ET Mi 109: Electrical Engineering for Autonomous Exploration Robots

EE3330TU Guiding & Radiating Structures

Entry Test

Name and study number: ______________________

COMPLEX NUMBERS

Notation:

$j = \sqrt{-1}$ (imaginary unit)

$z = Re\{z\} + jIm\{z\}$ (rectangular form)

$z = |z|e^{j\angle z}$ (polar form)

1. Given two complex numbers

$$V = 3 - j4$$
$$I = -(2 + j3)$$

   (a) express V and I in polar form
   (b) calculate VI
   (c) calculate VI^*
   (d) calculate V/I
   (e) calculate $\sqrt{I}$

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2. Express the following complex functions in polar form:

$$z_1 = (4 - j3)^2$$
$$z_2 = (4 - j3)^{1/2}$$

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3. Show that $\sqrt{2j} = \pm(1 + j)$

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4. Evaluate each of the following complex numbers and express the result in rectangular form
   (a) $z = 4e^{j\pi/3}$
   (b) $z = \sqrt{3}e^{j3\pi/4}$
   (c) $z = 6e^{-j\pi/2}$
   (d) $z = j^3$
   (e) $z = j^{-4}$
   (f) $z = (1 - j)^3$
   (g) $z = (1 - j)^{1/2}$

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5. If $z = -2 + j4$, determine the following quantities in polar form
   (a) $1/z$
   (b) z^3
   (c) $|z|^2$
   (d) $Im\{z\}$
   (e) $Im\{z^*\}$

VECTOR ANALYSIS

Notation:

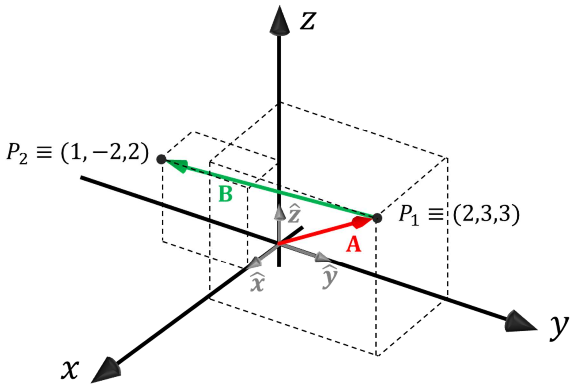
$\mathbf{A} = A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + A_z\hat{\mathbf{z}}$ (vector in Cartesian coord.)

$\hat{\mathbf{a}}$ (unit vector)

$A = |\mathbf{A}|$ (magnitude)

$\mathbf{A} \cdot \mathbf{B}$ (scalar or inner product)

$\mathbf{A} \times \mathbf{B}$ (vector product)



6. In Cartesian coordinates, vector $\mathbf{A}$ is directed from the origin to point $P_1(2,3,3)$, and vector $\mathbf{B}$ is directed from point P_1 to point $P_2(1,-2,2)$. Find
   (a) vector $\mathbf{A}$, its magnitude A, its unit vector $\hat{\mathbf{a}}$
   (b) the angle that $\mathbf{A}$ makes with the y-axis
   (c) vector $\mathbf{B}$
   (d) the angle between $\mathbf{A}$ and $\mathbf{B}$
   (e) the vector product $\mathbf{A} \times \mathbf{B}$

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7. Given $\mathbf{A} = 2\hat{\mathbf{x}} - 3\hat{\mathbf{y}} + \hat{\mathbf{z}}$ and $\mathbf{B} = B_x\hat{\mathbf{x}} + 2\hat{\mathbf{y}} + B_z\hat{\mathbf{z}}$,
   (a) Find B_x and B_z if $\mathbf{A}$ is parallel to $\mathbf{B}$
   (b) Find a relation between B_x and B_z if $\mathbf{A}$ is perpendicular to $\mathbf{B}$

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8. Given $\mathbf{A} = 3\hat{\mathbf{x}} + 4\hat{\mathbf{z}}$ and $\mathbf{B} = 4\hat{\mathbf{x}} - 10\hat{\mathbf{y}} + 5\hat{\mathbf{z}}$,
   (a) Find the vector component of $\mathbf{A}$ along $\mathbf{B}$
   (b) Determine a unit vector perpendicular to both $\mathbf{A}$ and $\mathbf{B}$