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A Review and Probabilistic Analysis of Limit State Functions of Corroded Pipelines

Zahiraniza Mustaffa[1,2] and *Pieter van Gelder*[1]
[1]Hydraulic Structures and Probabilistic Design, Hydraulic Engineering Section,
Civil Engineering and Geosciences, Delft University of Technology,
Delft, The Netherlands.

[2]Civil Engineering Department, Universiti Teknologi PETRONAS,
Tronoh, Perak, Malaysia.

ABSTRACT

This paper introduces a new approach in developing a limit state function used in a probabilistic method when analyzing an aging structure. The so called dimensionless limit state function was formulated from the application of the well known Buckingham π theorem coupled with the multivariate statistics. Its application was proven to be an easier solution for aging pipelines subjected to corrosions. Not only that, this method has the advantage to incorporate the *circumferential defect (w)* term, a parameter which is normally neglected by the existing codes and most limit state functions developed by literatures. For this, a review on the past and present limit state function developments was first discussed. The inclusion of w term in the dimensionless limit state function was found to be reasonable as it represented better estimation of the defect shape rather than its area (as seen) from the longitudinal view. Results from this new approach were compared with the literatures and codes. Its favorable results may overcome the conventionality of the existing codes and this is something to ponder.

KEY WORDS: dimensionless limit state function, defect circumferential width.

INTRODUCTION

The current design practice on defect assessment for corroded pipelines involves the application of *failure pressure* (PF) models. The models are able to compute the remaining strength of corroded pipelines subjected to internal pressure. Several common design codes that incorporate the PF models are the DnV RP F101, Modified ASME B31G, Shell, RSTRENG and PCORRC. It can be said that these codes were more or less originated from the original B21G criterion but later evolved using extensive series of full-scale tests results on corroded pipelines. Most of these codes are represented by safety factors and thus making it as deterministic in nature. An example of these codes can be found in Table 1. Each PF model is governed by input parameters of pipe outer diameter (D), wall thickness (t), minimum yield strength (SMYS) or ultimate tensile strength (SMYS), longitudinal extend of corrosion (l) and corrosion defect depth (d). Even though design codes have helped avoid unnecessary repairs and replacements, the excess conservatism continues to cause some unnecessary repairs (Escoe, 2006).

The existing codes use a single simple corrosion geometry and the corrosion circumferential width (w) is not considered (Fu and Kirkwood, 1995). Literatures agreed to the fact that the longitudinal extent of corrosion is always of greatest important and should be considered first compared to the circumferential width. Defects in this orientation have been reported to be the most severe since it alters the hoop stress distribution and promotes bulging. Longitudinal axis hoop stress is the maximum stress for internal pressure. With regard to this, the parameter d and l have always become the important inputs for the PF models, as they are aligned longitudinally.

On the other hand, literatures have shown that the influence of corrosion circumferential width (w) to failures was not that significant. Interested readers are advised to refer to work by Chouchaoui and Pick (1994), Fu and Kirkwood (1997) and Batte *et. al.* (1997) on this matter. Circumferential defect acting alone may not harm much of the pipeline remaining strength. However, defects in the circumferential direction would become more important when poor longitudinal stresses resulted from pipe bending presence (Chouchaoui and Pick, 1994).

The current practice tends to deal with the parameter w separately at the later stage when the most severe defect has been identified. The parameter w becomes important when qualitatively assessing the interaction of a colony of defects under the *Fitness-for-Purpose* approach. The circumferential extent of damage is only become priority when depth of the corrosion is greater than 50% of the original pipe wall thickness and the circumferential extent is greater than 1/12 (8.33%) of the circumference (Escoe, 2006).

To date, quantitative assessment incorporating d, l and w together can only be conducted either experimentally or numerically. The former is also known as the burst test while the latter utilizes finite element method. Literatures have showed that results from both methods conformed to each other.

Table 1. *Failure pressure* (PF) models used to compute remaining strength of pipeline subjected to corrosion

Failure pressure models	Failure pressure expression, *PF*	Bulging factor, *M*
Modified ASME B31G	$PF = \frac{2(SMYS + {}^{*}68.95)\,t}{D}\left[\frac{1-0.85\,(d/t)}{1-0.85\,(d/t)/M}\right]$	$M = \sqrt{1+0.6275\left(\frac{l^2}{Dt}\right)-0.003375\left(\frac{l^4}{D^2t^2}\right)}$ for $\sqrt{\frac{l^2}{Dt}} \le 50$; $M = 3.3+0.032\left(\frac{l^2}{Dt}\right)$ for $\sqrt{\frac{l^2}{Dt}} > 50$
DNV RP F101	$PF = \frac{2\,SMTS\,t}{D-t}\left[\frac{1-(d/t)}{1-(d/t)/M}\right]$	$M = \sqrt{1+0.31\left(\frac{l}{\sqrt{Dt}}\right)^2}$
SHELL-92	$PF = \frac{1.8\,SMTS\,t}{D}\left[\frac{1-(d/t)}{1-(d/t)/M}\right]$	$M = \sqrt{1+0.805\left(\frac{l^2}{Dt}\right)}$
RSTRENG	$PF = \frac{2SMTS\,t}{D}\left[1-(d/t)/M\right]$	$M = \sqrt{1+0.6275\left(\frac{l^2}{Dt}\right)-0.003375\left(\frac{l^4}{D^2t^2}\right)}$

However, it may not be practical and economical to depend on these methods every time when doing the assessment, especially when all corrosion defects in the pipeline need to be considered. This would consume too much time.

Although analysis on corrosions has always based on their science and physics, we should always admit that their occurrence is somewhat complicated, random and dependent on their environment and surroundings. It is believed that there is no single solution as proposed in any corrosion models that suit all corrosions, thus if possible their analysis may wise be tackled on a case-to-case basis. Owing to this, this paper proposed an equation that takes into account all defect geometries (*d*, *l* and *w* in particular). The idea was not to neglect or omit any less significant parameters as done in the present approach because their correlation between each other can never be guaranteed to be any less important. Because of their random and complicated in nature, it is wise to represent corrosion characteristics as a whole instead.

The probabilistic method has introduced researchers to apply the *limit state function* (LSF) approach as another mean to compute the reliability of corroded pipelines. A brief overview of past LSFs will be presented in the next section. With the aid of the LSF approach, the present work tried to incorporate the *w* parameter into the equation to compute reliability. Not only in the PF models, the *w* term has also been ignored in the past LSFs. Despite long arguments about its insignificance, this work tried to prove that incorporating the parameter can be easily done using a dimensionless LSF. The idea of introducing this term was to represent corrosion defect shape in a better way, instead of allowing the PF models to only visualize it along the longitudinal view of the pipeline.

OVERVIEW OF PAST LIMIT STATE FUNCTIONS

Most limit state functions (LSF) developed by literatures were originated from the original B21G criterion, as shown in Table 1. A comprehensive evaluation on the past, present and future assessment of corroded pipelines involving the design codes has been reported by Bjørnøy and Marley (2001). Thus it is not the intention of the present work to discuss these matters in more details.

Particularly for marine pipelines, several LSFs developed in the past are selected for discussions. The works include Ahammed and Melchers (1996), Pandey (1998), Ahammed (1998), De Leon and Macias (2005) and Teixeira *et. al.* (2008), as given in below equations:

Ahammed and Melchers (1996),

$$Z = \left[2\,m_f s_y \frac{t}{D}\frac{1-d/t}{1-d/(tM)}\right] - p_a \tag{1}$$

Pandey (1998),

$$Z = \left[2.3\,s_y \frac{t}{D}\frac{1-d/t}{1-d/(tM)}\right] - p_a \tag{2}$$

Ahammed (1998) and De Leon and Macias (2005),

$$Z = \left\{2(s_y + 68.95)\frac{t}{D}\frac{1-[d_0 + R_d(T-T_0)]/t}{1-[d_0+R_d(T-T_0)]/tM}\right\} - p_a \tag{3}$$

with, $d=d_o+R_d(T\text{-}T_o)$ and $l=l_o+R_L(T\text{-}T_o)$

Teixeira *et. al.* (2008),

$$Z=\left(\frac{1.1s_y 2\,t}{D}\right)[1-0.9435(d/t)^{1.6}(l/D)^{0.4}]-p_a \qquad (4)$$

For all expressions, the M term is given by,

$$M=\left(1+0.6275\frac{l^2}{Dt}-0.003375\frac{l^4}{D^2t^2}\right)^{1/2} \text{ for } \quad l^2/Dt \leq 50 \text{ or} \qquad (5)$$

$$M=0.032\frac{l^2}{Dt}+3.3 \quad \text{for} \quad l^2/Dt > 50 \qquad (6)$$

with,
- m_f = multiplying factor (1.10[10] and 1.15[10])
- s_y = yield stress
- d = defect depth
- l = defect longitudinal length
- t = wall thickness
- D = pipe outer diameter
- M = Folias/bulging factor
- p_a = applied/operating pressure
- d_o = defect depth measured at time T_o
- l_o = defect longitudinal length measured at time T_o
- T = any future time
- T_o = time of last inspection
- R_d = radial corrosion rate ($=\Delta d/\Delta T$)
- R_L = longitudinal corrosion rate ($=\Delta l/\Delta T$)

The Folias factor, M is a measure of stress concentration that is caused by radial deflection of the pipe surrounding a defect. There are other works involving LSF development as reported by Ahammed and Melchers (1994, 1995, 1997), Guan and Melchers (1999), Caleyo *et. al.* (2002), Lee *et. al.* (2003, 2006), Lee *et. al.* (2005), Santosh *et. al.* (2006), Khelif *et.al.* (2007) and many more. Since most of these works were either for onshore or buried pipelines, it is not the intention of the present work to further deliberate about them. Interested readers are recommended to their work for further understanding on the matter. However, it is interesting to report here that most of these LSF equations comprise too many parameters, which make their computation becomes more or less complicated.

DIMENSIONLESS LIMIT STATE FUNCTION

A limit state function, Z is an equation used in probabilistic method to determine the reliability of a structure by comparing its strength (R) and load (S), as given below:

$$Z = R - S \qquad (7)$$

The limit state is described by Z = 0. The probability of failure (P_f) is then given by Eqn. 8. Failures takes place when the failure surface falls in the region of $Z < 0$ while $Z > 0$ is a survival region.

$$P_f = \mathrm{P}(Z \leq 0) = \mathrm{P}(R \geq S) \qquad (8)$$

The objective of this section is to introduce an approach that can be used to develop a dimensionless LSF equation to assess the remaining strength of a corroded pipeline. Since the importance of the w term has been addressed in the earlier section, its value will be added into this equation.

Buckingham-π Theorem

The Buckingham-π Theorem is a method that forms dimensionless parameters from several possible governing parameters of a certain scenario under investigation. It enables us to select the most significant parameters describing the characteristics of the scenario while omitting the less ones. Interested readers are recommended to refer to book chapter on *Dimensional Analysis* from any *Hydraulics* or *Fluid Mechanics* books for further discussion about this method.

Discussion will first be given on how to create the *strength (R)* term of a dimensionless LSF. For the assessment of a corroded pipeline, the governing parameters required to compute its remaining reliability are:

i. corrosion geometry
ii. operating pressure
iii. burst pressure

Also needed in the assessment are the design paramateres:

iv. pipeline geometry (diameter and wall thickness)
v. strength

From the above parameter list, seven parameters has been selected in this study, namely burst pressure (P_b), specified minimum tensile strength (*SMTS*), pipeline wall thickness (t), diameter (D), defect depth (d), defect longitudinal length (l) and defect circumferential width (w). Except for P_b, other terms can be gathered from the design values as well as data reported by the intelligent pigging probe. The P_b is normally obtained from either experimental or numerical studies. Therefore, the P_b database for this study utilized DnV Technical Report (1995). This report is a compilation of laboratory tests of corroded pipelines from four institutions, namely American Gas Association (AGA), NOVA, British Gas and University of Waterloo. These participants have conducted many experimental tests for longitudinally corroded pipes under internal pressure for different corrosion defect depths, longitudinally lengths and circumferential widths. Out of the 151 burst pressure database reported, only 31 of them was utilized in this work after considering the suitability of the current and reported database.

The Buckingham-π Theorem addresses the dependency of the seven parameters as,

$$\pi_1 = f(SMTS, P_b, D, t, d, l, w) \qquad (9)$$

Eqn. 9 can then be refined by making it dimensionless according to their units, as given below,

$$\pi_2 = f\left(\frac{P_b}{SMTS}, \frac{t}{D}, \frac{d}{t}, \frac{l}{w}\right) \qquad (10)$$

Mustaffa *et al.* (2009) has statistically proven that strong correlation existed between d, w and l. The dependency between the four dimensionless parameters in Eqn. 10 was later formulated using the *multivariate regression analysis* and a nonlinear model was chosen to best describe the parameters, as given in Eqn. 11. This is the equation representing the remaining strength (R) of the corroded pipeline.

$$\frac{P_b}{SMTS}=\left(\frac{t}{D}\right)^{0.8442}\left(\frac{d}{t}\right)^{-0.0545}\left(\frac{l}{w}\right)^{-0.0104} \qquad (11)$$

To complete the limit state function, Z equation the load, *(S)* term is included into the equation. Since the R equation was made dimensionless, the S term was also made dimensionless by dividing the maximum allowable operating pressure (P_o) with the *SMTS*.

$$Z = \left[\left(\frac{t}{D}\right)^{0.8442} \left(\frac{d}{t}\right)^{-0.0545} \left(\frac{l}{w}\right)^{-0.0104} \right] - \frac{P_o}{SMTS} \tag{12}$$

Advantages

The advantages of using Eqn. 12 are discussed below:

i. In contradictory to the design codes which assumed the defect shape by focusing on the parameter *d* and *l* only, the inclusion of all defect geometries (*d*, *l* and *w*) has enabled it to be more realistic. This has paved the assumption that the shape and characteristics of a particular defect is actually representing the interaction between the pipeline characteristics (materials, geometry, strength), environment (sea water, reservoir properties) and operating (loading, pressure) conditions. These are the factors that contribute to the way the defects behave and spread. No single reservoir can be said to be 100% homogeneous to another reservoir and so thus the operating conditions. These are the governing factors that create the corrosion unique characteristics which cannot be thoroughly described in any design codes.

ii. The equation is able to provide a visual view of the defects with respect to the pipeline original geometry. While the expression (*t/D*) corresponds to the original geometry of the pipeline, the (*d/t*) term represents the wall loss that has taken place from a cross section view. The (*l/w*) term is important to describe the size/spread of the defect, as seen from a plan view. Thus this equation is comprehensive enough to illustrate the physical layout of a corroded pipeline.

iii. There have been many arguments on the corrosion shapes assumed by the design codes. Corroded area has been argued from as simple shape as a rectangular (*dl*) to parabolic (2/3 *dl*) and average of rectangular and parabolic (0.85 *dl*) shapes. Even until today, one can never be too sure of the assumptions made in those codes. Knowing this, it may be rationale to apply the dimensionless corrosion parameters without making any assumptions on the shapes.

iv. The fact that this equation was developed in a probabilistic way, all input parameters are treated as random variables and thus no single value (or safety factor) will be used but to apply the probability density function (pdf) instead.

v. As a pipeline can be several kilometers long, analysis on any interested section along the pipeline can be done easily using Eqn. 12 as long as its corrosion development is counted as a pdf of that particular section.

vi. In contradictory to the qualitative present assessment on corrosion defects where *d*, *l* and *w* are analyzed separately, Eqn. 12 has illustrated an easy approach to incldue the *w* parameter into one single equation. Careful attention, however, should be given when dealing with a scenario of interacting defects.

vii. The equation is simple and straightforward compared to the lengthy LSF as reported by literatures. Thus less computation is required to carry out the analysis.

Limitation

Fu and Kirkwood (1995) in their work has come out with a full list of classification of defects. The defects sizes in this study in particular can be classified as shallow ($d/t < 0.30$), short ($l/D < 0.20$) and broad ($w/t > 0.50$) type of corrosions. Therefore, the burst pressure (P_b) database taken from the DnV Technical Report (1995) was chosen based on these defects classification, making Eqn. 12 to be applicable only to those defect shapes. For classes of defects that might not be covered in the report, numerical analysis using ANSYS or ABAQUS needs to be carried out for P_b computation. Continuing research is needed for various corrosion shapes beyond the scope of this research.

SAMPLE OF APPLICATION

A 128 km steel pipeline type API 5LX-65 located at the east coast of Malaysia was selected for the analysis. The pipeline transports gas from offshore to onshore. 554 internal corrosion defects of various types were reported by the intelligent pigging. Descriptive statistics of the corrosion data is as shown in Table 2. The wall loss was calculated up to 30% from the actual wall thickness.

Table 2. Descriptive statistics of corrosion defects

Variable			Distribution	Mean value	Standard deviation
Symbol	Description	Unit			
d	Depth	mm	Weibull	1.90	1.19
l	Longitudinal length	mm	Exponential	32.64	23.52
w	Circumferential width	mm	Gamma	36.76	33.17

Table 3. Random variables of pipeline characteristics

Variable			Distribution	Mean value	Standard deviation
Symbol	Description	Unit			
D	Diameter	mm	Normal	711.2	21.3
t	Wall thickness	mm	Normal	25.1	1.3
SMTS	Specified minimum tensile strength	MPa	Normal	530.9	37.2
P_o	Operating pressure	MPa	Normal	14 - 30	1.4 – 3.0

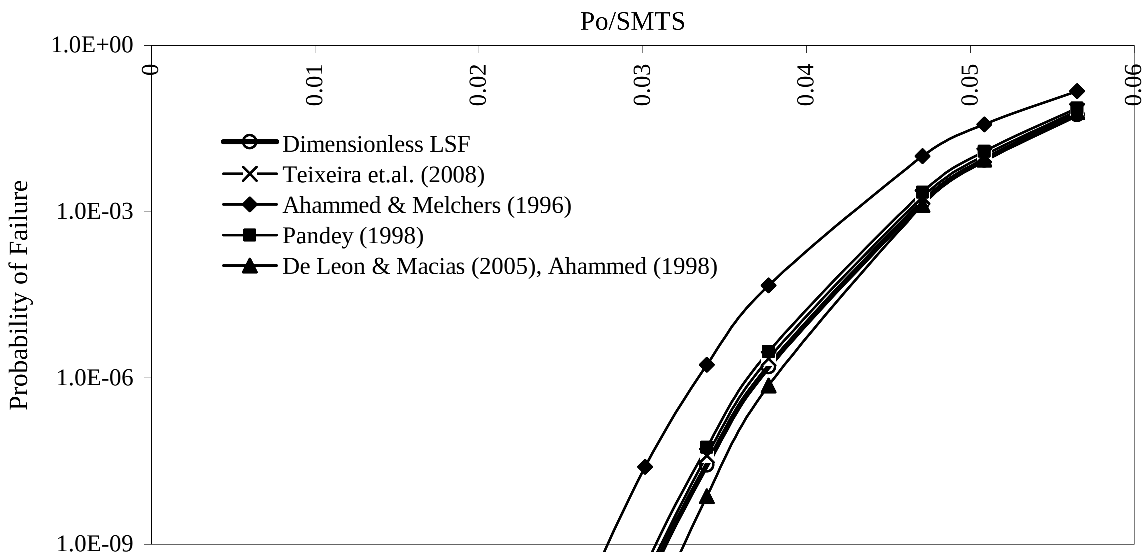

Figure 1. Probability of failure computed for all limit state functions

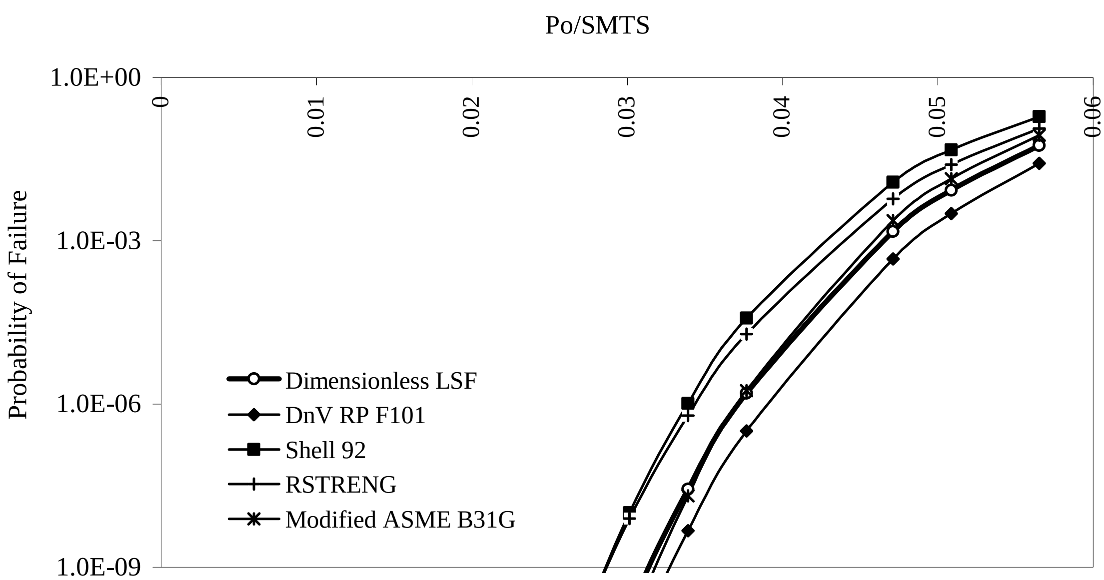

Figure 2. Probability of failure computed for all limit state functions

DISCUSSION

This section presents the computation of probability of failure (P_f) for pipeline under studied using the dimensionless LSF equation, Eqn. 12. Results from this equation were compared with (i) limit state functions reported in the literatures (Fig. 1) and (ii) design codes (Fig. 2). Data from Table 2-3 were applied into the analysis. The P_f in Eqn. 8 was simulated using the analytical approximation methods called First Order Reliability method (FORM).

It is important to highlight here that it was not the intention of the present work to distinguish the best model to determine the reliability of corroded pipelines. This discussion is meant at measuring the goodness and performance of Eqn. 12 by comparing it with other models. By investigating the closet behavior of Eqn. 12 to other models, it can then be said that they might exist similar physics or characteristics between Eqn. 12 and the assumptions made in those models.

Both Fig. 1 and 2 revealed that the probability of failure (P_f) increases as the loads increases. This is true because as higher loadings exerted to the pipeline, its capability to withstand that load decreases and thus prone to failure. It was obvious that some models tend to be over or underestimated than one another. Arguments about such behavior will not be reported in this paper but interested readers are recommended to refer to the list of references for further information on this matter.

In Fig. 1, results from the Eqn. 12 were compared with other LSF literatures. It can be seen from this figure that work by Pandey (1998) and Teixeira *et. al.* (2008) were very close to the dimensionless LSF results. Fig. 2 on the other hand provides comparison between the dimensionless LSF and existing PF models taken from different design codes, namely DnV RP F101, Modified ASME B31G, Shell 92 and RSTRENG. Results from the present work were found to be very close to the Modified ASME B31G.

It is favorable to find out that result from the present work has provided good comparison with the work by Teixeira *et. al.* (2008). This was partly contributed by the similar approach *i.e.* regression analysis used when developing the LSF equation. Therefore, the approach used to develop Eqn. 12 can be said to be acceptable. Also, similar performance to the Modified ASME B31G was expected as that design code assumed an arbitrary corrosion shape. The assumption on an arbitrary corrosion shape is somehow reasonable as corrosion shapes are random, unique and complicated to describe. By describing a defect using the dimensionless corrosion parameters of d, l and w, one is looking at a more detail description of the corrosion shape. It is true that the selection of those three parameters are still not sufficient to represent an actual corrosion size as they are only measured at maximum values *i.e.* d_{max}, l_{max} and w_{max}. Even though its shape can not be thoroughly presented this way but the inclusion of w in a dimensionless way is hoped to be able to provide the least estimation on the spread of the defect as seen from a plan view (*i.e.* pipeline is cut open).

CONCLUSIONS

Corrosions in marine pipelines are random, unique, complicated and depending on the surrounding environment, in which their characteristics can not always be described by the design codes. It may be wise to treat them as a case-by-case basis. This paper introduces an easier approach to assess the reliability of a corroded marine pipeline without depending on the design codes. The ideology of the present paper was to making full use of corrosion data reported by the intelligent pigging (IP) during inspection. Corrosion is a product of many physical and chemical interactions, thus it can be said that its development is subjected to its surrounding environment. By utilizing more parameters (d, l and w) to describe corrosion shape, one is looking at a more detail description about its shape as well as addressing its interaction with the surrounding environment. The present paper applied the probabilistic method thru multivariate regression analysis to explain the dependency of defect d, l and w. Having them together in an expression was assumed to be able to provide better estimation on defect characteristics. The so called dimensionless limit state function was easy, straightforward and its application was found to be in favorable when compared with other design codes as well as literatures on limit state functions. This approach is hoped to avoid unnecessary estimation when computing the reliability of corroded pipelines.

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