

Probabilistic Description of Scour Hole Downstream of Flip Bucket Spillway of Large Dams

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Abstract: Risk and reliability analysis is presently being performed in almost all fields of engineering depending upon the specific field and its particular area. Probabilistic risk analysis (PRA), also called quantitative risk analysis (QRA) is a central feature of hydraulic engineering structural design.

Actually, probabilistic methods, which consider resistance and load parameters as random variables, are more suitable than conventional deterministic methods to determine the safety level of a hydraulic structure. In fact, hydraulic variables involved in plunge pools, such as discharge, flow depth, and velocity, are stochastic in nature, which may be represented by relevant probability distributions. Therefore, the optimal design of a plunge pool needs to be modelled by probabilistic methods.

The main topic of this paper is concerned with the reliability-based assessment of the geometry of the plunge pool downstream of a ski jump bucket. Experimental data obtained from a model of a flip bucket spillway has been used to develop a number of equations for the prediction of scour geometry downstream from a flip bucket spillway of a large dam structure. The accuracy of the developed equations was examined both through statistical and experimental procedures with satisfactory results. In addition, reliability computations have been carried out using the Monte Carlo technique.

The main conclusions are that structural reliability analysis can be used as a tool in the dam safety risk management process and that the most important factors for further analysis are erosion, friction coefficient, uplift and self-weight.

Keywords: risk analysis, reliability, plunge pool, Monte Carlo simulation, flip bucket, large dams

Introduction

Probabilistic design approach is a powerful tool in reliability of civil hydraulic engineering. Uncertainty and risk are central features of hydraulic engineering. Hydraulic design is subject to uncertainties due to the randomness of natural phenomena, data sample limitations and errors, modelling reliability, and operational variability. Uncertainties can be measured in terms of the probability density function, confidence interval, or statistical moment such as standard deviation or coefficient of variation of the stochastic parameters.

In recent years, reliability analysis and probabilistic methods have found wide application in hydraulic engineering. Development of reliability based analysis methods for engineering application can be found in many references (Tung and Mays 1980; Yen and Tung 1993). Several applications of the methods to hydraulic design have been reported in the literature (Tang and Yen 1972; Yen and Tung 1993; Vrijling 2001; Ang and Tang 2007). This allows us to determine the true probability of the component failure and of the whole system.

Energy dissipation downstream of large dams is a serious concern. Trajectory spillways or so-called ski jumps are employed whenever the velocity at the dam foot is in excess of typically 20 (m/ s) because of problems with stilling basins in terms of cavitations, abrasion and uplift

(Vischer and Hager 1998). Ski jumps are currently widely used because they appear to be the only hydraulic element allowing for the technically sound and the hydraulically

safe control of large quantities of excess hydraulic energy during flood events.

In the present study, the probabilistic method will be used for estimating plunge pool erosion downstream of a ski jump bucket. The Probabilistic Design method is an approach that can provide a better understanding of the failure mechanisms and their occurrence probabilities as well as the consequences of failure of such important infrastructure.

Background and theory

Literature Review

A plunge pool is a natural phenomenon caused by the erosive effects of flowing water in alluvial streams. Excessive scouring can progressively undermine the foundations of hydraulic structures such as bridges and embankments and the areas downstream of outlet facilities.

Large-scale plunge pools downstream of outlet structures which are induced by jets of various configurations constitute an important field of research because of the possibility of structural failure at the foundation level. The jet can dissipate its excessive energy by excavating a large scour hole. Drastic examples of this phenomenon were experienced downstream of the Alder and Nacimiento Dams in the USA, Picote Dam in Portugal, Kariba Dam in Zambia, and Tarbela Dam in Pakistan (Mason, 1985).

The plunge pool mechanism is relatively complex because of difficulties arising from the modeling of sediment-laden flow in and around the scour hole induced by the jet action of the flow downstream of outlet facilities. Therefore, the experimental study of plunge pool has been limited to the consideration of governing variables involved in the phenomenon while keeping the other parameters of secondary importance constant.

Energy dissipation in such spillways may be in the form of a ski-jump which throws the water jet away from the bucket lip into the air and then into the plunge pool formed at the point of impact on the tail water (Fig.1). Because of the impact of the high velocity jet, scour takes place both upstream and downstream of the point impingement.

Many theoretical and experimental studies have been conducted about problems of ski-jumps and flip bucket plunge pools. The relationships between geotechnical and hydraulic parameters are very complicated and they change with different local plunge pool levels. The geometric parameters that are used in the plunge pool formulae contained in this paper are defined in Figure 1.

Several researchers in the past have produced empirical formulas based on laboratory as well as prototype observation in order to predict the plunge pool depth downstream of the ski-jump bucket. Veronese (1937), Wang (1987), Yildaz (1994), Llopardo (2002), Yanmaz (2003), Rajaratnam (2002) and Azmathullah (2005).

A relationship evolved by Mason (1985) has the form.

$$d_s = 3.27 \frac{q^{0.6} H_1^{0.05} d_w^{0.15}}{g^{0.3} d_{50}^{0.06}} \quad (1)$$

Also he demonstrated that the head drop H_1 might not directly affect the scour depth, other than by varying the amount of air entrained in the plunge pool. An alternative expression that better described the plunge pool process was therefore proposed.

For free jet spillways such as those on the crests of arch dams, Veronese formula is generally applied in the plunge pool is as defined by Ervine.

$$d_s = 1.90 q^{0.54} H_1^{0.225} \quad (2)$$

Attempts have been made to generalize this formula as:

$$d_s = K_r q^{0.54} H_1^{0.225} \quad (3)$$

K_r is related to the rock mass characteristics and takes values between 0.25 and 2.5.

Yildiz et al (1994) suggest that for applying this formula to spillways with flip buckets etc, it should be modified to: (Khatsuria, 2005)

$$d_s = 1.90 q^{0.54} H_1^{0.225} \sin \theta \quad (4)$$

Where θ is the jet impact angle at tail water surface. Schoklisch suggest this formula for estimate plunge pool depth.

$$d_s = 3.15 q^{0.57} H_1^{0.2} / D_{90}^{0.32} \quad (5)$$

Basics of plunge pool flow

In a trajectory basin or ski jump, the energy is mainly dissipated by jet dispersion. The trajectory basin is composed of five reaches. (Figure 1)

- a: approach chute
- b: deflection and takeoff
- c: dispersion of water jet in air
- d: impact and scour of jet
- e: tail water zone

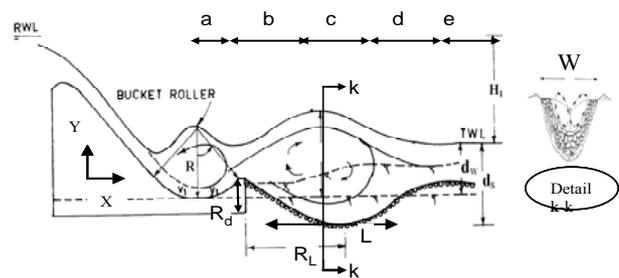


Figure 1 Scour profile and plunge pool characteristic

Plunge pool Characteristics

Hydraulic, geotechnical and morphologic factors control the depth of plunge pool. The governing parameters, which influence the local plunge pool phenomenon downstream of a flip bucket, are illustrated in Figure (1):

Control structure characteristics are, width of the structure W_s , upstream water depth H_1 (m), length of the apron L_s (m), bucket radius R (m), lip angle θ (rad). Bed material characteristics are, median material diameter by weight d_{50} (mm), standard deviation of bed material particle size σ_{50} and specific gravity G_s .

Fluid characteristics are density of water ρ_w (kg/m^3), dynamic viscosity μ_w (pas).

Flow characteristics are depth of tail water depth d_w (m), plunge pool depth d_s (m), average velocity of water jet v (m/s) and discharge q ($\text{m}^3/\text{s.m}$).

Hydrodynamic Equations

Continuity Equation

The continuity equation in tensor form for steady flow is given by: Eqn. 6 can be written for a two dimensional form as:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (6)$$

In which U is velocity in X direction (flow direction) and V is velocity in Y direction.

Momentum Equation

The Reynolds equation for turbulent flow in tensor form is given by :

$$\rho \left(\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial X_j} \right) = - \frac{\partial P}{\partial X_i} + \frac{\partial}{\partial X_i} \left(\mu \frac{\partial U_i}{\partial X_j} - \rho U_i U_j \right) + \rho g_i \quad (7)$$

In which $\rho U_i U_j$ is Reynolds stress and P is pressure. Neglecting Reynolds stress and for two dimensional flow and assuming steady state flow Eqn (7) can be simplified and rewritten, taking the bed in X direction, as follows Eqn(8):

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = - \frac{1}{\rho} \left(\frac{\partial P}{\partial X} - \frac{\partial \tau}{\partial Y} \right) + g \sin \theta \quad (8)$$

In which τ is a shear stress and θ is bed slope.

Data Collection

The data that we used in this assessment was experimental data reported by Azmathullah (2005). This data was drawn from past measurement of scour parameters at the Central Water and Research Station (CWPRS), pune, India.

The hydraulic model studies were also conducted on three different bucket designs. The three hydraulic models simulated the dams across the Subarnarekha, Ranganadi and Parbati Rivers in India. Azmathullah (2005)

The first dam was 52(m) high and 720(m) long. Its spillway consisted of 13 spans of 15(m) wide each with crest at elevation 177(m). Radial gates of size 15(m)×15(m) regulated the flow over this spillway. The design outflow flood was 26150 (m³/s) which corresponded to a maximum water level at an elevation of 192.37(m). The ski-jump bucket with bucket radius of 25(m) and lip angle of 32.5° was provided at the toe for energy dissipation.

The second dam was 60(m) high, made up of concrete with a rock fill portion on its right side. It had an overflow spillway with seven spans of 10(m) width and 12(m) height. The spillway catered to a maximum outflow flood of 12500(m³/s). This corresponded to the maximum water level of 568.3(m) and the full reservoir level of 567(m) with the crest level of the spillway at 544(m). It had a bucket radius of 18 m with 35° as the lip angle.

The third dam spillway is 85(m) high. It was designed to pass a maximum discharge of 1,850 (m³/s) at the full reservoir level of 2,198(m) elevation. It had three spans, 6 m wide and 9(m) high, separated by 6 m thick piers, and is fitted with radial gates. An apron and a plunge pool along the downstream side fronted the bucket, which had a bucket radius of 28(m) with the lip angle of 30°.

Equation for Scour Hole

An important objective of this model study was to develop an equation to describe the geometry of the scour hole. The development and the significance of the dimensionless variables (π -terms) and some individual variables describing the geometry of the scour hole due to the flip bucket spillway were discussed in previous

sections. The aim of this section is to discuss and present the equation that resulted from this model study.

Development of the Equation

The dependent variables d_s , L_s , W_s may be expressed in terms of other variable by the following functional.

$$d_s = f_1(q, H_1, d_{50}, G_s, V, \rho_w, \sigma_{50}, g, \mu_w, d_w, R, \phi, \theta) \quad (9)$$

$$L_s = f_2(q, H_1, d_{50}, G_s, V, \rho_w, \sigma_{50}, g, \mu_w, d_w, R, \phi, \theta) \quad (10)$$

$$W_s = f_3(q, H_1, d_{50}, G_s, V, \rho_w, \sigma_{50}, g, \mu_w, d_w, R, \phi, \theta) \quad (11)$$

The general form selected in this study for the equation relating a π -term with a number of independent π -term is in the product of powers of relevant π -term, i.

$$\pi_1 = c \pi_2^{a_2} \pi_3^{a_3} \pi_4^{a_4} \dots \pi_m^{a_m} \quad (12)$$

The value of parameters c, a_1, a_2, \dots, a_n which were obtained from a multiple regression analysis, can then be replaced back into Eq (12). Applying the Buckingham π -theorem, with V, d_w, ρ_w, g as repeating variables, equations above are transformed to:

$$\frac{d_s}{d_w} = f_1 \left(\frac{V}{\sqrt{g d_w}}, \frac{H_1}{d_w}, \frac{d_{50}}{d_w}, \frac{\sigma_{50}}{d_w}, G_s, \frac{\mu}{\rho V d_w}, \frac{R}{d_w}, \phi, \theta \right) \quad (13)$$

$$\frac{L_s}{d_w} = f_2 \left(\frac{V}{\sqrt{g d_w}}, \frac{H_1}{d_w}, \frac{d_{50}}{d_w}, \frac{\sigma_{50}}{d_w}, G_s, \frac{\mu}{\rho V d_w}, \frac{R}{d_w}, \phi, \theta \right) \quad (14)$$

$$\frac{W_s}{d_w} = f_3 \left(\frac{V}{\sqrt{g d_w}}, \frac{H_1}{d_w}, \frac{d_{50}}{d_w}, \frac{\sigma_{50}}{d_w}, G_s, \frac{\mu}{\rho V d_w}, \frac{R}{d_w}, \phi, \theta \right) \quad (15)$$

Since the flow was in the fully turbulent flow, the viscosity had a negligible effect on the local plunge pool scour; hence, Equations above can be simplified to the form:

$$\frac{d_s}{d_w} = a_0 (Fr_{(N)})^{a_1} \left(\frac{H_1}{d_w} \right)^{a_2} \left(\frac{d_{50}}{d_w} \right)^{a_3} (G_s)^{a_4} \left(\frac{R}{d_w} \right)^{a_5} (\theta)^{a_6} \quad (16)$$

$$\frac{L_s}{d_w} = b_0 (Fr_{(N)})^{b_1} \left(\frac{H_1}{d_w} \right)^{b_2} \left(\frac{d_{50}}{d_w} \right)^{b_3} (G_s)^{b_4} \left(\frac{R}{d_w} \right)^{b_5} (\theta)^{b_6} \quad (17)$$

$$\frac{W_s}{d_w} = c_0 (Fr_{(N)})^{c_1} \left(\frac{H_1}{d_w} \right)^{c_2} \left(\frac{d_{50}}{d_w} \right)^{c_3} (G_s)^{c_4} \left(\frac{R}{d_w} \right)^{c_5} (\theta)^{c_6} \quad (18)$$

In which $Fr_{(N)}$ is the Froude number at the end of the apron at the location of d_w .

Formulation and Statistical Regression

Equation 16, 17, 18 can be fitted using ordinary least squares. Goodness-of-fit statistics, we can estimate of correlation coefficient (r) and the standard error (σ), from this equation. In assessing the accuracy and applicability of prediction equations, the characteristics of the residuals are important. (McCuen 1990)

The above dimensionless groups of parameters were related to each other in the present study based on nonlinear regression. The set of dimensionless equation

with their original exponents obtained from regression analysis is as follows. This yielded the following equations in order to estimate the maximum scour depth, maximum scour width, and distance of maximum scour location from the bucket lip, respectively:

$$\frac{d_s}{d_w} = 15.89 (Fr_N)^{1.1308} \left(\frac{H_1}{d_w}\right)^{-0.3019} \left(\frac{R}{d_w}\right)^{-0.2597} \left(\frac{d_{30}}{d_w}\right)^{0.1002} (\sin\theta)^{0.4014} \quad (19)$$

$$\frac{L_s}{d_w} = 7.20 (Fr_N)^{0.3581} \left(\frac{H_1}{d_w}\right)^{0.3918} \left(\frac{R}{d_w}\right)^{0.0774} \left(\frac{d_{30}}{d_w}\right)^{0.0203} (\sin\theta)^{0.2986} \quad (20)$$

$$\frac{W_s}{d_w} = 0.942 (Fr_N)^{0.0223} \left(\frac{H_1}{d_w}\right)^{0.8182} \left(\frac{R}{d_w}\right)^{0.1351} \left(\frac{d_{30}}{d_w}\right)^{0.0071} (\sin\theta)^{-1.3839} \quad (21)$$

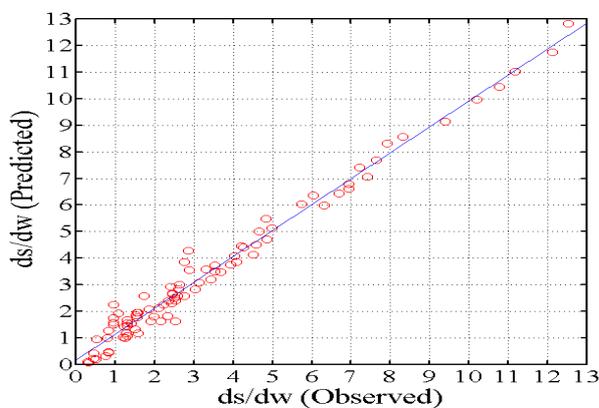


Figure3. Observed versus predicted relative

Statistical characteristic and comparison between observed (X) and predicted (Y) values, standard residual and density function and multivariate density function of plunge pool depth, length and width for the validation set is qualitatively shown in table 1 and figures 2-10 respectively.

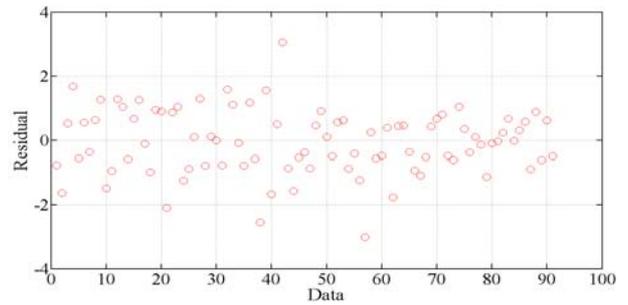


Figure2. Depth Residual

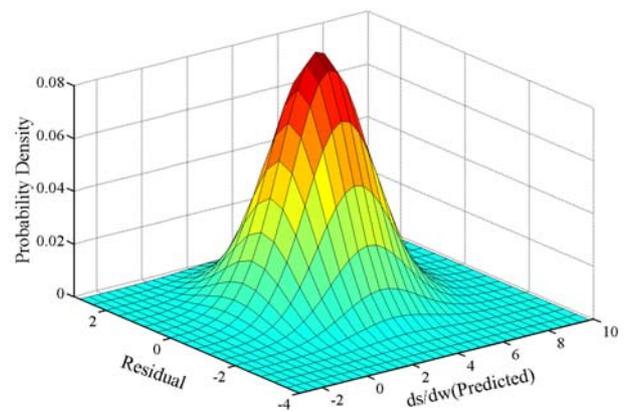


Figure4. Multivariate density function

Table 1. Statistical characteristic (X observed, Y predicted)

Y \ X	d_s/d_w	L_s/d_w	W_s/d_w	R-Square	RMSE	μ_x	σ_x
d_s/d_w	$Y=0.98 X+0.15$	--	--	0.99	0.4008	3.41	2.844
L_s/d_w	--	$Y=0.97X+0.48$	--	0.976	0.998	11.94	6.516
W_s/d_w	--	--	$Y=0.95X+1.05$	0.95	1.823	12.58	8.399
μ_Y	3.491	12.08	12.97	--	--	--	--
σ_Y	2.807	6.409	8.166	--	--	--	--

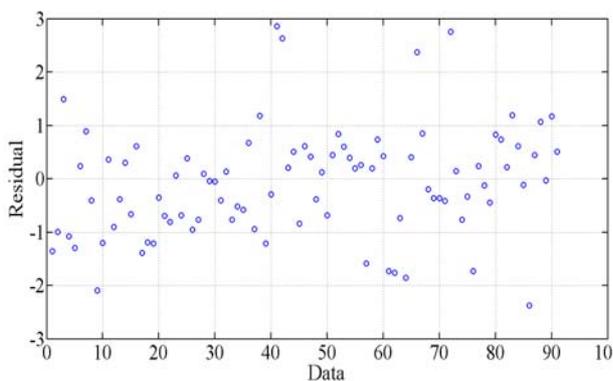


Figure 5. Length Residual

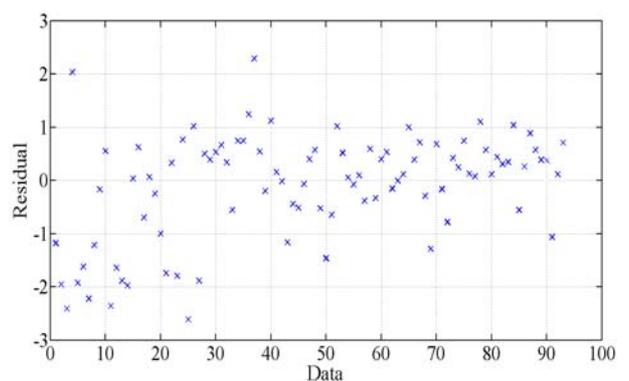


Figure 8. Width Residual

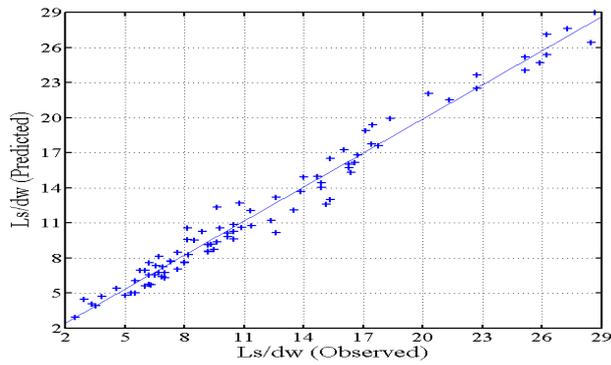


Figure 6. Observed versus predicted relative

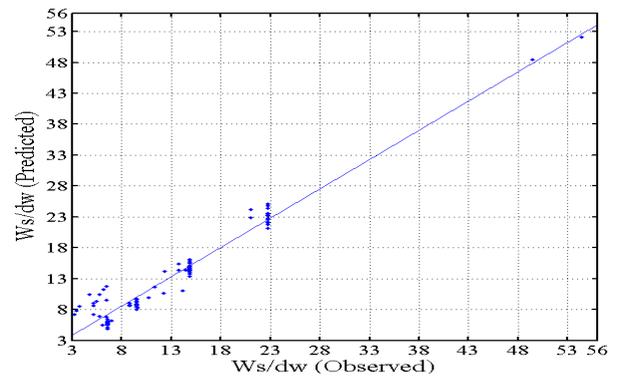


Figure 9. Observed versus predicted relative

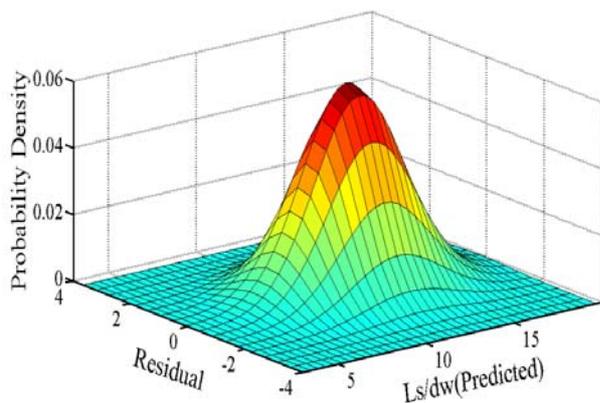


Figure 7. Multivariate density function

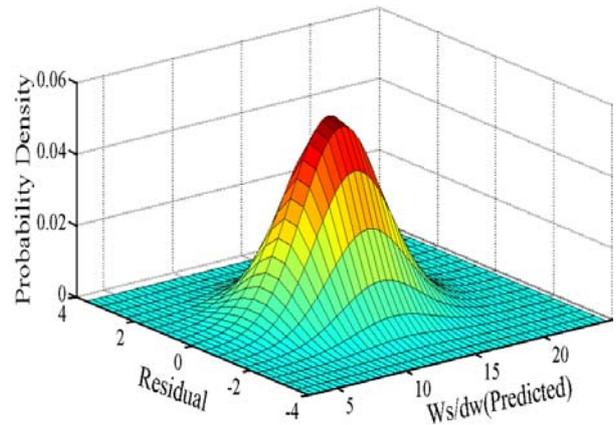


Figure 10. Multivariate density function

Hydrosystems Reliability

Tools of Reliability Analysis

Probabilistic design methods are well known but the application is generally limited to difficult cases and to the development of design codes. The application of the probabilistic design methods offers the designer a way to unify the design of structures, dikes, dunes, mechanical, equipment and management systems. For this reason, there is a growing interest in the use of these methods (Van Gelder 2000).

The tools available to the engineer for performing a reliability analysis fall into three broad categories. First are the methods of direct reliability analysis. These propagate the uncertainties in properties, geometries, loads, water levels, etc. through analytical models to obtain probabilistic descriptions of the behaviour of a structure or system. The second includes event trees, fault trees, and influence diagrams, which describe the interaction among events and conditions in an engineering system. The third includes other statistical techniques. In particular, some problems are so poorly defined that it is useless to try to formulate mechanical models and the engineer must rely on simple statistics. (Van Gelder 1996)

Limit State Function

A model that applies to the failure of an engineering system can be described as the load L (external forces or demands) on the system exceeding the resistance R

(strength, capacity, or supply) of the system Eqn.25. (Vrijling 2001)

$$Z = \text{Strength} - \text{Load} = R_i - L_j \quad (22)$$

The reliability P_s is described as the probability of safe operation, in which the resistance of the structure exceeds or equals to the load, that is,

$$P_s = P(Z > 0) = P(L \leq R) \quad (23)$$

In which P_f denotes the failure probability and can be computed as:

$$P_f = P(Z < 0) = P(R < L) = 1 - P_s \quad (24)$$

The definitions of reliability and failure probability, Eqn. (26) and (27) are equally applicable to individual system components as well as total system reliability. The graph of figure 11 shows the reliability function.

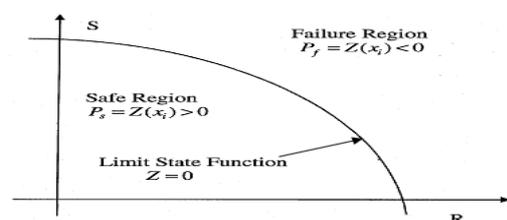


Figure 11 Reliability function

In the reliability function, the strength and load variables are assumed stochastic variables. A stochastic variable is a variable, which is defined by a probability distribution and a probability density function. (Figure 12)

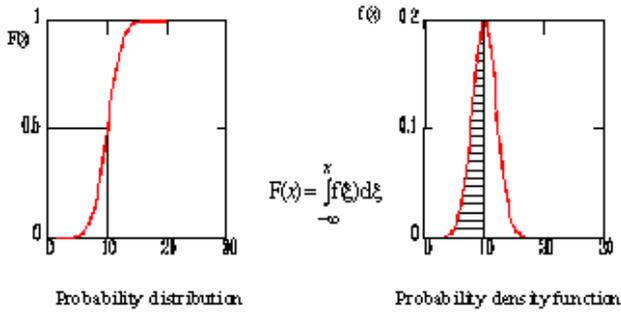


Figure 12

The probability distribution $F(x)$ returns the probability that the variable is less than x . The probability density function is the first derivative of the probability distribution (Van Gelder 1996). If the distribution of the density of all the strength and load variables is known it is possible to estimate the probability that the load has a value x and that the strength has a value less than x (see Figure 13).

The failure probability is the probability that $S=x$ and $R<x$ for every value of x . So

$$\left. \begin{aligned} P(S=x) &= f_s(x) dx \\ P(R \leq x) &= F_R(x) \end{aligned} \right\} \Rightarrow P(S=x \cap R \leq x) = f_s(x) F_R(x) dx \quad (25)$$

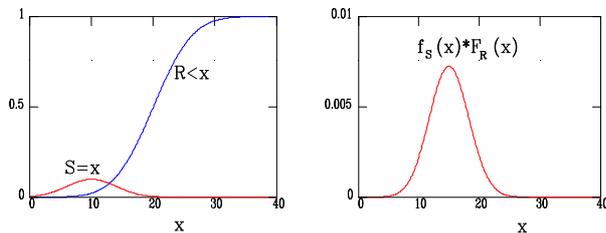


Figure 13. Components of the failure probability

We have to compute the sum of the probabilities for all possible values of x :

$$P_f = \int_{-\infty}^{\infty} f_s(x) F_R(x) dx \quad (26)$$

This method can be applied when the strength and the load are independent of each other. Figure 14 gives the joint probability density function for the strength and the load for a certain failure mode in which the strength and the load are not independent. The strength is plotted on the horizontal axis and the load is plotted on the vertical axis. The contours give the combinations of the strength and the load with the same probability density. In the area ($Z < 0$) the value of the reliability function is less than zero and the element will fail. (Van Gelder 1996).

The failure probability can be determined by summation of the probability density of all the combinations of strength and load in this area.

$$P_f = \iint_{Z < 0} f_{RS}(r, s) dr ds \quad (27)$$

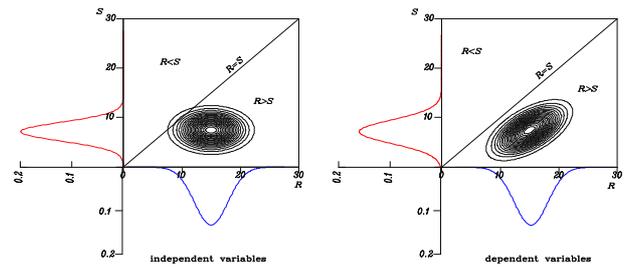


Figure 14. Joint probability density function

In a real case the strength and the load in the reliability function are nearly always functions of multiple variables. For instance the load can consist of the water level and the significant wave height. In this case, the failure probability is less simple to evaluate. Nevertheless with numerical methods like numerical integration and Monte Carlo simulation it is possible to solve the integral:

$$P_f = \iint_{Z < 0} \dots \iint_{r_1, s_1, \dots, r_n, s_n} f_{r_1, s_1, \dots, r_n, s_n}(r_1, r_2, \dots, r_n, s_1, s_2, \dots, s_n) dr_1 dr_2 \dots r_n ds_1 ds_2 \dots ds_n \quad (28)$$

These methods which take into account the real distribution of the variables are called level III probabilistic methods. If the reliability function (Z) is a sum of a number of normal distributed variables then Z is also a normal distributed variable. The mean value and the standard deviation can easily be computed with these equations:

$$Z = \sum_{i=1}^n a_i x_i \quad (29)$$

$$\mu_z = \sum_{i=1}^n a_i \mu_{x_i} \quad (30)$$

$$\sigma_z = \sqrt{\sum_{i=1}^n (a_i \sigma_{x_i})^2} \quad (31)$$

This is the base of the level II probabilistic calculation. The level II methods approximate the distributions of the variables with normal distributions and they estimate the reliability function with a linear first order Taylor polynomial, so that the Z -function is normal distributed. If the distribution of the Z -function is normal and the mean value and the standard deviation are known it is easy to determine the failure probability.

By computing β as μ divided by σ it is possible to use the standard normal distribution to estimate the failure probability. There are tables available of the standard normal distribution in the handbooks for statistics. In case of a non linear Z -function it will be estimated with a Taylor polynomial:

$$Z(\bar{x}) \approx Z(\bar{x}^*) + \sum_{i=1}^n \frac{\partial Z}{\partial x_i} \cdot (x_i - x_i^*) \quad (32)$$

The function is depending of the point where it will be linearised. The mean value and the standard deviation of the linear Z-function are:

$$\mu_z \approx Z(\bar{x}^*) + \sum_{i=1}^n \frac{\partial Z}{\partial x_i} \cdot (\mu_{x_i} - x_i^*) \quad (33)$$

$$\sigma_z = \sqrt{\sum_{i=1}^n \left(\frac{\partial Z}{\partial x_i} \cdot \sigma_{x_i} \right)^2} \quad (34)$$

If the reliability function is estimated by a linear Z-function in the point where all the variables have their mean value ($X_i^* = \mu_{x_i}$), we speak of a Mean Value Approach. The so-called design point approach estimates the reliability function by a linear function for a point on $Z=0$ where the value β has its minimum. Finding the design point is a minimising problem. For this problem there several numerical solutions which will not be discussed here. (Van Gelder 1996)

Non Normally Distributed Basic Variables

If the basic variables of the Z-function are not normally distributed, the Z-function will be unknown and probably not normally distributed. To cope with this problem the not normally distributed basic variables in the Z-function can be replaced by a normally distributed variable. For the design point, the adapted normal distribution must have the same value as the real distribution. Because the normal distribution has two parameters (μ and σ) one condition is not enough to find the right normal distribution. Therefore the value of the adapted normal probability density function must also have the same value as the real probability density function (figure 15-16).

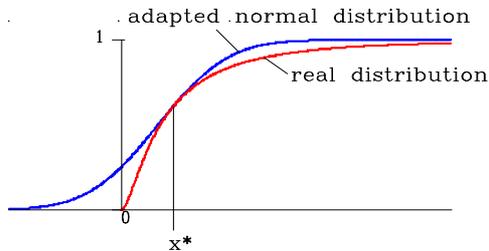


Figure 15 Adapted normal distribution

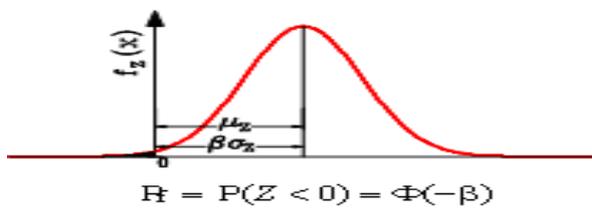


Figure 16 Probability density of the Z-function

The two conditions give a set of two equations with two unknown which can be solved:

$$\left. \begin{aligned} F_N(x^*) &= F_x(x^*) \\ f_N(x^*) &= f_x(x^*) \end{aligned} \right\} \Rightarrow \mu_N, \sigma_N \quad (35)$$

This method is known as the Approximate Full Distribution Approach (AFDA).

For the general case, where the basic random variable can be correlated, the probability of failure can determine by solving the following integral: (Van Gelder 1996)

$$P_f = \int \int_{Z(0)} f_{rs}(r,s) dr ds = \int \dots \int_{Z(0)} (x_1, x_2, \dots, x_n) dx_1 dx_2 \dots dx_n \quad (36)$$

First order reliability method (FOSM)

The idea here is that, if we know the means and the variances (the second moments) of the variables that enter into the evaluation of a function such as Z, we can estimate the mean and variance of Z using only first order terms in a Taylor expansion, in which the x_i are uncertain variables.

$$\mu_z \approx Z(\mu_{x_1}, \mu_{x_2}, \dots, \mu_{x_n}) \quad (37)$$

$$\sigma_z^2 \approx \sum_{i=1}^n \sum_{j=1}^n \left(\frac{\partial Z}{\partial X_i} \right)_\mu \left(\frac{\partial Z}{\partial X_j} \right)_\mu \text{cov}(X_i, X_j) \quad (38)$$

When the variables are uncorrelated, a somewhat more complicated expression is used. When it is difficult to evaluate the partial derivatives directly, central divided partial differences usually provide sufficient accuracy.

First order second moment (FOSM) approach is that the result depends on the particular values of the variables x_i at which the partial derivatives are calculated. Hasofer and Lind (1974) proposed to resolve this difficulty by evaluating the derivatives at the critical point on the failure surface.

Finding this point usually requires iteration, but the process tends to converge rapidly. If the variables are all normalized by dividing them by their respective standard deviations, the distance between the failure point and the point defined by their normalized means is the reliability index β . This method assumes normal distributions and must be modified to accommodate other distributions.

$$\beta = \frac{\mu_z}{\sigma_z} \quad (39)$$

$$P_f = \Phi(-\beta) \quad (40)$$

Monte Carlo Method

Monte Carlo simulation is a powerful analysis tool that involves a random number generation and simulates the behaviour of a variable when the data is insufficient to make decisions. The random number generation is based on a probability density function that defines the variable variation. Randomness is used to describe events whose outcomes are uncertain; random variables count or measure that which is of interest to analyse.

The first step to the Monte Carlo process is to build a mathematical model with a set of relationships that simulates a real system. Then it is necessary to define the inputs and outputs variables. When the inputs and outputs are restricted to one value (each parameter takes only one value), we are dealing with a deterministic model. On the

other hand, when the inputs and outputs are represented by random numbers or a probability density function, the model is known as stochastic or probabilistic. Monte Carlo simulation combines the principles of probability and statistics with the expert opinion and data sources to quantify the uncertainty associated with the real systems.

The Monte Carlo simulation method uses the possibility of drawing random number from a uniform probability density function between zero and one. Each continuous variable is replaced by a large number of discrete values generated from the underlying distributed, these values are used to compute a large number of values of function Z and its distribution.

$$F_X(x) = X_u \quad (41)$$

X_u is the uniformly distributed variable between zero and one, $F_X(x)$ is the non-exceedence probability $P(x < X)$. Thus, for the variable X :

$$X = F_X^{-1}(x_u) \quad (42)$$

$F_X^{-1}(x_u)$ is inverse of the probability distribution function of X .

To draw a value out of a joint probability density function, the function must be formulated as the product of the conditional probability distributions of the base variables. In formula:

$$F_X(\vec{X}) = F_{X_1}(X_1) \cdot F_{X_2|X_1}(X_2|X_1) \cdots F_{X_n|X_1, X_2, \dots, X_{n-1}}(X_n|X_1, X_2, \dots, X_{n-1}) \quad (43)$$

By taking n realizations of the uniform distribution between zero and one, a value can be determined for every X_i .

$$X_1 = F_{X_1}^{-1}(X_{u_1})$$

$$X_2 = F_{X_2|X_1}^{-1}(X_{u_2}|X_1)$$

$$X_n = F_{X_n|X_1, X_2, \dots, X_{n-1}}^{-1}(X_{u_n}|X_1, X_2, \dots, X_{n-1}) \quad (44)$$

This corresponds to Eqn (47). By inserting the values for the reliability functions one can check whether the obtained vector (X_1, X_2, \dots, X_n) is located in the safe area. By repeating this procedure a large number of times, the probability of failure can be estimated with:

$$P_f = n_f / n \quad (45)$$

In which, n is the total number of simulations and n_f is the number of simulations, for which $Z < 0$. (Van Gelder 1996). There are also several serious questions of convergence and of randomness in the generated variables. Several so-called variance reduction schemes can be effective in improving convergence and reducing computational effort. Fishman (1995) provides one of many treatments of the method. Monte Carlo simulation with variance reduction is particularly helpful in improving the accuracy of first order reliability method (FORM) results.

Fault Tree

Fault tree, and influence diagrams are techniques for describing the logical interactions among a complex set of events, conditions. Formal calculation of the failure risk can be determined by incorporating the fault tree analysis (Henly and Kumamoto 1981; Ang and Tang 1984; Yen and Tung 1993). The fault tree contains the conditions that must be met for the failure to occur. There are four main intermediate events on top geotechnical, Hydrological, Structure, and Mechanical.

The analyst develops the tree from the top down, moving from condition to condition. In the usual formulation, the conditions at each stage must be independent and must encompass all the conditions that could lead to the next stage. Fault trees have also been used in Science Engineering practice. The Fault tree diagram displays the relations between various events and conditions in a system. The direction of the arrows and other conventions represent the dependencies between the objects as shown by Vrijling, J. K. et al., (1996).

Reliability Simulation

Application of the simulation method for uncertainty analysis requires formulation of the performance function.

In this case of a hydraulics structure, the performance function Z expresses in terms of the margin safety is presented in Eqn. (49-51)

where d_s is depth, L_s length and W_s is width of the plunge pool, and R_d , R_l , R_w are resistance, respectively depth, length and width then, Considering Esq. (25), the performance function can be written as:

$$Z_d = R_d - 15.89 (Fr_N)^{1.1308} \left(\frac{H_1}{d_w}\right)^{-0.3019} \left(\frac{R}{d_w}\right)^{-0.2597} \left(\frac{d_{50}}{d_w}\right)^{0.1002} (\sin\theta)^{0.4014} \quad (46)$$

$$Z_L = R_L - 7.20 (Fr_N)^{0.3381} \left(\frac{H_1}{d_w}\right)^{0.3918} \left(\frac{R}{d_w}\right)^{0.0774} \left(\frac{d_{50}}{d_w}\right)^{0.0203} (\sin\theta)^{0.2386} \quad (47)$$

$$Z_w = R_w - 0.942 (Fr_N)^{0.0223} \left(\frac{H_1}{d_w}\right)^{0.8182} \left(\frac{R}{d_w}\right)^{0.1351} \left(\frac{d_{50}}{d_w}\right)^{0.0071} (\sin\theta)^{-1.339} \quad (48)$$

In this case, estimates of the reliability of the geometry of plunge pool are generated for different resistances between (0.2- 20), for depth, length and width, respectively. The main resistance and load parameter are as presented in Table 2.

The main analysis was made considering the above random variables. The possible failure mechanism is a scour hole with the above given limit state functions. Equation (49-51) is nonlinear; therefore, a simulation technique, such as Monte Carlo analysis can be used to determine reliability. In Monte Carlo simulation, random numbers between zero and one are generated for the variables having uniform distribution (Ang and Tang, 1984). The summarize of results indicated in Tables 3 to 5.

Table 2 Basic variable characterizes plunge pool

Variable	q	H ₁	R	d ₅₀	Θ	ds	L _s	W _s
μ	0.068	0.633	0.259	0.0069	0.590	0.211	0.915	0.874
σ	0.042	0.459	0.162	0.0039	0.180	0.099	0.508	0.419
distribution	LN	N	N	LN	N	N	N	N

Table 3 design points and reliability for Z_d=R_d-(ds/dw)

R _d	H ₁	R	Θ	d ₅₀	dw	q	ds	P(z)	β
0.2	0.711	0.274	0.515	0.006	0.283	0.023	0.0588	0.997	-2.7
3	0.52	0.222	0.562	0.006	0.084	0.055	0.2520	0.55	-0.128
4	0.503	0.217	0.567	0.006	0.074	0.061	0.2976	0.44	0.146
6	0.48	0.21	0.575	0.006	0.062	0.07	0.3729	0.3	0.533
10	0.453	0.202	0.584	0.006	0.049	0.083	0.4943	0.154	1.02
14	0.435	0.197	0.591	0.006	0.042	0.092	0.5896	0.09	1.34
16	0.429	0.195	0.593	0.006	0.04	0.097	0.6384	0.07	1.47
20	0.418	0.191	0.598	0.006	0.036	0.104	0.7199	0.047	1.68
25	0.407	0.188	0.602	0.006	0.033	0.112	0.8109	0.029	1.89
30	0.398	0.185	0.606	0.006	0.03	0.119	0.9003	0.02	2.07
40	0.385	0.181	0.611	0.006	0.027	0.132	1.0576	0.0096	2.34

Table 4 design points and reliability for Z_L=R_L-(Ls/dw)

R _L	H ₁	R	Θ	d ₅₀	dw	q	Ls	P(z)	β
0.2	0.146	0.18	0.461	0.006	1.805	0.024	0.365	1.000	-5.58
4	0.365	0.208	0.534	0.006	0.184	0.046	0.741	0.930	-1.51
6	0.413	0.212	0.545	0.006	0.135	0.05	0.814	0.830	-0.954
10	0.484	0.218	0.559	0.006	0.092	0.056	0.920	0.600	-0.257
12	0.511	0.22	0.564	0.006	0.08	0.058	0.959	0.503	-0.009
13	0.524	0.22	0.566	0.006	0.075	0.059	0.977	0.460	0.1
15	0.548	0.222	0.57	0.006	0.067	0.061	1.012	0.3840	0.296
20	0.599	0.225	0.579	0.006	0.054	0.064	1.078	0.246	0.688
25	0.641	0.227	0.585	0.006	0.046	0.068	1.141	0.160	0.993
30	0.679	0.229	0.59	0.006	0.04	0.07	1.187	0.107	1.24
40	0.742	0.232	0.599	0.006	0.032	0.075	1.274	0.051	1.63

Table 5 design points and reliability for Z_w=R_w-(Ws/dw)

R _w	H ₁	R	Θ	d ₅₀	dw	q	Ws	P(z)	β
0.2	0.103	0.179	0.998	0.006	0.499	0.056	0.102	1.000	-4.28
1	0.196	0.194	0.794	0.006	0.239	0.057	0.239	0.995	-2.56
5	0.376	0.211	0.63	0.006	0.113	0.057	0.566	0.800	-0.825
6	0.405	0.213	0.614	0.006	0.104	0.058	0.623	0.740	-0.628
9	0.477	0.218	0.579	0.006	0.086	0.058	0.775	0.580	-0.189
10	0.498	0.219	0.57	0.006	0.082	0.058	0.820	0.530	-0.075
12	0.536	0.221	0.555	0.006	0.075	0.058	0.905	0.450	0.122
15	0.587	0.223	0.538	0.006	0.068	0.058	1.018	0.360	0.363
20	0.66	0.227	0.516	0.006	0.059	0.058	1.189	0.250	0.675
25	0.722	0.23	0.499	0.006	0.054	0.058	1.342	0.180	0.916
40	0.874	0.232	0.467	0.006	0.043	0.059	1.720	0.080	1.42

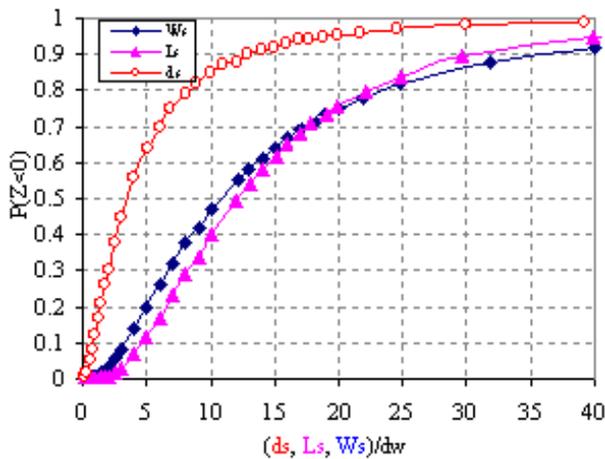


Figure16. Probability of failure in scour hole

Discussion

It is very important that any designer should keep the downstream and upstream flow characteristics in mind before designing a conveyance structure after the flip bucket spillway structure. For safety and stability of the structure, special attention must be given to monitor downstream scour holes.

The fitting of the power models of equations 16, 17 and 18 using the logarithmic transformation and leads to an equation that has unbiased partial regression coefficients and provides unbiased estimates of d_s , L_s and W_s .

The residual prediction for depth and length and width shows in Figures 2, 5, and 8 respectively. It shows the range number of residual for depth is between (-4, 4), length and width (-3, 3) respectively.

The statistical analysis of the data predicted by the equation developed in this study and comparison with observed values in this model (table 1) show high values of the correlation coefficient (0.99, 0.98 and 0.95), and less value of the root mean square error (0.4, 0.998 and 1.823) which indicate that the equations developed herein accurately predict model scour geometry.

Comparisons between 5 hydraulic models for design are shown in Fig. 17. It shows that correlation coefficient between observed (X) and predicted (Y) values in this method are more than other methods.

Conclusions

In this paper, we presented the traditional empirical formulas used to obtain prediction of geometry scour hole with the probabilistic multivariate regression, and the analysis of reliability estimation of scour hole downstream of flip bucket spillway.

It is based on the approach of probabilistic methods and it involved analysis of an extensive data base in order to obtain the geometry of scour hole after the flip bucket spillway of the given parameters of q , H_1 , R , Θ , d_w and d_{50} .

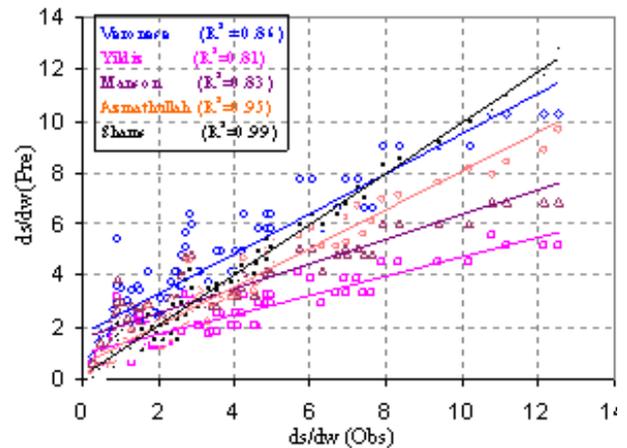


Figure 17.comparison between five methods

Reliability analysis can provide a formal approach to the analysis of the performance of hydraulic structures, taking all uncertainties in models, load- and strength variables into account.

Further analysis of the other components and prototype data should be carried out in order to find the weak links of the hydraulic structure. The vulnerability of a hydraulic structure can then be improved by strengthening these weak elements.

References

- Amiri-Simkooei AR (2007). "Least-squares variance component estimation." PhD thesis, Delft Univ., The Netherlands, ISBN 10-90-804147-5-1.
- Ang, A.H.S., and Tang, W., (2007). "Probability concept in engineering Emphasis on Application to Civil and Environmental Engineering." John Wiley & Sons, New York.
- Azmathullah, H.Md., Deo, M.C., and Deolalikar, P.B., (2005). "Neural Networks for Estimation of Scour Downstream of a Ski-Jump Bucket." J. Hydr. Eng., Vol. 131, No. 10, ASCE, ISSN 0733-9429/2005/10-898-908.
- Fishman, G. S. (1995). "Monte Carlo: Concepts, algorithms, and applications." Springer, New York.
- Heller, V., Hager, W.H., and Minor, H.E (2005). "Ski Jump Hydraulics." J. Hydr. Eng., Volume 131, Issue 5, pp. 347-355
- Henley, E. J., and Kumamoto, H. (1981). "Reliability engineering and risk assessment." Prentice-Hall, Englewood Cliffs, N.J.
- Khaturia, R. M., (2005). "Hydraulics of spillways and energy dissipators". New York. Dakker, ISBN 0-8247-5789-0
- Lopardo, R.A., Lopardo, M.C., and Casado, J.M. (2002). "Local rock scour downstream large dams." Proc., Int. Workshop on Rock Scour Due to High Velocity Jets, Lausanne, Switzerland, 55-58.
- Mason, P. J., and Arumugam, K. (1985). "Free jet scour below dams and flip buckets." J. Hydraul. Eng., 111(2), 220-235.

McCuen, R.H., Leahy, R.B., and Johnson, P.A., 1990. "Problems with logarithmic transformations in regression." *J. Hydr. Eng-ASCE*, 116(3) 414-428 .

Rajaratnam, N. "Erosion by Submerged Circular Jets", *J. Hydr. Eng, ASCE*, 108(HY2), 262-267, 1982.

Rajaratnam, N., and Mazurek, K. A. _2002_. "Erosion of a polystyrene bed by obliquely impinging circular turbulent air jets." *J. Hydraul.Res.*, 40_6_, 709-716.

Tung, Y. K., and Mays, L. W. (1980). "Risk analysis for hydraulic design." *J. Hydr. Div., Am. Soc. Civil. Eng.*, 106(5), 93-913.

Van Gelder, P.H.A.J.M.(2000). "Statistical Methods for the Risk-Based Design of Civil Structures." PhD thesis, Delft Univ., The Netherlands, ISBN 90-9013452-3.

Van Gelder, P.H.A.J.M., 1996b. "How to deal with wave statistical and model uncertainties in the design of vertical breakwaters." In H.G. Voortman, editor, *Probabilistic Design Tools for Vertical Breakwaters; Proceedings Task 4 Meeting, Hannover, Germany*, pages 1-13.

Vischer, D. L., and Hager, W. H. (1998). *Dam hydraulics*, Wiley, Chichester, England.

Vrijling J.K. and Bruinsma J., (1980). "Hydraulic Boundary Conditions". *Proc., Int. Hydraulic Aspects of Coastal Structures; developments in hydraulic engineering related to the design of the Oosterschelde*

storm surge barrier in the Netherlands. Editors: A. Paape, J. Stuij, W.A.Venis, Delft Univ Press 1980, 109-133, ISBN 90-6275-0540.

Vrijling, J. K. (2001). "Probabilistic design of water defense systems in the Netherlands." *Journal of Reliability Engineering & System Safety*, 74(3), 337-344.

Vrijling, J. K. et al., (1996). "Probability in Civil Engineering." Publication of the faculty of the civil engineering, CT 4130, TU Delft, The Netherlands.

Wang, S. (1987). "Scouring of river beds below sluices and dams." *Design of Hydraulic Structures—Proc., International Symp. on Design of Hydraulic Structures, Colorado State Univ., Fort Collins, Colo.*, 295-304.

Yanmaz, A.M. (2003). "Reliability Simulation of Scouring Downstream of Outlet Facilities ", *Turkish .J. Eng. Env. Sci.* ,65-7127

Yen, B. C., and Tung, Y. K. (1993). "Some recent progress in reliability analysis for hydraulic design." *Reliability and uncertainty analysis in hydraulic design*, B. C. Yen and Y. K. Tung, eds., ASCE, Reston, VA., 35-79.

Yildiz, D., and Üzücek, E. (1994). "Prediction of scour depth from free falling flip bucket jets." *J. Water Power Dam Construction*, 46(11), 50-56.

Appendix

In this paper the following notation was used:

d_s =Scour depth [m]
 L_s =Length of apron [m]
 W_s =Width of structure [m]
 H_1 =Upstream water depth [m]
 R = Bucket radius [m]
 Θ =Lip angel [rad]
 d_{50} =Median bed material diameter [mm]
 σ_{50} =Standard deviation bed material [mm]
 G_s =Specific gravity
 ρ_w =Density of water [kg/m³]
 ρ =Density of bed particle [kg/m³]
 μ_w =Dynamic viscosity [pas]
 v =Average velocity [m/s]
 q =Discharge [m³/s.m]
 t =Time [s]
 g =Gravity acceleration [m/s²]
 d_w =Normal water depth [m]
 Fr =Froude number
 \hat{Y} = predicted value of the criterion (dependent)variable Y

α_i = sample estimates
 X =Observed value
 Y =Predicted value
 Z =reliability function
 μ = Mean value
 σ = Standard deviation
 R_i =Strength
 L_j =Load
 P_f =Probability of failure
 P_s =Probability of safe
 $F_{(x)}$ =Probability distribution
 $f_{(x)}$ =Density function
 β =Reliability index
 R_d =Resistance for depth
 R_L =Resistance for length
 R_W =Resistance for width
 U =Velocity in X direction [m/s]
 V =Velocity in Y direction [m/s]
 n_f =Number of simulations leading to failure
 n =Total number of simulations