	2.169593	-0.994408	0.285117

Table 2. Approximations of the first six eigenvalues $\lambda = \lambda_1 + i\lambda_2$ for some specific values of the parameter $\delta > 2$.

From table 2 it follows that for each specific value $\delta > 2$ (as listed in table 2) the first eigenvalue always has a positive real part. From (8) it can readily be seen that for these first eigenvalues there exist nontrivial solutions $X(x)T(t)$ of (3) which become unbounded for increasing times t. So for this first vibration mode there certainly is no energy dissipation.

5 The case $0 < \delta < 2$

In this section we will study the boundary value problem (9) with $0 < \delta < 2$. The characteristic equation for the differential equation in (9) has the form (16). It can be shown elementarily that for $\lambda_2 = 0$ the boundary value problem (9) has only trivial solutions. For $\lambda_2 \neq 0$ it follows from the characteristic equation (16) that

$$k^2 = \lambda\Big(\frac{\delta}{2} + i\frac{\sqrt{4-\delta^2}}{2}\Big), \quad \text{or} \quad k^2 = \frac{\lambda}{\frac{\delta}{2} + i\frac{\sqrt{4-\delta^2}}{2}}. \tag{22}$$

Putting $a = \frac{\delta}{2} + i\frac{\sqrt{4-\delta^2}}{2}$ it follows from (22) that $k^2 = \lambda a$, or $k^2 = \frac{\lambda}{a}$. And so, the roots of the characteristic equation (16) are

$$ap_1, \quad -ap_1, \quad p_1, \quad -p_1,$$

where $p_1 = \eta_1 + i\eta_2$ with

$$\begin{aligned}
\eta_1 &= \frac{1}{2}\left(\sqrt{\frac{\sqrt{\lambda_1^2+\lambda_2^2}+\lambda_1}{2}(2+\delta)} - \sqrt{\frac{\sqrt{\lambda_1^2+\lambda_2^2}-\lambda_1}{2}(2-\delta)}\ \right), \quad \text{and} \\
\eta_2 &= \frac{1}{2}\left(\sqrt{\frac{\sqrt{\lambda_1^2+\lambda_2^2}-\lambda_1}{2}(2+\delta)} + \sqrt{\frac{\sqrt{\lambda_1^2+\lambda_2^2}+\lambda_1}{2}(2-\delta)}\ \right).
\end{aligned} \tag{23}$$

As in section 4 the solution of the differential equation in (9) can be written in the form (19). Again we obtain a system of four linear, homogeneous equations for C_1, C_2, C_3, and C_4 by substituting (19) into the boundary conditions in (9). To have a nontrivial solution the determinant of the coefficient matrix has to be zero, yielding (20). The only difference now with the previous section is that a and p_1 are both complex valued. Taking apart the real and imaginary parts in equation (20) we obtain a system of two nonlinear equations for η_1 and η_2 (note that $p_1 = \eta_1 + i\eta_2$)

$$\begin{aligned}
&(\delta^2-2)^2 + \\
&(\delta^2/2-1)(\delta-2)\cosh\Big(((\delta/2+1)\eta_1-\eta_2\sqrt{4-\delta^2}/2)\pi\Big)\cos(((\delta/2+1)\eta_2+\eta_1\sqrt{4-\delta^2}/2)\pi\Big) - \\
&\delta\sqrt{4-\delta^2}(\delta-2)/2\sinh\Big(((\delta/2+1)\eta_1-\eta_2\sqrt{4-\delta^2}/2)\pi\Big)\sin(((\delta/2+1)\eta_2+\eta_1\sqrt{4-\delta^2}/2)\pi\Big) - \\
&(\delta^2/2-1)(\delta+2)\cosh\Big(((\delta/2-1)\eta_1-\eta_2\sqrt{4-\delta^2}/2)\pi\Big)\cos(((\delta/2-1)\eta_2+\eta_1\sqrt{4-\delta^2}/2)\pi\Big) + \\
&\delta\sqrt{4-\delta^2}(\delta+2)/2\sinh\Big(((\delta/2-1)\eta_1-\eta_2\sqrt{4-\delta^2}/2)\pi\Big)\sin(((\delta/2-1)\eta_2+\eta_1\sqrt{4-\delta^2}/2)\pi\Big) = 0, \\
&\text{and} \\
&\delta\sqrt{4-\delta^2}(\delta^2-2) + \\
&\delta\sqrt{4-\delta^2}(\delta-2)/2\cosh\Big(((\delta/2+1)\eta_1-\eta_2\sqrt{4-\delta^2}/2)\pi\Big)\cos(((\delta/2+1)\eta_2+\eta_1\sqrt{4-\delta^2}/2)\pi\Big) - \\
&(\delta^2/2-1)(\delta-2)\sinh\Big(((\delta/2+1)\eta_1-\eta_2\sqrt{4-\delta^2}/2)\pi\Big)\sin(((\delta/2+1)\eta_2+\eta_1\sqrt{4-\delta^2}/2)\pi\Big) - \\
&\delta\sqrt{4-\delta^2}(\delta+2)/2\cosh\Big(((\delta/2-1)\eta_1-\eta_2\sqrt{4-\delta^2}/2)\pi\Big)\cos(((\delta/2-1)\eta_2+\eta_1\sqrt{4-\delta^2}/2)\pi\Big) - \\
&(\delta^2/2-1)(\delta+2)\sinh\Big(((\delta/2-1)\eta_1-\eta_2\sqrt{4-\delta^2}/2)\pi\Big)\sin(((\delta/2-1)\eta_2+\eta_1\sqrt{4-\delta^2}/2)\pi\Big) = 0.
\end{aligned} \tag{24}$$

Using the formula manipulation package Maple numerical approximations of the solution of (24) can easily be obtained. Using these approximations and (23) the eigenvalues $\lambda = \lambda_1 + i\lambda_2$ can be approximated. The first six approximations of the eigenvalues λ of the boundary value problem (9) are listed in table 3 for some specific values of the parameter $0 < \delta < 2$.

	$\delta = 0.01$		$\delta = 1$		$\delta = 1.999$	
Nr.	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
1	0.000438	0.356246	0.041226	0.347651	0.072445	0.327575
2	-0.006735	2.232556	-0.669480	2.183093	-1.305498	2.005905
3	-0.023255	6.251193	-2.352370	5.821765	-4.927529	4.259369
4	-0.050113	12.249825	-5.049603	11.194332	-10.651827	6.780897
5	-0.086926	20.249820	-13.444049	27.132464	-18.429408	9.490139
6	-0.133743	30.249707	-19.141120	37.699619	-28.239893	12.347001

Table 3. Approximations of the first six eigenvalues $\lambda = \lambda_1 + i\lambda_2$ for for some specific values of the parameter $0 < \delta < 2$.

From table 3 it follows that for each specific value $0 < \delta < 2$ (as listed in table 3) the first eigenvalue always has a positive real part. From (8) it can readily be seen that for these first eigenvalues there exist nontrivial solutions $X(x)T(t)$ of (3) which become unbounded for increasing times t. So for this first vibration mode there certainly is no energy dissipation.

6 Conclusion

In this paper a phenomenological dissipation model for a cantilevered beam has been studied. This new model has been introduced in [1, 2] as a model for a beam where the damping is assumed to be proportional to the bending rate of the beam. For the cantilevered beam the relationship between the damping rates and the frequencies has been obtained by using the recently developed, adapted version of the method of separation of variables (see [6]). It should be remarked that this relationship also can be obtained by applying the Laplace transformation method to (3). The boundary value problem (9) then also is obtained.

It has been shown that this phenomenological model for the cantilevered beam does not always generate damping. For some significant values of the "damping" parameter δ it has been shown numerically that the first (that is, the lowest) vibration mode is unstable. So for this mode there certainly is no energy dissipation. From this point of view we have to conclude that the elastic dissipation model (as proposed in [1, 2]) for the cantilevered beam is not an adequate dissipation model.

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