



# Economic order quantity and purchasing price for items with imperfect quality when inspection shifts from buyer to supplier

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## ABSTRACT

Traditional Economic Order Quantity (EOQ) models, implicitly assume that all items that are received are perfect. Although recent EOQ models for items with imperfect quality, which relax this assumption, are closer to real-world problems, they implicitly assume that suppliers do not conduct a full inspection. In this paper, we study the relationship between buyer and supplier with regard to conducting the inspection and resulting in a change the buyer's economic order quantity and purchasing price. We model and analyze the problem under two conditions: (1) assuming there is no relationship between the buyer's selling price, buyer's purchasing price, and customer demand; (2) assuming there is relationship between the buyer's selling price, buyer's purchasing price, and customer demand. Numerical examples are provided to illustrate the models.

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## 1. Introduction

Traditional economic order quantity (EOQ) models offer a mathematical approach to determine the optimal number of items a buyer should order to a supplier each time. One major implicit assumption of these models is that all the items are of perfect quality. However, this is not always the case, as in some situations a percentage of the items is imperfect. [Porteus \(1986\)](#) and [Rosenblatt and Lee \(1986\)](#) were among the first to study the effect of imperfect items on EOQ and EPQ (economic production quantity) models. Following them, several studies have incorporated the effect of imperfect items into different EOQ and EPQ models, most notably [Salameh and Jaber \(2000\)](#), who considered a situation in which an average  $p\%$  of all items ordered are imperfect. The buyer conducts an inspection of all the items to separate the imperfect items from the perfect ones, after which the imperfect items are assumed to be sold as a single batch at the end of inspection process. Formulating the problem, the optimal order quantity is derived. [Salameh and Jaber's \(2000\)](#) model has been extended in several directions.

For example [Goyal and Cárdenas-Barrón \(2002\)](#) corrected a small error in the original model. [Rezaei \(2005\)](#), [Papachristos and](#)

[Konstantaras \(2006\)](#), [Wee et al. \(2007\)](#) and [Chang and Ho \(2010\)](#) studied the problem by considering the occurrence of shortage. [Chung and Huang \(2006\)](#) identified the optimal order quantity with imperfect items for retailers when delay in payment is allowed. [Hsu and Yu \(2009\)](#) formulated the problem to determine the optimal order quantity under a one-off discount. [Chan et al. \(2003\)](#) divided imperfect items into three categories: imperfect items that can be sold at a lower price, imperfect items that can be reworked and imperfect items that should be rejected, after which they devised a mathematical model to determine the EPQ. [Wang \(2005\)](#) proposed a mathematical model to determine the EPQ and optimal inspection. [Haji and Haji \(2010\)](#) formulated and solved the problem in situations where imperfect items are reworked with a random rate. In a recent study, [Maddah and Jaber \(2008\)](#) corrected some of the flaws of the original study by [Salameh and Jaber \(2000\)](#), which, although mathematically interesting, led to no significant changes in the final results.

Some researchers considered imperfect items in the context of buyer-supplier relationships. [Huang \(2002, 2004\)](#) and [Goyal et al. \(2003\)](#), for example, formulated models to determine the optimal integrated buyer-supplier inventory policy for imperfect items and found that joint decision-making can reduce the expected annual cost of inventory significantly. [Chen and Kang \(2007, 2010\)](#) formulated the problem after considering the delay in payment. They assumed that the supplier can increase the warranty cost to maintain the long-term relationship. [Rezaei and Davoodi \(2008\)](#) formulated a model to determine the optimal

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lot-size including imperfect items and select the suppliers simultaneously, while Lin (2009) formulated a model for a single-supplier/single-buyer relationship to determine the optimal lot-size when some items are imperfect. Quality of received items has been also recognized in the literature of supplier selection as one of the most important criteria (see for example Rezaei and Davoodi, 2008, 2011). For a detailed review and discussion of the extensions of EOQ model for imperfect quality items, see Khan et al. (2011).

A review of existing literature reveals that several extensions of the problem have been proposed. However, in all the relevant studies, the inspection process is assumed to be conducted by the buyer. In this paper, we study the problem from a different perspective and propose a suitable framework to determine how the buyer can help the supplier carry out the inspection and reduce the imperfect rate. The rest of this paper is organized as follows. Section 2 contains the mathematical modeling of the problem under two different conditions. In Section 3, numerical examples are presented and Section 4, finally, contains the conclusion and suggestions for future research.

## 2. Mathematical modeling

### 2.1. Notations

The following notations are used in this paper.

|        |  |
|--------|--|
| $K$    | Fixed ordering cost  |
| $c$    | Regular purchasing price per unit (if imperfect rate $p > 0$ ) |
| $Mc$   | Maximum purchasing price per unit (if imperfect rate $p = 0$ ) |
| $s$    | Selling price per perfect unit                                 |
| $h$    | Holding cost per unit per time unit                            |
| $x$    | Inspection rate per time unit                                  |
| $p$    | Imperfect rate   |
| $E[p]$ | Expected imperfect rate  |
| $d$    | Screening (inspection) cost per unit                           |
| $D$    | Demand rate per time unit                                      |
| $v$    | selling price per imperfect unit                               |
| $T$    | Ordering cycle duration  |
| $y$    | Order quantity   |

### 2.2. Problem definition and research questions

In contrast to traditional EOQ models, which implicitly assume that all items are completely perfect, Salameh and Jaber (2000) have formulated the problem in situations where not all items are perfect. The imperfect items are separated from perfect ones by a full inspection and are used in another inventory situation. As the authors indicate, the electronic industry is a good example of such a situation.

The implicit assumption of Salameh and Jaber's (2000) model, however, is that the supplier does not perform a full inspection; otherwise the received batches are expected to be completely perfect. In fact, the very presence of imperfect items in a batch depends on whether or not the supplier carries out a full inspection, which is why we outline two different possible scenarios here:

**Scenario 1.** The supplier does not perform a full inspection and, as a result, the batches received by the buyer contain some imperfect items. This implies that the buyer should conduct a full inspection.

**Scenario 2.** The supplier performs a full inspection and, as a result, the batches received by the buyer contain no imperfect items.

The first scenario was formulated and analyzed in Salameh and Jaber (2000), while the second scenario is the implicit assumption of traditional EOQ models.

Based on these two scenarios, we examine the following research questions:

1. If the buyer is able to select between two suppliers, one of whom fits the first scenario, while the other one fits the second scenario (performs a 100% inspection), which one is preferred by the buyer?
2. If the supplier belongs to the former category (does not perform a 100% inspection), does it make sense for the buyer to be of assistance the supplier (by paying more for each item) to improve the supplier's production quality or to help the supplier perform a full inspection?

At face value, it appears that the answer to the first question is simply to select a supplier belonging to the second scenario. However, if the conditions are the same, it is to be expected that, in the second scenario, the unit price of items offered by the supplier are higher than those in the first scenario, which means there is trade-off between quality and price. To answer this question, we need to identify the maximum purchasing price ( $Mc$ ) the buyer is willing to pay to a supplier belonging to the second scenario. If the unit price is less than  $Mc$ , the second scenario is preferred, if not the first scenario is preferred. However, the comparison problem may be more complicated when a supplier belonging to the first scenario also agrees to conduct a full inspection (see Section 2.5 for more details).

To answer the second question, knowing  $Mc$ , we can also determine how much more the buyer is willing to pay for a full inspection by the supplier. To improve production quality and reduce the percentage of imperfect items, the buyer can also pay more for each item, which is discussed later in this section (here it is still the buyer who conducts the full inspection). Note that the additional payment to improve the supplier's production quality is different from the additional payment designed to help the supplier carry out a full inspection.

As we can see, the purchasing price changes when a supplier belonging to the second scenario in the first research question is selected or when the goal is to improve the supplier's production quality. Changes in buyer's purchasing price usually influence the buyer's selling price which in turn, assuming the relationship between the buyer's selling price and customer demand, affects the customer demand and economic order quantity.

We model and solve the problem under two different conditions. First, (in Section 2.3) we assume that buyer's selling price does not depend on the purchasing price, and second, (in Section 2.4), in a situation that is closer to real-world problems, we assume that the purchasing price influences the buyer's selling price and consequently customer demand.

### 2.3. Determining $Mc$ assuming there is no relationship between the buyer's selling price, the purchasing price, and customer demand

According to Salameh and Jaber (2000) and Maddah and Jaber (2008), the buyer's expected profit per ordering cycle is as follows.

$$\begin{aligned}
 TP(y) = & \text{total sales of good quality items} \\
 & + \text{total sales of imperfect quality items} \\
 & - \text{ordering cost} - \text{purchasing cost} - \text{inspection cost} \\
 & - \text{holding costs}
 \end{aligned}$$

or equivalently:

$$TP(y) = sy(1-p) + vyp - K - cy - dy - h \left( \frac{[y(1-p)]^2}{2D} + \frac{py^2}{x} \right) \quad (1)$$

Then the buyer's expected total profit per time unit is as

$$ETPU(y) = \frac{((s(1-E[p]) + vE[p] - c - d)D - KD/y - hy(E[(1-p)^2]/2 + E[p]D/x))}{1-E[p]} \quad (2)$$

Consequently, the optimal order quantity would be:

$$y^{SMJ*} = \left( \frac{2KD}{h(E[(1-p)^2] + 2E[p]D/x)} \right)^{1/2} \quad (3)$$

In this model<sup>2</sup>, the buyer purchases each unit at price  $c$  knowing that, on average,  $p\%$  of each received batch will be imperfect. However, it is clear that purchasing price increases when the percentage of imperfect items decreases. Given completely perfect batches ( $p=0$ ), which result to zero inspection for the buyer, the total profit per time unit is:

$$TPU_{p=0}(y) = Ds - Dc' - \frac{KD}{y} - \frac{hy}{2} \quad (4)$$

In this case, we expect the unit purchasing price ( $c'$ ) paid for completely perfect batches to be higher than that of a batch with some imperfect items;  $c' > c$ . As we know, the optimal order quantity for this condition is the traditional EOQ:

$$y^{trad*} = \sqrt{\frac{2KD}{h}} \quad (5)$$

Now the maximum purchasing price for batches without imperfect items should be determined. To this end, we first determine the difference between the total profit per time unit when there are no imperfect items and the expected profit per time unit when there are  $p\%$  imperfect items on average in each batch. We consider  $c'$  as a variable here.

$$TPU(y^{trad*}, c') - ETPU(y^{SMJ*}) = D(s - c') - \frac{KD}{y^{trad*}} - \frac{hy^{trad*}}{2} - ETPU(y^{SMJ*}) \quad (6)$$

The buyer accepts to pay more if and only if:

$$TPU(y^{trad*}, c') - ETPU(y^{SMJ*}) \geq 0 \quad (7)$$

Or equivalently:

$$\Rightarrow c' \leq s - \frac{K}{y^{trad*}} - \frac{hy^{trad*} + 2ETPU(y^{SMJ*})}{2D} \quad (8)$$

The right-hand side of this equation determines the maximum unit purchasing price ( $Mc$ ) the buyer is willing to pay for batches without imperfect items.

#### 2.4. Determining $Mc$ assuming there is a relationship between buyer's selling price, purchasing price, and customer demand

In Section 2.3, we determined the maximum purchasing price ( $Mc$ ) the buyer is willing to pay if there are no imperfect items, keeping the buyer's selling price fixed. However, in most real situations, changing the purchasing price influences the buyer's selling price. Taking this into account and assuming there is a relationship between the buyer's selling price and customer demand, we model the problem and find the optimal buyer's selling price and order quantity in cases where there are imperfect items, after which we determine the maximum purchasing

price ( $Mc$ ) when there are no imperfect items. We model and solve this problem in two phases.

##### 2.4.1. Phase 1. Determining the optimal buyer's selling price and order quantity

To determine the optimal buyer's selling price and order quantity, we consider the expected total profit equation (Eq. (2)) as an objective function, while both the buyer's selling price  $s$ , and the order quantity  $y$ , are decision variables. The expected total profit  $\bar{\pi}$  of the inventory problem should be maximized subject to the price-demand relationship function.

$$\text{Maximize } \bar{\pi}(y, s) = \frac{((s(1-E[p]) + vE[p] - c - d)D - KD/y - hy(E[(1-p)^2]/2 + E[p]D/x))}{1-E[p]}$$

s.t.

$$D = f(s) \quad (9)$$

To obtain the optimal values of  $y$ ,  $s$  and  $D$ , we first make the following Lagrangian function.

$$L = \bar{\pi}(y, s) - \lambda(D - f(s)) \quad (10)$$

Then we should set the partial derivation of the Lagrangian function with respect to  $y$ ,  $s$ ,  $D$  and  $\lambda$  to zero and solve the resulting simultaneous equation system as follows.

$$\frac{\partial L}{\partial D} = \frac{1}{1-E[p]} \left( (s(1-E[p]) + vE[p] - c - d) - \frac{K}{y} - \frac{hyE[p]}{x} \right) - \lambda = 0 \quad (11)$$

$$\frac{\partial L}{\partial s} = D + \lambda f'(s) = 0 \quad (12)$$

$$\frac{\partial L}{\partial y} = \frac{1}{1-E[p]} \left( \frac{KD}{y^2} - h(E[(1-p)^2]/2 + E[p]D/x) \right) = 0 \quad (13)$$

$$\frac{\partial L}{\partial \lambda} = -D + f(s) = 0 \quad (14)$$

Which results in:

$$y^{**} = \left( \frac{2KD}{h(E[(1-p)^2] + 2E[p]D/x)} \right)^{1/2} \quad (15)$$

$$s^{**} = -\frac{D}{f'(s)} - \left( vE[p] - c - d - \frac{K}{y} - \frac{hyE[p]}{x} \right) / (1-E[p]) \quad (16)$$

$$D^{**} = f(s) \quad (17)$$

In Eq. (16), there is no a specific price-demand relationship, which means it is a general formula designed to obtain the optimal value of buyer's selling price  $s$ . Given a specific price-demand relationship function, we can find the optimal values of  $y$  and  $s$  using Eqs. (15) and (16) and then replacing the optimal value of buyer's selling price in Eq. (17) to obtain the optimal value of  $D$ .

##### 2.4.2. Phase 2. Determining the maximum purchasing price

To determine the maximum purchasing price ( $Mc$ ) we first calculate the difference between the total profit when there are no imperfect items and the expected total profit when there are  $p\%$  imperfect items on average in every batch:

$$\pi(s, y, c') - \bar{\pi}(y^{**}, s^{**}) = D \left( s - c' - \frac{K}{y} \right) - \frac{hy}{2} - \bar{\pi}(y^{**}, s^{**}) \quad (18)$$

here, the buyer agrees to pay more for each item in batches without imperfect items if and only if:

$$\pi(s, y, c') - \bar{\pi}(y^{**}, s^{**}) \geq 0 \quad (19)$$

<sup>2</sup> SMJ is an abbreviation of the authors of Salameh and Jaber (2000) and Maddah and Jaber (2008).

Consequently, we have:

$$\Rightarrow c' \leq s - \frac{K}{y} - \frac{hy + 2\pi(y^{**}, s^{**})}{2D} \quad (20)$$

The right-hand side of this equation is the maximum purchasing price. To determine the highest value of  $c'$ , we should find its maximum value ( $R$ ). As  $R$  is a function of variables  $s$  and  $y$ , we consider it a function that should be maximized subject to the price-demand relationship function.

$$\begin{aligned} \text{Maximize } R(s, y) &= s - \frac{K}{y} - \frac{hy + 2\pi(y^{**}, s^{**})}{2D} \\ \text{s.t.} \\ D &= f(s) \end{aligned} \quad (21)$$

To obtain the optimal values of  $s$ ,  $y$  and  $D$ , we first make the following Lagrangian function.

$$L = s - \frac{K}{y} - \frac{hy + 2\pi(y^{**}, s^{**})}{2D} - \lambda(D - f(s)) \quad (22)$$

Next, we need to set the partial derivation of the Lagrangian function with respect to  $y$ ,  $s$ ,  $D$  and  $\lambda$  to zero and solve the resulting simultaneous equations system as follows:

$$\frac{\partial L}{\partial D} = \frac{hy + 2\pi(y^{**}, s^{**})}{2D^2} - \lambda = 0 \quad (23)$$

$$\frac{\partial L}{\partial s} = 1 + \lambda f'(s) = 0 \quad (24)$$

$$\frac{\partial L}{\partial y} = \frac{K}{y^2} - \frac{h}{2D} = 0 \quad (25)$$

$$\frac{\partial L}{\partial \lambda} = -D + f(s) = 0 \quad (26)$$

As a result, we have:

$$s_p = \frac{(-f'(s)(2\pi(y^{**}, s^{**}) + hy)/2)^{1/2} - f(0)}{f'(s)} \quad (27)$$

$$y_p = \left(\frac{2KD}{h}\right)^{1/2} \quad (28)$$

This is a general model to determine the values of  $s$  and  $y$ , which lead to determine the maximum value of  $c'$ ,  $Mc$ . Specifying a suitable price-demand relationship function, we find the optimal values of  $s$  and  $y$ . Putting the value of  $s_p$  and  $y_p$  on (20), we can determine the maximum value of  $c'$ .

### 2.5. Decision rules

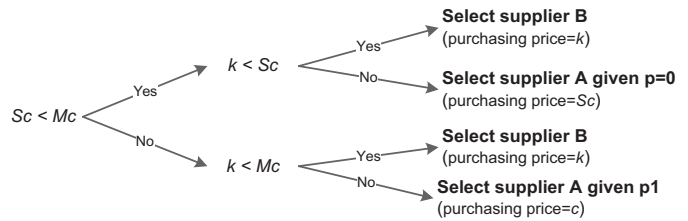
In Sections 2.3 and 2.4, the maximum purchasing price  $Mc$ , that the buyer agrees to pay in order to receive batches with no imperfect items is determined. As mentioned before,  $Mc$  is greater than  $c$ , and it is the *maximum* price the buyer is willing to pay. However, the supplier may agree to conduct a full inspection offering the items with a negotiated price between  $c$  and  $Mc$  or request a price greater than  $Mc$ . We refer to the item price (proposed by the supplier) given a full inspection is conducted (by the supplier) as  $Sc$ .

With regard to the first research question, taking the designed scenarios into account, the buyer may face the following suppliers:

Supplier A: purchasing price from this supplier given the imperfect rate  $p1 (> 0)$  on average =  $c$ ; purchasing price from this supplier given a full inspection =  $Sc$ ; maximum purchasing price from this supplier given a full inspection =  $Mc$ .

Supplier B: purchasing price from this supplier =  $k$ ; its imperfect rate =  $0$ .

The following sourcing policy is supported based on the analyses conducted.



As can be seen, when the negotiated supplier item price  $Sc$  is less than the maximum purchasing price  $Mc$ , comparing the negotiated supplier item price  $Sc$  and  $k$ , the buyer makes the decision. That is to say, if  $k < Sc$ , it is wise to select supplier B, otherwise supplier A (who carries out a full inspection) is preferred. However, when the negotiated supplier item price  $Sc$  is greater than the maximum purchasing price  $Mc$ , it is no longer wise to pay extra money to supplier A to conduct a full inspection (the case in which the supplier is unwilling to conduct the full inspection can be included in this condition, as in that case  $Sc = \infty$ ). So the buyer, comparing  $k$  and  $Mc$  decides to select supplier A (here the inspection is conducted by the buyer), or supplier B. Please note that in the above decision tree for  $Sc = Mc$ ,  $k = Sc$ , and  $k = Mc$  it is identical to follow one of the two corresponding branches.

With regard to the second research question, the following policies are supported based on the analyses conducted.

**IF supplier A agrees to conduct a 100% inspection THEN pay an extra up to  $Mc - c$  to the supplier per item.**

The buyer pays  $c$  per item to the supplier while receiving batches that include  $p\%$  imperfect items on average.  $Mc$  is the maximum price the buyer is willing to pay to the supplier to receive batches without imperfect items. This means that maximum additional payment per item would be  $Mc - c$ .

**IF supplier A wants to improve its production quality reducing imperfect rate from  $E[p2]$  to  $E[p1]$  THEN pay an extra up to  $Mc(E[p2]) - Mc(E[p1])$  to the supplier per item.**

When the batches received from the supplier contains  $p2$  percent imperfect items on average, the buyer is willing to pay the maximum item price  $Mc(E[p2])$ , shifting the full inspection to the supplier. While if the received batches contains  $p1$  percent imperfect items on average ( $p1 < p2$ ), the maximum purchasing price for shifting the full inspection to the supplier would be  $Mc(E[p1])$ . Therefore, if the supplier could reduce its average imperfect rate from  $E[p2]$  to  $E[p1]$ , the buyer is willing to pay at the most the difference between the corresponding maximum purchasing prices or  $Mc(E[p2]) - Mc(E[p1])$ .

### 3. Numerical examples

**Example 1.** Determining  $Mc$  assuming there is no relationship between the buyer's selling price, the purchasing price, and customer demand

To illustrate the proposed model in Section 2.3, we adopt the same data as used in Salameh and Jaber (2000) and Maddah and Jaber (2008) as follows:

$$f(p) = \begin{cases} 25, & 0 \leq p \leq 0.04, \\ 0 & \text{otherwisw.} \end{cases} \Rightarrow E[p] = 0.02 \text{ and } E[(1-p)^2] = 0.96$$

$D = 50,000$  units/year,  $c = \$25$ /unit,  $K = \$100$ /cycle,  $h = \$5$ /unit/year,  $x = 1$  unit/min,  $d = \$0.5$ /unit,  $s = \$50$ /unit,  $v = 20$ /unit, and the inventory operation operates on an 8 h/day, for 365 day a year.

Using (2) and (3), we calculate the optimal order quantity and optimal expected total profit per time unit as follows (here each received batch contains an average of 2% imperfect items):

$$y^* = 1434.48 \quad \text{and} \quad \text{ETPU}^* = 1212,274.30$$

Also, using (4) and (5), we calculate the optimal order quantity and optimal total profit per time unit as follows (here, the received batches contain no imperfect items):

$$y^{\text{trad}^*} = 1414.21 \quad \text{and} \quad \text{TPU}^{\text{trad}^*} = 1242,929$$

According to (8) the maximum purchasing price for this condition is:

$$Mc = 25.61$$

Table 1 and Fig. 1 show the corresponding maximum purchasing price ( $Mc$ ) for different average rates of imperfect items. As becomes clear, the higher the average rate of imperfect items  $E[p]$ , the higher the maximum purchasing price,  $Mc$ . For instance, compare the maximum purchasing price for two cases  $E[p]=0.02$  and  $E[p]=0.2$  (see Fig. 1). When  $E[p]=0.02$  the buyer agrees to pay at most 25.61, while, if  $E[p]=0.2$ , the buyer will be ready to pay much more (26.89) to avoid receiving imperfect items. However, it is important to note that for all  $E[p]$ , the buyer agrees to pay more than the screening cost per unit  $d$ . For instance, when  $E[p]=0.02$ , the buyer agrees to pay 0.61 (25.61–25) more to purchase an item while the screening cost per unit is 0.5, although this is the maximum extra amount per unit the buyer may be willing to pay.

**Table 1**  
Average rate of imperfect items in each batch and the corresponding maximum purchasing price ( $Mc$ ).

| $E[p]$      | $Mc$         | $E[p]$      | $Mc$         |
|-------------|--------------|-------------|--------------|
| 0.01        | 25.56        | 0.14        | 26.40        |
| <b>0.02</b> | <b>25.61</b> | 0.15        | 26.48        |
| 0.03        | 25.67        | 0.16        | 26.56        |
| 0.04        | 25.73        | 0.17        | 26.64        |
| 0.05        | 25.79        | 0.18        | 26.72        |
| 0.06        | 25.85        | 0.19        | 26.80        |
| 0.07        | 25.92        | <b>0.20</b> | <b>26.89</b> |
| 0.08        | 25.98        | 0.21        | 26.98        |
| 0.09        | 26.05        | 0.22        | 27.07        |
| 0.10        | 26.12        | 0.23        | 27.16        |
| 0.11        | 26.19        | 0.24        | 27.25        |
| 0.12        | 26.26        | 0.25        | 27.35        |
| 0.13        | 26.33        |             |              |

**Example 2.** Determining  $Mc$  assuming there is a relationship between the buyer's selling price, the purchasing price, and customer demand

To illustrate the proposed model in Section 2.4, we consider the same data presented in example 1. Commonly, two demand functions have been considered in literature: (1) the constant price-elasticity function, and (2) the linear demand function (e.g., Abad, 2003; Khouja, 2006; Rezaei and Davoodi, in press). Here, we suppose a linear price–demand relationship function as  $D = 100,000 - 1000s$ . According to (9) and (15), (16) and (17), we have:

$$s^{**} = 62.8477; \quad y^{**} = 1238.39; \quad D^{**} = 37,152.32$$

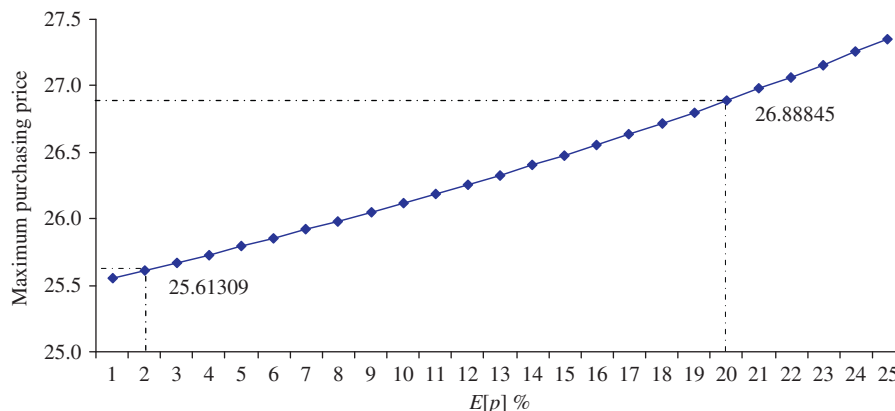
$$\text{and} \quad \bar{\pi}^*(s,y) = 1377,260.25$$

Each batch received contains 2% imperfect items on average that after a 100% inspection these imperfect items are separated and sold at a lower price. However according to (20), (27) and (28) we have:  $c' = 25.61$ ,  $s_p = 62.8475$ , and  $y_p = 1219.06$ , which means that the buyer agrees to pay 0.61 (25.61–25) extra to purchase each item if the supplier is willing to conduct the in-house inspection and delivers the lots with no imperfect items. Due to this increase in purchasing price, we usually expect the buyer to increase the selling price. However, as can be seen, the buyer even reduces the selling price (from 62.8477 to 62.8475). In addition, the economic order quantity decreases from 1238.39 to 1219.06. Fig. 2 shows the values of  $y^{**}$ ,  $y_p$ ,  $s^{**}$  and  $s_p$  for different  $E[p]$ s. As can be seen, the higher the  $E[p]$  the higher the  $y^{**}$  and the less the  $y_p$ . Moreover, although both  $s^{**}$  and  $s_p$  increase when  $E[p]$  increases,  $s_p \leq s^{**}$  for all  $E[p]$ . Since the values of  $s^{**}$  and  $s_p$  are very close to each other, the difference between them cannot be seen in Fig. 2, which is why we show the differences ( $s_p - s^{**}$ ) in Fig. 3.

Table 2 shows the optimal value of these variables ( $s^{**}$ ,  $y^{**}$ ,  $D^{**}$ ,  $\bar{\pi}^*(s,y)$ ,  $s_p$ ,  $y_p$  and  $Mc$ ) for different values of  $E[p]$ .

### 3.1. Marginal $Mc$

As we discussed before, the maximum purchasing price ( $Mc$ ) is a price the buyer agrees to pay to the supplier in order to receive the batches without any imperfect items, or  $p=0$ . This means that, instead of the buyer, is the supplier who conducts a full inspection. However, the buyer may want to be of assistance to the supplier in reducing the number of imperfect items, while it is still the buyer carrying out the full inspection. For instance, the buyer may want to assist the supplier to reduce the imperfect rate from 3% to 2%. For this reason, we introduce the *Marginal  $Mc$*



**Fig. 1.** The relationship between average imperfect rate  $E[p]$ % and maximum purchasing price ( $Mc$ ).

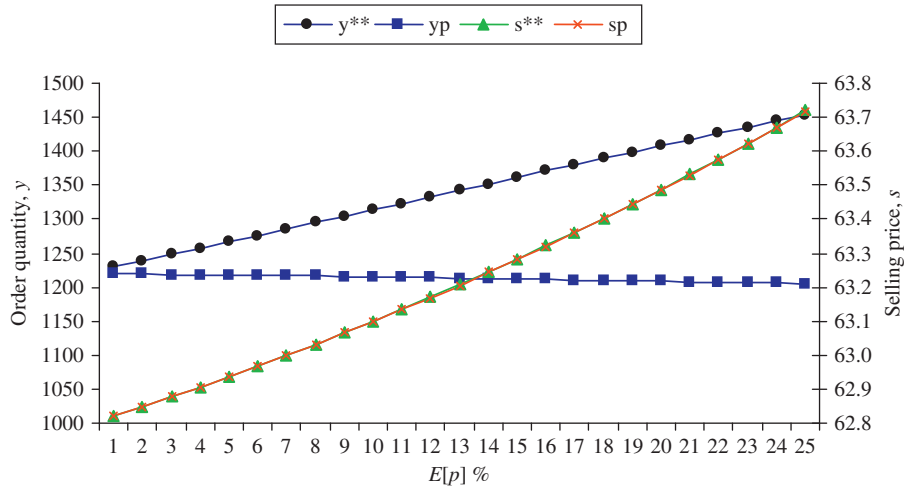


Fig. 2.  $y^{**}$ ,  $y_p$ ,  $s^{**}$  and  $s_p$  for different values of  $E[p]$ %.

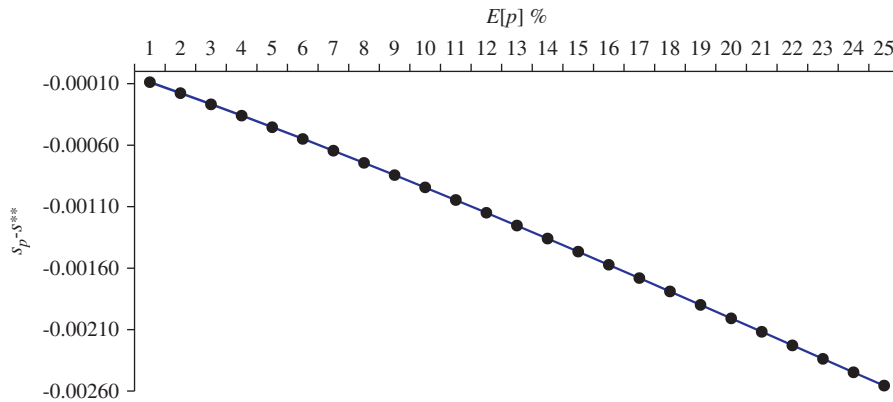


Fig. 3.  $s_p - s^{**}$  for different values of  $E[p]$ %.

Table 2

The optimal value of  $s^{**}$ ,  $y^{**}$ ,  $D^{**}$ ,  $\bar{\pi}^*(s,y)$ ,  $s_p$ ,  $y_p$  and  $Mc$  for different values of  $E[p]$ .

| $E[p]$      | $s^{**}$      | $y^{**}$       | $D^{**}$         | $\bar{\pi}^*(s,y)$ | $s_p$         | $y_p$          | $Mc$         | $\Delta Mc$  |
|-------------|---------------|----------------|------------------|--------------------|---------------|----------------|--------------|--------------|
| 0.01        | 62.819        | 1229.16        | 37,180.96        | 1379,381.21        | 62.819        | 1219.53        | 25.56        | 0.556        |
| <b>0.02</b> | <b>62.848</b> | <b>1238.39</b> | <b>37,152.32</b> | <b>1377,260.25</b> | <b>62.848</b> | <b>1219.06</b> | <b>25.61</b> | <b>0.057</b> |
| 0.03        | 62.877        | 1247.66        | 37,123.08        | 1375,096.78        | 62.877        | 1218.58        | 25.67        | 0.058        |
| 0.04        | 62.907        | 1256.98        | 37,093.23        | 1372,889.51        | 62.906        | 1218.09        | 25.73        | 0.059        |
| 0.05        | 62.937        | 1266.33        | 37,062.75        | 1370,637.11        | 62.937        | 1217.59        | 25.79        | 0.061        |
| 0.06        | 62.968        | 1275.72        | 37,031.61        | 1368,338.19        | 62.968        | 1217.08        | 25.85        | 0.062        |
| 0.07        | 63.000        | 1285.14        | 36,999.80        | 1365,991.29        | 63.000        | 1216.56        | 25.92        | 0.063        |
| 0.08        | 63.033        | 1294.59        | 36,967.28        | 1363,594.91        | 63.032        | 1216.03        | 25.98        | 0.065        |
| 0.09        | 63.066        | 1304.06        | 36,934.05        | 1361,147.48        | 63.065        | 1215.48        | 26.05        | 0.066        |
| 0.10        | 63.100        | 1313.54        | 36,900.07        | 1358,647.34        | 63.099        | 1214.92        | 26.12        | 0.068        |
| 0.11        | 63.135        | 1323.04        | 36,865.32        | 1356,092.80        | 63.134        | 1214.35        | 26.19        | 0.069        |
| 0.12        | 63.170        | 1332.54        | 36,829.77        | 1353,482.06        | 63.169        | 1213.77        | 26.26        | 0.071        |
| 0.13        | 63.207        | 1342.05        | 36,793.40        | 1350,813.26        | 63.205        | 1213.17        | 26.33        | 0.072        |
| 0.14        | 63.244        | 1351.55        | 36,756.17        | 1348,084.47        | 63.242        | 1212.56        | 26.40        | 0.074        |
| 0.15        | 63.282        | 1361.03        | 36,718.06        | 1345,293.63        | 63.280        | 1211.93        | 26.48        | 0.076        |
| 0.16        | 63.321        | 1370.50        | 36,679.03        | 1342,438.63        | 63.319        | 1211.29        | 26.56        | 0.078        |
| 0.17        | 63.361        | 1379.94        | 36,639.06        | 1339,517.24        | 63.359        | 1210.63        | 26.64        | 0.080        |
| 0.18        | 63.402        | 1389.34        | 36,598.10        | 1336,527.14        | 63.400        | 1209.96        | 26.72        | 0.082        |
| 0.19        | 63.444        | 1398.71        | 36,556.12        | 1333,465.88        | 63.442        | 1209.27        | 26.80        | 0.084        |
| 0.20        | 63.487        | 1408.02        | 36,513.09        | 1330,330.93        | 63.485        | 1208.55        | 26.89        | 0.086        |
| 0.21        | 63.531        | 1417.28        | 36,468.96        | 1327,119.59        | 63.529        | 1207.83        | 26.98        | 0.088        |
| 0.22        | 63.576        | 1426.47        | 36,423.68        | 1323,829.08        | 63.574        | 1207.08        | 27.07        | 0.090        |
| 0.23        | 63.623        | 1435.58        | 36,377.22        | 1320,456.45        | 63.620        | 1206.31        | 27.16        | 0.093        |
| 0.24        | 63.670        | 1444.61        | 36,329.53        | 1316,998.61        | 63.668        | 1205.52        | 27.25        | 0.095        |
| 0.25        | 63.719        | 1453.55        | 36,280.56        | 1313,452.31        | 63.717        | 1204.71        | 27.35        | 0.098        |

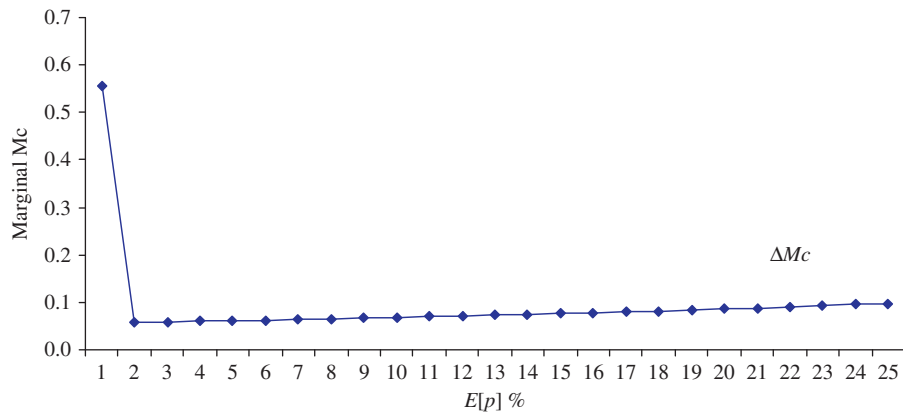


Fig. 4. Marginal  $Mc$  for different values of  $E[p]\%$ .

( $\Delta Mc$ ) as follows:

$$\Delta Mc = \frac{dMc}{dE[p]} \times 100$$

For example for  $E[p]=0.03$  we have:

$$\Delta Mc(E[p] = 0.03) = \frac{25.671-25.613}{0.03-0.02} \times 100 = 0.058$$

This means that the buyer can be of assistance to the supplier to reduce the imperfect items from 3% to 2% by paying 0.058 more for each item.

The *Marginal Mc* for different imperfect rates are presented in Table 2 and Fig. 4. As can be seen, the *Marginal Mc* for  $p=0.01$  is significantly higher than other *Marginal Mcs*. The reason is that, by reducing  $E[p]$  from 0.01 to 0.0, the buyer no longer needs to carry out the inspection (the inspection cost in this example is 0.5 per item), while reducing  $E[p]$  to any lower amount other than 0.0 means that the buyer cannot get rid of the full inspection.

#### 4. Conclusion and future research

In this paper, we have formulated and solved a problem to determine the maximum purchasing price a buyer is willing to pay to a supplier to avoid receiving imperfect items under two conditions. First, we assumed that the buyer's selling price is independent of the buyer's purchasing price, while under the second condition, we assumed that changing the buyer's purchasing price influences the buyer's selling price and customer demand. We solved a number of numerical examples, the results of which show that, in both cases, in addition to the purchasing price, the buyer agrees to pay more than the screening cost to avoid receiving imperfect items. Paying this additional amount implies that the supplier conducts the inspection process. We have also shown how the buyer can help the supplier to improve production quality by paying some more than the usual purchasing price.

It is clear that the screening process costs the supplier much less than the buyer, as the supplier is more familiar with the product and its deficiencies. Therefore the results of this paper show that having the inspection process conducted by the buyer is no longer cost-efficient. For future research, we suggest studying situations where buyer and supplier conduct the inspection together. While existing literature focuses on imperfection due to production quality, incorporating other causes of imperfection into the problem, such as transition, is suggested. We also suggest taking the findings of this study into account when formulating the supplier selection problems. Finally, we believe that the proposed model in this paper can be extended to be used in the

context of supplier development, where the main concern is improving the supplier's performance and capabilities. That is, while this paper considers the reasonability of improving the quality of supplied items, studying the improvement of other features of the items (e.g., delivery) as a way to develop suppliers may be considered as well.

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