International Journal of Production Research
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713696255

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First published on: 27 January 2010

To cite this Article Rezaei, Jafar and Dowlatshahi, Shad(2010) 'A rule-based multi-criteria approach to inventory classification', International Journal of Production Research, 48: 23, 7107 — 7126, First published on: 27 January 2010 (iFirst)

To link to this Article: DOI: 10.1080/00207540903348361

URL: http://dx.doi.org/10.1080/00207540903348361

International Journal of Production Research
Vol. 48, No. 23, 1 December 2010, 7107–7126

A rule-based multi-criteria approach to inventory classification

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(*Received 24 June 2009; final version received 15 September 2009*)

The traditional inventory classification method classifies stock keeping units (SKUs) to three classes based on their annual dollar usage, while in real world problems, other criteria are important as well. In this paper, considering multi-criteria situations, a simple, effective and practical rule-based method is designed and implemented in a real world case, using MATLAB software. The most important characteristic of the proposed method is taking into account the inherent ambiguities that exist in the reasoning process of the system of classification. The methodology and the method proposed here may be easily implemented by inventory managers. The results obtained from the case study in this paper are compared with the analytic hierarchy process (AHP) method. Finally concluding remarks and suggestions for future work are provided.

Keywords: inventory management; ABC classification; multi-criteria decision-making; fuzzy logic; fuzzy inference system

1. Introduction

The ABC inventory classification process is an analysis of a range of distinct items, referred to as stock keeping units (SKUs), such as finished products into three categories: A – outstandingly important; B – of average importance; C – relatively unimportant, as a basis for an inventory control scheme. Each category can and sometimes should be handled in a different way, with more attention being devoted to category A, less to B, and even less to C. The larger firms, with larger inventory investments, will often use a 12-class system (Martin and Stanford 2007).

The traditional ABC classification has generally been based on just one criterion – the annual dollar usage of the items. However, depending on what part of the organisation is concerned; the criterion of what is most important with respect to inventory items can change (Flores and Whybark 1986). There are other criteria that represent important considerations for management such as lead time, obsolescence, availability, substitutability, criticality, repairability, commonality, certainty of supply, impact of stock-out, inventory cost, number of requests for the item in a year, scarcity, durability, order size requirement, stock ability, and demand distribution (Flores and Whybark 1986, 1987, Vollmann *et al.* 1997, Ramanathan 2006, Rezaei 2007).

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ISSN 0020–7543 print/ISSN 1366–588X online

DOI: 10.1080/00207540903348361
http://www.informaworld.com

In this paper using fuzzy logic developed by Zadeh (1965) a simple and applicable method for multi-criteria inventory classification is presented. The remainder of this paper is structured in four sections. This introduction is followed by a literature review. Section 2 presents the research methodology. The case study and comparisons and assessment of the results are discussed in Section 3. Finally conclusions of the results and future research directions are provided in Section 4.

1.1 *Review of literature*

Multi-criteria inventory classification (MCIC) was introduced by Flores and Whybark (1986, 1987). Although they introduced several criteria such as obsolescence, lead times, substitutability, repairability, criticality and commonality, their concept of a 'joint criteria matrix' was developed for two criteria. This concept is not a suitable method for considering more than two criteria. They stated: 'The greater the number of criteria that are viewed as important, the more complex the task of developing the classification becomes. If all criteria are important and need to be incorporated in the analysis, the task may be very hard' (Flores and Whybark 1986). The 'joint criteria matrix', therefore, can be considered as a bi-criteria inventory classification. However, based on the authors' multi-criteria concept, several multi-criteria decision-making (MCDM) methods have been proposed to solve this problem.

Cohen and Ernst (1988) used a statistical technique called cluster analysis to group items across many dimensions. The main advantage of this approach is that it can accommodate a large number of combinations of attributes, which are significant for both strategic and operational reasons. However, this requires substantial data and the use of factor analysis and a clustering procedure may render this technique impractical in a typical stockroom environment. Furthermore the clusters themselves must be re-evaluated in order to classify new stock items. Therefore, there is a chance that previously classified stock may end up being reclassified differently every time new items are added. This may disturb the inventory control procedure. In short, their method may pose difficulties for inventory managers (Partovi and Anandarajan 2002).

Flores *et al*. (1992), and Partovi and Burton (1993) presented similar approaches to the ABC classification problem. The proposed methods based on Saaty's analytic hierarchy process (AHP) (Saaty 1982), rated items on both qualitative and quantitative criteria. The main advantage of the AHP method is that it is able to consider several criteria. However, when the number of criteria is increased, the consistency rate will be very sensitive and reaching a consistent rate will be very difficult.

Reynolds (1994) provided a classification scheme, appropriate for process industries. This classification assisted managers to focus their attention on important items even if they are rarely used. However, the application of this method may not be suitable in other industries.

Guvenir and Erel (1998) proposed a method to learn the weight vector along with the cut-off values for multi-criteria inventory classification. The proposed method called genetic algorithm for multi-criteria inventory classification (GAMIC) used a genetic algorithm to learn the weights of criteria along with AB and BC cut-off points from pre-classified items. Once the criteria weights are obtained, the weighted scores of the items in the inventory are computed similarly to the approach with AHP. Then the items with scores greater than the AB cut-off value are classified as class A; those with scores between

AB and BC as class B; and the remaining items are classified as class C. This method had the advantages and disadvantages of the AHP method. In addition, the classification results, to some extent, depended on the pre-classified items.

Puente *et al.* (2002) presented a fuzzy method of classifying different productive items of a company. Whereas the rankings obtained using the classical method were based on information about costs and demand over a period of time in the past. This new method allowed new fuzzy information about the future to be included, thus allowing stricter control of the fuzzy 'A-items' that resulted from this new classification. The authors, however, only considered two criteria of demand and cost in their study. The authors' model was in fact a bi-criteria rather than a multi-criteria model.

Partovi and Anandarajan (2002) presented an artificial neural network for ABC classification of inventory. They utilised two learning methods in their approach: back propagation and genetic algorithm. The reliability of their proposed methods was tested by comparing their classification ability with two data sets. The methods were compared with the multiple discriminant analysis technique. Their results showed that both proposed methods had higher predictive accuracy than discriminant analysis. There was no significant difference between the two learning methods used to develop the artificial neural network. However, the application of these methods could become cumbersome for inventory managers.

Ramanathan (2006) proposed a weighted linear optimisation method for classifying inventory items with multiple criteria. In the proposed approach, a weighted additive function was used to aggregate the performance of an inventory item in terms of different criteria to a single score, called the optimal inventory score of an item. The weights were chosen using an optimisation method subject to the constraints that the weighted sum for all the items must be less than or equal to one. The weighted sum was computed using the same set of weights. This method used a maximisation objective function. To obtain the optimal scores of all inventory items, proposed method should be solved repeatedly by changing the objective function. These scores can then be used to classify the inventory items. Zhou and Fan (2007) presented an extended version of Ramanathan's model. They incorporated some balancing features for MCIC using two sets of weights that are most favourable and least favourable for each item.

Rezaei (2007), by using a fuzzy analytic hierarchy process (FAHP), presented a MCIC method. The weights of the criteria were calculated by using FAHP; then a six-step algorithm was presented to calculate the final normalised weighted score of each item. Finally, using a principle of comparison for fuzzy numbers, the final scores were compared with one another and all items were classified into three classes according to their final scores. Cakir and Canbolat (2008) presented a methodology for MCIC by using FAHP as well. The difference between this method and Rezaei's (2007) method was that this method was web-based and used a decision support system.

Bhattacharya *et al.* (2007) using the TOPSIS (technique for order preferences by similarity to the ideal solution) proposed a multi attribute inventory classification (MAIC) method. They illustrated this technique in a pharmaceutical company by considering these criteria: unit cost, lead time, consumption rate, perishability of items, and cost of storing of raw materials in a crisp format. They concluded that constructing fuzzy models such as fuzzy TOPSIS and neuro-fuzzy hybrid model would be suitable by taking the vagueness in attribute values into account.

Chu *et al.* (2008) proposed an inventory control approach combining ABC and fuzzy classification. They applied this method to an example with 159 SKUs and surprisingly classified 59 items in class A, 69 items in class B and 64 items in class C which is not consistent with the basic concept of ABC classification. However, it does not seem logical to classify roughly the same number of SKUs at three classes A, B and C.

Chen *et al*. (2008) introduced a case-based MCIC based on the 'right' distance based preference expression. Using the decision-maker's assessment of case sets as input, preferences over alternatives were represented intuitively by using weighted Euclidean distances. Then a quadratic optimisation program finds optimal classification thresholds. Although this method is a robust one, it requires some complicated implementation steps which may make it difficult for the decision-maker. This method classified items to just three classes and for the cases requiring more than three classes the complication of the method will be dramatically increased.

1.2 *The implications and assessment of the literature review*

Table 1 provides an overview of the literature review and the methods used therein.

The literature review revealed that a large number of MCDM techniques have been applied to inventory classification problems. However, only three of these references, Puente *et al*. (2002), Rezaei (2007), and Cakir and Canbolat (2008) have considered the inventory classification problem in a fuzzy sense. In fact the proposed model in Puente *et al*. (2002) is a limited bi-criteria model in which only two criteria work in the fuzzy environment. Both of the other works, Rezaei (2007), and Cakir and Canbolat (2008), applied fuzzy AHP.

The advantages and disadvantages of various methods to inventory classification were explored in the literature review. Also, as mentioned by Zadeh (1996) humans employ mostly words in 'computing' and 'reasoning'. Fuzzy AHP makes the problem closer to the real world by using words (and equivalent fuzzy numbers) in computing, but it does not apply words in reasoning. Fuzzy logic enables us to use words (or natural language) for both computing and reasoning. Therefore, considering the inherent vagueness in measuring the relevant criteria in most real world situations, applying fuzzy logic is a more suitable and logical approach. Additionally this approach is capable of handling both quantitative and qualitative criteria. Also, as the output of this method is a set of defuzzified scores in the range zero to one, the decision-maker can classify the SKUs to a desired number of classes. In other words, the decision-maker determines the cut-off points. As construction of the rules is independent from the characteristics of each specific SKU, to classify the new SKUs, no change is necessary in the rule base. Although in most real world situations choosing a limited number of criteria is important for inventory classification purposes, this approach has the ability to consider a large number of criteria. In this case one may think that the number of rules could be increased exponentially. In order to avoid this possible case, we can use the 'rule base reduction' methods (see, for example, Setnes *et al*. 1998, Yam *et al*. 1999).

Precise data and weighting factors are not usually always available in practical situations. The criteria that affect ABC classification are not independent. Volumes of demand, for example, have a large impact on lead times. Bearing this in mind, managers prefer linguistic to numerical values in measuring criteria affecting ABC classification. In many cases, researchers have utilised natural language expressions such as high, low, or fair in their attempt to evaluate these factors. Generally, the representation of managerial knowledge by linguistic rules performs better when there are no units of

Table 1. Multi-criteria ABC inventory classification methods.

Author(s)	Methodology	Description
Flores and Whybark (1986, 1987)	Joint criteria matrix	The 'multi-criteria' concept of ABC inventory classification is introduced by using this method for the first time. However, the proposed methodology is a bi-criteria rather than multi-criteria method.
Cohen and Ernst (1988)	Cluster analysis	By using cluster analysis, SKUs are clustered into different groups based on different criteria. This method can consider a large number of criteria. However, it is too sophisticated and is also vulnerable to the introduction of new criteria.
Flores *et al*. (1992); Partovi and Burton (1993)	Analytic hierarchy process (AHP)	This methodology is able to classify SKUs into different number of classes using different criteria. However, it is vulnerable to the number of criteria used.
Reynolds (1994)	Heuristic	This method is only appropriate for process industries.
Guvenir and Erel (1998)	AHP and genetic algorithm (GA)	The AHP classification method has most of the advantages and disadvantages of AHP. Here, GA is used to learn the weight vector along with the cut-off values for multi-criteria inventory classification.
Puente *et al*. (2002)	Fuzzy set theory	This method considers the predicted criteria as fuzzy numbers. Although this method is a bi-criteria method it can be extended to a multi-criteria approach.
Partovi and Anandarajan (2002)	Artificial neural network (ANN)	Two learning methods have been used in this approach: back propagation and genetic algorithm. This method is largely dependent on the data set used for learning.
Ramanathan (2006); Zhou and Fan (2007)	Weighted linear optimisation	This method is a data envelopment analysis method, which ranks SKUs based on their optimal scores. This method does not utilise the specialist knowledge to determine the criteria weights.
Rezaei (2007); Cakir and Canbolat (2008)	Fuzzy analytic hierarchy process (FAHP)	For comparison of the criteria, this method uses fuzzy numbers instead of crisp numbers. This method is able to classify the SKUs to different number of classes.
Bhattacharya *et al*. (2007)	Technique for order preferences by similarity to the ideal solution (TOPSIS)	By using experts' knowledge, this method ranks SKUs and classifies them into different groups. This method has some of the disadvantages of AHP as it uses the pairwise comparisons of criteria.
Chu *et al*. (2008)	Fuzzy classification analysis	This method uses fuzzy classification analysis to classify SKUs into three classes. This method is able to apply both nominal and non-nominal criteria and attributes. Using this method to classify SKUs to more than three classes is very difficult.
Chen *et al*. (2008)	Case-based	This method is dependent on the data set. It has complicated phases and classifies SKUs to only three classes.

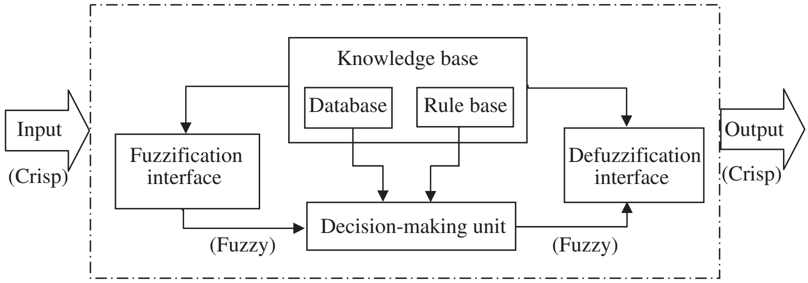

Figure 1. Fuzzy inference system.

measurement for the attributes of the system and no quantitative criteria for the values of such attributes (Zadeh 1983). Fuzzy logic offers a systematic base in dealing with such cases.

2. Methodology

Motivated by the ideas about fuzzy logic, we introduce a rule-based measurement scheme for MCIC. The most important characteristic of the proposed method is taking into account the inherent ambiguities that exist in the reasoning process of the system of classification. Rules contain already known facts but in compact form, such as, 'IF durability is *low*, and . . . THEN the item belongs to class A' in which the linguistic value (*low*) here is represented by the appropriate fuzzy set. The class of item is the result of a fuzzy or approximate reasoning procedure.

2.1 *Fuzzy inference systems*

Fuzzy inference systems are also known as *fuzzy-rule-based systems*, *fuzzy models*, *fuzzy associative memories* (*FAMs*), or *fuzzy controllers* when used as controllers. Basically a fuzzy inference system is composed of five functional blocks as described in Figure 1 by Jang (1993):

- A rule base containing a number of fuzzy IF–THEN rules.
- A database which defines the membership functions of the fuzzy sets used in the fuzzy rules.
- A decision-making unit which performs the inference operations on the rules.
- A fuzzification interface which transforms the crisp inputs into degrees of match with linguistic values.
- A defuzzification interface, which transforms the fuzzy results of the inference into a crisp output.

Usually, the rule base and the database are jointly referred to as the *knowledge base*.

2.1.1 *Fuzzification inference*

Figure 2 illustrates the intended fuzzy inference system in which we define four input linguistic variables for each inventory item. They are unit price (UP), annual

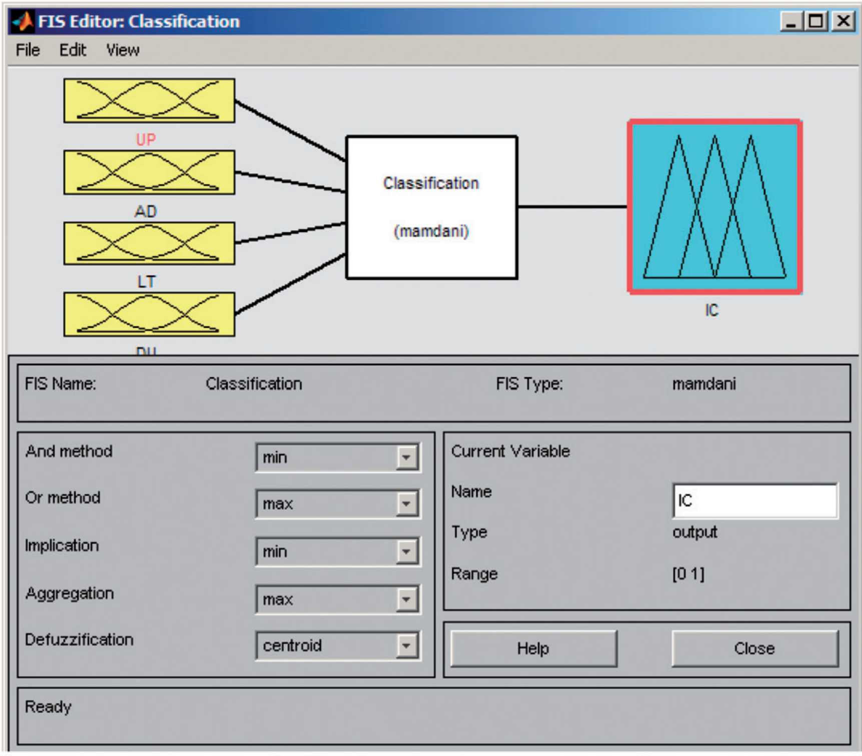

Figure 2. The intended fuzzy inference system.

demand (AD), lead time (LT), and durability (DU). These criteria were selected by the inventory managers of the company selected for this study. In this case to fuzzify the input and output variables, using the expert's knowledge, the following fuzzy subsets (linguistic values) are used: low (L), with corresponding fuzzy number: $(0, 0, 0.4)$; medium (M), with corresponding fuzzy number: $(0.1, 0.5, 0.9)$; and high (H), with corresponding fuzzy number: $(0.6, 1, 1)$. These subsets are known as triangular membership functions. If F is a triangular fuzzy number in R and $a, b, c, x \in R$, its membership function $\mu_F : R \to [0, 1]$ is:

$$\mu_F(x) = \begin{cases} (x-a)/(b-a), & a \le x \le b \\ (c-x)/(c-b), & b \le x \le c \, . \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

It should be noted that the linguistic values are commonly used by all variables but they are scaled into the interval $[0, 1]$ (see Equations (4) and (5) in Section 3). The physical domain of the linguistic variables is defined by managers. For example, the physical domain of LT in this paper is $[1, 7]$ days.

2.1.2 *Knowledge base*

As mentioned before, the rule base and the database are jointly referred to as the knowledge base. The knowledge acquisition phase comprises experts' knowledge of the application domain and the decision rules that govern the relationships between inputs and outputs. The membership functions of inputs and outputs are designed by inventory managers of the company studied based on their knowledge of the system and their experience. However, the main purpose of the knowledge base is to provide a fuzzy rule base needed for the fuzzy processor.

2.1.2.1 *Fuzzy rule base*. The fuzzy rule base contains a set of IF–THEN rules developed by the experience and knowledge of one or more experts. A typical fuzzy rule has the form: IF antecedent THEN consequent. The rule base of our study contains $3^4 = 81$ rules, which include all variations of the linguistic values, i.e., three linguistic values for each of the four linguistic variables. The rules were constructed based on inventory managers' knowledge. The rules representing these experts' knowledge showing how the variables affect classification have the following form:

$$\begin{array}{llll} \text{IF} & \text{unit price} & \text{is} & up \in UP \\ \quad\text{AND} & \text{annual demand} & \text{is} & ad \in AD \\ \quad\text{AND} & \text{lead time} & \text{is} & lt \in LT \\ \quad\text{AND} & \text{durability} & \text{is} & du \in DU \\ \text{THEN} & \text{item class} & \text{is} & ic \in IC. \end{array}$$

Let $T = \{\text{L, M, H}\}$ be the set of linguistic values for all the five input and output variables UP, AD, LT, DU and IC, and let T_{UP}, T_{AD}, T_{LT}, T_{DU} and $T_{\text{IC}} \in T$ be the linguistic value sets for UP, AD, LT, DU and IC, respectively. Then the above rule can be rewritten compactly as follows:

$$\begin{array}{ll} \text{IF} & UP \text{ is } T_{\text{UP}} \text{ AND } AD \text{ is } T_{\text{AD}} \text{ AND } LT \text{ is } T_{\text{LT}} \text{ AND } DU \text{ is } T_{\text{DU}} \\ \text{THEN} & IC \text{ is } T_{\text{IC}}, \end{array}$$

where T_{UP}, T_{AD}, T_{LT}, T_{DU} and $T_{\text{IC}} \in T$.

Table 2 shows a sample of the developed rule base (with four inputs).

Table 2. Fuzzy rule base for four inputs comprising 81 antecedent-consequent pairs.

	Fuzzy rule base input				Fuzzy rule base output
Rule no.	UP	AD	LT	DU	Item class
1	L	L	L	L	L
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
39	M	M	L	H	H
40	M	M	M	L	M
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
81	H	H	H	H	H

2.1.3 *Decision-making unit*

The decision-making unit, also known as fuzzy inference engine, is similar to simulating human decision-making in inferring fuzzy control actions based on the rules of inference in fuzzy logic. The evaluation of a rule is based on computing the truth value of its premise part and applying it to its conclusion part. This results in assigning one fuzzy subset to each output variable of the rule. This component interacts with the knowledge base and performs mathematical computations based on the above fuzzy numbers. In particular, they take place according to mathematical operators that are defined, on the basis of the expert's knowledge, for the connectives AND/OR, ELSE and the IF/THEN implication. In this paper, the type of inference engine developed by Mamdani and Assilian (1975) was used by employing a compositional minimum operator, which represents a conservative attitude towards ABC classification (see Figure 2). In minimum inferencing the entire strength of the rule is considered as the minimum membership value of the input variables' membership values:

$$\mu_{\text{output}} = \min\{\mu_{\text{input1}}, \mu_{\text{input2}}, \ldots, \mu_{\text{input}N}\}. \tag{2}$$

2.1.4 *The defuzzification interface*

The output of the fuzzy inference engine is a fuzzy number while the decision-maker needs a crisp number. Defuzzification inference is the final operation that converts this fuzzy output into a crisp output. In general, there are five methods of defuzzification described in the literature (Yen and Langari 1999). These include the centre of gravity (COG) defuzzification or the mean of maximum (MOM) defuzzification. The COG is the one that is most commonly used. This technique calculates the centre of the area of the combined membership function as:

$$y_0 = \frac{\int_i \mu_F(y_i)\, y_i \mathrm{d}y_i}{\int_i \mu_F(y_i)\mathrm{d}y_i}, \tag{3}$$

where y_i is the representative value of the fuzzy subset member i of the output, and $\mu_F(y_i)$ is the confidence in that member (membership value), and y_0 is the crisp value of the output.

3. The case study

The proposed methodology has been applied to the case study of a food manufacturing company (XYZ Company). XYZ is a medium-sized company that produces cooking and table margarines. The majority of XYZ's products are sold and consumed in The Netherlands, Belgium, France, and Germany. XYZ produces 13 product families (170 products under five brands). The number of XYZ's SKUs (including raw materials, work-in-process goods and completely finished goods, and MRO – maintenance, repair, and operations – goods) is more than 4200. To build a fuzzy expert system for an inventory classification method that is based on fuzzy logic, the researchers have captured, organised and used human expert knowledge by interviewing the inventory managers.

Today food manufacturers are struggling in a very competitive environment, especially for the highly perishable products. The selection of a food manufacturer is a good case study because it uses perishable materials. Other criteria such as durability, lead time, etc. would become important as well. The only two criteria of annual demand and unit price that are typically considered in traditional inventory classification would not work for food manufacturers.

For our study we chose a sample of 54 SKUs denoted as S1 through S54 (see Table 4, presented in the Appendix). As the XYZ's inventory managers were familiar with the inventory classification, we requested them to carefully select a sample of SKUs that would be a suitable representative of all SKUs of the company.

Four criteria of UP, AD, LT, and DU based on the degree of their importance as indicated by company inventory managers through personal interviews were used. The first three criteria were positively related to the score of the inventory items, while the DU had a negative relationship with the score of the inventory items. Table 4 (presented in the Appendix) shows the original and normalised measures (columns 2–9) by normalising the UP, AD, and LT criteria measures using Equation (4) and the DU criterion measure using Equation (5) in the scale of 0-1:

$$F_{\text{pos}}^{\text{norm}} = \frac{F_i - F_{\min}}{F_{\max} - F_{\min}} \tag{4}$$

$$F_{\text{neg}}^{\text{norm}} = \frac{F_{\max} - F_i}{F_{\max} - F_{\min}}, \tag{5}$$

where F_i, $F_{\max}$ and $F_{\min}$ are the ith value, the maximum value, and the minimum value of the factor under normalisation.

Using the proposed methodology by MATLAB's Fuzzy Logic Toolbox (Jang and Gulley 1995, The MathWorks n.d.), the defuzzified value for each item is calculated. For example consider item S1 (in Table 4). Normalised inputs for this item are: 0.387 (UP), 0.548 (AD), 0.5 (LT) and 0.706 (DU). After entering these values into the system (as shown at the bottom of Figure 3) and based on their fuzzy sets, the decision rules are applied and the fuzzy results of the output variable IC are composed and defuzzified using the COG method, (see Equation (3)). Then the final output (defuzzified number) is calculated to be 0.553. Table 4 (column 11) shows these results for all 54 SKUs.

Finally, the resulted defuzzified values are sorted in the descending order and the inventory classification is conducted based on the traditional ABC principle. The cut-off points are determined based on the inventory managers' point of view. Therefore, we classify nine SKUs of the top list (approximately 16%) in class A, next 15 SKUs (approximately 28%) in class B, and the last 30 SKUs (approximately 56%) in class C (see Table 5, presented in the Appendix).

Here is the decision process a manager needs to follow when s/he decides to apply the proposed methodology:

(1) Determine the most important criteria for the SKUs classification.
(2) Make the IF-THEN rules in the MATLAB software.
(3) Enter the measures of each item in the system. The output would be the defuzzified score for each item.
(4) Sort the SKUs in descending order and determine the cut-off points based on the desired number of classes. For example for three classes, determine two cut-off points.

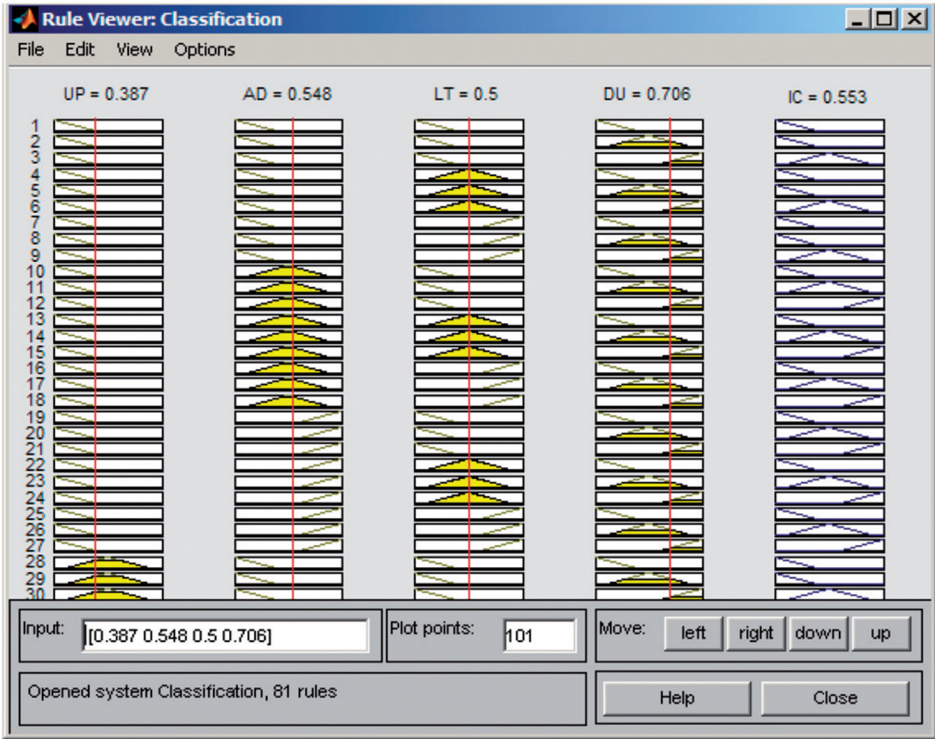

Figure 3. Rules output for an example (S1) with inputs (0.387 0.548 0.5 0.706) and output 0.553.

3.1 *Comparisons of the results*

AHP is one of the most common techniques to solve MCDM problems in general and it is also a dominant technique in MCIC (see Flores *et al*. 1992, Partovi and Burton 1993, Guvenir and Erel 1998 for AHP, and Rezaei 2007, Cakir and Canbolat 2008 for fuzzy AHP). For comparison purposes, we have applied the AHP method to the case data. Similar to the proposed methodology in this paper, AHP is constructed based on the user judgment and would be a suitable technique for our comparison purpose.

As the objective of this paper is not the application of the AHP method, we only report the scores that we have obtained for the criteria based on the judgment of the company inventory managers. For this case study the resulted weights are: 0.157 (UP), 0.41 (AD), 0.056 (LT), and 0.377 (DU). These values are used to calculate the score of each item. The AHP score of items can be seen in Table 4 (column 10). We sorted these scores in descending order and classified the items to three classes (see Table 5, presented in the Appendix). We also adopted the number of items used in our proposed classification method.

Table 3 shows the comparison of the results of the proposed classification method with the AHP results. This comparison shows that:

- There are two cases of different classification of SKUs in class A.
- There are four cases of different classifications of SKUs in class B.
- There are four cases of different classifications of SKUs in class C.

Table 3. Comparison of the proposed method and AHP.

		AHP			
		A	B	C	Total
Proposed method	A	7	1	1	9
	B	1	11	3	15
	C	1	3	26	30
	Total	9	15	30	54

3.2 *Assessment of the results and discussion*

Herein we discuss the advantages of the proposed approach in comparison to AHP in classification of inventory.

The proposed rule-based method is completely constructed based on the inventory managers' reasoning. Using the proposed method, the inherent interdependencies between criteria are implicitly considered (in the phase of rule making). However the AHP is not capable of taking these interdependencies into account. For instance, it is expected that SKUs with low durability have relatively shorter lead time than more durable SKUs. Therefore, although both approaches are based on the managers' judgment, the proposed method results in more accurate and reliable classification.

In this paper, we have four criteria and 54 items. The final score of each item in the AHP is calculated by the sum of the product of item measures by criteria weights. The criteria weights are calculated based on a four by four pair-wise comparison matrix; while the final score of each item in the proposed method is based on 81 rules. Although both methods, as most MCDM methods, depend on experts knowledge and judgment, changing a few rules in the proposed method has little impact on the final classification, while changing even one cell of pair-wise comparison matrix in the AHP may dramatically change the final classification. In AHP, changing a cell results in changing all weights and consequently the final score of all alternatives (here SKUs). In a rule-based system, the antecedents (IF ...) are pre-defined and the decision-maker (DM) decides about the consequences (THEN ...). Changing one consequence *only* affects the final score of SKUs, which follow the antecedents of that specific rule (and not the final score of all SKUs). Since the occurrence of errors in conducting the pair-wise comparison matrix or IF–THEN rules, in practical situations, is expected, the proposed method can become more suitable and less vulnerable than AHP.

In AHP the weights of criteria are determined based on a pair-wise comparison. In a pair-wise comparison, the DM has to determine the preference of a criterion over another criterion in absence of the other criteria. In most real-world situations, however, the preference of a criterion over another one depends on other criteria. For example, if we ask the DM to assign a value between 1 and 9 to show his/her preference of unit price over lead time, s/he may state that this decision is dependent upon durability. The DM may believe that 'if' the SKU is durable then unit price is preferred over lead time; otherwise lead time is preferred. If the DM believes that other criteria can influence his/her preference, this comparison will become conditional depending on the number of other criteria. However, AHP is not capable of considering the potential influence of other criteria on each pair-wise comparison. The proposed rule-based, however, is completely capable of considering this issue, as in each rule a specific combination of the importance of criteria is considered.

The final score of each item in AHP is calculated by the sum of the product of item measures by criteria weights. In other words, the final score of each item is obtained by a linear function. Therefore, a change in each measure has a linear impact on final scores in AHP. However in some real-world situations changing a measure may have an exponential impact on the final score. This issue can be mitigated by the proposed method in the phase of rule-building.

Additionally, there have been some other serious concerns raised about the theoretical basis underlying the AHP. Harker and Vargas (1987) challenged the AHP and expressed four areas of concerns as follows: (1) lack of an axiomatic foundation; (2) the ambiguity of the question the decision-maker must answer; (3) the scale used to measure the intensity of preferences; and (4) the principle of hierarchical composition and rank reversal. Perez (1995) also focused on criteria weights and the rank reversal phenomenon. The author argued that this undesirable effect does not, per se, invalidate the AHP, but it does make it necessary to identify the kind of situations in which the method is suitable. Dyer (1990) also argued that the AHP is flawed as a procedure for ranking alternatives in that the rankings obtained by this method are arbitrary. This paper further focused on the operational difficulty encountered by the AHP.

The proposed methodology provides a robust decision support system (DSS) that is developed based on the linguistic judgment of managers. Humans mostly employ words in computing and reasoning (Zadeh 1996) and therefore the proposed method makes the classification problem more accurate and easy to understand and apply. Here are the features of the proposed method in general:

(1) It mimics the ability of the human mind to effectively employ modes of reasoning (Zadeh 1994).
(2) It is constructed based on the managers' natural language and therefore more acceptable and understandable to inventory managers.
(3) It is capable of considering a large number of criteria to classify a large number of SKUs.
(4) It is capable of classifying the SKUs to the desired number of classes.
(5) The results are not considerably changed due to a few changes in rules.

The proposed methodology has, however, some disadvantages. For example aggregating the rules from different experts is sometimes difficult when these experts do not agree on a specific output of a rule.

4. Conclusion and future research directions

In this paper using fuzzy logic, we constructed a rule-based inference system for classifying inventories into different classes according to their multi-criteria importance. While the traditional inventory classification considers only one criterion (annual dollar usage) and classifies items into only three classes, the proposed method in this paper considers several criteria for its classification. The proposed method also has an ability to classify items to more than three classes. Implementation of this method in real world situations is simple and easier to understand by inventory managers because it is constructed based on natural language. We also used the AHP for comparison purposes and discussed the advantages and disadvantages of the proposed method.

Here are some suggestions and ideas for future work. In this paper, we implemented our method based on a set of *crisp* data of 54 SKUs from a real case study. This was made possible because our four criteria were measurable quantitatively. However, collecting the relevant data, in a crisp format in some real world situations where qualitative criteria have to be considered as well, is difficult. For example, in some real world situations, managers prefer to use linguistic values such as 'low' or 'very high' for a qualitative criterion such as 'critical'. Linguistic values of linguistic variables are usually embodied by fuzzy numbers. In this situation the proposed methodology is not suitable. Instead, we suggest exploring the proposed method by using data as input of the system that is not accessible and/or measurable in crisp format. The suggested method would then work with fuzzy input (instead of crisp input).

In the literature of inventory classification (both traditional and multi-criteria approaches), SKUs are classified into different classes according to their *importance*. The importance of items is defined based on the managers' point of view of the firms; while nowadays most firms make most of their decisions in supply chains. Supply chain management (SCM) is the management of materials and information which flow both in and between facilities such as vendors, manufacturing and assembly plants, and distribution centres (Thomas and Griffin 1996). Therefore, we suggest considering the impact of these relationships in supply chain management on inventory classification. For instance we can consider the impact of information sharing in SCM on inventory classification. Information sharing can reduce ordering costs, inventory costs, and supply lead times (Seidmann and Sundararajan 1997). Therefore, it is clear that the importance of items with long lead time will diminish and these items will be moved from their classes to less important classes. As an example, joint planning in the SCM can lead to the 'integrated importance' for inventories across the supply chain.

Finally, while all methods of ABC classification have been proposed in a multi-criteria framework, we believe that inventory classification should be considered as a multi-objective problem. For example we have to take into account the objectives of carrying inventories as outlined by Dowlatshahi (2007):

- To meet variations (fluctuations) in demand.
- To hedge against inflation or sudden increases in price.
- To allow flexibility for product and operational scheduling.
- To maintain independence of operation.
- To provide a safeguard against delivery problems (e.g., supplier strike, weather conditions, custom delays),
- To take advantage of quantity discounts.

Considering these objectives, we are also able to determine the 'optimal' solution for the problem, which does not depend on the managers' 'judgment'. This optimal solution can be used as a benchmark to compare the MCIC methods.

Acknowledgements

The authors are grateful to Prof. C. Whybark (co-author of the first paper in multi-criteria ABC classification) and anonymous referees for their constructive criticism of the initial version of this paper.

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Appendix

Table 4. The normalised measures and defuzzified scores of the inventory criteria measures.

Item no.	Unit price (€)	Annual demand	Lead time (day)	Durability (month)	Normalised measures: Unit price	Annual demand	Lead time	Durability	AHP score	Deffuzzified score
S1	14	2500	4	6	0.387	0.548	0.5	0.706	0.580	0.553
S2	2.2	2750	1	2	0.006	0.604	0	0.941	0.604	0.862
S3	8.5	350	1	12	0.21	0.061	0	0.353	0.191	0.155
S4	26	3200	2	3	0.774	0.706	0.167	0.882	0.753	0.775
S5	6.7	850	1	6	0.152	0.174	0	0.706	0.362	0.411
S6	2	1200	1	1	0	0.253	0	1	0.481	0.592
S7	16	500	3	12	0.452	0.095	0.333	0.353	0.261	0.471
S8	25	750	2	6	0.742	0.152	0.167	0.706	0.454	0.466
S9	11.5	2270	3	12	0.306	0.495	0.333	0.353	0.403	0.479
S10	4.5	850	1	12	0.081	0.174	0	0.353	0.217	0.315
S11	13	570	2	3	0.355	0.111	0.167	0.882	0.443	0.522
S12	15	660	2	8	0.419	0.131	0.167	0.588	0.351	0.303
S13	15	2150	3	2	0.419	0.468	0.333	0.941	0.632	0.853
S14	3.1	360	2	8	0.035	0.063	0.167	0.588	0.263	0.147
S15	3.5	860	1	1	0.048	0.176	0	1	0.457	0.534
S16	5	3350	2	1	0.097	0.74	0.167	1	0.705	0.839
S17	33	480	2	12	1	0.09	0.167	0.353	0.336	0.481
S18	9.5	900	1	6	0.242	0.186	0	0.706	0.380	0.442
S19	6	1100	3	3	0.129	0.231	0.333	0.882	0.466	0.564
S20	8.5	650	3	3	0.21	0.129	0.333	0.882	0.437	0.548
S21	9	580	3	3	0.226	0.113	0.333	0.882	0.433	0.56
S22	12	4500	4	6	0.323	1	0.5	0.706	0.755	0.667
S23	7.5	1280	1	3	0.177	0.271	0	0.882	0.472	0.601
S24	13	880	2	12	0.355	0.181	0.167	0.353	0.272	0.327
S25	17	1470	2	8	0.484	0.314	0.167	0.588	0.436	0.46
S26	2.5	1120	2	9	0.016	0.235	0.167	0.529	0.308	0.396

(*continued*)

Table 4. Continued.

Item no.	Unit price (€)	Annual demand	Lead time (day)	Durability (month)	Normalised measures: Unit price	Normalised measures: Annual demand	Normalised measures: Lead time	Normalised measures: Durability	AHP score	Deffuzzified score
S27	2.8	860	4	6	0.026	0.176	0.5	0.706	0.371	0.412
S28	7.5	2600	1	1	0.177	0.57	0	1	0.639	0.851
S29	8.5	950	5	1	0.21	0.197	0.667	1	0.528	0.556
S30	23	670	2	2	0.677	0.133	0.167	0.941	0.525	0.529
S31	15	760	3	6	0.419	0.154	0.333	0.706	0.414	0.521
S32	11.5	890	1	8	0.306	0.183	0	0.588	0.345	0.335
S33	12	4500	2	12	0.323	1	0.167	0.353	0.603	0.51
S34	11	250	4	3	0.29	0.038	0.5	0.882	0.422	0.622
S35	8.5	870	1	2	0.21	0.179	0	0.941	0.461	0.539
S36	15	1350	2	1	0.419	0.287	0.167	1	0.570	0.629
S37	4.7	740	6	12	0.087	0.149	0.833	0.353	0.255	0.275
S38	8.6	170	3	12	0.213	0.02	0.333	0.353	0.193	0.371
S39	7.4	220	2	6	0.174	0.032	0.167	0.706	0.316	0.406
S40	6.5	750	5	2	0.145	0.152	0.667	0.941	0.477	0.522
S41	16	150	4	8	0.452	0.016	0.5	0.588	0.327	0.5
S42	11	650	1	12	0.29	0.129	0	0.353	0.232	0.25
S43	3.6	270	3	9	0.052	0.043	0.333	0.529	0.244	0.147
S44	8.3	470	1	18	0.203	0.088	0	0	0.068	0.154
S45	5.5	860	1	6	0.113	0.176	0	0.706	0.356	0.412
S46	7.5	80	4	8	0.177	0	0.5	0.588	0.278	0.32
S47	9.5	140	3	3	0.242	0.014	0.333	0.882	0.395	0.573
S48	13	1240	2	3	0.355	0.262	0.167	0.882	0.505	0.592
S49	9.5	660	3	1	0.242	0.131	0.333	1	0.488	0.582
S50	27	560	7	1	0.806	0.109	1	1	0.604	0.848
S51	3.8	380	1	6	0.058	0.068	0	0.706	0.303	0.362
S52	16	860	2	12	0.452	0.176	0.167	0.353	0.286	0.318
S53	15	1250	5	8	0.419	0.265	0.667	0.588	0.433	0.5
S54	9.2	340	3	3	0.232	0.059	0.333	0.882	0.412	0.565

Table 5. Final classification results of the proposed method and the AHP.

Item no.	Descending order of defuzzified values	Fuzzy logic classification	AHP classification
S2	0.862	A	A
S13	0.853	A	A
S28	0.851	A	A
S50	0.848	A	A
S16	0.839	A	A
S4	0.775	A	A
S22	0.667	A	A
S36	0.629	A	B
S34	0.622	A	C
S23	0.601	B	B
S6	0.592	B	B
S48	0.592	B	B
S49	0.582	B	B
S47	0.573	B	C
S54	0.565	B	C
S19	0.564	B	B
S21	0.56	B	C
S29	0.556	B	B
S1	0.553	B	A
S20	0.548	B	B
S35	0.539	B	B
S15	0.534	B	B
S30	0.529	B	B
S11	0.522	B	B
S40	0.522	C	B
S31	0.521	C	C
S33	0.51	C	A
S41	0.5	C	C
S53	0.5	C	C
S17	0.481	C	C
S9	0.479	C	C
S7	0.471	C	C
S8	0.466	C	B
S25	0.46	C	B
S18	0.442	C	C
S27	0.412	C	C
S45	0.412	C	C
S5	0.411	C	C
S39	0.406	C	C
S26	0.396	C	C
S38	0.371	C	C
S51	0.362	C	C
S32	0.335	C	C
S24	0.327	C	C
S46	0.32	C	C
S52	0.318	C	C
S10	0.315	C	C
S12	0.303	C	C
S37	0.275	C	C
S42	0.25	C	C

(*continued*)

Table 5. Continued.

Item no.	Descending order of defuzzified values	Fuzzy logic classification	AHP classification
S3	0.155	C	C
S44	0.154	C	C
S14	0.147	C	C
S43	0.147	C	C