

DELFT UNIVERSITY OF TECHNOLOGY

REPORT 06-11

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ISSN 1389-6520

Reports of the Delft Institute of Applied Mathematics

Delft 2006

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INDONESIAN OPTIONS

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ABSTRACT. Jakarta Stock Exchange Indonesia has started to trade Indonesian options at September 9th, 2004. An Indonesian option can be considered as an American style barrier option with immediate (forced) exercise if the price hits or crosses the barrier before maturity. The payoff of the option is based on a moving average of the price of the underlying stock. The barrier is fixed at the strike price plus or minus a 10 percent. The option is automatically exercised when the underlying stock hits or crosses the barrier and the difference between strike and barrier is paid immediately. We will refer to type of this option as Indonesian option.

In this paper we study the pricing of the Indonesian option in a Black-Scholes economy. We will derive analytic approximations for the option price. We will discuss volatility and it turns out that expression we cannot calculate the implied volatilities.

1. INTRODUCTION

September 9, 2006, the Jakarta Stock Exchange (JSX) Indonesia¹ introduced the trading of options on stocks of five companies: Telekomunikasi Indonesia Tbk (TLKM), Astra International Tbk (ASII), HM Sampoerna Tbk (HMSP), Bank Central Asia Tbk (BBCA) and Indofood Sukses Makmur Tbk (INDF). The regulations for trading in options are available on the JSX website, unfortunately these regulations are only available in Indonesian. These regulations contain, among others, descriptions of put and call option contracts with the stock of one of the above-mentioned companies as underlying. Since these contracts are rather special, we will refer to them as Indonesian put or call option. As usual, a strike price K and a maturity date T are specified in these contracts. For example an AHMSP4100 contract is a call option contract with maturity date January (A = January, B=February,...,L=December) and with strike 4100 written on HMSP stocks (HM Sampoerna Tbk). The letters O,P,...,Z are used to code

1991 *Mathematics Subject Classification.* Primary 47A15; Secondary 46A32, 47D20.

Key words and phrases. Indonesian option, barrier option, Black-Scholes model, one-touch option, Variance Gamma process.

This work was completed with the support of the Royal Netherlands Academy of Arts and Sciences (KNAW).

¹For more information, see the JSX homepage : <http://www.jsx.co.id>

the maturity date for put option contracts. So the put option contract with maturity January on the same stock and with the same strike is coded by OHMSP4100. Every 15 minutes, a Weighted Moving Average Price (WMA) of the stock-price over the last 30 minutes is published: Monday, starting at 10.00.01 the WMA over the time-period 9.30-10.00

$$P_{(10.01-10.15)} = \frac{\sum_{t=9.30}^{t=10.00} P_t Q_t}{\sum_{t=9.30}^{t=10.00} Q_t},$$

where P_t is the stock price at time t , and Q_t the transaction volume. At 10.15.01 the next WMA price over the time-period 9.45-10.15 is published, and so on. The last WMA price of the morning (section 1) is published at 12.00.01, except on Friday. On Friday, the last WMA price publication in the morning is at 11.30.01. In the afternoon (section 2) the first publication is at 14.15.01 and the last one at 16.00.01. So Monday-Thursday are divided in 17 periods of 15 minutes between consecutive publications of WMA prices. These periods are numbered 1,2,...,17. Friday has 15 periods. The option contract can be exercised anytime during the period of stock option trading, till maturity T or till the WMA price hits or crosses the barrier $B = 0.9K$ for a put, $B = 1.1K$ for a call. If the option contract is exercised at time t and if the WMA price has not hit or crossed the barrier, then the payoff is equal to the difference between the strike and the last published WMA price, $K - \text{WMA}$ for a put, $\text{WMA} - K$ for a call. This payoff is payed the next day. If the WMA price hits or crosses the barrier, then the option contract is automatically exercised and the owner of option contract gets immediately $K - B$ in case of a put and $B - K$ in case of a call. To calculate prices for these option contracts, we have to model the WMA price. This is not easy. Even if we adopt a Black-Scholes model for the price of the underlying stock, and assume that the weights are constant, we have to face the problem of convolution of lognormal distributions. It is argued in [3] that a generalized type of lognormal distribution can be used as a closed form approximation for the WMA. But in general, the weights will certainly not be constant. We present in Section 2 the case where the WMA price is replaced by the price of the underlying stock, which will be modelled as a geometric Brownian Motion. The option contract can then be considered as an American barrier option. In the next Section, we will approximate the WMA price with the price of the underlying stock at the beginning of the corresponding 15 minute period. The price of the underlying stock will again be modelled as a geometric Brownian motion. The WMA price of the next period can then be obtained by multiplying the old price with an independent factor.

2. BLACK-SCHOLES PRICE

In this Section we will assume that the price process of the underlying stock follows a geometric Brownian Motion. So if $S = \{S(t) : t \geq 0\}$ denotes the price process of the underlying stock, we assume that the dynamics of S under the risk-neutral measure are given by the stochastic differential equation

$$dS(t) = rS(t) dt + \sigma S(t) dW(t), \quad S(0) = s_0, \quad (1)$$

where r is the risk-free interest rate and $\sigma > 0$ the volatility, see [1]. We consider the following modified Indonesian call option contract: the holder of the option contract has the right, at any time till an in advance specified maturity date T , to buy the stock for a certain strike K unless the price of the stock hits a barrier $B > K$ and $B > s_0$. In that case the option has to be exercised immediately and the owner of the option gets a payoff $(B - K)$. So the option contract can be considered as an American up-and-out call, where the holder exercises just if the the stock price is near to the barrier. A modification of the usual argument shows that it is never optimal to exercise before maturity T , unless we are forced to exercise since the price hits the barrier. So the value of this option must be the same as the European version where we can only exercise at time T unless the barrier is hit in which case we have to exercise immediately.

Let τ_B denote the hitting time of the barrier B :

$$\tau_B = \min\{t : S(t) = B\}. \quad (2)$$

For the Black-Scholes model, where the price process follows the dynamics given in equation (1), we have

$$S(t) = s_0 \exp(\mu t + \sigma W(t)), \quad (3)$$

where $\mu = r - \frac{1}{2}\sigma^2$. It follows that we can represent τ_B as a hitting time of Brownian motion with drift as follows:

$$\tau_B = \min\{t : \mu t + \sigma W(t) = b\}, \quad (4)$$

where $b = \ln(B/s_0) > 0$. This representation of τ_B is useful to find its probability density. For the maximum

$$M(t) = \max\{\mu s + \sigma W(s) : s \in [0, t]\}$$

we have

$$F_{M(t)}(x) = \mathbb{P}(M(t) \leq x) = \mathcal{N}\left(\frac{x - \mu t}{\sigma\sqrt{t}}\right) - e^{2\mu x/\sigma^2} \mathcal{N}\left(\frac{-x - \mu t}{\sigma\sqrt{t}}\right), \quad (5)$$

where \mathcal{N} denotes the cumulative distribution function of the standard normal distribution with density

$$\phi(x) = \frac{1}{\sqrt{2\pi}} \exp(-x^2/2),$$

see for example [1]. Since $\mathbb{P}(\tau_B \leq t) = \mathbb{P}(M(t) \geq b)$, τ_B has a probability density given by

$$f_{\tau_B}(t) = \frac{d}{dt} (1 - F_{M(t)}(b)) = \frac{b}{\sigma\sqrt{2\pi}t^{3/2}} \exp\left\{-\frac{1}{2}\left(\frac{b-\mu t}{\sigma\sqrt{t}}\right)^2\right\}. \quad (6)$$

In Section 4, we also have to consider the case $B < s_0$, i.e. $b < 0$. It follows from the symmetry of Brownian motion that

$$\tau_B \stackrel{d}{=} \min\{t : -\mu t + \sigma W(t) = -b\},$$

hence, for $b < 0$,

$$f_{\tau_B}(t) = \frac{d}{dt} (1 - F_{M^-(t)}(-b)) = \frac{-b}{\sigma\sqrt{2\pi}t^{3/2}} \exp\left\{-\frac{1}{2}\left(\frac{-b+\mu t}{\sigma\sqrt{t}}\right)^2\right\}, \quad (7)$$

where

$$F_{M^-(t)}(x) = \mathcal{N}\left(\frac{x+\mu t}{\sigma\sqrt{t}}\right) - e^{-2\mu x/\sigma^2} \mathcal{N}\left(\frac{-x+\mu t}{\sigma\sqrt{t}}\right). \quad (8)$$

The payoff Φ at time T of an Indonesian option with strike K , barrier B and maturity T , can now be expressed as follows:

$$\Phi = (B - K)e^{r(T-\tau_B)} 1_{\{\tau_B \leq T\}} + (S(T) - K)^+ 1_{\{\tau_B > T\}}, \quad (9)$$

where we discounted the early exercise payoff to time T . Note that the payoff is path-dependent. It follows that the price of the option at time 0 is given by:

$$V = (B - K)\mathbb{E}(e^{-r\tau_B}; \tau_B \leq T) + e^{-rT}\mathbb{E}((S(T) - K)^+ 1_{\{\tau_B > T\}}), \quad (10)$$

where the first term of the RHS is the value $V_{ot}(B)$ of a type of one-touch option that exercises automatically if the price of the underlying hits the barrier B with immediate payoff. The second term of the RHS is the value $V_{uoc}(B, K)$ of a standard up-and-out call with barrier B and strike K .

To calculate $V_{ot}(B)$, note first that

$$\mathbb{E}(e^{-r\tau_B}; \tau_B \leq T) = \int_0^T e^{-ry} f_{\tau_B}(y) dy. \quad (11)$$

Completing squares, the integrand $e^{-ry} f_{\tau_B}(y)$ can be written as

$$\begin{aligned} & \frac{s_0}{B} \frac{b}{\sigma\sqrt{2\pi}y^{3/2}} \exp\left\{-\frac{1}{2}\left(\frac{b-\tilde{\mu}y}{\sigma\sqrt{y}}\right)^2\right\} \\ &= \frac{s_0}{B} \frac{d}{dt} \left\{ 1 + e^{2\tilde{\mu}b/\sigma^2} \mathcal{N}\left(\frac{-b-\tilde{\mu}t}{\sigma\sqrt{t}}\right) - \mathcal{N}\left(\frac{b-\tilde{\mu}t}{\sigma\sqrt{t}}\right) \right\}, \end{aligned} \quad (12)$$

where $\tilde{\mu} = \mu + \sigma^2$. It follows that

$$V_{ot}(B) = \frac{s_0(B-K)}{B} \left\{ 1 + e^{2\tilde{\mu}b/\sigma^2} \mathcal{N}(\tilde{c}_{-b}) - \mathcal{N}(\tilde{c}_b) \right\}, \quad (13)$$

where

$$c_x = \frac{x - \mu T}{\sigma \sqrt{T}}, \quad \tilde{c}_x = \frac{x - \tilde{\mu} T}{\sigma \sqrt{T}}. \quad (14)$$

Standard barrier contracts like an up-and-out call are well-known. The value of the contract at time 0 is given by

$$\begin{aligned} V_{uoc}(B, K) &= s_0 \left(\mathcal{N}(\tilde{c}_b) - \mathcal{N}(\tilde{c}_k) + e^{2\tilde{\mu}b/\sigma^2} \mathcal{N}(\tilde{c}_{k-2b}) - e^{2\tilde{\mu}b/\sigma^2} \mathcal{N}(\tilde{c}_{-b}) \right) \\ &\quad - K e^{-rT} \left(\mathcal{N}(c_b) - \mathcal{N}(c_k) + e^{2\mu b/\sigma^2} \mathcal{N}(c_{k-2b}) - e^{2\mu b/\sigma^2} \mathcal{N}(c_{-b}) \right) \end{aligned} \quad (15)$$

where $k = \ln(K/s_0)$. In an Appendix we will give a derivation of this formula, because it cannot be found directly in the literature. The price of the Indonesian option with stock price instead of WMA price is now given by

$$\begin{aligned} V &= V_{ot}(B) + V_{uoc}(B, K) \\ &= \frac{s_0(B - K)}{B} \left\{ 1 + e^{2\tilde{\mu}b/\sigma^2} \mathcal{N}(\tilde{c}_{-b}) - \mathcal{N}(\tilde{c}_b) \right\} \\ &\quad + s_0 \left(\mathcal{N}(\tilde{c}_b) - \mathcal{N}(\tilde{c}_k) + e^{2\tilde{\mu}b/\sigma^2} \mathcal{N}(\tilde{c}_{k-2b}) - e^{2\tilde{\mu}b/\sigma^2} \mathcal{N}(\tilde{c}_{-b}) \right) \\ &\quad - K e^{-rT} \left(\mathcal{N}(c_b) - \mathcal{N}(c_k) + e^{2\mu b/\sigma^2} \mathcal{N}(c_{k-2b}) - e^{2\mu b/\sigma^2} \mathcal{N}(c_{-b}) \right) \end{aligned}$$

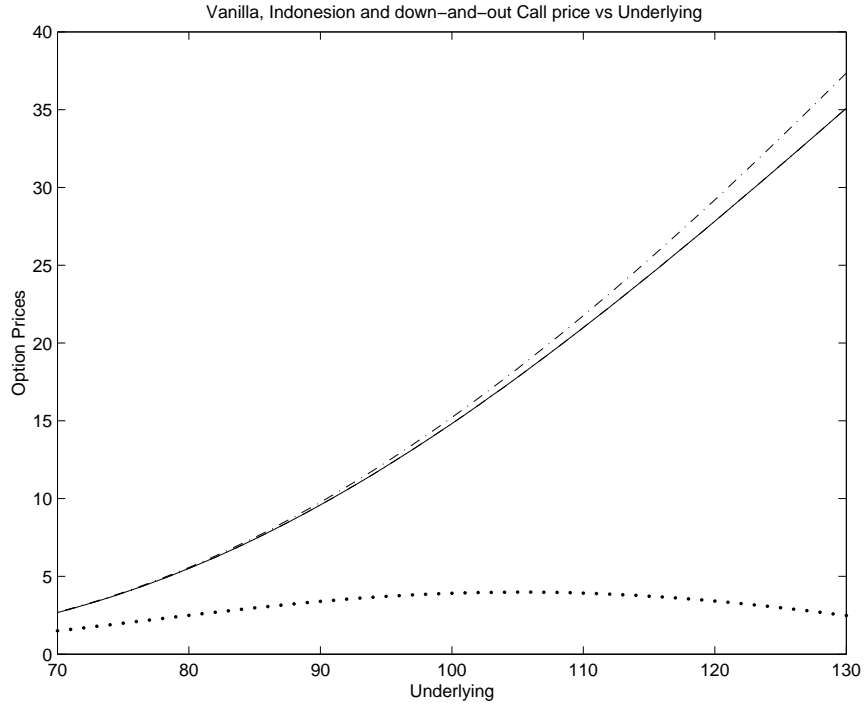


FIGURE 1. Numerical Example for Indonesian Call Option

Numerical Example. The figure shows a numerical example for the Indonesian Call option with strike 100, barrier 150, volatility 35% and interest rate 3.5%. The lower dotted line is the amount of the value of Indonesian option that consists of the up-and-out call. The solid line is the value of the Indonesian Option, so the difference between the solid and the dotted line, is the contribution of getting paid the difference between the barrier and the strike at the moment the barrier is hit. The upper dash-dotted line is the value of the vanilla call option and is - maybe surprisingly - not so far off from the value of the Indonesian call.

3. VOLATILITY

The regulations² of the Indonesian option contain an Appendix about the calculation of the volatility. The translation is as follows:

Calculation of volatility

There are 5 steps to calculate the average daily volatility for a period of one year,

- (1) calculate the average of the intra day stock price,

$$\bar{h}_{Qi} = \frac{\sum_{j=1}^n h_{Qij}}{n}$$

where

Q is a stock Q

h_{Qij} is the price of stock Q at day i and at transaction j

n is the number of transactions in one day.

- (2) calculate the intra day standard deviation³ of the stock price,

$$\sigma_{Qi} = \sqrt{\frac{\sum_{j=1}^n (h_{Qij} - \bar{h}_{Qi})^2}{n - 1}}$$

where

Q is a stock Q

h_{Qij} is the price of stock Q at day i and at transaction j

n is the number of transactions in one day.

²Peraturan nomor II-D: tentang perdagangan opsi saham (Kep-310/BEJ/09-2004), web site JSX

³Apparently there is a mistake in the formula in step 2 of the Appendix II-D.1. We present a corrected formula

- (3) calculate the average of the intra day stock price in one year,

$$\bar{h}_Q = \frac{\sum_{i=1}^N \bar{h}_{Qi}}{N}$$

where

Q is a stock Q

\bar{h}_{Qi} is the average of the intra day stock price at day i

N is the number of trading days in one year.

- (4) calculate the intra day standard deviation of the stock price in one year,

$$\bar{\sigma}_Q = \sqrt{\frac{\sum_{i=1}^N \sigma_{Qi}^2}{N}}$$

where

Q is a stock Q

σ_{Qi} is the average of the intra day standard deviation of the stock price at day i

N is the number of trading days in one year.

- (5) calculate the percentage of the intra day standard deviation of the stock price in one year,

$$Volatility = \frac{\bar{\sigma}_Q}{\bar{h}_Q} \times 100$$

Sofar the regulations. The method that is proposed in the regulations gives a historical average volatility. The analytical formula presented in Section 2 enables us in principle to calculate an implied volatility by equating the price formula for an option contract on a given stock to its market price. For example, for vanilla European options there is always a unique solution. This is not necessarily true for the Indonesian options. The next Figure contains plots of the call option contract GASII1950 on stocks Astra International Tbk with strike price $K = 11,950$ IDR⁴ and with barrier $B = 1.1K = 13,145$ IDR. The next Figure contains the prices for Indonesian call option contracts for several values of s_0 and time to maturity 3 month's. The lower plot corresponds to $s_0 = 11,950$ and then in steps of 100 IDR up to $s_0 = 12,950$.

It is clear that, in general, we cannot calculate the implied volatility from the market prices.

The calculation of the historical volatility is based on the following considerations. Suppose that historical stock price data are available

⁴Indonesian Rupiah

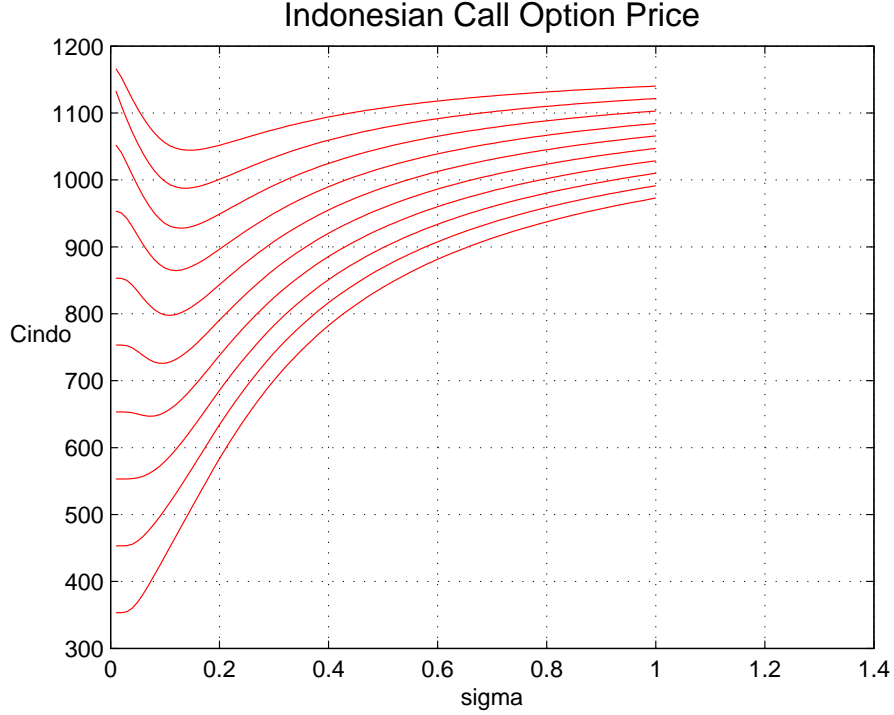


FIGURE 2

at equally spaced time values $t_i = i\Delta t$, so $S(t_i)$ is the stock price at time t_i . We define the log ratios

$$R_i = \log \frac{S(t_i)}{S(t_{i-1})}. \quad (16)$$

We assume that the R_i are independent normal random variables with mean $(\mu - \sigma^2/2)\Delta t$ and variance $\sigma^2\Delta t$. Suppose the current time is $t = t_k$, and the $n + 1$ most recent stock prices $S(t_{k-n}), S(t_{k-n+1}), \dots, S(t_k)$ are available. The maximum likelihood estimates for μ and σ are now given by

$$\hat{\sigma} = \frac{b_n}{\sqrt{\Delta t}}, \quad (17)$$

$$\hat{\mu} = \frac{a_n}{\Delta t} + \hat{\sigma}^2/2, \quad (18)$$

where a_n and b_n denote the sample mean and the sample standard deviation respectively. Example

We want to estimate the historical volatility at April 28, 2006, based on three month historical stock price data, i.e. January, 27 - April 28, 2006. It turns out that $\hat{\sigma} = 0.175$. We also inspected the data for normality of the log-returns.

The Figure shows that the normality assumption may be doubtful.

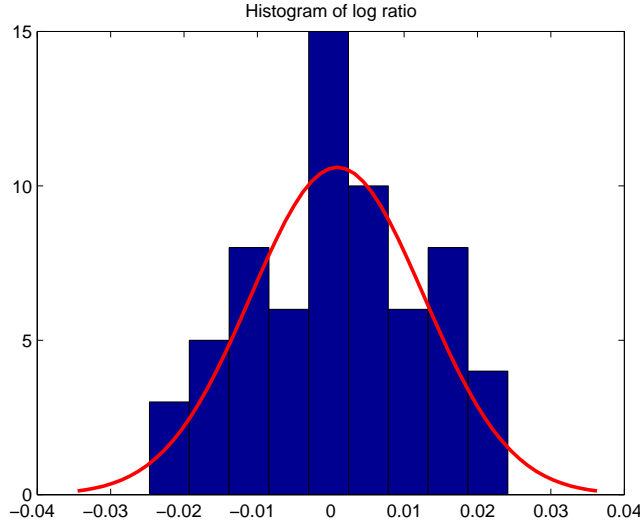


FIGURE 3

4. PUT OPTION CONTRACTS

So far we computed prices for the Indonesian up call. Now we derive prices for the Indonesian down put, which has a pay-off Ψ_d given by,

$$\Psi_d(\{S_t\}_{0 \leq t \leq T}) = (K - B)e^{r(T-\tau_B)}1_{\{\tau_B \leq T\}} + 1_{\{\tau_B > T\}}(K - S_T)^+,$$

where $K > B = 0.9K$. So for the option price V_d we have a similar decomposition as in the call case,

$$\begin{aligned} V_d &= (K - B)\mathbb{E}(e^{-r\tau_B}; \tau_B \leq T) + e^{-rT}\mathbb{E}((S(T) - K)^+ 1_{\{\tau_B > T\}}) \\ &= V_{ot} + V_{dop}(B, K), \end{aligned}$$

where again $V_{ot}(B)$ is the value of a type of one-touch option that exercises automatically if the price of the underlying hits the barrier B with immediate payoff. The term $V_{dop}(B, K)$ is the value of a standard down-and-out put with barrier B and strike K .

We can compute V_{ot} for $B < s_0$ in the same way as in Section 2 for the case $B > s_0$. Using now formula (7) we get

$$V_{ot} = \frac{s_0(K - B)}{B} \left(1 - \mathcal{N}\left(\frac{-b + \tilde{\mu}T}{\sigma\sqrt{T}}\right) + e^{2\tilde{\mu}b/\sigma^2} \mathcal{N}\left(\frac{b + \tilde{\mu}T}{\sigma\sqrt{T}}\right) \right). \quad (19)$$

It turns out that the values of the down-and-out put and the up-and-out call, which is given by (15), are identical, we refer to the Appendix for the calculation.

Numerical Example. In figure (4) we give a numerical example for the Indonesian put option. We have the same parameters for the stock price process and the option as in the previous example, except for the barrier, this is now set to 60. We remark that the Indonesian put price

exceeds the European put price. Intuitively this is correct, because the value of a European put can be less than intrinsic when it is in the money. The Indonesian put pays the intrinsic value at the barrier. Of course the value of the Indonesian Put option varies with the level of the barrier. Dependent on this level, the value of the Indonesian put can be below or above the value of the European put price, but it will never exceed the value of the American put price. Figure (5) illustrates this behaviour.

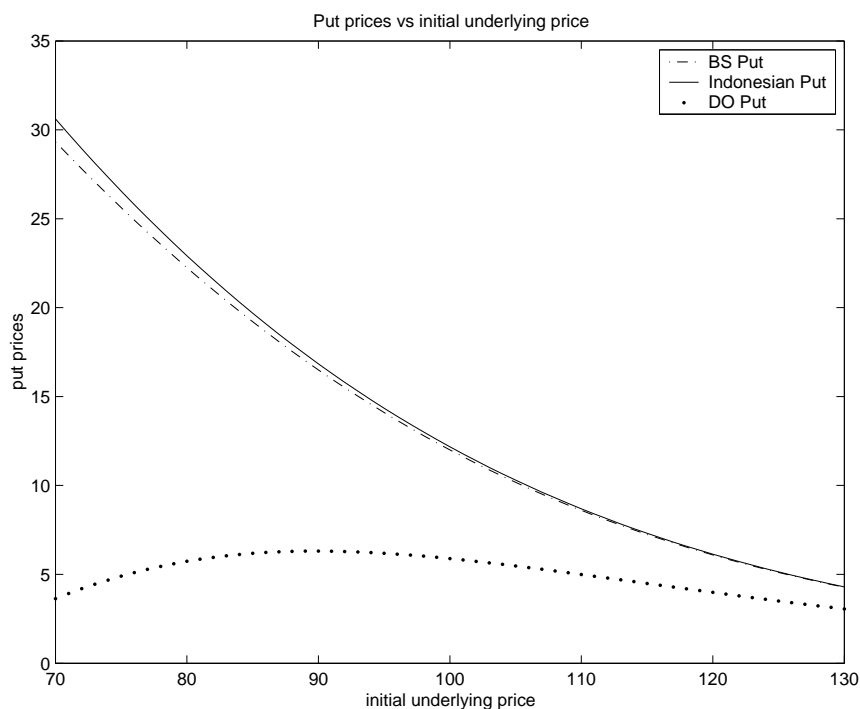


FIGURE 4. Numerical Example for Indonesian Put Option

5. PDE APPROACH

In this Section we study the Indonesian option from a PDE point of view.

5.1. Call Option. The Black-Scholes partial differential equation (PDE) is given by,

$$\frac{\partial V}{\partial t} + a(S, t) \frac{\partial^2 V}{\partial S^2} + b(S, t) \frac{\partial V}{\partial S} + c(S, t)V = 0, \quad (20)$$

where

$$a(S, t) = \frac{1}{2}\sigma^2 S^2, \quad b(S, t) = rS, \quad c(S, t) = -r \quad (21)$$

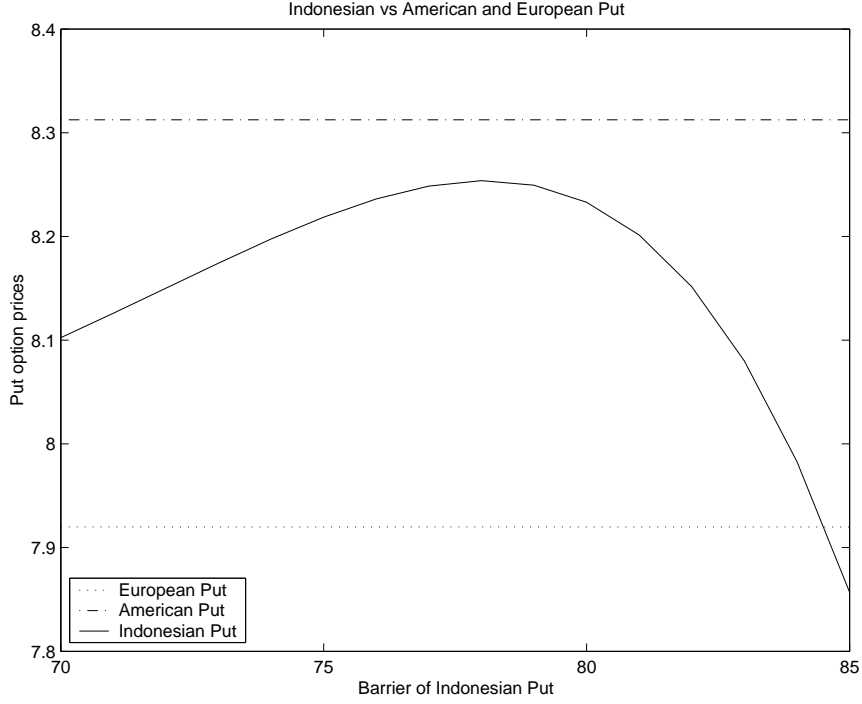


FIGURE 5. Numerical Example for Indonesian Put Option

First we apply an explicit difference method to solve the equation for the call option contract. The approximation to the derivatives are substituted in (20) yields

$$\begin{aligned} & \frac{V_i^k - V_i^{k+1}}{\delta t} + a_i^k \frac{V_{i+1}^k - 2V_i^k + V_{i-1}^k}{\delta S^2} \\ & + b_i^k \frac{V_{i+1}^k - V_{i-1}^k}{2\delta S} + c_i^k V_i^k = O(\delta t, \delta S^2). \end{aligned} \quad (22)$$

Rearrange, put the $(k+1)$ term in the left side, we have

$$V_i^{k+1} = A_i^k V_{i-1}^k + (1 + B_i^k) V_i^k + C_i^k V_{i+1}^k, \quad (23)$$

where

$$\begin{aligned} A_i^k &= \nu_1 a_i^k - \frac{1}{2} \nu_2 b_i^k, \\ B_i^k &= -2\nu_1 a_i^k + \delta t c_i^k, \\ C_i^k &= \nu_1 a_i^k + \frac{1}{2} \nu_2 b_i^k, \\ \nu_1 &= \frac{\delta t}{\delta S^2}, \\ \nu_2 &= \frac{\delta t}{\delta S}. \end{aligned}$$

The error is $O(\delta t^2, \delta t \delta S^2)$.

We use $S = i\delta S$ and substitutes (21) to the coefficients (23), we have

$$\begin{aligned} A_i^k &= \frac{1}{2}(\sigma^2 i^2 - r)\delta t, \\ B_i^k &= -(\sigma^2 i^2 - r)\delta t, \\ C_i^k &= \frac{1}{2}(\sigma^2 i^2 + ri)\delta t. \end{aligned}$$

Payoff: For $S \leq B$, the payoff of Indonesian call option is

$$H(S) = \max(S - K, 0),$$

or in the finite difference notation is

$$H(S) = \max(i\delta S - K, 0).$$

Final Condition: At expiry we have

$$V(S, T) = H(S),$$

or

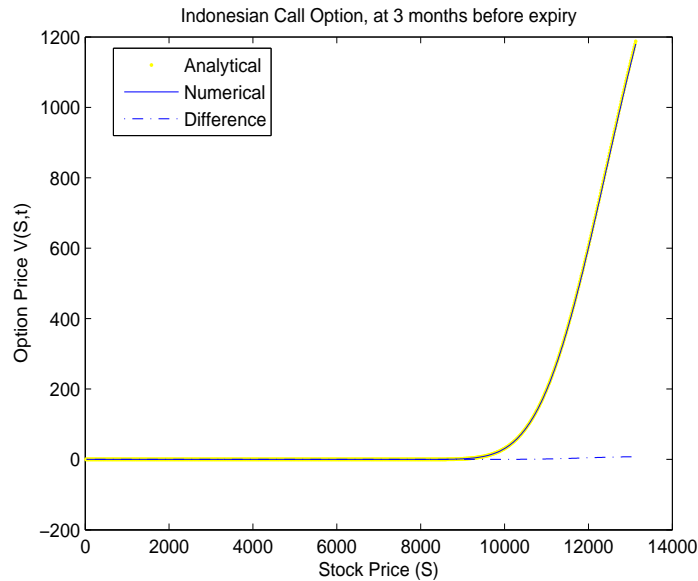
$$V_i^0 = H(i\delta S) = \max(i\delta S - K, 0).$$

Boundary Condition: We must specify the option value at the extremes of the region i.e. at $S = 0$ (for small S) and at $S = I\delta S$ (for large S). The boundaries are

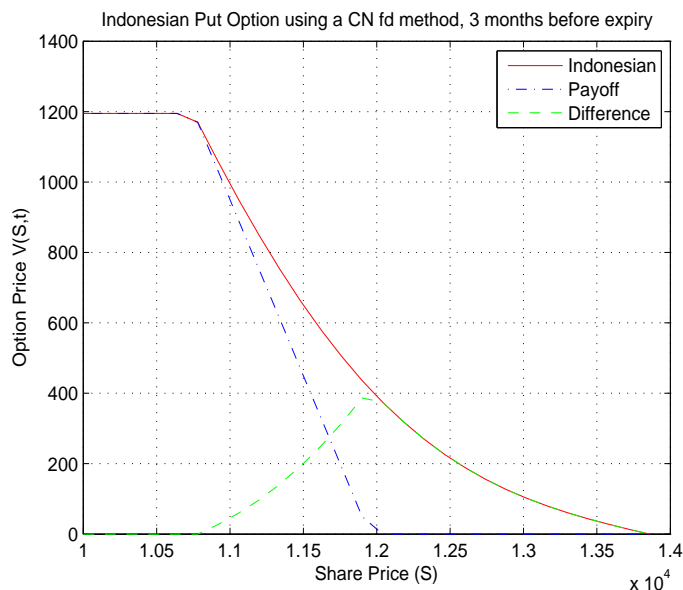
$$V_0^k = 0$$

and

$$V_I^k = B - K.$$



5.2. Put option. The valuation of the Indonesian put option needs more effort because of the early-exercise constraint. How to handle the early-exercise constraint, see [4]. The Indonesian put option can be considered as a type of American put option with a barrier, see also [2]. The value of the Indonesian option is always greater than its payoff.



In Wilmott [4] two ideas are proposed to solve the PDE in this case. First, implement the early-exercise into an explicit finite difference as before. A second method consists of applying the Crank-Nicolson method. The accuracy of the first method is of order $O(\delta t, \delta S^2)$. The accuracy of the second method is better, it is of order $O(\delta t^2, \delta S^2)$, but it is an implicit method. If we apply an implicit method we cannot simply replace the option value by the payoff, because the accuracy would reduce to $O(\delta t)$. To handle this problem, the replacement must be done at the same time as the values are found by projected Successive Over-Relaxation (SOR) methods, see [4].

Payoff: For $S \geq B$, the payoff of Indonesian put option is

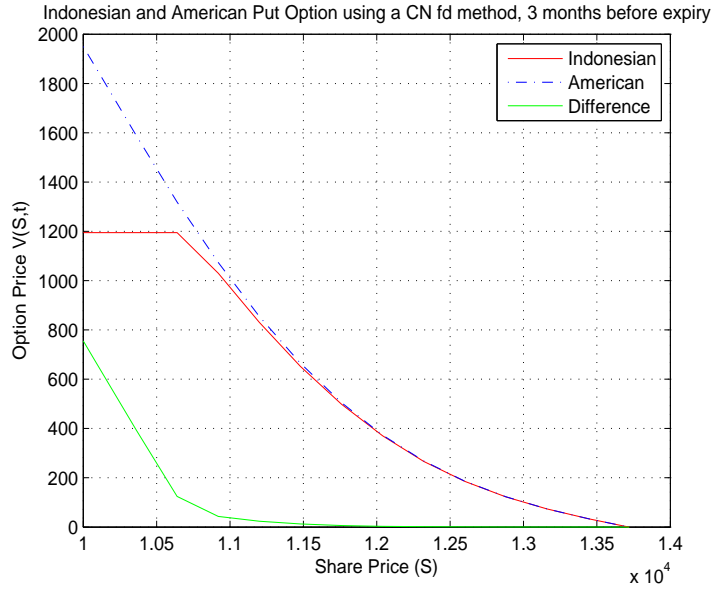
$$H(S) = \max(K - S, 0),$$

or in the finite difference notation is

$$H(S) = \max(K - i\delta S, 0).$$

Final Condition: At expiry we have

$$V(S, T) = H(S),$$



or

$$V_i^0 = H(i\delta S) = \max(K - i\delta S, 0).$$

6. INDONESIAN OPTIONS ARE AMERICAN

In the derivation of the previous Indonesian option price formulae we assumed that either the contract is exercised at maturity or it is automatically exercised before maturity at the first time the stock price equals L . However, the contract specification of the options traded on the JSX dictate that it is possible to exercise the option at any moment. Therefore the Indonesian option value is equivalent to the value of an American knock-out option, which you would exercise ϵ before the stock hits the barrier, giving you $L - K - \epsilon$. In case of a call option, a basic argument of short-selling stock instead of exercising the option shows that the early-exercise feature of the Indonesian call option will never be used. Therefore the closed for solution of the "European" Indonesian option is valid for both the Indonesian call option and the American up-and-out call. For the Indonesian put we expect that the early-exercise feature will matter, as it does for the standard American put option.

In order to price these Indonesian put options with early-exercise feature we use a binomial tree method. It is common practice to use the recombining tree, which assures that if we have N layers we have k stock prices $S_{k,i}$, $1 \leq i \leq k$ at layer k . The calculation procedure is backwards, starting from the pay-off at layer N prices $f_{N-1,i}$, $1 \leq i \leq N-1$ are calculated for layer $N-1$ and so on to the beginning of the tree. In order to adapt the calculation method for the Indonesian option,

we have to check in every node i at layer k if the stock price process is below L . As hitting L means automatic exercise of the put we have to replace in each step all the calculated option prices belonging to stock price levels below L . This replacement of calculated prices $f_{k,i}$, $1 \leq i \leq k$ at layer k can be done in two ways:

- For each k such that $S_{k,i} \leq L$ we put $f_{k,i} = K - L$ or,
- $f_{k,i} = K - S_{k,i}$.

Although these approaches will converge to the same limit, we have different behaviour for a finite number of layers. In figure (6) we show this behaviour where we calculated the Indonesian European Put option (no early exercise feature) using a different number of layers in the tree. The parameters used are the same as in the other examples except for the barrier, this is set to 80. The jumps in the graph are a result of the discretisation of the stock price process. The down-factor depends on N and for some integer values N a row of stock prices is just below the level L . Increase N by one and this row will be just above this level, leading to a completely different behaviour. It is possible to calculate these values for N , which we will use to price the American Indonesian put in order to be reasonable close to the true price.

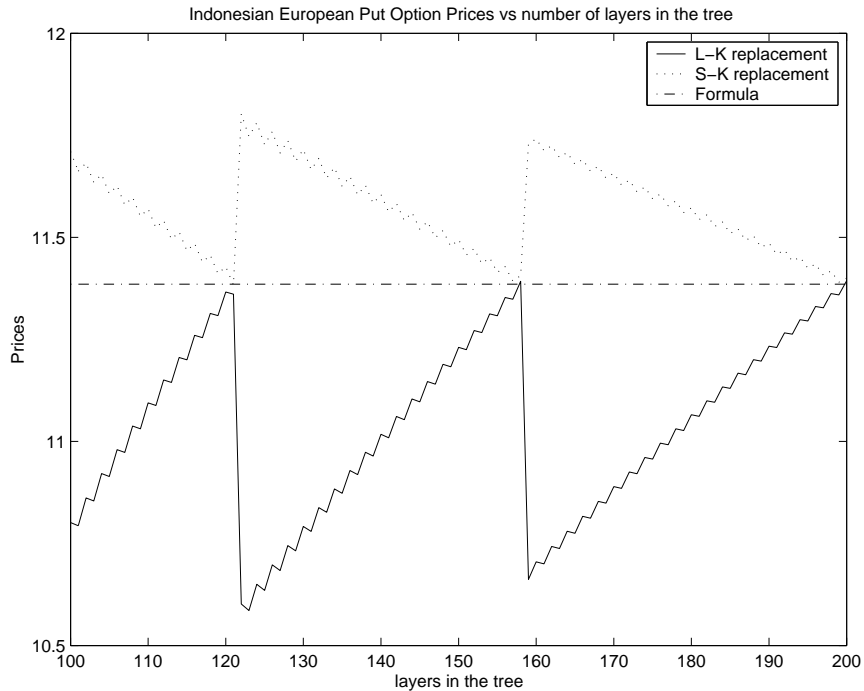


FIGURE 6. Numerical Example for Indonesian Put Option

In order to calculate the American Indonesian put we make the following usual replacement of the calculated values $f_{k,i}$, $1 \leq i \leq k$ at layer k

by,

$$f_{k,i} = \max(f_{k,i}, K - S_{k,i}).$$

It is now interesting to compare the American Indonesian Put with the European Indonesian Put, the standard European put and the standard American put. Figure (7) show the different prices. The Indonesian American put value is not so different from the Indonesian European put if the barrier is not too low. Where from some point the Indonesian European put is decreasing if the barrier is decreasing, its American counterpart is not, because by decreasing the barrier, the number of possible exercise strategies increases and therefore its value.

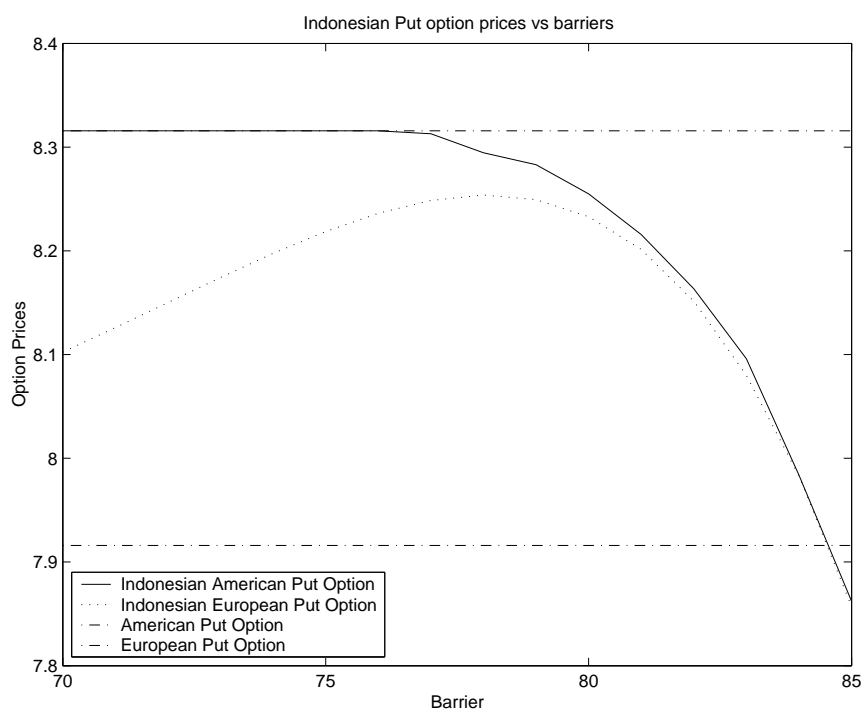


FIGURE 7. Numerical Example for Indonesian Put Option

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7. APPENDIX

Let the stochastic process $(X(t))_{t \geq 0}$ be given by

$$X(t) = \mu t + \sigma W(t), \quad t \geq 0.$$

The marginal distributions of the process $X_b(t) = X(t \wedge \tau_b^X)$ absorbed at b are given by

$$f_b(x; t) = \phi(x; \mu t, \sigma\sqrt{t}) - e^{2\mu b/\sigma^2} \phi(x; \mu t + 2b, \sigma\sqrt{t}), \quad (24)$$

where $\phi(x; \mu, \sigma)$ denotes the probability density of the normal distribution with mean μ and variance σ^2 ,

$$\phi(x; \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp \left\{ -\frac{(x - \mu)^2}{2\sigma^2} \right\},$$

see [1]. Defining $b = \ln(B/s_0)$ and $k = \ln K$, we have

$$\begin{aligned} V_{uoc}(B, K) &= e^{-rT} \mathbb{E} \left((S(T) - K)^+ 1_{\{\tau_B > T\}} \right) \\ &= e^{-rT} \mathbb{E} \left((s_0 e^{X(T)} - K)^+ 1_{\{\tau_b^X > T\}} \right) \\ &= e^{-rT} \mathbb{E} \left((s_0 e^{X_b(T)} - K)^+ 1_{\{X_b(T) < b\}} \right) \\ &= e^{-rT} \int_k^b (s_0 e^x - K) f_b(x; T) dx. \end{aligned} \quad (25)$$

Since

$$e^x \phi(x; \mu T, \sigma\sqrt{T}) = e^{rT} \phi(x; \tilde{\mu} T, \sigma\sqrt{T}) \quad (26)$$

and

$$e^x \phi(x; \mu T + 2b, \sigma\sqrt{T}) = e^{rT+2b} \phi(x; \tilde{\mu} T + 2b, \sigma\sqrt{T}), \quad (27)$$

we get

$$\int_k^b \phi(x; \tilde{\mu} T, \sigma\sqrt{T}) dx = \mathcal{N}(\tilde{c}_b) - \mathcal{N}(\tilde{c}_k) \quad (28)$$

$$\int_k^b \phi(x; \tilde{\mu} T + 2b, \sigma\sqrt{T}) dx = \mathcal{N}(\tilde{c}_{-b}) - \mathcal{N}(\tilde{c}_{k-2b})$$

$$\int_k^b \phi(x; \mu T, \sigma\sqrt{T}) dx = \mathcal{N}(c_b) - \mathcal{N}(c_k) \quad (29)$$

$$\int_k^b \phi(x; \mu T + 2b, \sigma\sqrt{T}) dx = \mathcal{N}(c_{-b}) - \mathcal{N}(c_{k-2b}).$$

It follows that

$$\begin{aligned} V_{uoc}(B, K) &= s_0 \left(\mathcal{N}(\tilde{c}_b) - \mathcal{N}(\tilde{c}_k) + e^{2\tilde{\mu}b/\sigma^2} \mathcal{N}(\tilde{c}_{k-2b}) - e^{2\tilde{\mu}b/\sigma^2} \mathcal{N}(\tilde{c}_{-b}) \right) \\ &\quad - K e^{-rT} \left(\mathcal{N}(c_b) - \mathcal{N}(c_k) + e^{2\mu b/\sigma^2} \mathcal{N}(c_{k-2b}) - e^{2\mu b/\sigma^2} \mathcal{N}(c_{-b}) \right), \end{aligned}$$

For for $B < S_0$ and $K > B$,

$$V_{dop} = e^{-rT} \int_l^k (K - s_0 e^x) f_l(x) dx,$$

which is very similar to the integral for the valuation of the up-and-out call, except for the interchange of both the K and s_0 -term and the integration bounds resulting in,

$$V_{dop} = -\frac{S_0}{\sqrt{2\pi\sigma^2T}} \left(\int_l^k e^{-\frac{(x-\tilde{\mu}T)^2}{2\sigma^2T}} dx - e^{\frac{2\tilde{\mu}l}{\sigma^2}} \int_l^k e^{-\frac{(x-\tilde{\mu}T-2l)^2}{2\sigma^2T}} dx \right) +$$

$$\frac{Ke^{-rT}}{\sqrt{2\pi\sigma^2T}} \left(\int_l^k e^{-\frac{(x-\mu T)^2}{2\sigma^2T}} dx - e^{\frac{2\mu l}{\sigma^2}} \int_l^k e^{-\frac{(x-\mu T-2l)^2}{2\sigma^2T}} dx \right).$$

The minus signs and the interchange of integration bounds cancel, so we get the same formula for the value of the down-and-out put as for the up-and-out call.

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