In the design of civil structures, such as dikes, storm surge barriers, bridges, buildings, offshore platforms, etc., cost-benefit analyses have become common to determine the safety target of the structural design. The designer should model the trade-off mechanism between structural costs and benefit due to a reduction in the failure probability. In this study statistical methods for the risk-based design of civil structures have been evaluated, compared and extended. For that purpose, the study has been divided into four main topics:

- the structuring and analysis of various kinds of uncertainties,
- the treatment of the statistical estimation methods,
- the analysis of the homogeneity aspects of data,
- the investigation of design philosophies for civil structures.

The study contains several case studies where high degrees of reliability are demanded and where statistical information is scarce.

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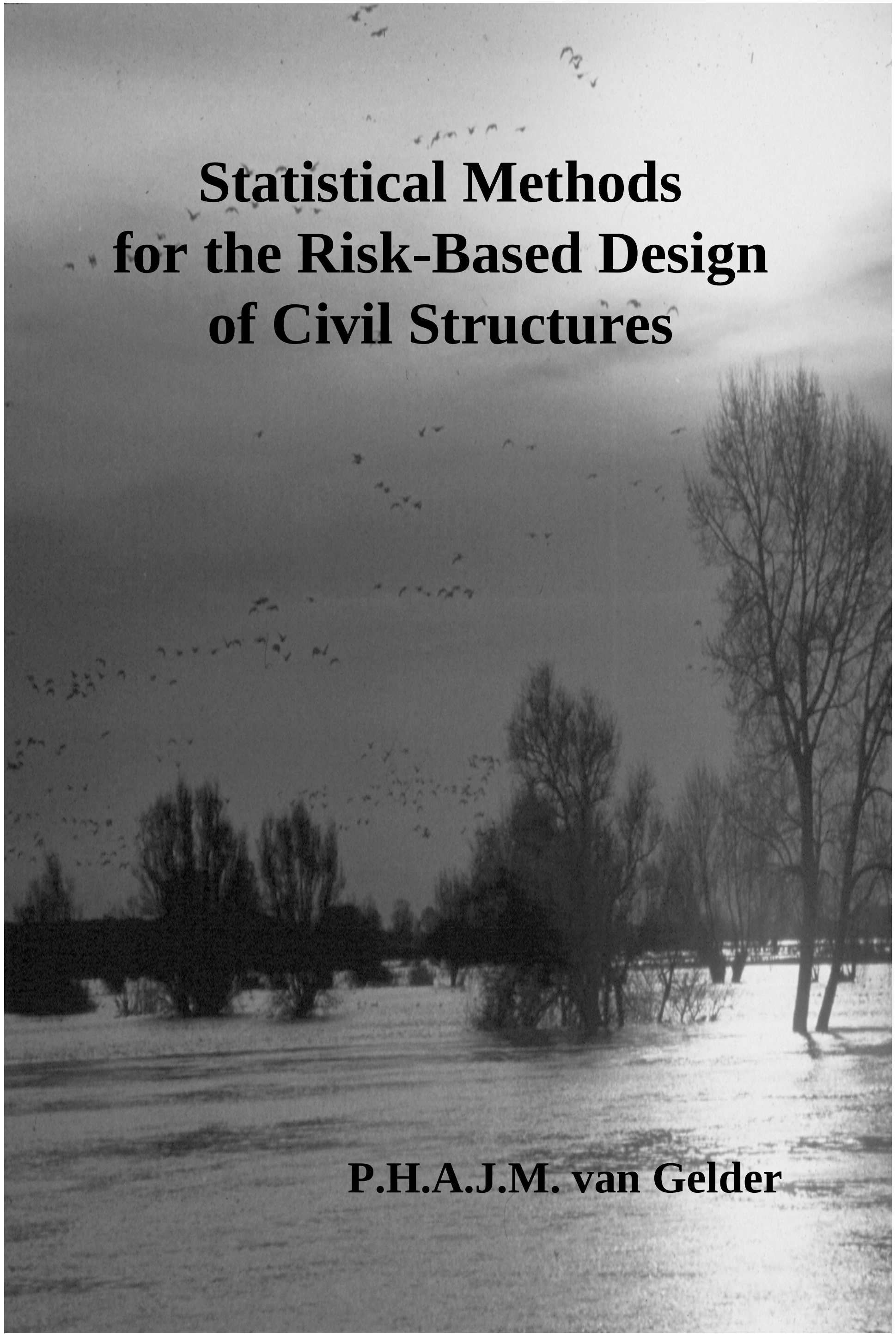

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Statistical Methods for the Risk-Based Design of Civil Structures

PROEFSCHRIFT

ter verkrijging van de graad van doctor
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in het openbaar te verdedigen ten overstaan van een commissie,
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door

Petrus Hermanus Antonius Johannes Maria VAN GELDER

wiskundig ingenieur

geboren te Oss

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Front cover: high water levels along the river IJssel at Ravenswaarden in December 1993, photo by Rijkswaterstaat - Dienst Weg en Waterbouwkunde, Delft

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The present PhD-thesis has been reviewed by the PhD-committee as given on the inside of the cover. However, also two additional reviewers, namely Jan van Noortwijk and Bernard Bobée, have commented in detail on the thesis. I appreciate their time and advises.

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Zoetermeer, December 1999 Pieter van Gelder

Summary

Statistical Methods for the Risk-Based Design of Civil Structures

Probabilistic methods are with increasing frequency used in the design of civil structures such as dikes, storm surge barriers, bridges, buildings, etc. The methods are being applied directly, or are translated to relatively simple design rules with safety coefficients. In both cases the foundation of the calculations is given by the statistical distribution functions of the strength and load variables.

In the application of probabilistic methods, the availability of useful calculation models and adequate statistical distributions is required. The development of calculation models has had a lot of attention during the last years, and has therefore, besides some exceptions, no need for further care. A larger problem in the application of probabilistic calculation techniques is the lack of good argued statistical models. It is on this area that the current thesis would like to give a contribution to the knowledge. The question is not how to determine the distributions themselves, but is more directed towards the methodology to develop a determination strategy. Of importance is the characteristic that usually only few observations are available and that particular interest is given in the tails of the distribution. In this thesis the most common estimation methods are presented with respect to their use in civil engineering problems. Apart from the standard methods such as the method of moments, maximum likelihood, bootstrap and least-squares, also Bayesian, L-Moments, Entropy, and non-parametric methods are shown. As for the distribution functions, the normal, lognormal, uniform, exponential, Rayleigh, Gumbel, Gamma (Pearson type III), Weibull, Fréchet, Generalised Extreme Value, (Generalised) Pareto, (Generalised) Logistic, Kappa, and Wakeby are considered. In all cases, methods for finding the best estimate, as well as a quantification of the uncertainty in the estimate, are described. How to deal with uncertainties is an essential part of this thesis. In many applications the errors in one direction (e.g. overdesign) are less severe than errors in another direction (e.g. underdesing). In order to analyse if a certain method is sufficiently robust also artificially disturbed series (disturbed with measurement errors or from a discordant distribution) will be generated. The sensitivity of the method against such deviations is investigated.

The following three situations can be discerned:

- there is an abundance of statistical data;
- there are only few observations;
- there is no data.

In the first case, which in civil engineering practice only rarely occurs, there is no theoretical problem: all available techniques lead to the same theoretical result. Practically, disturbance or pollution of the data can be a problem, or the fact that no simple mathematical model can be considered as sufficiently accurate. In the third case one relies on the judgements of experts. The last years there is a strong development to translate expert uncertainty to probability distributions. Although these methods can enlarge the applicability of probabilistic design methods, they cannot be a substitution for physical measurements. The thesis focusses in particular on the second case: there are only few observations and it is not possible, or not economically feasible, to enlarge the amount of data. In civil engineering practice this is the most common case. The current thesis examines this case with different statistical methods, under which the Bayesian methods. These Bayesian methods are developed to optimally use background information in a decision problem with a scarce amount of data.

The goal of this thesis is to explore, compare, and critically evaluate the various statistical methods, to identify methods which are applicable in civil engineering, and to operationalise the chosen methods. In the exploration of the statistical methods various types of uncertainties, such as inherent, model and statistical, play an important role. There is an interaction with the final design decisions which have to be taken on the basis of the estimates: a too low estimate of a load parameter can have more severe consequences than a too high estimate. There exist also inverse situations. These kinds of interactions are treated in this thesis with Bayesian decision theory.

Also attention to the distribution choice is given in this thesis. Since small probabilities of failure are commonly required in civil engineering practice, the tails of the distribution are extremely crucial. Sometimes theoretical considerations can be made for a certain distribution, but in any case, as much as possible information should be derived from the available observations. Methods such as χ–square and Kolmogorov-Smirnov tests are not suitable for this purpose. In this thesis other techniques have been developed. Apart from Bayesian solutions, Entropy methods and L-Kurtosis approaches appear to perform satisfactory. Also the earlier mentioned interaction with the design problem shows up at this point.

In the recommendations of the thesis also issues such as cleaning and homogenisation of data are mentioned. Which part of the dataset is useful, depends on the application. Sometimes, the whole distribution function is of interest; sometimes only the tails. The thesis examines this aspect as well.

The insights and tools of part I of the thesis are applied to a few case studies. A number of case studies is presented in part I, such as the reliability analysis of dikes along Lake IJssel, and the frequency analysis of more than 200 measurement stations along the West- and Middle European rivers. In part II, the statistical analysis of high sea water levels along the Dutch coast and the wave heights on the North Sea are investigated. In the first case study the issue was how to include historical information about floods in the Middle Ages in the statistical analysis of the instrumental dataset of the last century. In the second case study the issue was how to include regional information about wave heights at neighbouring measurement stations in the statistical analysis of a particular location. The acquired theory of part I was very suitable to perform such kind of investigations.

Samenvatting

Statistische Methoden voor het Risico-Gebaseerd Ontwerpen van Civiele Constructies

In toenemende mate wordt in de civiele techniek bij het ontwerpen van constructies, zoals dijken, stormvloedkeringen, bruggen, gebouwen, etc., gebruik gemaakt van probabilistische methoden. Soms worden deze methoden direct toegepast, soms wordt eerst in een voorschrift een vertaling gegeven naar relatief eenvoudige ontwerpregels met behulp van veiligheidscoëfficiënten. In beide gevallen liggen aan de berekening statistische verdelingen van de sterkte en de belastingparameters ten grondslag.

Voor de toepassing van probabilistische methoden is het nodig dat men beschikt over bruikbare rekenmodellen en adequate statistische verdelingen. De ontwikkeling van rekenmodellen heeft daarbij in de afgelopen jaren het meest de aandacht gehad en vormt, uitzonderingen daargelaten, niet de eerste zorg. Een groter probleem voor toepassing van de probabilistische berekeningstechnieken is het ontbreken van goed onderbouwde statistische modellen. Het is op dit terrein dat dit proefschrift een bijdrage aan de kennis wil geven. Daarbij gaat het niet in de eerste plaats om deze verdelingen zelf te bepalen, maar om de methodiek van de bepaling te ontwikkelen. Van belang is daarbij het kenmerk dat er meestal weinig waarnemingen zijn en men bijzondere interesse heeft voor de staarten van de verdeling. In het proefschrift zullen de meest bekende schattingsmethoden worden getoond op hun bruikbaarheid in problemen die typerend zijn in de civiele techniek. Naast de standaard-methoden zoals momenten methode, maximum likelihood, bootstrap en kleinste kwadraten-methode, zullen ook Bayesiaanse, L-Momenten, Entropy, en niet-parametrische methodes getoond worden. Als verdelingen zullen de normale, de lognormale, de uniforme, de exponentiële, de Rayleigh, de Gumbel, de Gamma (Pearson type III), de Weibull, de Frechet, de Gegeneraliseerde Extreme Waarde, de (Gegeneraliseerde) Pareto, de (Gegeneraliseerde) Logistische, de Kappa, en de Wakeby verdeling worden meegenomen. In alle gevallen gaat het zowel om methoden om de beste schatting te vinden, als om een kwantificering van de onzekerheid in de schatting. Het omgaan met de onzekerheid is een essentieel onderdeel van dit proefschrift. In veel toepassingen zijn bijvoorbeeld fouten in de ene richting minder ernstig dan fouten in de andere richting. Om na te gaan of een methode voldoende robuust is wordt in het proefschrift ook gewerkt worden

met kunstmatige verstoorde reeksen. Bijvoorbeeld gegenereerd op basis van een afwijkende verdeling of toevoeging van meetfouten. De gevoeligheid van de methode voor dit soort verstoringen is onderzocht.

Voor de discussie is het doelmatig onderscheid te maken tussen de volgende drie gevallen:

- er is een overvloed aan statistische gegevens;
- er zijn enkele waarnemingen;
- er is geen enkele waarneming.

In het eerste geval, dat met name in de civiele techniek meer uitzondering dan regel is, heeft men theoretisch geen probleem: alle beschikbare technieken leiden naar theoretisch hetzelfde resultaat. Practisch kan echter verstoring of vervuiling van het waarnemingsmateriaal een probleem zijn of komt men tot de ontdekking dat geen enkel mathematisch eenvoudig hanteerbaar model als voldoende nauwkeurig aangewezen kan worden. In het derde geval is men overgeleverd aan meningen van experts. De laatste jaren laten een sterke ontwikkeling zien in methoden om expert onzekerheid te vertalen in waarschijnlijkheidsverdelingen. Alhoewel deze methoden de toepasbaarheid van probabilistische ontwerpmethoden kunnen vergroten, vormen ze geen vervanging voor fysische metingen. Het proefschrift spitst zich met name toe op de tweede categorie: er zijn enkele waarnemingen, maar het is niet mogelijk of niet economisch haalbaar om dit aantal uit te breiden. In de civiele techniek is dit de meest voorkomende situatie. Het proefschrift behandelt deze situatie met verschillende statistische methoden, waaronder de Bayesiaanse methoden. Deze Bayesiaanse methoden zijn met name ontwikkeld om optimaal gebruik te maken van achtergrondkennis bij het onttrekken van informatie uit schaarse gegevens voor een beslisprobleem.

Doelstelling van dit proefschrift is het exploreren, vergelijken en kritisch evalueren van de diverse statistische methoden, het identificeren van methoden die in aanmerking komen voor toepassing in de civiele techniek, en het operationeel maken van de gekozen methoden. Bij het exploreren van de statistische methoden spelen diverse typen onzekerheden, zoals inherente -, model - en statistische onzekerheden een belangrijke rol. Binnen het ontwerp dient hiermee correct te worden omgegaan. Hier treedt een interactie op met de uiteindelijke ontwerpbeslissing die op grond van de schatting moet worden genomen: een te lage schatting van een belastingparameter kan bijvoorbeeld veel vervelender consequenties hebben dan een te hoge. Er zijn echter ook gevallen waarin het omgekeerde geldt. In het proefschrift is met dit soort interacties Bayesiaanse beslistheorie toegepast.

Tevens wordt in het proefschrift aandacht gegeven aan de keuze van het verdelingstype.

Met name omdat in de civiele techniek kleine faalkansen geeist worden zijn staarten van verdelingen van groot belang. Meestal vindt extrapolatie plaats tot ver buiten het waarnemingsmateriaal. De keuze van het verdelingstype kan dan cruciaal zijn. Soms zijn er theoretische overwegingen voor een bepaald type verdeling, maar in ieder geval moet zoveel mogelijk informatie uit het waarnemingsmateriaal gehaald worden. Methoden als χ-kwadraat en de Kolmogorov-Smirnov test blijken daarbij in de praktijk vaak niet te voldoen. In het proefschrift is hierbij naar nieuwe wegen gezocht. Naast Bayesiaanse oplossingen blijken ook Entropy-methoden en L-Kurtosis benaderingen zeer wel te werken. Ook de eerder genoemde interactie met het ontwerpprobleem komt hierbij weer om de hoek.

Bij de praktische aanbevelingen in het proefschrift horen evenzeer zaken als het "opschonen en homogeniseren" van de data en het bepalen van de keuze omtrent het "bruikbare deel" van het data-bestand. Dit "bruikbaarheidscriterium" hangt op zijn beurt weer samen met het toepassingsgebied; soms is men geinteresseerd in de hele verdeling, soms alleen in een van de staarten.Het proefschrift gaat hier ondermeer op in.

Met behulp van de inzichten en de gereedschappen verworven in deel I van het proefschrift zijn enkele practische gevallen onderzocht. Een aantal case studies is in deel I behandeld, zoals de betrouwbaarheidsanalyse van de dijken langs het IJsselmeer en de frequentie-analyses van meer dan 200 meetstations aan de West- en Midden Europese rivieren. In deel II zijn verder de statistische analyse van hoogwaterstanden langs de Nederlandse kust en de golfhoogten op de Noordzee aan bod gekomen. In de eerste case study is onderzocht hoe gegevens over overstromingen uit de Middeleeuwen meegenomen konden worden bij de statistische analyse van de instrumentele dataset uit deze eeuw. In de tweede case study is onderzocht hoe gegevens van nabij liggende locaties meegenomen konden worden bij de statistische analyse van golfhoogten voor een gegeven locatie. De theorie verworven uit deel I blijkt zeer geschikt voor het uitvoeren van deze analyses.

Contents

Chapter 1

Introduction

All our knowledge is but the
knowledge of schoolchildren.
- A. Einstein.

This thesis is concerned with mathematical models of risks and safety issues with respect to protection measures against natural hazards. The natural hazards that will be considered in this thesis will mainly be floods from sea and rivers. However, the modeling approaches will also be applied to other types of natural hazards, such as earthquakes, extreme winds, – precipitation, – wave heights, etc. With protection measures against flooding we mean the use of civil structures, such as dikes, storm surge barriers, etc. Especially in the Netherlands the safety of structures protecting against sea, river and lake floods is extremely important. The so-called flood frequency models form the basis of the design methods of flood protection structures. These design methods are based on probabilistic techniques.

1.1 Flooding in the Netherlands

The Netherlands is a low-lying country which has to protect itself against flooding from the sea and its rivers. Reliable flood defenses are essential for the safety of the country. Without the dikes and dunes, more than half of the Netherlands would be regularly inundated. So the extensive system of dikes and dunes is essential to the safety and habitability of the country and an absolute precondition for good economic development. In a country like the Netherlands, flood protection must never be neglected. The management and maintenance of flood defences must always be a top priority. Climate changes may soon lead to higher design water levels. Our water systems need room to evolve if they are to cope with uncertain and unforeseen future developments. For the rivers, this could mean water conservation throughout the entire catchment area, enlarging the flow area of the river and also dike strengthening. Where the coast is concerned, it could mean extensive sand nourishment and the design of engineering structures. Room for water also means that we may sometimes need to take a step backwards and, for instance, stop building in the flood plains of the rivers, on the beaches and in the dunes facing the sea and to reserve land now for possible future use to maintain flood protection.

The flood protection structures in the Netherlands are the dikes, dunes and storm surge barriers against the sea, lakes and rivers. In Figure 1.1, a topographical map of the Netherlands with its main rivers Rhine, Meuse, Waal and IJssel and the sea Noordzee is shown. River - and sea dikes with a total length of approximately 3000 km are depicted in the figure.

Figure 1.1: Dike Map of the Netherlands

Figure 1.2: River dike “in action”

Especially in the south-western part of the Netherlands (the estuary of the Rhine River), as part of the Delta Plan, quite some storm surge barriers have been built. One of the most complicated works is the construction of the Rotterdam storm surge barrier. The storm surge barrier alone is unable to guarantee the safety of South Holland, as seawater can enter the Europort area freely through the seaports. For this reason a supplementary dike reinforcement programme is implemented, with a further barrier. This runs through the Europort area and is known as the Europort Barrier. Thanks to the New Waterway Rotterdam Storm Surge Barrier the chance of exceedance of a water level in Rotterdam of 3.60 m above Amsterdam Ordnance Datum (NAP) is reduced from once every 100 years on average to once every 10,000 years on average. The failure risk of the barrier itself is only once in 10,000,000 years. Where flood defences are concerned, measures related to the sea defences have the highest priority (risk to human life, little advance warning of flooding), followed by those in the diked sections of the rivers (risk to human life, more advance warning). Measures along the undiked sections of the rivers have a lower priority because they present no risk to human life. But there is no such thing as absolute safety. Whatever we do, we may at some time face a water-level which our flood defences are simply not designed to withstand. We must learn to live with the awareness of that residual risk and be prepared to cope with such circumstances if they occur (Van Gelder, '96b).

During the floods of 1993 and 1995, many cities along the Rivers Rhine, Waal, IJssel and Meuse were threatened by high water (Figure 1.2). In 1995 some dikes were at risk of bursting in the Netherlands. As a matter of precaution, several hundreds of thousands of people were evacuated. Damages were estimated to several billion ECU. Therefore, the Ministers of Environment of France, Germany, Belgium, Luxembourg and the Netherlands declared on February 4, 1995 in Arles that they deemed necessary to reduce flood-related risks as rapidly as possible (Moyen, 1998). It was not acceptable to them that situations as came up at that time put people’s lives and property and the environment at such great risks.

Floodings are natural phenomenons. The natural variation of water levels is part of the feature of rivers. It is the basis for river flow dynamics and the development of a typical floodplain profile. Extreme floodings occur when intensive precipitation falls on soils, which are already saturated due to former precipitation or which are frozen and can thus not absorb any water. Extreme floodings may only be influenced to a limited extent. Various human interferences have clearly altered the river regime. Human interferences should continue to increase water storage on the surfaces and in the floodplains, but also to reduce the damage risks in flood-prone areas (Van Manen et al., 1994).

Flood damages are created by the interplay of two independent mechanisms. Nature causes the high-water levels. At the same time, man increases the values along the river and the damage risks. At a given time, the combination of floodings and the accumulation of values in areas at risk create a greater damage. In order to reduce the risk due to floods, three main approaches are taken to flood prediction. First, statistical studies can be undertaken to determine the probability and frequency of high discharges of streams that cause flooding. Second, floods can be modeled and maps can be made to determine the extent of possible flooding when it occurs (Visser, 1998) in the future. And third, since the main causes of flooding are extreme amounts of rainfall or sudden thawing of snow or ice, storms and snow levels can be monitored to provide short-term flood prediction. In this thesis, the accent will be on the statistical studies for determining the probabilities of high water levels.

1.2 Statistical studies

Flood frequencies can be determined for any given high water level if data is available over an extended period of time. Such data allows statistical analysis to determine how often a given discharge or sea level is expected. From this analysis a recurrence interval can be determined and a probability calculated for the likelihood of a given discharge in the stream or high sea level for any year. The data needed to perform this analysis are the yearly maxima or peaks over a certain threshold from one or several gaging station over a long enough period of time.

In order to determine the recurrence interval, many methods are available from literature. In order to familiarize the reader with the subject a simple method is explained here. Let us assume that yearly maximum discharge data is given at the gaging station of Lobith along the river Rhine. The data should first be ranked. Each discharge is associated with a rank, m, with m = 1 given to the maximum discharge over the years of record, m = 2 given to the second highest discharge, m = 3 given to the third highest discharge, etc. The smallest discharge will receive a rank equal to the number of years over which there is a record, n. Thus, the discharge with the smallest value will have m = n.

The number of years of record, n, and the rank for each peak discharge are then used to calculate recurrence interval, RI by using, for instance, the following equation, called the Weibull equation:

$$RI = (n+1)/m \tag{1.1}$$

A graph is then made plotting discharge for each year of the record versus recurrence interval. The graph usually plots the recurrence interval on a logarithmic scale since quite some distribution functions (in particular the tail of the distribution) then appear as a straight line. An example of such a plot is shown below for the Rhine river of the Lobith gaging station (Chick et al., 1995).

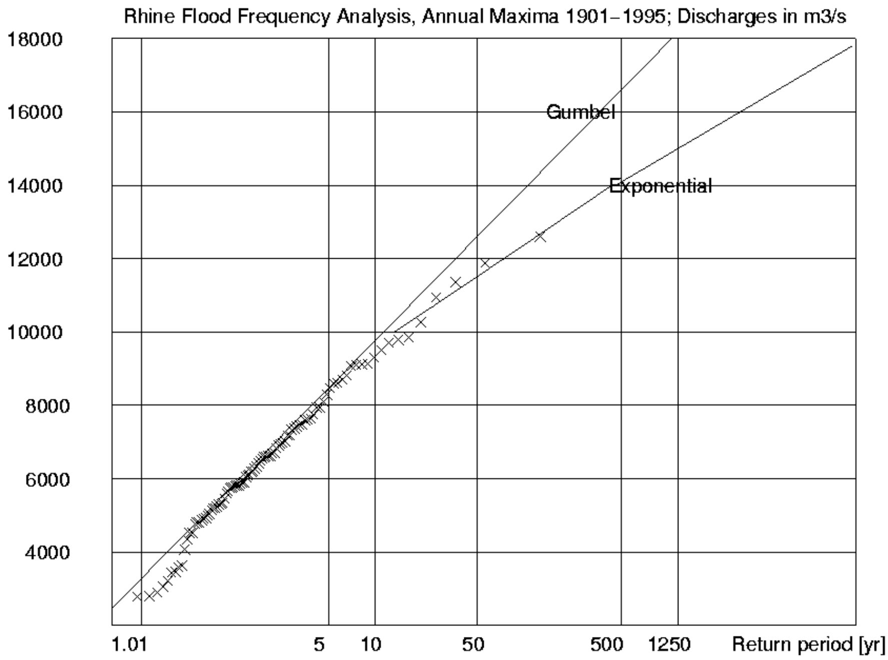


Figure 1.3: A flood frequency analysis of the River Rhine

The Dutch river dikes are designed to withstand floods with a discharge that can occur once every 1,250 years, called the design discharge. The Boertien Committee (1993) made an analysis in which they used as data sets the annual maxima (AM) and peaks over treshold (POT) data of the Rhine discharges. As distributions they used Gumbel, Pearson III (shifted Gamma), Lognormal and Generalised Pareto. Subsequently a piecewice exponential distribution to the average of the above four fitted probability distributions is determined. Boertien's advise is adopted by the politics and nowadays used to determine the design discharges. As a comparison the Gumbel fit is also shown in Figure 1.3. The dots show the annual maxima of the past 100 years as the empirical distribution form. Notice that the 1/1,250 years discharge level is estimated by the exponential distribution about 3000 m^3/s lower than with the Gumbel distribution. This corresponds with a few decimeters lower design level for the height of the river dikes (Van Gelder et al., 1995). In Chapter 4, other methods will be described to determine the 1/1,250 years discharge level at Lobith by including flood data from neighbouring locations (a level of 16,000 m^3/s is obtained (Van Gelder, 1999g)). The 1926 flood on the River Rhine had the highest discharge that has been measured so far (12,600 m^3/sec, which is equivalent to a 150-year flood). Two floods that reached a similar stage occurred on the Rhine River in the years 1995 and 1993.

The estimation of the annual exceedance probability has been a popular study object for many researchers during the last decades. The amount of statistical estimation methods (non-parametric and parametric), available for this purpose, is huge. The question therefore arises which estimation method and/or which probability distribution function should be prefered. Can we rank the performances of the estimation methods by some sort of criterion? Which uncertainties are encountered and what is the relation with the design of flood protection structures? What happens when the datasets are suspected to be inhomogeneous? Would it still be possible to use the data in that situation? These are just a few questions (depicted in Fig. 1.4) on which this thesis will focus.

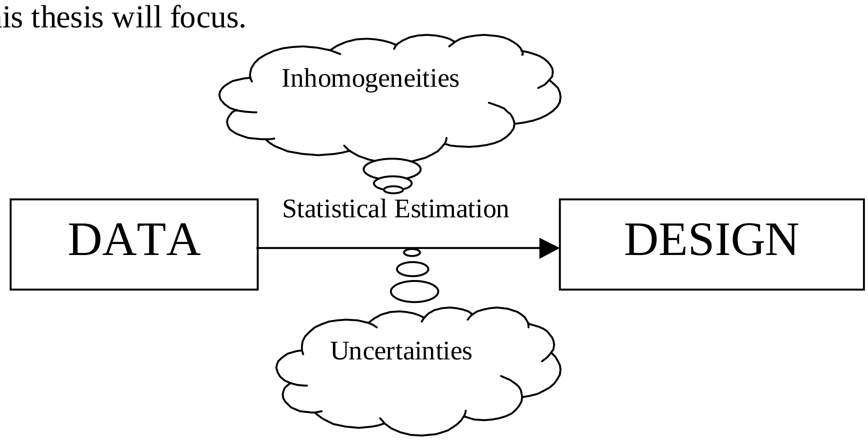


Figure 1.4: The five key-elements in the Thesis: Data, Design, Statistical Estimation, Uncertainty and Homogeneity

1.3 Overview of the Thesis

The thesis is organized as follows. The mathematical foundations for uncertainty analyses, statistical estimation methods, homogeneity analyses and design philosophies for civil structures are laid down in Chapters 2 to 5. In these chapters the theory is often applied on small case studies. The case studies that will have our attention is summarized in the following list:

Sea level data at Hook of Holland

Close to the industrial districts of Europoort and Rotterdam lies Hook of Holland. It is literally situated on a corner - along the North Sea coast and along the Nieuwe Waterweg; the access river to the harbour of Rotterdam. Sea level set-up (corrected for the astronomical tide) data at Hook of Holland has been provided by RIKZ (Dutch National Institute for Coastal and Marine Management, The Hague). The data consists of POT data over the period 1887 to 1985 with a threshold level of 30 cm. From this data base both AM-data and POT-data with higher threshold levels (150 cm to 280 cm) are filtered out.

Significant wave height data at Pozzallo

Pozzallo (a Sicilian fishing village) is situated on the coast of the Ibleo Territory between the Mediterranean Sea and the hills of Modica. The coast of Pozzallo forms a wide inlet that goes from the point of Raganzino to the sandy coast of Marza. Significant wave height data at the location Pozzallo Harbour (Lat. 36°43'39" N - Long.14°50'40" E) has been provided by PIANC (1991) (Permanent International Navigation Commission). The data consists of POT: 31 data points over 3.2 years. The treshold level is 2.0 m with a mean significant wave height level of 2.50 m, a standard deviation of 0.52m and a maximum observed significant wave height of 4.48m.

River flow data in North-Western and Central Europe

From the GRDC (Global Runoff Data Centre, Koblenz, Germany) a 18MB database was received with daily river discharges of 213 locations over a period of time varying from location to location from 1 year to almost 200 years (Fig. 1.5). Characteristics of the datasets such as station ID, river name, location name, country, latitude and longitude of the location, size of the basin area, starting year and month of the daily measurements, ending year and month of the daily measurements were given by the GRDC. Statistics of the annual average flows, annual maximum flows, annual minimum flows, and the mean annual precipitation, population density and elevation of the location, were obtained from the screening of the data and the analysis of various maps from BosAtlas (1998).

Figure 1.5: Distribution of the river gauges

Reliability-based model of Lake IJssel

From RIZA (Dutch Institute for Inland Water Management and Waste Water Treatment, Lelystad), a physical- and reliability-based model were obtained for the storm surges on Lake IJssel (1200 km^2). The physical model is based on a two-dimensional water flow model (WAQUA) and a wave model (HISWA). The reliability model is concerned with the 2% wave run-up at the dike of Rotterdamsche Hoek on the Eastern side of the Lake.

Wave rider observations at the Bay of Bengal

The Center for Ocean-Land-Atmosphere Studies in Calverton, MD, USA provided the following map containing different Bay of Bengal tropical systems (Fig. 1.6). Data consists of wave rider observations near the city of Madras during the South-West monsoons from Mid-April to Mid-August 1993, provided by Haskoning. The set is POT with 144 observations and threshold level 39cm.

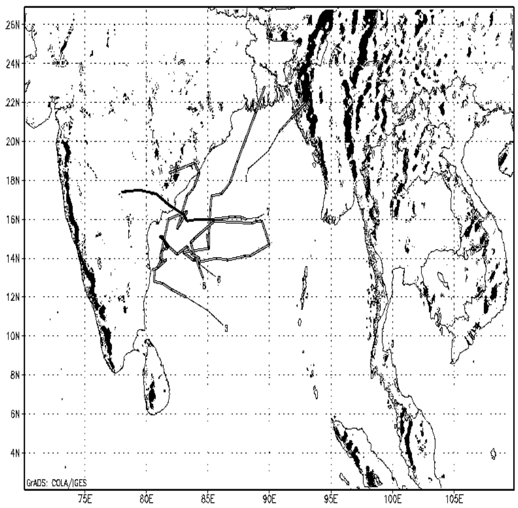


Figure 1.6: Bay of Bengal

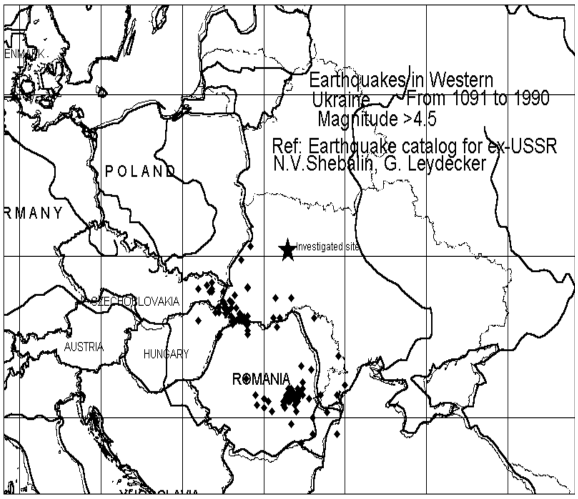


Figure 1.7: Seismicity map of Eastern Europe

Hydrologic Homogeneity Analysis Southern France

Daily river discharges along 34 sites in Southern France were provided by GRDC. Using the discordance-based homogeneity approach of Chapter 4, a homogeneous region was derived.

Seismic hazard assessment in Eastern Europe

Earthquake data (Fig. 1.7) from the Vrancea Source in Romania has been provided by UTCB (Technical University of Civil Engineering, Bucharest, Romania); data from the Uzhgorod and Russian Plain Sources were provided by IVO Power Engineering, Vantaa, Finland. The data consisted of Gutenberg-Richter magnitudes, coordinates of the epicenters and focal depths during the last century and estimated values during the last millennium.

Finally, two large case studies on dike design and wave height estimation are discussed in Chapters 6 and 7. Conclusions and recommendations are given in Chapter 8. An index of notation and abbreviations is included. The thesis finishes apart from a detailed reference list with an extensive overview of all familiar probability models and their estimation methods.

Part 1: Theory

Chapter 2

Uncertainties

Probability is common sense
reduced to calculation.
- Laplace.

2.1 Introduction

Uncertainties are everywhere. They surround us in everyday life. Among the numerous synonyms for "uncertainty" one finds unsureness, unpredictability, randomness, hazardness, indeterminacy, ambiguity, variability, irregularity and so forth. Recognition of the need to introduce the ideas of uncertainty in civil engineering today reflects in part some of the profound changes in civil engineering over the last decades.

Recent advancements in statistical modelling have provided civil engineers with an increasing power for making decisions under uncertainty. The process and information involved in the engineering problem-solving are, in many cases, approximate, imprecise and subject to change. It is generally impossible to obtain sufficient statistical data for the problem at hand, reliance must be placed on the ability of the engineer to synthesize existing information when required. Hence, to assist the engineer in making decisions, analytical tools should be developed to effectively use the existed uncertain information. In this chapter problem-solving techniques will be presented for managing and dealing with uncertainties.

The area of uncertainty analysis has its roots within the mathematical sciences and has lately received a lot of attention. Engineering textbooks such as Hald (1952), Benjamin and Cornell (1970) and Ang and Tang (1990) treat the subject of uncertainty analysis very well. The Delta Committee included uncertainty analysis in their investigations on flood prevention of the Netherlands already in the late fifties. Cooke (1991) gives an excellent overview of describing and modelling uncertainties by expert opinions and Freeman and Smith (1994), contains a collection on different aspects of uncertainty. The uncertainties that are involved in a flood frequency analysis (FFA) have been recognized in the work of Wood and Rodriguez-Iturbe (1975a and b), Martins and Clarke (1993), and Lambert and Haimes (1994b). Bernier (1987b, 1993) improved their Bayesian approach to deal with these uncertainties. Numerous authors (Duckstein et al. (1975), Rasmussen and Rosbjerg (1991), Stevens (1992), Durrans (1994)) have investigated the uncertainty models in FFA's. Also in the area of wave height frequency analysis, the uncertainties have received quite some attention (Goda (1988 and 1992), Andrew and Hemsley (1990)). Recently the uncertainties that arise in failure mechanism models (as described in for instance De Groot et al., 1996) have been approached by Vrouwenvelder (1997), Slijkhuis et al. (1998) and Van Noortwijk et al. (1999). The consequences of the uncertainties on the structural design is treated in Vrijling et al. (1997b, 1998b) and Van Gelder et al. (1996b) and will also be the subject in Chapter 5 of this thesis.

In this chapter, we will concentrate on the distinction between different types of uncertainty in civil engineering practice. In particular, when studying the probabilistic design of structures, such as flood defences like dikes or breakwaters, it is important to develop a philosophy to discern between different types of uncertainty. The difference between the so-called inherent uncertainty in time and in space will appear to play an important role. Inherent uncertainty in time means that the realisations of the process in the future remain uncertain. For a dike design for instance, we may have inherent uncertainty in time for the individual wave heights, water levels, etc. Unlimited data will not reduce this inherent uncertainty. Inherent uncertainty in space is from a different kind. The properties of the foundation and the strength of the dike have only one realisation per lifetime. An important aspect of this type of uncertainty is that without further investigations the knowledge of the foundation increases with time. During the life of the structure information will be gained as each storm exceeding the previous that is survived by the structure, pushes the lower limit of the strength upward (Bayesian updating of the strength).

The above principles will be illustrated in this chapter by developing a general model in which the influence of all kinds of uncertainties on the probability of failure can be analysed. This model will then be refined to a model in which the probability distribution of the life time to failure of the structure is given. In this model the resistance of the structure contains inherent uncertainty in space and the yearly maximal load contains inherent uncertainty in time. From this distribution the conditional failure rate can be derived. The refined model can show the influence when the standard deviation of the resistance will be varied against the standard deviation of the yearly maximal load. The consequence on the conditional failure rate is a direct output of the model.

The chapter is organized as follows. First the overview of the different types of uncertainties will be given, including a detailed description of the inherent uncertainty in time and inherent uncertainty in space. The influence of the uncertainties on the probability of failure and the effect on the lifetime reliability of flood protection will be described in the main part of the chapter. The chapter ends with a discussion on how to deal with uncertainties in civil engineering practice.

2.2 Types of uncertainty

Uncertainties in decision and risk analysis can primarily be divided in two categories: uncertainties that stem from variability in known (or observable) populations and therefore represent randomness in samples (inherent uncertainty), and uncertainties that come from basic lack of knowledge of fundamental phenomena (epistemic uncertainty).

Inherent uncertainties represent randomness or the variations in nature. For example, even with a long history of data, one cannot predict the maximum water level that will occur in, for instance, the coming year at the North Sea. It is not possible to reduce inherent uncertainties.

Epistemic uncertainties are caused by lack of knowledge of all the causes and effects in physical systems, or by lack of sufficient data. For example, it might only be possible to obtain the type of the distribution, or the exact model of a physical system, when enough research could and would be done. Epistemic uncertainties may change as knowledge increases.

In this chapter it is proposed to subdivide the inherent uncertainty and epistemic uncertainty in five types of uncertainty: inherent uncertainty in time and in space, parameter uncertainty and distribution type uncertainty (together also known as statistical uncertainty) and finally model uncertainty (Fig 2.1). Uncertainties such as construction costs uncertainties, damage costs uncertainties and financial uncertainties are considered as examples of model uncertainties.

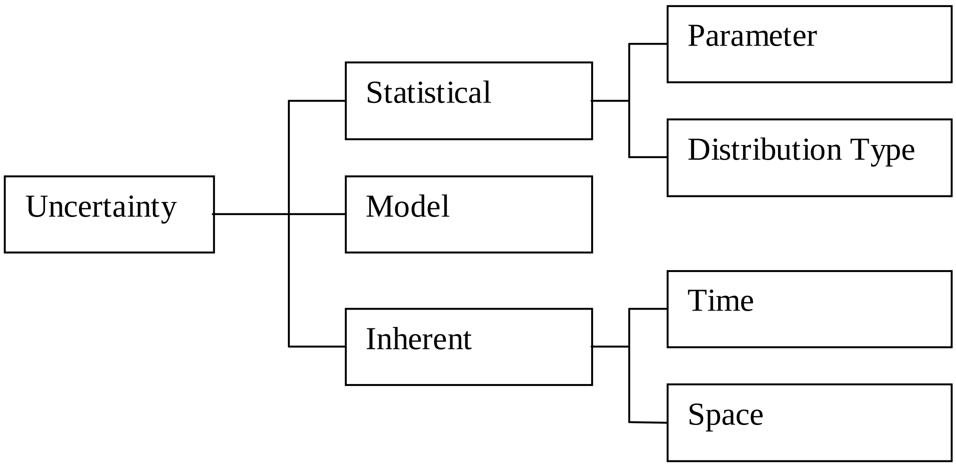


Figure 2.1: Types of uncertainty

2.2.1 Inherent uncertainty in time

When determining the probability distribution of a random variable that represents the variation in time of a process (like the occurrence of a water level), there essentially is a problem of information scarcity. Records are usually too short to ensure reliable estimates of low-exceedance probability quantiles in many practical problems. The uncertainty caused by this shortage of information is the statistical uncertainty of variations in time. This uncertainty can theoretically be reduced by keeping record of the process for the coming centuries.

Stochastic processes running in time (individual wave heights, significant wave heights, water levels, discharges, etc.) are examples of the class of inherent uncertainty in time. Unlimited data will not reduce this uncertainty. The realisations of the process in the future remain uncertain. The probability density function (PDF) or the cumulative probability distribution function (CDF) and the auto-correlation function describe the process.

In case of a periodic stationary process like a wave field the autocorrelation function will have a sinusoidal form and the spectrum, as the Fourier-transform of the autocorrelation function, gives an adequate description of the process. Attention should be paid to the fact that the well known wave energy spectra as Pierson-Moskowitz and Jonswap are not always able to represent the wave field at a site. In quite some practical cases, swell and wind wave form a wave field together. The presence of two energy sources may be clearly reflected in the double peaked form of the wave energy spectrum.

An attractive aspect of the spectral approach is that the inherent uncertainty can be easily transferred through linear systems by means of transfer functions. By means of the linear wave theory the incoming wave spectrum can be transformed into the spectrum of wave loads on a flood defence structure. The PDF of wave loads can be derived from this wave load spectrum. Of course it is assumed here that no wave breaking takes place in the vicinity of the structure. In case of non-stationary processes, that are governed by meteorological and atmospheric cycles (significant wave height, river discharges, etc.) the PDF and the autocorrelation function are needed. Here the autocorrelation function gives an impression of the persistence of the phenomenon. The persistence of rough and calm conditions is of utmost importance in workability and serviceability analyses.

If the interest is directed to the analysis of ultimate limit states e.g. sliding of the structure, the autocorrelation is eliminated by selecting only independent maxima for the statistical analysis. If this selection method does not guarantee a set of homogeneous and independent observations, physical or meteorological insights may be used to homogenise the dataset. For instance if the fetch in NW-direction is clearly maximal, the dataset of maximum significant wave height could be limited to NW-storms. If such insight fails, one could take only the observations exceeding a certain threshold (POT) into account hoping that this will lead to the desired result. In case of a clear yearly seasonal cycle the statistical analysis can be limited to the yearly maxima.

Special attention should be given to the joint occurrence of significant wave height H_s and spectral peak period T_p. A general description of the joint PDF of H_s and T_p is not known. A practical solution for extreme conditions considers the significant wave height and the wave steepness s_p as independent stochastic variables to describe the dependence. This is a conservative approach as extreme wave heights are more easily realised than extreme peak periods. For the practical description of daily conditions (service limit state: SLS) the independence of s_p and T_p seems sometimes a better approximation. Also the dependence of water levels and significant wave height should be explored because the depth limitation to waves can be reduced by wind set-up. Here the statistical analysis should be clearly supported by physical insight. Moreover it should not be forgotten that shoals could be eroded or accreted due to changes in current or wave regime induced by the construction of the flood defence structure.

2.2.2 Inherent uncertainty in space

When determining the probability distribution of a random variable that represents the variation in space of a process (like the fluctuation in the height of a dike), there essentially is a problem of shortage of measurements. It is usually too expensive to measure the height or width of a dike in great detail. This statistical uncertainty of variations in space can be reduced by taking more measurements (Vrijling and Van Gelder, 1998b).

Soil properties can be described as stochastic processes in space. From a number of field tests the PDF of the soil property and the (three-dimensional) autocorrelation function can be fixed for each homogeneous soil layer. Here the theory is further developed than the practical knowledge. Numerous mathematical expressions are proposed in the literature to describe the autocorrelation. No clear preference has however emerged yet as to which functions describe the fluctuation pattern of the soil properties best. Moreover, the correlation length (distance where correlation becomes approximately zero) seems to be of the order of 30 to 100m while the spacing of traditional soil mechanical investigations for flood defence structures is of the order of 500m. So it seems that the intensity of the soil mechanical investigations has to be increased considerably if reliable estimates have to be made of the autocorrelation function.

The acquisition of more data has a different effect in case of stochastic processes in space than in time. As structures are immobile, there is only one single realisation of the field of soil properties. Therefore the soil properties at the location could be exactly known if sufficient soil investigations were done. Consequently the actual soil properties are fixed after construction, although not completely known to man. The uncertainty can be described by the distribution and the autocorrelation function, but it is in fact a case of lack of info.

2.2.3 Parameter uncertainty

This uncertainty occurs when the parameters of a distribution are determined with a limited number of data. The smaller the number of data, the larger the parameter uncertainty. A parameter of a distribution function is estimated from the data and thus a random variable. The parameter uncertainty can be described by the distribution function of the parameter. In Van Gelder (1996b) an overview is given of the analytical and numerical derivation of parameter uncertainties for certain probability models (Exponential, Gumbel and Log-normal). The bootstrap method is a fairly easy tool to calculate the parameter uncertainty numerically. Bootstrapping methods are described in for example Efron (1982) and Efron and Tibshirani (1993). Given a dataset $x=(x_1,x_2,...,x_n)$, we can generate a bootstrap sample x^* which is a random sample of size n drawn with replacement from the dataset x. The following bootstrap algorithm can be used for estimating the parameter uncertainty:

1. Select B independent bootstrap samples x^{*1}, x^{*2}, ..., x^{*B}, each consisting of n data values drawn with replacement from x.
2. Evaluate the bootstrap corresponding to each bootstrap sample;

$$\theta^*(b)=f(x^{*b}) \text{ for } b=1,2,...,B \tag{2.1}$$

3. Determine the parameter uncertainty by the empirical distribution function of θ^*.

The algorithm has been applied in Van Gelder (1996b) to the location parameter A and scale parameter B of the Gumbel distribution with a Maximum Likelihood fit to the Hook of Holland data. The parameter uncertainty could be very well approximated by normal distributions with coefficient of variations of around 10%.
Other methods to model parameter uncertainties like Bayesian methods can be applied too (Van Gelder, 1996b). Bayesian inference lays its foundations upon the idea that states of nature can be and should be treated as random variables. Before making use of data collected at the site the engineer can express his information concerning the set of uncertain parameters Θ for a particular model $f(x|\Theta)$, which is a PDF for the random variable x. The information about Θ can be described by a prior distribution

$\pi(\Theta|I)$, i.e. prior to using the observed record of the random variable x. The basis upon which these prior distributions are obtained from the initial information I are described in for instance Carlin and Louis (1996). Non-informative priors can be used if we don't have any prior information available. If $p(\theta)$ is a non-informative prior, consistency demands that $p(\varsigma)d\varsigma=p(\theta)d\theta$ for $\varsigma=\varsigma(\theta)$; thus a procedure for obtaining the ignorance prior should presumably be invariant under one-to-one reparametrisation. A procedure which satisfies this invariance condition is given by the Fisher matrix of the probability model:

$$I(\theta)=-E_{x|\theta}[\partial^2/\partial\theta^2 \log f(x|\theta)] \tag{2.2}$$

giving the so-called non-informative Jeffrey's prior $p(\theta)=I(\theta)^{1/2}$.

The engineer now has a set observations x of the random variable X, which he assumes comes from the probability model $f_X(x|\Theta)$. Bayes' theorem provides a simple procedure by which the prior distribution of the parameter set Θ may be updated by the dataset X to provide the posterior distribution of Θ, namely,

$$f(\Theta|X,I)=l(X|\Theta)\pi(\Theta|I)/K \tag{2.3}$$

where

$f(\Theta\|X,I)$	posterior density function for Θ, conditional upon a set of data X and information I;
$l(X\|\Theta)$	sample likelihood of the observations given the parameters
$\pi(\Theta\|I)$	prior density function for Θ, conditional upon the initial information I
K	normalizing constant ($K=\Sigma l(X\|\Theta)\pi(\Theta\|I)$)

The posterior density function of Θ is a function weighted by the prior density function of Θ and the data-based likelihood function in such a manner as to combine the information content of both. If future observations X_F are available, Bayes' theorem can be used to update the PDF on Θ. In this case the former posterior density function for Θ now becomes the prior density function, since it is prior to the new observations or the utilization of new data. The new posterior density function would also have been obtained if the two samples X and X_F had been observed sequentially as one set of data. The way in which the engineer applies his information about Θ

depends on the objectives in analyzing the data. The Bayesian methods will be described in more detail in Chapter 3.

2.2.4 Distribution type uncertainty

This type represents the uncertainty of the distribution type of the variable. It is for example not clear whether the occurrence of the water level of the North Sea is exponentially or Gumbel distributed or whether it has another distribution. A choice was made to divide statistical uncertainty into parameter- and distribution type uncertainty although it is not always possible to draw the line; in case of unknown parameters (because of lack of observations), the distribution type will be uncertain as well.

Any approach that selects a single model and then makes inference conditionally on that model ignores the uncertainty involved in the model selection, which can be a big part in the overall uncertainty. This difficulty can be in principle avoided , if one adopts a Bayesian approach and calculates the posterior probabilities of all the competing models following directly from the Bayes factors. A composite inference can then be made that takes account of model uncertainty in a simple way with the weighted average model:

$$f(h)=\theta_1 f_1(h)+\theta_2 f_2(h)+...+\theta_n f_n(h) \tag{2.4}$$

where $\Sigma\theta_i = 1$.

The approach described above gives us some sort of Bayesian discrimination procedure between competing models. This area has become very popular recently. Theoretical research comes from Kass and Raftery, (1995), and applications can be found mainly in the biometrical sciences (Volinsky et al., 1996) and econometrical sciences (De Vos, 1996). The very few applications of Bayesian discrimination procedures in civil engineering come from Wood and Rodriguez-Iturbe (1975), Pericchi and Rodriguez-Iturbe (1983 and 1985) and Perreault et al. (1999).

Other methods to account for distribution type uncertainty are available. In Chapter 3, we will come back to these methods.

2.2.5 Model uncertainty

Many engineering models describing the natural phenomena like wind and waves are imperfect. They can be imperfect because the physical phenomena are not known (for example when regression models without the underlying physics are used), or they can be imperfect because some variables of lesser importance are omitted in the engineering model for reasons of efficiency.

Suppose that the true state of nature is X. Prediction of X may be modeled by X*. As X* is a model of the real world, imperfections may be expected; the resulting predictions will therefore contain errors and a correction N may be applied. Consequently, the true state of nature may be represented by Ang (1973):

$$X = NX^* \tag{2.5}$$

If the state of nature is random, the model X* naturally is also a random variable. The inherent variability is described by the coefficient of variation (CV) of X*, given by $\sigma(X^*)/\mu(X^*)$.The necessary correction N may also be considered a random variable, whose mean value $\mu(N)$ represents the mean correction for systematic error in the predicted mean value, whereas the CV of N, given by $\sigma(N)/\mu(N)$, represents the random error in the predicted mean value.

It is reasonable to assume that N and X* are statistically independent. Therefore we can write the mean value of X as:

$$\mu(X)=\mu(N)\mu(X^*) \tag{2.6}$$

The total uncertainty in the prediction of X becomes:

$$CV(X) = \sqrt{(CV^2(N) + CV^2(X^*) + CV^2(N)CV^2(X^*))} \tag{2.7}$$

In Van Gelder (1998), an example of model uncertainty is presented in fitting physical models to wave impact experiments.

We can ask ourselves if there is a relationship between model and parameter uncertainty. The answer is No. Consider a model for predicting the weight of an individual as a function of his height. This might be a simple linear correlation of the form W=aH+b. The parameters a and b may be found from a least squares fit to some sample data. There will be *parameter* uncertainty to a and b due to the sample being just that, a sample, not the whole population. Separately there will be *model* uncertainty due to the scatter of individual weights either side of the correlation line.

Thus parameter uncertainty is a function of how well the parameters provide a fit to the population data, given that they would have been fitted using only a sample from that population, and that sample may or may not be wholly representative of the population. Model uncertainty is a measure of the scatter of individual points either side of the model once it has been fitted. Even if the fitting had been performed using the whole population then there would still be residual errors for each point since the model is unlikely to be exact.

Parameter uncertainty can be reduced by increasing the amount of data against which the model fit is performed. Model uncertainty can be reduced by adopting a more elaborate model (e.g. quadratic fit instead of linear). There is, however, no relationship between the two.

2.2.6 Uncertainties related to the construction

To optimize the design of a hydraulic structure the total lifetime costs, an economic cost criterion, can be used. The input for the cost function consists of uncertain estimates of the construction cost and the uncertain cost in case of failure.

The construction costs consist of a part which is a function of the structure geometry (variable costs) and a part which can only be allocated to the project as a whole (fixed costs). For a vertical breakwater for instance the variable costs can be assumed to be proportional to the volumes of concrete and filling sand in the cross Section of the breakwater.

The costs in case of ULS (ultimate limit state) failure consist of replacement of (parts of) the structure and thus depend on the structure dimensions. The costs in case of SLS failure are determined by the costs of downtime and thus are independent of the structure geometry.

The total risk over the lifetime of the structure is given by the sum of all yearly risks, corrected for interest, inflation and economical growth. This procedure is known as capitalization. The growth rate expresses that in general the value of all goods and equipment behind the hydraulic structure will increase during the lifetime of the structure.

Several cost components can be allocated to the building project as a whole. Examples of these cost components are:

- Cost of the feasibility study;
- Cost of the design of the flood protection structure;
- Site investigations, like penetration tests, borings and surveying;
- Administration.

In principle there are two ways in which a structure can fail. Either the structure collapses under survival conditions after which there will be more wave penetration in the protected area or the structure is too low and allows too much wave generation in the protected area due to overtopping waves. In both cases possibly harbour operations have to be stopped, resulting in (uncertain amount of) damage (downtime costs).

If a structure fails to protect the area of interest against wave action, possibly the operations in this area will have to be stopped. The damage costs which are caused by this interruption of harbour operations are called downtime costs. The exact amount of downtime costs is very difficult to determine and therefore contain a lot of uncertainty. The downtime costs for one single ship can be found in literature, but the total damage in case of downtime does not depend solely on the downtime costs of ships. The size of the harbour and the type of cargo are also important variables in this type of damage. Furthermore, the availability of an alternative harbour is very important. If there is an alternative, ships will use this harbour. In that case the damaged harbour will lose income because less ships make use of the harbour and possibly because of claims of shipping companies. On a macro-economic scale however there is possibly minor damage since the goods are still coming in by way of the alternative harbour. This shows that also the availability of infrastructure in the area influences the damage in case of downtime. If an alternative harbour is not available the economic damage may be felt beyond the port itself.

The location of the structure in relation to the harbour also influences the damage costs. If the structure protects the entrance channel, the harbour can not be reached during severe storms, thus causing waiting times. These waiting times have the order of magnitude of hours to a few days. If the structure protects the harbour basin or a terminal damage to the structure can cause considerable amounts of extra downtime due to the fact that the structure only partly fulfils its task over a longer period of time.

If the load on a structure component exceeds the admissible load, the component collapses. Several scenarios are now possible:

- The component is not essential to the functionality of the structure. Repair is not carried out and there is no damage in monetary terms. This is the case if, for instance, an armour block is displaced in the rubble foundation. It should be noted that this kind of damage can cause failure if a lot of armour blocks are displaced (preceding failure mode);
- The component is essential to the functionality of the structure. The stability of the structure is however not threatened. This is the case if, for instance, the crown wall of a caisson collapses. The result is a reduction of the crest height of the structure which could threaten the functionality of the structure. Therefore repair has to be carried out and there is some damage in monetary

terms;

- The structure has become unstable during storm conditions. There is considerable damage to the structure, resulting in necessary replacement of (parts of) the structure. The damage in monetary terms is possibly even higher than the initial investment in the structure.

When optimizing a structural design, an estimate of the damage is needed. In the case of a structure component this could be the cost of rebuilding. If large parts of the caissons are collapsed the area will have to be cleared before rebuilding the structure. In that case the damage will be higher than in the case of rebuilding alone. Furthermore, collapse will in general lead to downtime cost which further increases the damage.

In Van Gelder and Vrijling (1997b) the structural design of vertical breakwaters is considered in which the uncertainty in construction and damage costs is taken into account. In Chapter 5 we come back to this issue.

2.3 Reduction of uncertainty

In Section 2.2, it was mentioned that inherent uncertainties represent randomness or variations in nature. Inherent uncertainties cannot be reduced.

Epistemic uncertainties, on the other hand, are caused by lack of knowledge. Epistemic uncertainties may change as knowledge increases. In general there are three ways to increase knowledge:

- Gathering data
- Research
- Expert judgement

Data can be gathered by taking measurements or by keeping record of a process in time. Research can, for instance, be undertaken with respect to the physical model of a phenomenon or into the better use of existing data. By using expert opinions, it is possible to acquire the probability distributions of variables that are too expensive or practically impossible to measure.

The goal of all this research obviously is to reduce the uncertainty in the model. Nevertheless it is also thinkable that uncertainty will increase. Research might show that an originally flawless model actually contains a lot of uncertainties. Or after taking some measurements the variations of the dike height can be a lot larger. It is also thinkable that the average value of the variable will change because of the research that has been done.

The consequence is that the calculated probability of failure will be influenced by future research. In order to guarantee a stable and convincing flood defence policy after the transition, it is important to understand the extent of this effect.

In the next Section will be discussed what influence the future reduction of uncertainty can have on the probability of failure and how to present this influence. It is assumed that research will rarely increase uncertainty or change the average value of a variable.

2.4 The effect of uncertainty due to lack of information

The problem of lack of information in hydraulic engineering models is studied in detail in this section. Data records are usually too short to ensure reliable estimates of low-exceedance probability quantiles in many practical problems (Van Noortwijk and Van Gelder, 1998).

The information about a random variable can be updated by the help of expert judgements. Cooke (1991) describes various methods to do so. Expert judgements can also be used to reduce the uncertainty of the quantiles. In order to include expert judgements in the quantile estimation of a certain quantity, the following approach is proposed in this chapter.

Consider S the random variable describing the loads on a structure with exceedance probability p per year ($p \ll 1$). Consider R the random variable describing the strength of the structure modelled with a normal distribution with mean μ_R and standard deviation σ_R.

The effect of the value of information on the random variable S may be modelled by correcting its original mean value μ_S to its new value $\mu_S + v\sigma_I$ in which v is the standard normal distribution and σ_I is the standard deviation of the expert information. Furthermore the standard deviation of S will be reduced from $\sqrt{(\sigma_S^2+\sigma_I^2)}$ to σ_S under the influence of the expert. Summarized in Table 2.1:

Table 2.1: The effect of information on the random variables S and R

Lack of Information			With Information	
	μ	σ	μ	σ
S	μ_S	$\sqrt{(\sigma_S^2+\sigma_I^2)}$	$\mu_S + v\sigma_I$	σ_S
R	μ_R	σ_R	μ_R	σ_R
$\beta_{ni}=(\mu_S-\mu_R)/\sqrt{(\sigma_S^2+\sigma_I^2+\sigma_R^2)}$			$\beta_{wi}=(\mu_S+v\sigma_I-\mu_R)/\sqrt{(\sigma_S^2+\sigma_R^2)}$	

The exceedance probability or reliability index β_{wi} after including expert opinion can be seen as a random variable with a normal distribution with the following mean and standard deviation:

$$\beta_{wi} \sim N\left((\mu_S-\mu_R)/\sqrt{(\sigma_S^2+\sigma_R^2)},\ \sigma_I/\sqrt{(\sigma_S^2+\sigma_R^2)} \right) \tag{2.8}$$

The idea to consider a reliability index as a random variable is also adopted in the work on imprecise probabilities (Walley, 1991 and Dubois et al., 1988). Using the notation $\beta_m = (\mu_S-\mu_R)/\sqrt{(\sigma_S^2+\sigma_R^2)}$, and $\sigma_\beta = \sigma_I/\sqrt{(\sigma_S^2+\sigma_R^2)}$, β_{wi} can be written as:

$$\beta_{wi} = \beta_m + v\,\sigma_\beta \tag{2.9}$$

in which v is the standard normal distribution.

In order to determine the uncertainty in the probability of failure and its influence factors, a FORM (First Order Reliability Method) analysis can be performed. An overview of FORM is given in Ang and Tang (1990). The following reliability function is considered:

$$Z = \beta_{wi} - u \tag{2.10}$$

in which u is a standard normal distribution (independent of v). Together with the expression for β_{wi} the reliability function can be seen as a function of the 2 standard normal distributions u and v:

$$Z = \beta_m + v\,\sigma_\beta - u \tag{2.11}$$

Because $\partial Z/\partial u = -1$ and $\partial Z/\partial v = \sigma_\beta$, the reliability index for this Z-function can be derived to:

$$\beta = \beta_m / \sqrt{(1+\sigma_\beta^2)} \tag{2.12}$$

which appears to be exactly the same as the reliability index without information: β_{ni}

$$\beta = \beta_m / \sqrt{(1+\sigma_\beta^2)} = (\mu_S-\mu_R)/\sqrt{(\sigma_S^2+\sigma_R^2)}/\sqrt{(1+\sigma_\beta^2)} = \tag{2.13}$$

$$= (\mu_S-\mu_R)/\sqrt{(\sigma_S^2+\sigma_R^2)}/\sqrt{(1+ \sigma_I^2/(\sigma_S^2+\sigma_R^2))} = (\mu_S-\mu_R)/\sqrt{(\sigma_S^2+\sigma_I^2+\sigma_R^2)} = \beta_{ni}$$

The α-factors are given by:

$$\alpha_u = 1/\sqrt{(1+\sigma_\beta^2)} \text{ and } \alpha_v = \sigma_\beta / \sqrt{(1+\sigma_\beta^2)} \tag{2.14}$$

α_u and α_v represent the influence of the uncertainties in u and v on the reliability index β respectively. In terms of β_m and σ_β this can also be written as:

$$\beta_m = \beta / \alpha_u \tag{2.15}$$

$$\sigma_\beta = \sqrt{(1 - \alpha_u^2)} / \alpha_u = \alpha_v / \alpha_u \tag{2.16}$$

The implications of the changes in β and the related flooding frequency can be analysed in this framework.

As discussed in Section 2.3 it is possible that some uncertainties might be reduced in the future. This means that the future probability of failure will be smaller. One could say that those uncertainties add to the 'uncertainty' of the probability of failure. The obvious problem is that it can not be predicted which uncertainties will be reduced in the future. There are several philosophies thinkable about what uncertainty will remain part of the future probability of failure.

It is important to realise that, for every philosophy, the difference between the 'true' probability of failure and the 'uncertainty' of the probability of failure, is an artificial difference. If all uncertainties are integrated out this will result in the same, current probability of failure.

Let us consider all different philosophies (options) about the uncertainties that might be reduced and form the 'uncertainty' of the probability of flooding, and the uncertainties that cause the 'true' probability of flooding, as given in Table 2.2:

Option 1

No uncertainties will be reduced. All uncertainties in the model will be integrated to determine the probability of failure. This probability of failure also represents the current, actual probability of failure.

Option 2

A practical division is made between the uncertainties that might be influenced by taking measurements in time (reducing the statistical uncertainty of variations in time) or doing research (reducing the model uncertainty) and the remaining uncertainties. Statistical uncertainty of variations in time will be a part of the probability of failure.

This uncertainty can only theoretically be reduced by keeping record of the underlying process for the coming centuries.

Table 2.2: Five different options
(P_f : uncertainty is not reduced
- : uncertainty is reduced)

	Option 1	Option 2	Option 3	Option 4	Option 5
Inherent uncertainty (in time)	P_f	P_f	P_f	P_f	P_f
Inherent uncertainty (in space)	P_f	P_f	P_f	P_f	-
Statistical uncertainty (of variations in time)	P_f	P_f	-	-	-
Statistical uncertainty (of variations in space)	P_f	-	-	-	-
Model uncertainty	P_f	-	-	-	-
Other uncertainties	P_f	P_f	P_f	-	-

Option 3
A theoretical division is made between inherent uncertainty and epistemic uncertainty. The assumption is that through research all epistemic uncertainty disappears. This would result in a probability of failure that is solely caused by inherent uncertainties.

Option 4
Assumed is that through research the three uncertainties, related to construction costs, damage costs and financial rates, disappear. This will influence the optimal design of the structure and in that way also the probability of failure.

Option 5
Assumed is that only inherent uncertainty in time causes the probability of failure. This is also the philosophy used in the present safety standards.

The method described in this chapter is a very practical and simple method to get insight in the problems concerned with the lack of information in hydraulic engineering models. It can be seen as a generalization of the approach of Pate-Cornell (1996) and Slijkhuis et al. (1998). The method in this chapter shows that the more uncertainty is expected to be reduced in the future, the higher the mean and the larger the standard deviation of the distribution of the reliability index will be.

The method is demonstrated for a simple reliability function in Slijkhuis et al. (1999). The advantage of the method is that it also works for a very complex reliability function with a large number of variables.

2.5 Effect of uncertainty on lifetime reliability

From an engineering point of view, the Bayesian approach that takes all uncertainties into account as PDF's reflects the designer's intuition very well. Keeping the physical structure unchanged an increase in uncertainty of any variable increases the formal probability of failure too.

From this point of view there is no difference between inherent, statistical and model uncertainty; all have to be incorporated in the probabilistic calculations. In the probabilistic calculations however a difference occurs between uncertainties that have many (e.g. yearly) realisations during the lifetime of the structure and those that have only one connected to the specific structure and the site. For instance, every storm season shows an independent maximum H_s every year. The properties of the foundation and the strength of the flood defence structure have only one realisation per structure (when the structure does not deteriorate). Consequently the probability of failure is not solely a property of the structure but also a result of our lack of knowledge.

The effect of uncertainty on the lifetime reliability will be illustrated by the following hypothetical example of the probability of failure of a flood defence structure. In the example, one type of load function and three types of resistance functions are considered (see Fig. 2.2).

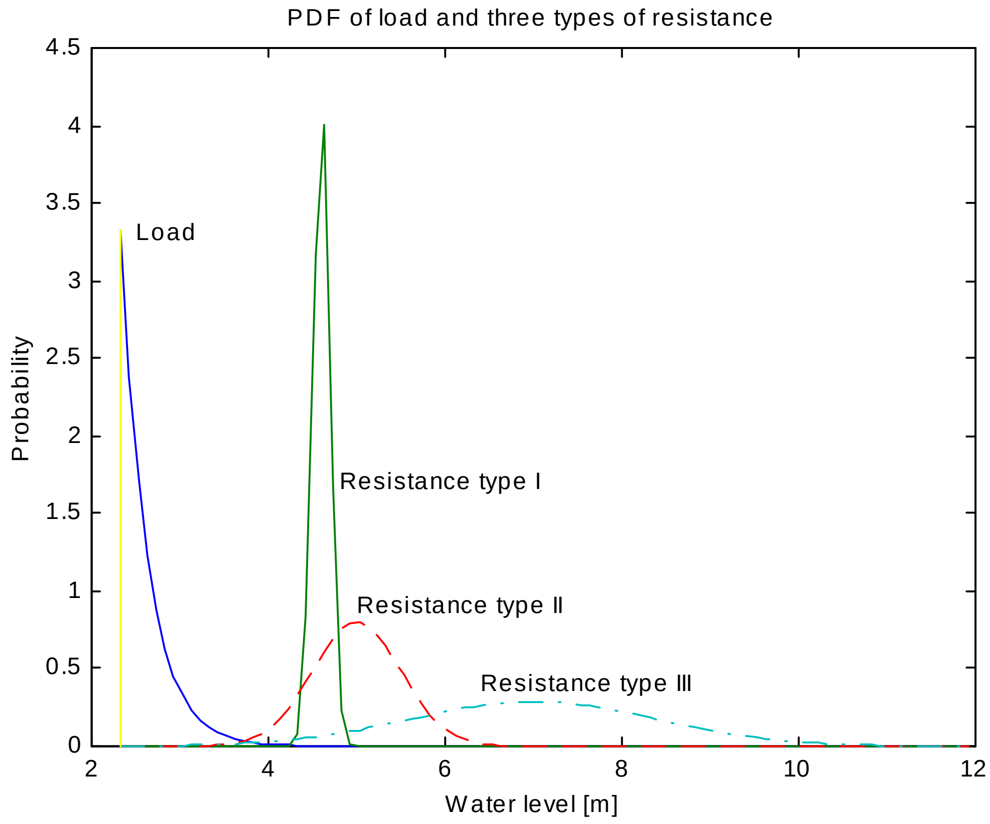


Figure 2.2. One type of load and three types of resistances.

For the load function one can think of the exponential PDF of the wave heights in front of the structure. For the three types of resistance functions one can think of the normal PDF's of the (quite certain; uncertain; very uncertain, resp.), crest height of the structure. Failure is defined by Z<0, in which Z=R-S with R the resistance function and S the load function. The mean values of the resistance types are chosen is such a way that the probabilities of failure are the same. A series of observations can be made in the following sections.

2.5.1 Correlation in two subsequent years

Let the failure in year i be defined as $\{Z_i<0\}$ and let S_i be the loads in year i. The loads S_i are mutually independent and the resistance R does not depend on time and is independent of S_i. Consider the correlation in the reliability in two subsequent years i and i+1.

$$\rho(Z_i,Z_{i+1}) = \sigma_R^2/(\sigma_R^2+\sigma_S^2) \tag{2.17}$$

This result can be derived from the calculation of

$$
\begin{aligned}
&COV(Z_i,Z_{i+1}) = E(((R-S_i)-(\mu_R-\mu_S))((R-S_{i+1})-(\mu_R-\mu_S))) = \\
&= E(((R-S_i)-\mu_Z) ((R-S_{i+1})-\mu_Z)) = E((R-S_i)(R-S_{i+1})) - \mu_Z E (R-S_{i+1}) - \\
&\quad \mu_Z E (R-S_i) + \mu_Z^2 = E(R^2) + E(S_iS_{i+1}) - E(RS_i) - E(RS_{i+1}) - \mu_Z^2 = \\
&= \sigma_R^2 + \mu_R^2 + \mu_S^2 - 2\mu_R\mu_S - \mu_Z^2 = \sigma_R^2 \qquad (2.18)
\end{aligned}
$$

In (2.18) some calculation rules of Section 3.7.2 are used such as E(XY)=E(X)E(Y) if X and Y are i.i.d. It can be noted from (2.17) that if σ_R approaches infinity, then ρ converges to one. In words: if the standard deviation of the yearly maximal wave load is large in relation to the standard deviation of the resistance, the dependence between failure in two subsequent years is low. If however, keeping the standard deviation of the reliability function unchanged, the opposite is true, the failure in subsequent years are dependent. With the load and resistance functions of Figure 2.2 the following correlation coefficients are obtained: ρ_1=0.0231, ρ_2=0.747, and ρ_3=0.964 (see Figure 2.3).

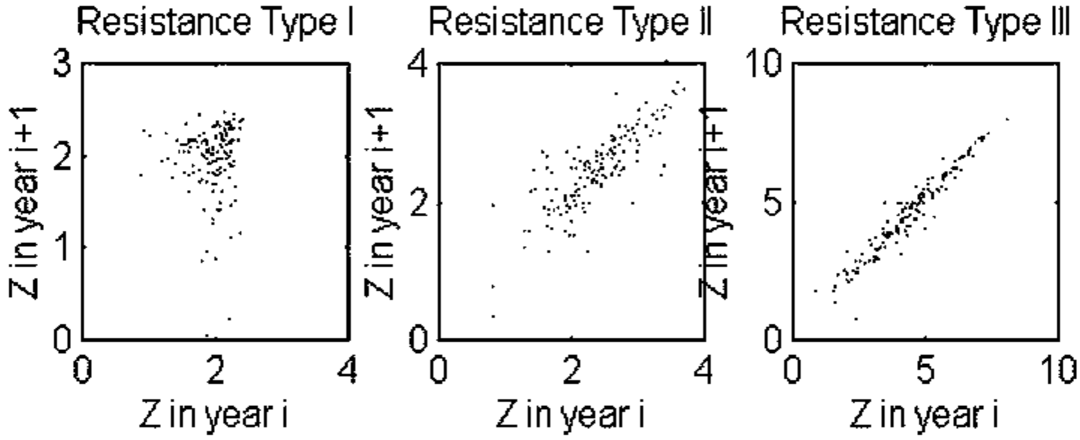


Figure 2.3. Monte Carlo simulation of the reliability in two subsequent years.

2.5.2 Failure rate

Consider the probability of failure at a certain moment (year number N), assuming that no failure occurred before that time. This probability is given by:

$$h(N) = f_L(N)/(1-F_L(N)) \qquad (2.19)$$

h(N) is called the failure rate (or hazard function). $F_L(N)$ the CDF of L (subscript L for lifetime). See for instance Høyland and Rausand (1994).

It can be noted that if $\sigma_R = 0$ (every year the same probability of failure), then h(N) remains constant. If σ_R increases, then h(N) converges to 0. This is illustrated in Figure 2.4, where the probabilities of failure $F_L(N)$ for the three types of resistances are depicted. Figure 2.5 shows the failure rates h(N).

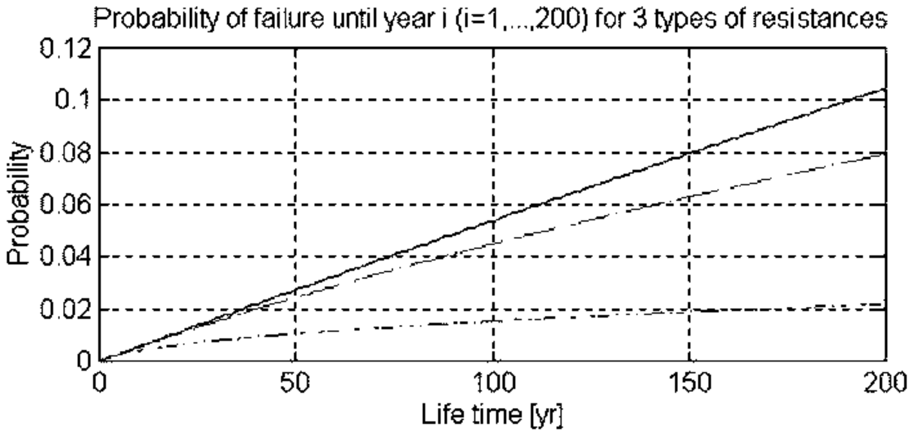


Figure 2.4: Probabilities of failure during lifetime
solid: resistance type I, dashed: type II, dash-dotted: type III

Failure rates are usually decreasing functions in the beginning of the lifetime of the structure. This phenomenon is also known as "infant mortality"; i.e. early failure of the structure that is attributable to construction defects (see Hoeg, 1996 with statistics of dam failures, occurring almost all in the beginning of their lifetimes (Figure 2.6)). Failure rates are increasing functions at the end of the lifetime of the structure, because of deterioration of the structure. Failure rates therefore have a U-shaped (or bath tub curved) form (see also Langley (1987) and Feld (1997)).

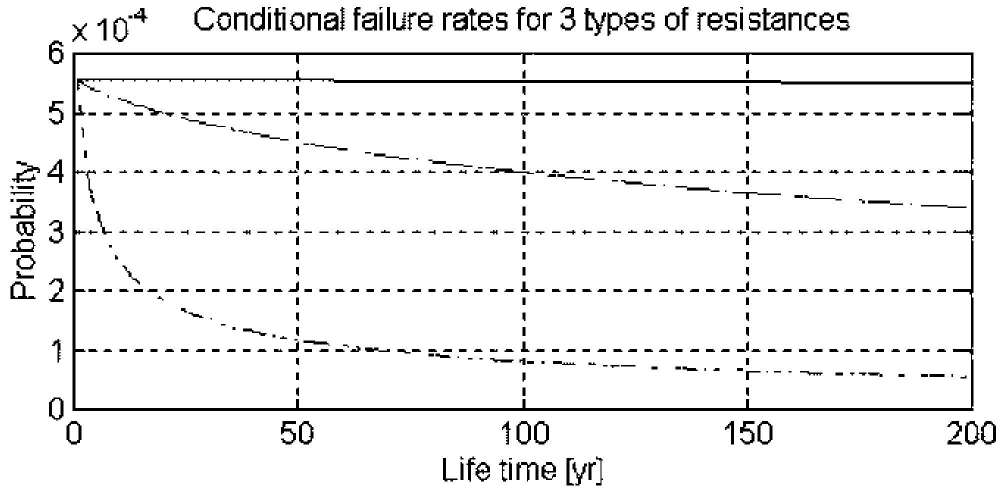


Figure 2.5: Failure rates
solid: resistance type I, dashed: type II, dash-dotted: type III

Over 75,000 dams in the United States form a critical part of their national infrastructure. Dams can pose a significant risk to the public if they are not maintained properly. Today, this risk is greater than in the past as a large number of the US dams approach and exceed their intended lifespan. According to the US National Performance of Dams Program (NPDP), more than 85 percent of the dams in the US will be more than 50 years old (the design life of a dam) by the year 2020. A recent report from the American Society of Civil Engineers notes that of the approximately 9,200 regulated dams in the United States that are classified as high hazard, i.e., their failure will likely cause a significant loss of life and property, 35 percent have not been inspected since 1990 or earlier. However, the cost to the public of a dam failure can be significant. When a dam fails, the potential energy of the water stored behind even a small dam can cause loss of life and great property damage if there are people downstream. Despite the strengthening of dam safety programs since the 1970's, dams continue to fail, causing loss of life and millions of dollars in property damage. In July 1994, the Tropical Storm Alberto caused over 200 dam failures in Georgia (FEMA, 1994). Nearly one-half of the deaths from the floods occurred when a series of unregulated earthen dams near the city of Americus burst, sending deadly walls of water through the town and surrounding areas and drowning 15 people. Between 1960 and 1997, there have been at least 23 dam failures causing 1 or more fatalities. Some failures also caused downstream dams to fail. There were 318 deaths as a result of these failures (FEMA, 1994).

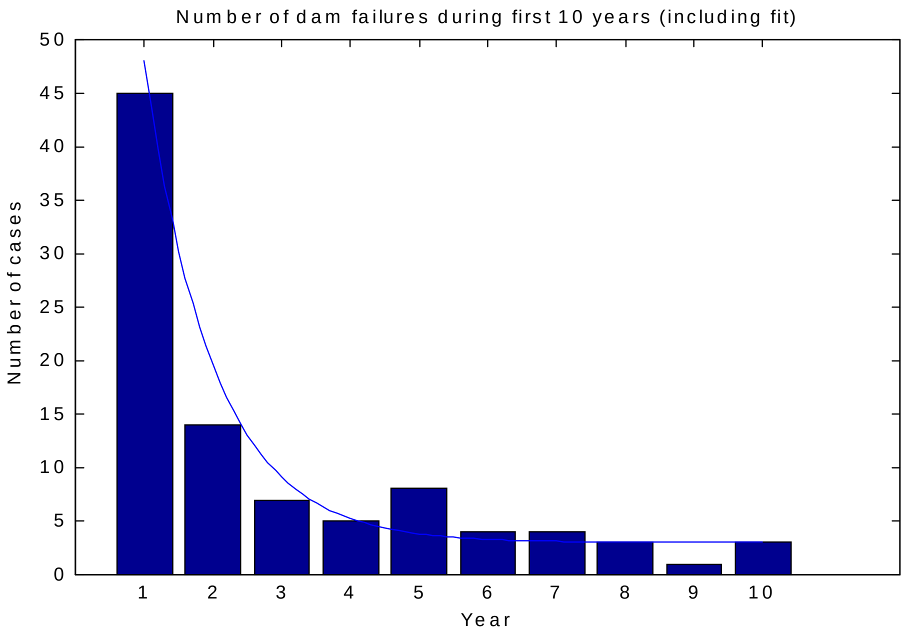


Figure 2.6: Failure statistics

2.5.3 Calculation methods

It was already noted that if the standard deviation of the yearly maximal wave load is large in relation to the standard deviation of the resistance the dependence between failure in two subsequent years is low. Therefore, if the probability of failure is say p per year then the failure probability is approximately N.p during the lifetime of N years. If however, keeping the standard deviation of the reliability function unchanged, the opposite is true, the failure in subsequent years are dependent and in that case the probability of failure in the first year is p and the probability of failure in the life time too. In the first case the failure rate is constant over time and equal to p. In the second case however the conditional failure rate equals p in the first year and falls to zero afterwards.

In terms of formulae: for small σ_R , the expression

$$F_L(N) = \int (1-F_S^N(x))\, f_R(x)\, dx \qquad (2.20)$$

can be approximated by the simple form:

$$1-(1-p)^N \qquad (2.21)$$

or

$$Np \qquad (2.22)$$

for small p, in which p is the probability of failure in the first year.

In case of larger σ_R this approximation is not allowed and only use of the integral expression of $F_L(N)$ can be made (Eqn. (2.20)). In Figure 2.7, the integral expression and the two approximation formulae (Eqn. (2.21) and (2.22)) are compared with each other.

Note that $P(R<S) = \iint\limits_{r<s} f_R(r) f_S(s) dr ds = \int_0^\infty \{1-F_S(r)\} f_R(r) dr = \int_0^\infty F_R(s) f_S(s) ds$, so that for N years we have:

$$P(\max(S_i | i=1..N) < R) = \int_0^\infty F_{\max(S_i|i=1..N)}(s) f_R(s) ds = \int_0^\infty F_S(s)^N f_R(s) ds$$

which explaines Eqn. (2.20).

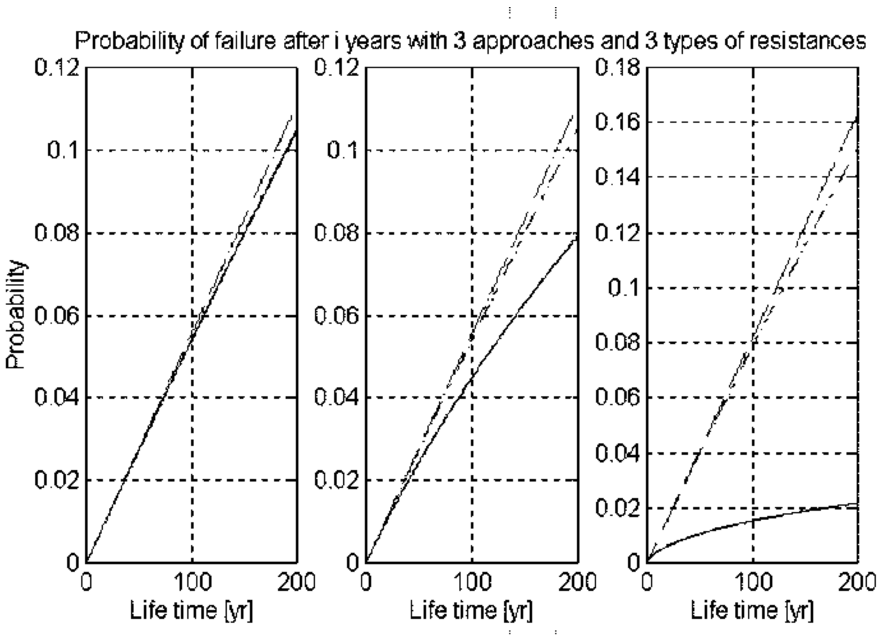


Figure 2.7. Comparison of three calculation methods
(Eqn. (2.20) solid, (2.21) dashed and (2.22) dash-dotted)

2.6 How to deal with uncertainties

If we are involved with calculating the expected probability distribution for a random variable X, then the inferences we make on X should reflect the uncertainty in the parameters θ. In the Bayesian terminology we are interested in the so-called predictive function:

$$F(x)=\int_{\theta} F(x|\theta)\, f(\theta)d\theta \qquad (2.23)$$

where $F(x|\theta)$ is the probabilistic model of X, conditional on the parameters θ and F(x) is the predictive distribution of the random variable x, now parameter free. In popular words: “the uncertainty in the θ parameters has to be integrated out”.

The predictive distribution can be interpreted as being the distribution $F(x|\theta)$ weighted by $f(\theta)$. *In making inferences on a random variable it is important to use the predictive function for x,* as opposed to the probabilistic model for x with some estimator for the parameter set θ, i.e. $f(x|\theta^*)$. This is because using point estimators for uncertain parameters underestimates the variance in the random variable X.

The techniques will be illustrated with an exponential distribution:

$$F(x)=1-e^{-\frac{x-\xi}{\alpha}} \qquad x\geq\xi \qquad (2.24)$$

The influence of statistical uncertainty in the shift parameter ξ will be considered by writing ξ as ξ + ϵ in which $\epsilon \sim N(0,\sigma_\xi)$. The PDF of ϵ is given by:

$$f(\varepsilon)=\frac{1}{\sigma_{\xi}\sqrt{2\pi}}e^{-\frac{\varepsilon^2}{2\sigma_{\xi}^2}} \tag{2.25}$$

According to Eqn. (2.23), we can write:

$$\begin{aligned} F(x) &= \int F(x|\varepsilon)f(\varepsilon)d\varepsilon = \int (1-e^{-\frac{x-\xi-\varepsilon}{\alpha}})\frac{1}{\sigma_{\xi}\sqrt{2\pi}}e^{-\frac{\varepsilon^2}{2\sigma_{\xi}^2}}d\varepsilon = \\ &= 1-\frac{1}{\sigma_{\xi}\sqrt{2\pi}}e^{-\frac{x-\xi}{\alpha}}\int e^{\frac{\varepsilon}{\alpha}-\frac{\varepsilon^2}{2\sigma_{\xi}^2}}d\varepsilon = 1-\frac{1}{\sigma_{\xi}\sqrt{2\pi}}e^{-\frac{x-\xi}{\alpha}}\int e^{-\frac{1}{2\sigma_{\xi}^2}\left(\varepsilon-\frac{\sigma_{\xi}^2}{\alpha}\right)^2}d\varepsilon\; e^{\frac{\sigma_{\xi}^2}{2\alpha^2}} = \\ &= 1-e^{-\frac{x-\xi}{\alpha}}e^{\frac{\sigma_{\xi}^2}{2\alpha^2}} = 1-e^{-\frac{x-\xi+\frac{\sigma_{\xi}^2}{2\alpha}}{\alpha}} \end{aligned} \tag{2.26}$$

Notice that the probability of exceedance curve is translated with $\sigma_{\xi}^2/2\alpha$.

Now we will consider the influence of statistical uncertainty in the scale parameter. For that purpose we rewrite the CDF as $F(x) = 1 - e^{-(ax - b)}$. Note that $a = 1/\alpha$ and $b = \xi/\alpha$. Assume a statistical uncertainty in the a parameter: $a = a + \epsilon$ in which $\epsilon \sim N(0,\sigma_a)$. Then:

$$F(x \mid \epsilon) = 1 - e^{-((a+\epsilon)x - b)} \tag{2.27}$$

and

$$f(\varepsilon)=\frac{1}{\sigma_{a}\sqrt{2\pi}}e^{-\frac{\varepsilon^2}{2\sigma_{a}^2}} \tag{2.28}$$

So:

$$\begin{aligned} F(x) &= \int F(x \mid \epsilon)\, f(\epsilon)\, d\epsilon = \int (1-e^{-((a+\epsilon)x-b)})½\sigma_a^{-1}\pi^{-1/2}\exp(-½(\epsilon/\sigma_a)^2)\, d\epsilon = \\ &= 1- e^{-(ax-b)}\exp(½\sigma_a^2 x^2) \end{aligned} \tag{2.29}$$

This can be written again in terms of ξ and α like:

$$F(x) = 1- \exp\{-(x-\xi-½\sigma_a^2 x^2 \alpha)/\alpha\} \tag{2.30}$$

Note that $\sigma_a = \sigma(1/\alpha) = f(\sigma_\alpha)$ is difficult to express as a function of σ_α. The approximation $\sigma(1/\alpha) \approx 1/\sigma(\alpha)$ may not be used. However the relation $CV(1/\alpha) \approx CV(\alpha)$ is quite good (as will be shown in Section 3.7.2 using Taylor expansions). We therefore use as a first approximation $\sigma_a = \sigma_\alpha/\alpha^2$. Substitution in Eqn. (2.30) leads to:

$$\begin{aligned} F(x) &= 1- \exp\{-(x-\xi-½\sigma_a^2 x^2 \alpha)/\alpha\} = 1 - \exp\{-(x-\xi-½\sigma_\alpha^2 x^2 \alpha^{-3})/\alpha\} = \\ &= 1-\exp\{-(x-\xi)/\alpha\}\exp\{\sigma_\alpha^2 x^2 / 2\alpha^4\} \end{aligned} \tag{2.31}$$

Notice that the probability of exceedance is translated as a function of σ_a and x. So apart from a shift also the slope of the survival function 1-F increases. Summarizing these results leads to the following Table 2.3:

Table 2.3: Multiplication factors

Exponential Distribution	Shift Parameter	Scale Parameter
Multiplication Factor	$e^{\frac{\sigma_\xi^2}{2\alpha^2}}$	$e^{\frac{\sigma_\alpha^2 x^2}{2\alpha^4}}$

From Table 2.3, we notice the influence of the x-value in the multiplication factor for the scale parameter. The influence of the x-value in the multiplication factor for the shift parameter has disappeared.

Summarized; if $F(x) = 1 - e^{-(x-\xi)/\alpha}$ has an uncertainty in the scale parameter (given by σ_α which should be not too large), then in making inferences on X the original exponential distribution should be "replaced" by Eqn.(2.31).

The equation (2.31) is applied to the data set of extreme water levels at Hook of Holland. This set can be modelled with an exponential distribution with parameters A=1.96 and B=0.33. Different levels of uncertainty in B will be discerned: $\sigma_B = 0.17$, $\sigma_B = 0.11$, and $\sigma_B = 0.05$. The influence of the uncertainty is depicted in Figure 2.9 and appears to be quite large in this particular case study. Notice the combination of translation and rotation of the frequency curves.

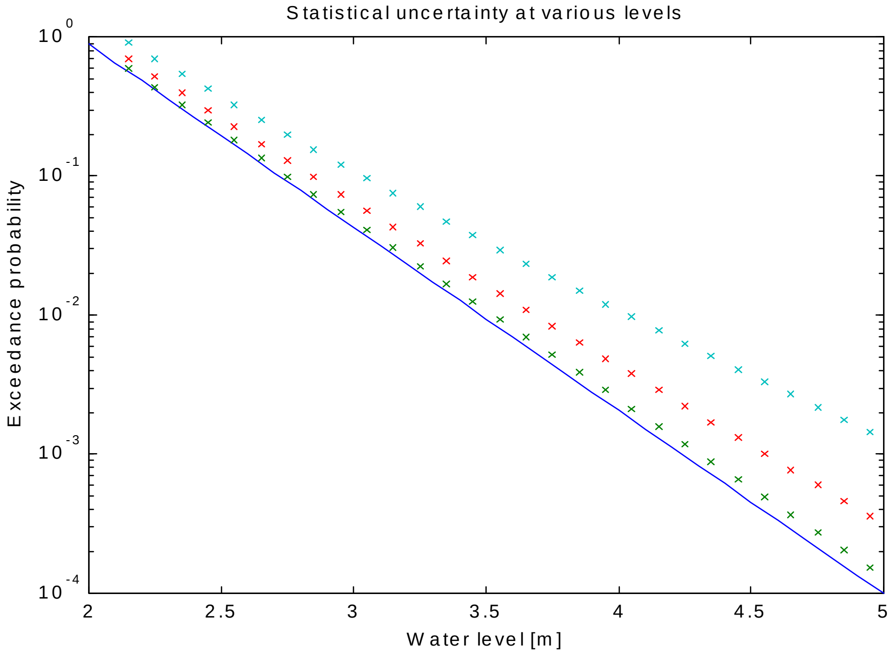


Figure 2.9: Translation and rotation of the frequency curves as σ_B increases from 5%, 11% to 17%

Uncertainty and sensitivity analyses are similar in that both strive to evaluate the variation in results arising from the variations in the assumptions, models, and data. However, they differ in scope, and the information they provide.

Uncertainty analysis attempts to describe the likelihood for different size variations and tends to be more formalized than sensitivity analysis. An uncertainty analysis explicitly quantifies the uncertainties and their relative magnitudes, but requires probability distributions for each of the random variables. The assignment of these distributions often involves as much uncertainty as that to be quantified.

Sensitivity analysis is generally more straightforward than uncertainty analysis, requiring only the separate (simpler) or simultaneous (more complex) changing of one or more of the inputs. Expert judgement is involved to the extent that the analyst decides which inputs to change, and how much to change them. This process can be streamlined if the analyst knows which variables have the greatest effect upon the results. Variation of inputs one at a time is preferred, unless multiple parameters are affected when one is changed. In this latter case, simultaneous variation is required.

Several studies involving detailed uncertainty/sensitivity analyses were performed by Varpasuo et al. (1999), Van Gelder and Varpasuo (1998), Choi et al. (1995) and Vrijling et al. (1999). In Lungu and Van Gelder (1996, 1997) and Van Gelder and Lungu (1997b), sensitivity studies were performed on the influence of the terrain roughness, the reference velocity and the height above the terrain on three different wind spectra of along wind gustiness. Attention was paid to their spectral density functions (in particular the bandwidth measure of Longuet-Higgens (1952), the zero-upcrossing rate (e.g. Pandey and Ariaratnam, 1996), and the mean and standard deviation of the gust factor), the notion of cut-off frequency, and integral scales of turbulence. In characterizing wind turbulence, the length scales of turbulence play an important role. The length scales of turbulence are a comparative measure of the average size of a gust in appropriate directions and are important scaling factors in determining how rapidly gust properties vary in space.

In Van Gelder and Varpasuo (1998) and Varpasuo et al. (1999), a seismic hazard assessment (SHA) for nuclear power plants (NPPs) contains the so-called seismic hazard curve and uniform hazard spectra for mean return periods of earthquakes of up to 10^6 years. A major uncertainty in a SHA is the lack of earthquake data. If one wants to estimate the magnitudes of earthquakes with mean return periods up to 10^6 years, one has to deal with statistical - as well as model uncertainty of the probabilistic model of the occurrence relations. In Van Gelder and Varpasuo (1998) a Bayesian type of SHA was suggested and applied to a case study of a NPP in Khmelnitsky in the western part of the Ukraine which is under the influence of the subcrustal seismic activities of the Vrancea source in Romania (Lungu et al., 1995 and 1997). In Van Gelder and Lungu (1997a), a statistical analysis was performed of the occurrences of earthquakes in the Vrancea source by combining, with Bayesian methods, the short instrument catalogue of 100 years with the much longer historical catalogue of 900 years. The results of their analysis were used to perform a Bayesian analysis of the SHA of Khmelnitsky. From Figure 2.10 it can be seen that the seismic hazard slightly increases for the return periods between 1,000 and 100,000 years.

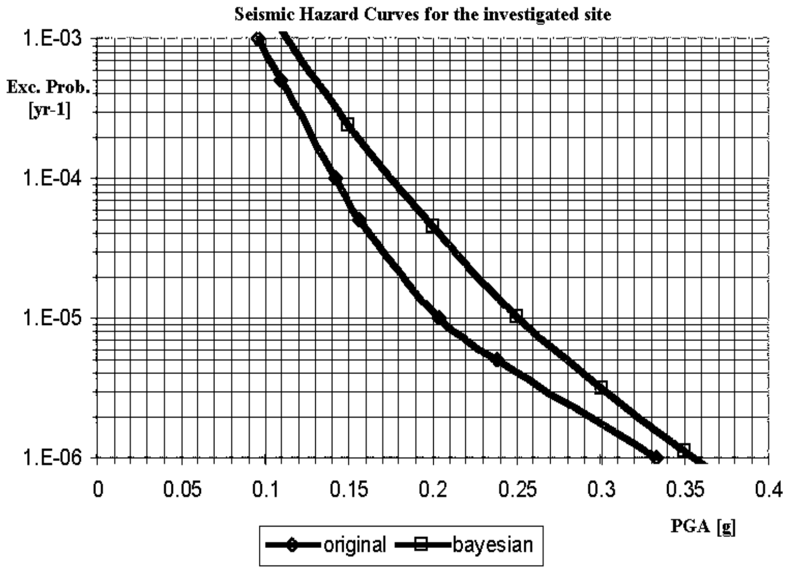


Figure 2.10: Uncertainty analysis of the seismic hazard in Khmelnitsky

2.7 Discussion

In probabilistic calculations of structures a difference occurs between uncertainties that have many realisations during the structure's lifetime (e.g. wave heights) and uncertainties that have only one realisation (e.g. soil parameters). This difference leads for example to the observation that every storm that was survived by the structure improves the knowledge of the PDF of the resistance (pushing the lower tail to the right). Every year the knowledge of the owner of the structure grows and the probability of failure of the structure decreases if it survived another year. In exactly the same way the failure probability of the structure can be improved by investigating e.g. the quality of the foundation assuming that this leaves the average unchanged and reduces the uncertainty. These ideas have been illustrated in this chapter by a simple hypothetical example of a flood protection structure. An analysis of the effect of inherent uncertainty in time and space on the probability model w.r.t. the failure probability and the failure rate has also been given. The way how to deal with uncertainties (by "integrating out") has been shown in Sec. 2.6.

Chapter 3

Statistical Estimation Methods

The book of the universe is written
in the language of mathematics.
- Gallileo.

3.1 Introduction

In designing civil engineering structures use is made of probabilistic calculation methods. Stress and load parameters are described by statistical distribution functions. The parameters of these distribution functions can be estimated by various methods. An extensive comparison of these different estimation methods is given in this chapter. The main point of interest is the behaviour of each method for predicting p-quantiles (the value which is exceeded by the random variable with probability p), where $p \ll 1$. The estimation of extreme quantiles corresponding to a small probability of exceedance is commonly required in the risk analysis of hydraulic structures. Such extreme quantiles may represent design values of environmental loads (wind, waves, snow, earthquake), river discharges, and flood levels specified by design codes and regulations (TAW, 1990). In this chapter the performance of the parameter estimation methods with respect to its small sample behaviour is analyzed with Monte Carlo simulations, added with mathematical proofs.

In civil engineering practice many parameter estimation methods for probability distribution functions are in circulation. Well known methods are for example:

- the method of moments (Johan Bernoulli, 1667-1748),
- the method of maximum likelihood (Daniel Bernoulli, 1700-1782),
- the method of least squares (on the original or on the linearized data), (Gauss, 1777-1855),
- the method of Bayesian estimation (Bayes, 1763),
- the method of minimum cross entropy (Shannon, 1949),
- the method of probability weighted moments (Greenwood et al., 1979),
- the method of L-moments (Hosking, 1990).

Textbooks, such as Benjamin and Cornell (1970), Berger (1980), treat the traditional methods in detail. The methods will be briefly reviewed in this chapter.

Many attempts (for instance, Goda and Kobune, (1990), Burcharth and Liu, (1994), Yamaguchi, (1997), and Van Gelder and Vrijling, (1997a)), have been made to find out which estimation method is preferable for the parameter estimation of a particular probability distribution in order to obtain a reliable estimate of the p-quantiles. In this chapter, we will in particularly investigate the performance of the parameter estimation method with respect to three different criteria; (i) based on the relative bias and root mean squared error (RMSE), (ii) based on the over- and underdesign, (iii) based on entropy measures.

It is desirable that the quantile estimate be unbiased, that is, its expected value should be equal to the true value. It is also desirable that an unbiased estimate be efficient, i.e., its variance should be as small as possible. The problem of unbiased and efficient estimation of extreme quantiles from small samples is commonly encountered in the civil engineering practice. For example, annual flood discharge data may be available for past 50 to 100 years and on that basis one may have to estimate a design flood level corresponding to a 1,000 to 10,000 years return period (Van Gelder et al., 1995).

The first step in quantile estimation involves fitting an analytical probability distribution to represent adequately the sample observations. To achieve this, the distribution type should be judged from data and then parameters of the selected distribution should be estimated. Since the bias and efficiency of quantile estimates are sensitive to the distribution type, the development of simple and robust criteria for fitting a representative distribution to small samples of observations has been an active area of research. In this chapter three different methods for the selection of the distribution type will be reviewed, extended and tested. The first method is based on Bayesian statistics, the second one on linear regression, and the third one on L-moments. Certain linear combinations of expectations of order statistics, also referred to as L-moments by Hosking (1990), have been shown to be very useful in statistical parameter estimation. Being a linear combination of data, they are less influenced by outliers, and the bias of their small sample estimates remains fairly small. A measure of kurtosis derived from L-moments, referred to as L-kurtosis, was suggested as a useful indicator of distribution shape (Hosking, 1992).

Hosking (1997) proposed a simple but effective approach to fit 3-parameter distributions. The approach involves the computation of three L-moments from a given sample. By matching the three L-moments, a set of 3-parameter distributions can be fitted to the sample data. In this chapter, a distribution type selection which is based on the the 4^{th} L-moment is suggested to be the most representative distribution, which should be used for quantile estimation. In essence, the L-kurtosis, which is related to the 4th L-moment, can be interpreted as a measure of resemblance between two distributions having common values of the first three L-moments.

The concept of probabilistic distance or discrimination between two distributions is discussed in great detail in modern information theory (Kullback 1959, Jumarie 1990). Mathematically sound measure of probabilistic distance, namely, the divergence, has been used to establish resemblance between two distributions or conversely to select the closest possible posterior distribution given an assumed prior distribution. The divergence is a comprehensive measure of probabilistic distance, since it involves the computation of departure of a distribution from the reference parent distribution over an entire range of the random variable. Apart from the performance of estimation methods based on bias and RMSE, and the performance based on under- and overdesign, also the performance based on entropy measures is suggested in this chapter.

Furthermore, this chapter will focus on evaluating the robustness of the L-kurtosis measure in the distribution selection and extreme quantile estimation from small samples. The robustness is evaluated against the benchmark estimates obtained from the information theoretic measure, namely, the divergence. For this purpose, a series of Monte Carlo simulation experiments were designed in which probability distributions were fitted to the sample observations based on L-kurtosis and divergence based criteria, and the accuracies of quantile estimates were compared. The simulation study revealed that the L-kurtosis measure is fairly effective in quantile estimation.

Finally, this chapter shows some analytical considerations concerning statistical estimation methods and probability distribution functions. The chapter ends with a discussion.

3.2 Classical estimation methods

To make statements about the population on the basis of a sample, it is important to understand in what way the sample relates to the population. In most cases the following assumptions will be made:

1. Every sample observation x is the outcome of a random variable X which has an identical distribution (either discrete or continuous) for every member of the population;
2. The random variables X_1, X_2, ..., X_n corresponding to the different members of the sample are independent.

These two assumptions (abbreviated to i.i.d. (independent identically distributed)) formalize what is meant by the statement of drawing a random sample from a population.

We have now reduced the problem to one which is mathematically very simple to state: we have i.i.d. observations x_1, x_2, ..., x_n of a random variable X with probability function (in the discrete case) or probability density function (in the continuous case) f, and we want to estimate some aspect of this population distribution (for instance the mean or the variance).

It is helpful here to stress again the notation that we are using in this thesis: small case letters x_i denote actual sample values. But each x_i is the realisation of a random variable X_i, denoted by capitals. Thus $X=(X_1, X_2, ..., X_n)$ denotes a random sample, whose particular value is $x=(x_1, x_2, ..., x_n)$. The distinction helps us to distinguish between a random quantity and the outcome this quantity actually realises.

A statistic, T(X), is any function of the data (note that T(X) denotes that this is a random quantity which varies from sample to sample; T(x) will denote the value for a specific sample x). If a statistic is used for the purpose of estimating a parameter then it is called an estimator and the realised value T(x) is called an estimate. The basis of our approach will be to use T(x) as the estimate of θ, but to look at the sampling properties of the estimator T(x) to judge the accuracy of the estimate. Since any function of the sample data is a potential estimator, how should we determine whether an estimator is good or not? There are, in fact, many such criteria: we will focus on the two most widely used:

- Though we cannot hope to estimate a parameter perfectly, we might hope that 'on average' the estimation procedure gives the correct result.
- Estimators are to be preferred if they have small variability; in particular, we may require the variability to diminish as we take samples of a larger size.

These concepts are formalized as follows.
The estimator T(X) is unbiased for θ if :

$$E(T(X)) = \theta \tag{3.1}$$

Otherwise, $B(T) = E(T(X)) - \theta$ is the bias of T.
If $B(T) \to 0$ as the sample size $n \to \infty$, then T is said to be asymptotically unbiased for θ.
The mean-squared error of an estimator is defined by:

$$MSE(T) = E((T(X) - \theta)^2) \tag{3.2}$$

Note that MSE(T)=var(T)+B^2(T). Indeed MSE(T) = E(T^2(X)-2θT(X)+θ^2) = E(T^2(X)) –2θE(T(X)) + θ^2 = E(T^2(X)) - 2θ(B(T)+θ) + θ^2 = E(T^2(X)) - 2θB(T) - θ^2 and var(T) = E(T^2(X)) – E^2(T(X)) = E(T^2(X)) – (B(T) + θ)^2 = E(T^2(X)) – B^2(T) - 2θB(T) - θ^2 . This proves the equality.

The root mean-squared error of an estimator is defined as:

$$RMSE = \sqrt{MSE} \tag{3.3}$$

An estimator T is said to be mean-squared consistent for θ if MSE(T) → 0 as the sample size n→∞.

Ideally, estimators are both unbiased and consistent. We also prefer estimators to have as small a variance as possible. In particular, given two estimators T_1 and T_2 both being unbiased for θ, then T_1 is said to be more efficient than T_2 if :

$$var(T_1(X)) < var(T_2(X)) \tag{3.4}$$

If estimators are not unbiased it is not so straightforward to determine efficiency: we often have to make a choice between estimators that have low bias but high mean squared error and estimators that have high bias but low mean squared error.

Having established some possible criteria (in Section 3.6.6, we will introduce other criteria) by which to judge estimators, we now turn to general procedures for constructing estimators. In this Section 3.2 we will concentrate on three classical estimation methods, and derive their properties for a certain number of distribution functions. Furtheron in the chapter an extremely powerful procedure based on L-Moments estimations (Sec. 3.3), as well as the Bayesian and Entropy methods (Sec. 3.4 and 3.5) will be presented.

3.2.1 Method of Moments (MOM)

It is difficult to trace back who introduced the MOM, but Johan Bernoulli (1667-1748) was one of the first who used the method in his work. With the MOM, the moments of a distribution function in terms of its parameters are set equal to the moments of the observed sample. Analytical expressions can be derived quite easily (Appendix), but the estimators can be biased and not efficient. The moment estimators however, can be very well used as a starting estimation in an iteration process.

The central moments of a distribution are given by:

$$
\begin{aligned}
&m_r = E(X-m)^r = \int (x-m)^r f_X(x)dx\\
&Variance{:}\, s^2 = m_2\\
&Skewness{:}\sqrt{b_1} = \frac{m_3}{m_2^{3/2}}\\
&Kurtosis{:}\, b_2 = \frac{m_4}{m_2^2}
\end{aligned}
\qquad (3.5)
$$

The sample moments are given by:

$$
\begin{aligned}
&\bar{x} = n^{-1}\Sigma x_i\\
&m_r = n^{-1}\Sigma(x_i - \bar{x})^r
\end{aligned}
\qquad (3.6)
$$

The sample mean $\bar{x}$ is a natural estimator for μ. The higher sample moments m_r are reasonable estimators of the μ_r , but they are not unbiased. Unbiased estimators are often used. In particular σ^2, μ_3 and the fourth cumulant $\kappa_4=\mu_4 - 3\mu_2^2$ are unbiasedly estimated by:

$$
\begin{aligned}
&s^2 = (n-1)^{-1}\Sigma(x_i - \bar{x})^2\\
&m_3^* = \frac{n^2}{(n-1)(n-2)} m_3\\
&k_4^* = \frac{n^2}{(n-2)(n-3)}\left(\frac{n+1}{n-1} m_4 - 3m_2^2\right)
\end{aligned}
\qquad (3.7)
$$

The sample standard deviation $s = \sqrt{s^2}$ is an estimator of σ but is not unbiased. The sample estimators of CV (Coefficient of Variation), skewness and kurtosis are respectively:

$$
\begin{aligned}
&\hat{C}_v = s / \bar{x}\\
&g = m_3^* / s^3\\
&k = k_4^* / s^4 + 3
\end{aligned}
\qquad (3.8)
$$

Finding theoretical moments as a function of θ is not easy for all probability distributions. The method is difficult to generalize to more complex situations (dependent data, covariates, non-identically distributed data). Sample covariances may be used to estimate parameters that determine dependence. For some distributions (such as Cauchy), moments may not exist. In the Appendix moments are given for most familiar PDFs.

3.2.2 Method of Maximum Likelihood (MML)

Also with the MML it is difficult to say who discovered the method, although Daniel Bernoulli (1700-1782) was one of the first who reported about it (Kendall, 1961). The likelihood function gives the relative likelihood of the obtained observations, as a function of the parameters θ:

$$L(\theta,x)=\Pi f(x_i , \theta) \tag{3.9}$$

With this method one chooses that value of θ for which the likelihood function is maximized. In the Appendix an overview is given of the likelihood functions of some familiar distribution functions. The ML-method gives asymptotically unbiased parameter estimations and of all the unbiased estimators it has the smallest mean squared error. The variances approach asymptotically to:

$$Var(\Theta)= - E(\partial^2 \log L(\theta,x)/\partial \theta^2) \tag{3.10}$$

Furthermore these estimators are invariant, consistent and sufficient. For the definitions we refer to Hald (1952). Analytical expressions for the parameter estimators are sometimes difficult to derive. In those cases, numerical optimization routines have to be used to determine the maximum of the likelihood function, which can also be quite difficult since the optimum of the likelihood function can be extremely flat for large sample sizes. Optimization of the likelihood function may also be hampered by the presence of local maxima. Furthermore:

- MML is (usually) straightforward to implement,
- Maximum likelihood estimators (MLEs) may not exist, and when they do, they may not be unique or give a bias error (Koch, 1991),
- MLE may give inadmissible results (Lundgren, 1988),
- The likelihood function can be used for much more than just finding the MLE: values close to the MLE are more plausible than those further away, for example. This argument can be used to obtain an interval $[p_L,p_U]$ which comprises a plausible range of values for θ,
- ML is adaptable to more complex modeling situations, because the MLE satisfies a very convenient invariance property: If q=h(θ) where h is a bijective function then $q_{ML}=h(\theta_{ML})$. For example, if q=1/θ, then $q_{ML}=1/\theta_{ML}$. So, having found the MLE for one parameterization, the MLEs for other parameterizations are immediate,
- The maximum likelihood estimator is unbiased, fully efficient (in that it achieves the Cramér-Rao bound under regularity conditions), and normally distributed; all of them in asymptotical sense. Regularity conditions are not fulfilled if the range of the random variable X depends on unknown parameters as is the case for many distributions in the present thesis.

The MML is extremely useful since it is often quite straightforward to evaluate from the MLE and the observed information. Nonetheless it is an approximation, and should only be trusted for large values of n (though the quality of the approximation will vary from model to model).

If the available sample sizes are large, there seems little doubt that the maximum-likelihood estimator is a good choice. It should be emphasized, however, that the properties above are asymptotic (large n), and better estimators may be available when sample sizes are small.

3.2.3 Method of Least Squares (MLS)

Least Squares were introduced by Gauss (1777-1855). Given the observations $x=(x_1, x_2, ..., x_n)$ and $y=(y_1, y_2, ..., y_n)$, a regression model can be fitted. For the general case:

$$E(Y|x) = \alpha + \beta\ x \qquad (3.11)$$

with σ^2 the assumed constant variance of Y around its regression line, the parameter estimates are:

$$\begin{aligned} &\alpha^* = m_Y - \beta^*\ m_X \\ &\beta^* = s_{XY} / s_X^2 \\ &\sigma^2{}^* = n/(n-2)\ (1-r_{XY}^2)s_Y^2 \end{aligned} \qquad (3.12)$$

in which m_X, m_Y, s_X, s_Y, s_{XY} and r_{XY} are defined as:

$$\begin{aligned} &m_X = 1/n\ \Sigma_{i=1..n}x_i \\ &m_Y = 1/n\ \Sigma_{i=1..n}y_i \\ &s_X^2 = 1/n\ \Sigma_{i=1..n}(x_i-m_X)^2 \\ &s_Y^2 = 1/n\ \Sigma_{i=1..n}(y_i-m_Y)^2 \\ &s_{XY} = 1/n\ \Sigma_{i=1..n}(x_i-m_X)(y_i-m_Y) \\ &r_{XY} = s_{XY}/s_Xs_Y \end{aligned} \qquad (3.13)$$

The estimators α^* and β^* are linear functions of the Y_i 's and they are unbiased. Their variances are:

$$\begin{aligned} &\sigma_A^2 = \sigma^2/n\ (1+m_X^2/s_X^2) \\ &\sigma_B^2 = \sigma^2 / ns_X^2 \end{aligned} \qquad (3.14)$$

in which A and B are respectively, α^* and β^* treated as random variables.

With the above regression techniques, the MLS can be defined. Assume that we have n observations in sorted order given by $x_{1:n}$, $x_{2:n}$, ..., $x_{n:n}$. Define the plotting position $p_i = i/(n+1)$ of the i-th observation. We want to estimate the optimal value θ of the distribution function $F(x|\theta)$ in least squares sense. The following options are available:

1) Least squares error: $\Sigma_{i=1}^{n} [p_i-F(x_{i:n}|\theta)]^2$ is to be minimized over θ (sometimes also referred to as non-linear regression (Demetracopoulis, 1994)).

Assume that we can linearize the probability distribution F(x) under consideration (with scale parameter B and location parameter A). So we can find a function g s.t.:

$$g(F(x)) = (x-A)/B \tag{3.15}$$

For instance, for the Exponential distribution g is given by $g_E(\zeta) = -\ln(1-\zeta)$ and for the Gumbel distribution, we have $g_G(\zeta) = -\ln(-\ln(\zeta))$ (see Appendix for a complete overview of linearization functions).

2) Least squares linearized error $\Sigma_{i=1}^{n} [g(p_i)-g(F(x_{i:n}|\theta))]^2$ to be minimized over θ.

3) Weighted least squares error $\Sigma_{i=1}^{n} [w_i(p_i)-w_i(F(x_{i:n}|\theta))]^2$ to be minimized over θ, where for instance w_i can weigh the extreme observations to be better fitted by the distribution function (for instance $w_i=1/(1-p_i)^4$).

4) Least squares to the ordered observations themselves: $\Sigma_{i=1}^{n} [F^{-1}(p_i)- x_{i:n}]^2$ as suggested by Moharram et al. (1993).

In Van Gelder (1996b), several LS-methods are applied to wave data at the location of Pozzallo in Southern Italy. In Section 3.9.1 it will be shown that the parameter estimation of the scale of various 2-parameter distributions with LS always leads to larger p-quantiles than with the method of moments. Furthermore, note that under the assumption that the errors in a regression model are independent and normally distributed with zero mean and a fixed standard deviation, the ML-estimators of the error-distribution are exactly the same as the LS-estimators (Gauss, 1777-1855). Problems with LS-estimators are described by McCuen et al. (1990), in which they showed that logarithmic transformations may lead to a biased model.

3.3 Method of L-Moments

Hosking, (1990), introduced the L-moments. They have become popular tools for solving various problems related to parameter estimation, distribution identification, and regionalization. It can be shown that L-moments are linear functions of probability weighted moments (PWM's, see Section 3.3.1) and hence for certain applications, such as the estimation of distribution parameters, serve identical purposes (Hosking, 1986). In other situations, however, L-moments have significant advantages over PWM's, notably their ability to summarize a statistical distribution in a more meaningful way. Since L-moment estimators are linear functions of the ordered data values, they are virtually unbiased and have relatively small sampling variance. L-moment ratio estimators also have small bias and variance, especially in comparison with the classical coefficients of skewness and kurtosis. Moreover, estimators of L-moments are relatively insensitive to outliers. These often-heard arguments in favor of estimation of distribution parameters by L-moments (or PWM's) should, nevertheless, not be accepted blindly. *In for instance a wave height frequency analysis, the interest is the estimation of a given quantile*, not in the L-moments themselves. Although the latter may have desirable sampling properties, the same does not necessarily apply to a function of them, such as a quantile estimator. In fact, several simulation studies have demonstrated that for some distributions, *other estimation methods may be superior in terms of mean square errors of quantile estimators* (Hosking and Wallis, 1987; Rosbjerg et al., 1992). As compared with for example the classical method of moments, the robustness vis-à-vis sample outliers is clearly a characteristic of L-moment estimators. However, estimators can be “too robust” in the sense that large (or small) sample values reflecting important information on the tail of the parent distribution are given too little weight in the estimation. Hosking (1990) assessed that L-moments weigh each element of a sample according to its relative importance.

In this section first the theory of L-Moments will be briefly described, followed with an overview of papers with applications of L-moments. The literature review has shown that the theory of L-moments have mostly been applied in a regionalized setting combining data from more than one site. However, in univariate settings the method of L-moments has not been investigated so much. Therefore furtheron in this chapter (Sec. 3.7.5) a Monte Carlo experiment is designed in a univariate setting in order to compare the L-moments method with the classical parameter estimation methods (MOM, MML, and MLS). The performance of these methods will also be analyzed w.r.t. inhomogeneous data.

Finally, L-moments are in fact nothing else than summary statistics for probability distributions and data samples. They are analogous to ordinary moments -- they provide measures of location, dispersion, skewness, kurtosis, and other aspects of the shape of probability distributions or data samples -- but are computed from linear combinations of the ordered data values (hence the prefix L). Hosking and Wallis (1997) give an excellent overview on the whole theory of L-Moments.

3.3.1 L-Moments for data samples

Probability weighted moments, defined by Greenwood et al. (1979), are precursors of L-moments. Sample probability weighted moments, computed from data values $x_{1:n}$, $x_{2:n}$, ... $x_{n:n}$, arranged in increasing order, are given by:

$$\begin{aligned} b_0 &= n^{-1}\sum_{j=1}^{n} x_{j:n} \\ b_r &= n^{-1}\sum_{j=r+1}^{n} \frac{(j-1)(j-2)\ldots(j-r)}{(n-1)(n-2)\ldots(n-r)} x_{j:n} \end{aligned} \qquad (3.16)$$

L-moments are certain linear combinations of probability weighted moments that have simple interpretations as measures of the location, dispersion and shape of the data sample. A sample of size 2 contains two observations in ascending order $x_{1:2}$ and $x_{2:2}$. The difference between the two observations $x_{2:2} - x_{1:2}$ is a measure of the scale of the distribution. A sample of size 3 contains three observations in ascending order $x_{1:3}$, $x_{2:3}$ and $x_{3:3}$. The difference between the two observations $x_{2:3} - x_{1:3}$ and the difference between the two observations $x_{3:3} - x_{2:3}$ can be subtracted from eachother to have a measure of the skewness of the distribution. This leads to: $x_{3:3} - x_{2:3} - (x_{2:3} - x_{1:3}) = x_{3:3} - 2x_{2:3} + x_{1:3}$. A sample of size 4 contains four observations in ascending order $x_{1:4}$, $x_{2:4}$, $x_{3:4}$ and $x_{4:4}$. A measure for the kurtosis of the distribution is given by: $x_{4:4} - x_{1:4} - 3(x_{3:4} - x_{2:4})$. In short: the above linear combinations of the elements of the ordered sample contain information about the location, scale, skewness and kurtosis of the distribution from which the sample was drawn. A natural way to generalize the above approach to samples of size n, is to take all possible sub-samples of size 2 and to average the differences $(x_{2:2} - x_{1:2})/2$:

$$l_2 = \frac{1}{2}\binom{n}{2}^{-1} \sum\sum_{i>j} (x_{i:n} - x_{j:n}) \qquad (3.17)$$

Furthermore, the skewness and kurtosis are related with:

$$l_3 = \frac{1}{3}\binom{n}{3}^{-1} \sum\sum\sum_{i>j>k} (x_{i:n} - 2x_{j:n} + x_{k:n}) \tag{3.18}$$

$$l_4 = \frac{1}{4}\binom{n}{4}^{-1} \sum\sum\sum\sum_{i>j>k>l} (x_{i:n} - 3x_{j:n} + 3x_{k:n} - x_{l:n}) \tag{3.19}$$

Hosking (1990) showed that the first few L-moments follow from PWMs via:

$$\begin{aligned} l_1 &= b_0 \\ l_2 &= 2b_1 - b_0 \\ l_3 &= 6b_2 - 6b_1 + b_0 \\ l_4 &= 20b_3 - 30b_2 + 12b_1 - b_0 \end{aligned} \tag{3.20}$$

The coefficients in Eqn. (3.20) are those of the shifted Legendre polynomials. The first L-moment is the sample mean, a measure of location. The second L-moment is (a multiple of) Gini's mean difference statistic (Johnson et al., 1994), a measure of the dispersion of the data values about their mean. By dividing the higher-order L-moments by the dispersion measure, we obtain the L-moment ratios,

$$t_r = l_r/l_2 \tag{3.21}$$

These are dimensionless quantities, independent of the units of measurement of the data; t_3 is a measure of skewness and t_4 is a measure of kurtosis -- these are respectively the L-skewness and L-kurtosis. They take values between -1 and +1 (exception: some even-order L-moment ratios computed from very small samples can be less than -1). The L-moment analogue of the coefficient of variation (standard deviation divided by the mean), is the L-CV, defined by:

$$t = l_2/l_1 \tag{3.22}$$

It takes values between 0 and 1 (if $X \geq 0$).

3.3.2 L-Moments for probability distributions

For a probability distribution with cumulative distribution function F(x), probability weighted moments are defined by:

$$b_r = \int x\{F(x)\}^r dF(x), \qquad r = 0,1,2,\ldots \tag{3.23}$$

L-moments are defined in terms of probability weighted moments, analogously to the sample L-moments:

$$
\begin{aligned}
l_1 &= b_0 \\
l_2 &= 2b_1 - b_0 \\
l_3 &= 6b_2 - 6b_1 + b_0 \\
l_4 &= 20b_3 - 30b_2 + 12b_1 - b_0
\end{aligned}
\tag{3.24}
$$

L-moment ratios are defined by:

$$\tau_r = \lambda_r / \lambda_2 \tag{3.25}$$

The L-moment analogue of the coefficient of variation, is the L-CV, defined by:

$$\tau = \lambda_2 / \lambda_1 \tag{3.26}$$

Examples (for a complete overview, see the Appendix):
Uniform (rectangular) distribution on (0,1):

$$l_1 = 1/2, \quad l_2 = 1/6, \quad t_3 = 0, \quad t_4 = 0. \tag{3.27}$$

Normal distribution with mean 0 and variance 1:

$$l_1 = 0, \quad l_2 = 1/\sqrt{p}, \quad t_3 = 0, \quad t_4 \approx 0.123. \tag{3.28}$$

The theory of L-moments has been applied in numerous papers. The following work is worth to mention: Rao and Hamed (1997), Duan et al. (1998), Ben-Zvi and Azmon (1997), Van Gelder and Neykov (1998), Demuth and Kuells (1997), Pearson et al. (1991), Ruprecht and Karafilis (1994), Anctil et al. (1998), Lin and Vogel (1993) and Gingras and Adamowski (1994).

3.3.3 Relation of L-Moments with order statistics

Consider a sample consisting of *n* observations $\{x_1, x_2, \ldots, x_n\}$ randomly drawn from a statistical population. If the sample values are rearranged in a non-decreasing order of magnitude, $x_{1:n} \le x_{2:n} \le \ldots \le x_{n:n}$, then the *r*-th member ($x_{r:n}$) of this new sequence is called the *r*-th *order statistic* of the sample (Harter, 1969). When all the sample values come from a common parent population with cumulative distribution function $F(x)$, the probability distribution (CDF) of the *r*-th *order statistic*, i.e., Prob[$X_{r:n} \le x$], means that at least *r* observations in a sample of *n* do not exceed a fixed value, *x*.

A sample randomly drawn from a distribution is analogous to a Bernouilli experiment in which the success is defined by the sampled value being less than the threshold, x. Naturally, the probability of success in such an experiment is given as $p = F(x)$, and the number of successes, a random variable, follows the binomial distribution. Based on this argument, the CDF of the r-th order statistic, $F_{(r)}(x)$, can be mathematically expressed as

$$F_{(r)}(x) = \sum_{k=r}^{n} \binom{n}{k} F^{k}(x)[1-F(x)]^{(n-k)} \tag{3.29}$$

The incomplete Beta function $I_x(a,b)$ (Kendall and Stuart 1977) is defined via the Beta function B(a,b) as:

$$I_x(a,b) = \frac{1}{B(a,b)} \int_0^x t^{a-1}(1-t)^{b-1}\,dt$$

in which: (3.30)

$$B(a,b) = \int_0^1 t^{a-1}(1-t)^{b-1}\,dt = \frac{(a-1)!(b-1)!}{(a+b-1)!} \quad if\ a,b>0$$

So, the expression (3.29) can be written in terms of an incomplete Beta function as:

$$F_{(r)}(x) = r\binom{n}{r}\int_0^{F(x)} u^{r-1}(1-u)^{n-r}\,du \quad = \quad I_{F(x)}(r,n-r+1) \tag{3.31}$$

Indeed; note that $\dfrac{1}{B(r,n-r+1)} = \dfrac{n!}{(r-1)!(n-r)!} = r\dbinom{n}{r}$.

The probability density function of $X_{r:n}$ is given by the first derivative of Eqn. (3.31):

$$f_{(r)}(x) = r\binom{n}{r} F^{r-1}(x)[1-F(x)]^{n-r} f(x) \tag{3.32}$$

Now, the expected value of r-th order statistics can be obtained as

$$E[X_{r:n}] = \int_{-\infty}^{\infty} x f_{(r)}(x)\,dx \tag{3.33}$$

Substituting from eqn.(3.32) into (3.33) and introducing a transformation, $u = F(x)$ or $x = F^{-1}(u)$, $0 \le u \le 1$, leads to

$$E[X_{r:n}] = r\binom{n}{r}\int_0^1 x(u)u^{r-1}(1-u)^{n-r}\,du \tag{3.34}$$

Note that $x(u)$ denotes the quantile function of a random variable. The expectation of the maximum and minimum of a sample of size n can be easily obtained from eqn.(3.34) by setting $r = n$ and $r = 1$, respectively.

$$E[X_{n:n}] = n\int_0^1 x(u)u^{n-1}du\,, \quad \text{and} \quad E[X_{1:n}] = n\int_0^1 x(u)(1-u)^{n-1}du \tag{3.35}$$

The probability weighted moment (PWM) of a random variable was formally defined by Greenwood et al. (1979) as

$$M_{i,j,k} = E[X^i u^j (1-u)^k] = \int_0^1 x(u)^i u^j (1-u)^k \, du \tag{3.36}$$

The following two forms of PWM are particularly simple and useful:

Type 1: $a_k = M_{1,0,k} = \int_0^1 x(u)(1-u)^k \, du \qquad (k = 0, 1, \ldots n)$ (3.37)

and

Type 2: $b_k = M_{1,k,0} = \int_0^1 x(u) u^k \, du \qquad (k = 0, 1, \ldots n)$ (3.38)

Comparing eqns.(3.37) and (3.38), it can be seen that α_k and β_k, respectively, are related to the expectations of the minimum and maximum in a sample of size k

$$\alpha_k = \frac{1}{k+1} E[X_{1:k+1}] \ , \qquad \beta_k = \frac{1}{k+1} E[X_{k+1:k+1}] \qquad (k \geq 1) \tag{3.39}$$

In essence, PWM's are the normalized expectations of maximum/minimum of k random observations; the normalization is done by the sample size (k) itself. From Eqn. (3.39), we notice that $E(X_{n:n}) = n\beta_{n-1}$ and from Eqn. (3.23) we have $b_{n-1} = \int xF^{n-1}(x) f(x) dx$. So $E(X_{n:n}) = \int xnf(x) F^{n-1}(x) dx$. On the other hand, using Eqn. (3.16), we have $b_{n-1} = n^{-1} \sum_{j=n}^{n} x_j = n^{-1} x_{n:n}$. From this it indeed follows that b_{n-1} is an unbiased estimator of β_{n-1}. Landwehr et al. (1979) gave a proof that b_r is an unbiased estimator of β_r for other values of r.

The expression for β_r in Eqn. (3.23) can numerically calculated by using a plotting-position formula as follows:

$$b_r = \int x\{F(x)\}^r dF(x) \approx \sum_{j=1}^{n} x_{j:n} \left(\frac{j}{n+1} \right)^r \frac{1}{n} = n^{-1} \sum_{i=1}^{n} l_i x_{i:n} \tag{3.40}$$

Notice that the expression looks almost the same as Eqn. (3.16) by writing:

$$b_r = n^{-1} \sum_{j=r+1}^{n} \frac{(j-1)(j-2)\ldots(j-r)}{(n-1)(n-2)\ldots(n-r)} x_{j:n} = n^{-1} \sum_{i=r+1}^{n} k_i x_{i:n} \tag{3.41}$$

The terms l_i and k_i in Eqn. (3.40) and (3.41) are compared in Figure 3.1 and we notice indeed a very close similarity. Reiss (1989) derived more approximate distributions of order statistics and Durrans (1992a) derived distributions of fractional order statistics.

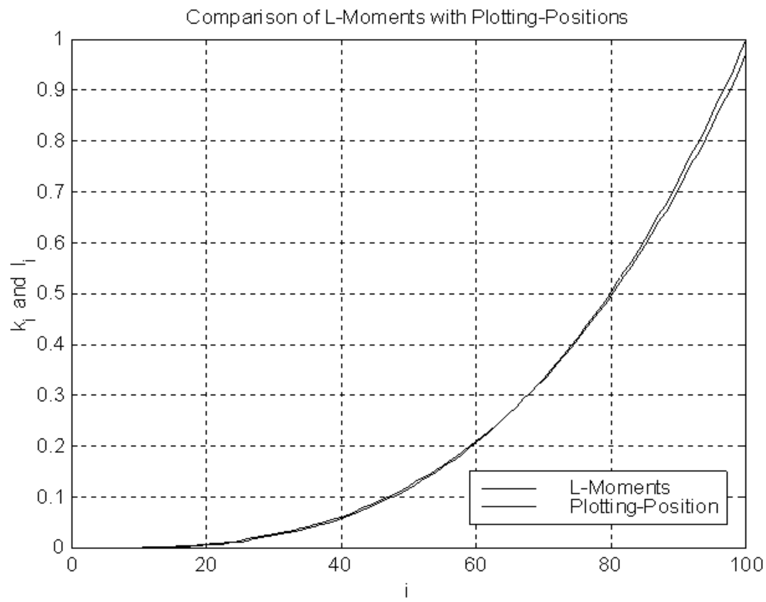


Figure 3.1: Comparison of Eqn. (3.40) and (3.41).

Summarizing: L-moments are certain linear combinations of probability weighted moments that are analogous to ordinary moments in a sense that they also provide measures of location, dispersion, skewness, kurtosis, and other aspects of the shape of probability distributions or data samples. An r^{th} order L-moment is mathematically defined as:

$$\lambda_r = \sum_{k=1}^{r} p^*_{r-1,k-1}\beta_{k-1} \tag{3.42}$$

where $p^*_{r,k}$ represents the coefficients of shifted Legendre polynomials (Hosking 1990). The following normalized form of higher order L-moments is convenient to work with:

$$t_r = \frac{l_r}{l_2} \quad , \quad r = 3,4,... \quad \text{and } |\tau_r| < 1 \tag{3.43}$$

The normalized fourth order L-moment, τ_4, is referred to as the L-kurtosis of a distribution. Hosking and Wallis (1997) showed that L-moments are very efficient in estimating parameters of a wide range of distributions from small samples. The required computation is fairly limited as compared with other traditional techniques, such as maximum likelihood and least squares. In the Appendix the L-moment formulae are given for a selection of PDFs. Apart from the well-known moments diagrams, also L-Moment diagrams exist in which L-skewness and L-kurtosis are plotted against eachother. However, the L-Moment diagrams do not form a complete class; that is to say points in the diagram may correspond to more than one probability distribution. This is in contrast to the ordinary moment diagram and also to the δ_1-δ_2 diagram of Halphen distributions (Bobee et al., 1993).

3.4 The Bayesian method

3.4.1 An introduction

As seen in Section 2.2.3, a consistent method to model parameter uncertainty is given by the Bayesian approach (see also Box and Tiao, 1973). As we will see later, also model uncertainty can be modeled by the Bayesian approach. As an introduction to the Bayesian approach, the following Gumbel model will be analyzed. The Gumbel likelihood model with location λ and scale δ is given by:

$$l(x|\lambda,\delta) = (1/\delta)^n \exp(-(\Sigma x_i-\lambda)/\delta) \exp(-\Sigma \exp(-(x_i-\lambda)/\delta)) \tag{3.44}$$

Where $x=(x_1, x_2, ..., x_n)$. If, for instance, we assume a normal distribution for the location parameter $p(\lambda_G) = N(\lambda_G |\mu_G,\sigma_G)$ then, the posterior distribution of λ becomes:

$$p(\lambda|x)= C\ N(\lambda|\mu_G,\sigma_G)\ \ (1/\delta)^n\ \ \exp(-(\Sigma x_i-\lambda)/\delta)\exp(-\Sigma\exp(-(x_i-\lambda)/\delta)) \tag{3.45}$$

in which C a normalisation constant. Figure 3.2 shows the prior and posterior of the λ-parameter when the following values are given: $\lambda_G = 20$, $\delta_G = 20$, $(\mu_\lambda,\sigma_\lambda) = (17, 2)$, n=150.

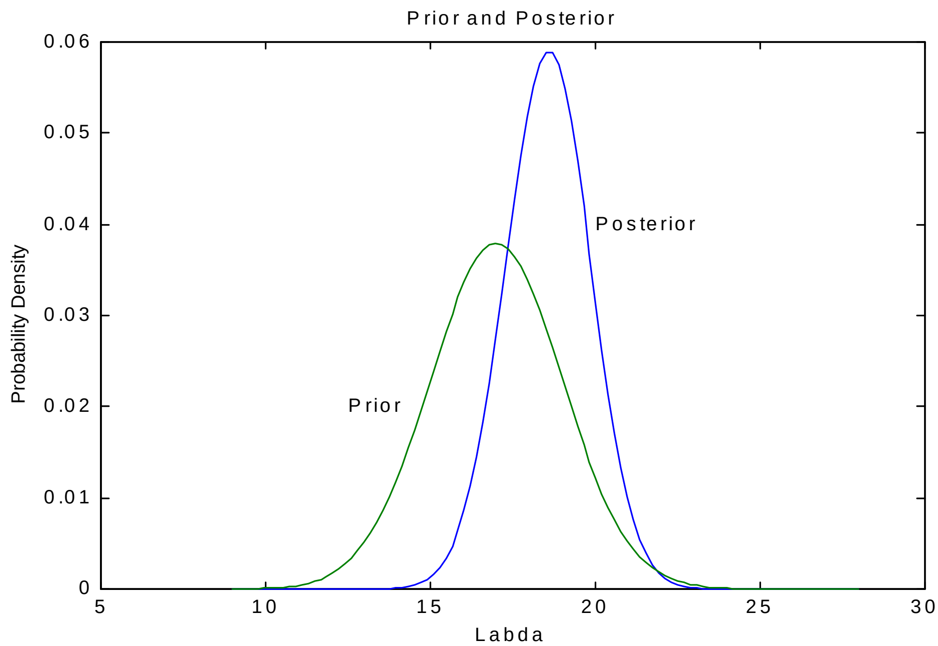


Figure 3.2: Prior and posterior of the λ-parameter of the Gumbel likelihood model

Notice that the updating process of 150 observations has led to a more peaked and shifted distribution function of the λ-parameter.

The way in which an engineer applies his information about a parameter Θ depends on the objectives in analyzing the data. If he is involved for instance in calculating the frequency of wave heights H, then the inferences he makes on H should reflect the uncertainty in Θ. In the Bayesian framework we are interested in the so-called predictive probability density function (Sec. 3.4.3):

$$f_H(h)=\int_\Theta f_H(h|\Theta)\, f(\Theta|h_1, h_2, ..., h_n ,I)d\Theta \tag{3.46}$$

where $f_H(h|\Theta)$ is the probabilistic model of wave heights, conditional on the parameters Θ, $f_H(h)$ is the predictive density of the wave heights (now parameter free), and $f(\Theta|h_1, h_2, ..., h_n ,I)$ is the posterior density of Θ when both the prior information I and the observations $h_1, h_2, ..., h_n$ are given. In popular words: “the uncertainty in the Θ parameters has been integrated out”.

The predictive distribution can be interpreted as being the density $f_H(h|\Theta)$ weighted by $f(\Theta|h_1, h_2, ..., h_n, I)$. Inferences made by combining new information are achieved by updating the distributions of the uncertain parameters through Bayes’ theorem and then by calculating the updated predictive function $f_H(h)$.

If we want to “summarize” the posterior distribution of Θ by one parameter we can use the Bayes estimator $\Theta^* = E(f(\Theta))$ or $\Theta^* = \max(f(\Theta))$ (associated with a quadratic loss function and 0-1 loss function respectively). *In making inferences on wave heights it is important to use the predictive function for h*, as opposed to the probabilistic model for h with the Bayes estimator for the parameter set Θ, i.e. $f(q|\Theta^*)$. This is because using point estimators for uncertain parameters underestimates the variance in wave heights.

This is in short the Bayesian way of thinking (see also Kuczera, 1994). A note on the way of implementation is valuable. If we have to calculate an integral of the form $\int f(\theta)g(\theta)d\theta$ we can apply Riemann integration which says:

$$\int f(\theta)g(\theta)d\theta \approx \Sigma\ f(\theta_i)g(\theta_i)\Delta\theta_i \tag{3.47}$$

where θ_i is a suitable discretisation of θ with discretisation interval $\Delta\theta_i$.
If g is a probability distribution function, we can also apply Monte Carlo simulation. Draw θ_i (i=1..n) from g and we have:

$$\int f(\theta)g(\theta)d\theta \approx \lim_{n\to\infty} 1/n\Sigma_{i=1..n} f(\theta_i) \tag{3.48}$$

More advanced implementation techniques are available like Markov Chain Monte Carlo methods amongst others the Gibbs sampling method, Metropolis-Hastings method, and more (see Carlin and Louis, 1996).

3.4.2 Obtaining conjugate priors

Provided they are not in direct conflict with our prior beliefs, and provided such a family can be found, the simplicity obtained by using a conjugate prior is very large. But in which situations can a conjugate family be obtained?

It appears that the only case where conjugates can be easily obtained is for data models within the exponential family (Bernardo and Smith, 1994). That is,

$$f(x|q) = h(x)g(q)e^{t(x)c(q)} \tag{3.49}$$

for functions h,g,t and c such that

$$\int f(x|q)dx = g(q)\int h(x)e^{t(x)c(q)}dx = 1 \tag{3.50}$$

This might seem restrictive, but in fact Eqn. (3.49) includes the exponential distribution, the Poisson distribution, the one-parameter Gamma distribution, the Binomial distribution, the Normal distribution (with known variance), and the extreme value distributions. For instance, the Gumbel distribution can be written as:

$$\begin{aligned} f(x \mid \alpha, \xi) &= \alpha \ \exp\{-(x-\xi)\alpha\}\exp[-\exp\{-(x-\xi)\alpha\}] = \\ &= \alpha\exp(-\alpha x)\exp(\xi\alpha)\exp[-\exp(-\alpha x)\exp(\xi\alpha)] = h(x)g(\xi)\exp(t(x)c(\xi)) \\ &\textit{in} \quad \textit{which} \\ h(x) &= \alpha\exp(-\alpha x) \\ g(\xi) &= \exp(\xi\alpha) \\ t(x) &= \exp(-\alpha x) \\ c(\xi) &= -\exp(\alpha\xi) \end{aligned} \tag{3.51}$$

α is considered as a constant; ξ is the conjugate parameter. With a prior of θ, f(θ), we can write:

$$\begin{aligned} f(q|x) \propto f(q)l(x|q) = f(q)\prod_{i=1}^{n} h(x_i)g(q)^n e^{\sum_{i=1}^{n} t(x_i)c(q)} \propto \\ \propto f(q)g(q)^n e^{\sum_{i=1}^{n} t(x_i)c(q)} \end{aligned} \tag{3.52}$$

Thus if we choose:

$$f(q) \propto g(q)^{d} e^{bc(q)} \tag{3.53}$$

In case of the Gumbel distribution, $f(x) \propto \exp(dax)\exp(-b\exp(ax))$, we obtain:

$$f(q|x) \propto g(q)^{n+d} \exp\{c(q)\left(\sum_{i=1}^{n} t(x_i) + b\right)\} = g(q)^{\tilde{d}} e^{\tilde{b}c(q)} \tag{3.54}$$

giving a posterior in the same family as the prior, but with modified parameters $\tilde{d} = n + d, \quad \tilde{b} = \sum_{i=1}^{n} t(x_i) + b.$ Indeed, for the Gumbel likelihood model and conjugate prior with parameters d and b, we obtain:

$$\begin{aligned} f(x|x) &= e^{dxa} e^{-be^{xa}} \prod a e^{-a(x_i - x)} e^{-e^{-a(x_i - x)}} = a^n e^{dxa - \Sigma a(x_i - x)} e^{-be^{xa} - \Sigma e^{-a(x_i - x)}} = \\ &= a^n e^{dxa - a\Sigma x_i + nax} e^{-be^{xa}} e^{-e^{ax}\Sigma e^{-ax_i}} = a^n e^{-a\Sigma x_i} e^{dxa + nxa} e^{-e^{ax}(b + \Sigma e^{-ax_i})} = \\ &\propto e^{xa(n+d)} e^{-e^{ax}(b + \Sigma e^{-ax_i})} \end{aligned} \tag{3.55}$$

The use of conjugate priors should be seen for what it is: a convenient mathematical device. However, expression of one's prior beliefs as a parametric distribution is always an approximation. In many situations the richness of the conjugate family is great enough for a conjugate prior to be found which is sufficiently close to one's beliefs (see also Pannullo et al., 1993).

The conjugate priors for the scale parameters of other PDFs can be easily found with the techniques from this section, and they are summarized in the Appendix.

3.4.3 The predictive distribution

So far, we have focused on parameter estimation. That is, we have specified a probability model to describe the random process which has generated a set of data, and have shown how the Bayesian framework combines sample information and prior information to give parameter estimates in the form of a posterior distribution. Commonly the purpose of formulating a statistical model is to make predictions about future values of the process. This is handled much more elegantly in Bayesian statistics than in the corresponding classical theory. The essential point is that in making predictions about future values on the basis of an estimated model there are two sources of uncertainty (Chapter 2):

- Statistical uncertainty in the parameter values which have been estimated on the basis of past data; and
- Inherent uncertainty due to the fact that any future value is itself a random event.

In classical statistics it is usual to fit a model to the past data, and then make predictions of future values on the assumption that this model is correct, the so-called estimative approach. That is, only the second source of uncertainty is included in the analysis, leading to estimates which are believed to be more precise than they really are. There is no completely satisfactory way around this problem in the classical framework since parameters are not thought of as being random.

Within Bayesian inference it is straightforward to allow for both sources of uncertainty by simply averaging over the uncertainty in the parameter estimates (the information of which is completely contained in the posterior distribution).

So, suppose we have past observations of a variable with density function (or likelihood) $f(x|\theta)$ and we wish to make inferences about the distribution of a future value y from this process. With a prior distribution $f(\theta)$, Bayes' theorem leads to a posterior distribution $f(\theta|x)$. Then the predictive density function of y given the data x is:

$$f(y|x) = \int f(y|\theta)\, f(\theta|x)\, d\theta \tag{3.56}$$

Thus, the predictive density is the integral of the likelihood (of a single observation) times the posterior. Notice that this definition is simply constructed from the usual laws of probability manipulation, and the definition itself has a straightforward interpretation itself in terms of probabilities.

The corresponding approach in classical statistics would be, for example, to obtain the maximum-likelihood estimate θ^* of θ and to base inference on the distribution $f(y|\theta^*)$, the estimative distribution.

To emphasize again, this makes no allowance for the variability incurred as a result of estimating θ, and so gives a false sense of precision (the predictive density $f(y|x)$ is more variable by averaging across the posterior distribution for θ).

For data models within the exponential family, we obtain from (3.49):

$$f(y|x) = \int f(y|q)f(q|x)dq = \int h(y)g(q)e^{t(y)c(q)}g(q)^{\tilde{d}}e^{\tilde{b}c(q)}dq = h(y)\int g(q)^{\tilde{d}+1}e^{c(q)(t(y)+\tilde{b})}dq$$

Note:

$$\int q^{u-1}e^{-mq}dq = \frac{\Gamma(u)}{m^u} \qquad (3.57)$$

The latter expression can be used to solve the integral in case of simple h, g, c and t functions. In case of the Gumbel likelihood model, we can derive (by applying the substitution $u=e^{\alpha\theta}$):

$$f(y|x) \propto h(y)\int u^{\tilde{d}+1}e^{-u(e^{-ay}+\tilde{b})}\frac{du}{au} = h(y)\frac{1}{a}\int u^{\tilde{d}}e^{-u(e^{-ay}+\tilde{b})}du = e^{-ay}\frac{\Gamma(\tilde{d}+1)}{(e^{-ay}+\tilde{b})^{\tilde{d}+1}} \qquad (3.58)$$

The normalising constant follows from $\int f(y|x)dy = 1$. The posterior functions of the location and shape parameters of the exponential family (except for the normal distribution) cannot be expressed in explicit form. Numerical integration has to be performed. Uniform prior distributions can be used for the scale and location parameters. For a large selection of models, the following information is given in the Appendix: likelihood model, non-informative prior (and corresponding posterior), the conjugate prior, the conjugate posterior, and the conjugate posterior predictive.

3.4.4 Some comments on the Empirical Bayes Method and the Bayes Linear Estimation

The empirical Bayes method is a way of using sample information to assist in specifying the prior distribution. As such, the procedure is not strictly Bayesian, since in a proper Bayesian procedure the prior distribution must be formulated independently of the data. However, the technique is now widely used.

It is clearly very difficult to formulate one's prior information very accurately, and it may be possible only to specify means, variances and covariances with any real faith. However, the posterior distribution will depend on a complete specification of the prior. The Bayes linear estimator of a parameter is an estimator whose value depends only on means and covariances and so does not require a fuller prior specification.

3.5 Entropy Method

Information theory can be characterized as a quantitative approach to the notion of information, largely based on the probability theory and statistics. It is conceptualized that the realization of a random event provides information about the event itself, and this information is inversely proportional to the probability of the event occurring.

In mathematical terms, consider a random event, E, which can take one of the n different states, E_1, E_2, . . , E_n, with respective probabilities of p_1, p_2, . . ., p_n. The event probabilities satisfy the natural constraints, $p_k > 0$ and $\sum p_k = 1$. The self-information of the event E_k is defined as (Jones 1979):

$$S(E_k) = \ln\left(\frac{1}{p_k}\right) = -\ln(p_k) \tag{3.59}$$

The use of a logarithmic measure for information, firstly introduced by Hartley (1928), is intuitive in the following sense: the information provided by a deterministic event (i.e., $p_k = 1$) is zero, whereas the rarer an event is ($p_k \ll 1$), the more information is conveyed by its realization. It is clear from Eqn.(3.59) that the self-information of an event increases as its uncertainty grows, i.e., the probability of occurrence reduces. In this respect, S can also be regarded as a measure of uncertainty (Jones 1979).

If the probabilities of various outcomes of the random experiment E are known a priori, can we predict the outcome (E_k) of the experiment in advance? The degree of difficulty of this prediction is dependent on the overall uncertainty associated with E. So the question then is this: can we measure the amount of uncertainty associated with a random variable? Shannon defined a measure of uncertainty, referred to as entropy, similar to that used in thermodynamics and statistical mechanics (Shannon 1949). The entropy can be interpreted as a measure of uncertainty that prevailed before the experiment was accomplished, or as a measure of information expected from an experiment (Aczél and Daróczy 1975). Considering the definition of self-information from Eqn. (3.59), Shannon's entropy for a random event E can be expressed as mathematical expectation of the self-information:

$$H(p) = -\sum_{k=1}^{n} p_k \ln p_k \tag{3.60}$$

Several other mathematical properties of entropy are elaborated by Jumarie (1990) and Kapur and Kesavan (1992).

Jaynes (1957) presented the maximum entropy principle as a rational approach for choosing a probability distribution amongst all possible distributions. The principle states that the best estimate of a probability distribution is that which maximizes the entropy subject to constraints supplied by the available information, e.g., moments of a random variable.

In the context of prior and posterior probabilities, Kullback (1959) introduced the concept of cross-entropy, or directed divergence, of a posterior probability distribution $g(x)$ from a prior distribution $f(x)$:

$$I(g, f) = \int_R g(x) \ln\left(\frac{g(x)}{f(x)}\right) dx \tag{3.61}$$

The cross entropy is a positive, additive, unsymmetrical, and convex function of probabilities. Shore and Johnson (1980) proved that the cross-entropy minimization is a uniquely correct method of probabilistic inference that satisfies all the consistency axioms.

A symmetric measure of probabilistic distance between two distributions, also know as divergence, was formulated by Kullback (1959) as

$$J(g, f) = \int \big(g(x) - f(x)\big) \ln\left(\frac{g(x)}{f(x)}\right) dx \quad = \quad I(g, f) + I(f, g) \tag{3.62}$$

$J(g, f)$ can be interpreted as a general measure of the difficulty of discriminating a distribution $g(x)$ from $f(x)$, the larger the value of divergence, the worse is the resemblance between the two distributions under consideration (Kullback 1959). In essence, divergence integrates the departure of $g(x)$ from $f(x)$ over the entire range of random variable X. In pattern recognition and signal analysis problems, the divergence has been commonly used as a measure of power of discrimination (Kapur and Kesavan 1992, Kailath 1967).

Important properties of $J(g, f)$ are summarized as follows:

(1) *Finiteness*: $J(g, f) < \infty$

(2) *Additivity*: $J(g, f) = J(g_1, f_1) + J(g_2, f_2)$ for all probability densities of the form, $f(x_1, x_2) = f_1(x_1)\, f_2(x_2)$ and $g(x_1, x_2) = g_1(x_1)\, g_2(x_2)$

(3) *Positivity*: $J(g, f) \geq 0$

(4) *Semi-boundedness*: $J(g, f) \geq J(f, f)$, the equality holds only if $g = f$

(5) *Linear Invariance*: $J(g, f)$ is invariant under any linear coordinate transformation.

In fact, Johnson (1979) proved that $J(g, f)$ is a uniquely characterized measure that satisfies aforementioned mathematical properties.

The way in which entropy can be used in statistical parameter estimation is described, for instance in Rodriguez (1984), Lind et al. (1989), Woodbury and Ulrych (1993), Singh (1993), and Singh and Guo (1997). A brief summary of their approach is described here.

The entropy function of $f(x;\theta)$ with uncertain parameters θ can be expressed by:

$$H(f) = -\int_{-\infty}^{\infty} f(x;q) \ln f(x;q) dx \tag{3.63}$$

Given m linearly independent constraints C_i, i=1,2,...,m, in the form:

$$C_i = \int w_i(x) f(x) dx, \quad i = 1,2,\ldots,m \tag{3.64}$$

where $w_i(x)$ are functions whose averages over f(x) are specified by C_i.

Then the maximum of H(f), subject to the Eqn. (3.64) is given by the distribution:

$$f(x) = \exp\left[a_0 - \sum_{i=1}^{m} a_i w_i(x)\right] \tag{3.65}$$

where a_i , i=1,2,...,m are the Lagrange multipliers, which can be determined from Eqn. (3.64) and (3.65). The corresponding entropy for the distribution f(x) is given by:

$$H(x) = a_0 + \sum_{i=1}^{m} a_i C_i \tag{3.66}$$

In the Appendix an overview of entropy functions is given for a selection of PDFs.

3.6 Nonparametric methods

The basic idea of nonparametric density estimation is to relax the parametric assumptions about the data, typically replacing these assumptions with ones about the smoothness of the density. The most common and familiar nonparametric estimator is the histogram. Here the assumption is that the density is fairly smooth (as determined by the bin widths) and an estimate is made by binning the data and displaying the proportion of points in each bin (producing a necessarily non-differentiable, but still useful estimate).

The kernel density estimator is related to the histogram, but produces a smooth (differentiable) estimate of the density. It has been studied widely since its introduction in Rosenblatt (1956). Given i.i.d. data $x_1,\ldots,x_n$ drawn from the unknown density α, the standard kernel estimator (SKE) is the single bandwidth estimator:

$$\hat{a}(x) = \frac{1}{nh} \sum_{i=1}^{n} K\left(\frac{x - x_i}{h}\right) \tag{3.67}$$

The bandwidth h determines the amount of smoothing produced by the estimator. See the recent books by Silverman 1986, Scott 1992 and the bibliographies contained therein, for a good introduction to kernel estimators. Much work has been done on selecting the optimal bandwidth h under different assumptions on α or different optimality criteria.

An obvious problem with this kind of estimator for a finite data set is that it uses a single bandwidth (or smoothing parameter) throughout the entire support of the density. For densities with long tails, or modes with different variances, this can be a problem.

The group at Utah Water Research Laboratory, under the guidance of Prof. Upmanu Lall has been working on developing and applying nonparametric estimation techniques to a wide range of surface and groundwater hydrologic problems including time-series forecasting. Research by Upmanu Lall and his co-workers has focused on developing nonparametric statistical methods for the estimation of probabilities of rare floods that are more appropriate in such situations (Moon and Lall, 1994, Moon et al., 1993, and Lall et al., 1993).

For densities with long tails, or modes with different variances, there are modifications to the standard kernel estimator (3.67) proposed by Marchette (1995), and Marchette et al. (1994) which uses a small number of bandwidths rather than a single one as in (3.67), which allows local tuning of the density. As motivation for this estimator, consider a density which consists of a mixture of two normals with different variances which are very far apart. For concreteness, let:

$$a(x) = pj(x, -m, s_1^2) + (1-p)j(x, m, s_2^2) \tag{3.68}$$

Suppose we wished to use the standard (single bandwidth) kernel estimator if possible. It seems reasonable that near the left-hand mode one would wish to use a bandwidth appropriate to that normal, and similarly on the right. So a reasonable approach might be to filter the data into two distinct data sets, one from the right component and one from the left, and estimate these two components separately.

One way to do this (approximately) is to define

$$\begin{aligned} r_1(x) &= c_{\{x>0\}}(x) \\ r_2(x) &= c_{\{x\le 0\}}(x) \end{aligned} \tag{3.69}$$

and define our estimator to be:

$$\hat{a}(x) = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{r_1(x_i)}{h_1}K\left(\frac{x-x_i}{h_1}\right) + \frac{r_2(x_i)}{h_2}K\left(\frac{x-x_i}{h_2}\right)\right) \tag{3.70}$$

This allows us to use the bandwidths appropriate to the different components in the different regions where they are supported. Equation (3.69) is not quite right, however, for as we move the two components closer together, the overlapping region becomes more and more significant. What we really want to do is use the posteriors for each component as our ρ functions. This is the motivation of the filtered kernel estimator as proposed by Marchette (1995).

With the above example in mind, suppose we wish to have a small number of bandwidths where each bandwidth is associated with a region of the support of the density. To each bandwidth we associate a function which "filters" the data, as in (3.69). Basically, the filter will define the extent to which each local bandwidth is to be used for any particular data point. We can then construct a kernel estimator which is a combination of the kernel estimators constructed using each bandwidth, with the data filtered by the filtering functions.

Another method of nonparametric estimations is the following. Suppose we have independent random variables X_1 ; X_2 ; ... , all of them with distribution function F . Suppose that F is in the domain of attraction of some extreme value distribution G_{γ}. In other words: suppose 1-F is regularly varying at infinity, i.e.,

$$\lim_{t\to\infty} \frac{1-F(tx)}{1-F(t)} = x^{-1/g} \tag{3.71}$$

for x>0, where γ is a positive parameter.

Let $X_{1:n}$ $X_{2:n}$ .... $X_{n:n}$ be the n-th order statistics. For some m < n define:

$$\hat{g}_p = (\log 2)^{-1} \log \frac{X_{n-m:n} - X_{n-2m+1:n}}{X_{n-2m+1:n} - X_{n-4m+1:n}} \tag{3.72}$$

This is Pickands' estimator for γ (Pickands, 1975).

Another estimator for γ is the moment estimator (Dekkers and De Haan, 1989):

$$\hat{g}_M = M_n^{(1)} + 1 - \frac{1}{2}\left\{1 - \frac{(M_n^{(1)})^2}{M_n^{(2)}}\right\}^{-1} \tag{3.73}$$

in which:

$$M_n^{(i)} = \frac{1}{k}\sum_{i=0}^{k-1} (\log X_{n-i:n} - \log X_{n-k:n})^i \tag{3.74}$$

The estimators are consistent for any γ. Under appropriate smoothness conditions on F and a further bound on the rate of increase of k(n) the estimators for γ are asymptotically normal after normalization (see e.g. Dekkers and de Haan, 1989).

3.7 Performance of the statistical estimation methods

In the previous six sections an overview of statistical estimation methods has been given. The large number of distributions and estimation methods proposed in the literature may cause confusion which method and/or distribution to use (Bobee et al., 1993b). The World Meteorological Organization (1989) and Cunnane (1987-1988) published a few reports which compare current methodologies, and recommend a number of statistical distributions and estimation procedures. These reports are already ten years old and the last decade has been extremely busy with new results on this topic. In the present section we report the latest developments, show advantages and disadvantages of the various methods, and propose new ideas for a possible comparison strategy.

3.7.1 Variability of estimators

As written in Section 3.3 we prefer estimators which have low variability. But the question arises as to whether there is any limit to the accuracy that an estimator can achieve (Beard, 1994). Intuitively, we would expect that there is, since the variation from one sample to another means that any estimator is bound to have some degree of variation. However, is it possible to bound the accuracy which an estimator can achieve? In fact, provided we restrict ourselves to unbiased estimators there is a remarkable theorem which provides a bound on the variance of any (unbiased) estimator (Barnett, 1973).
The quantity :

$$I(p)=-1/nE(\partial^2 \log L(p,x)/\partial p^2) \qquad (3.75)$$

is extremely important in statistical theory. It is termed the Expected Information (and related with the Fisher matrix of Eqn. (2.2) up to a factor n^{-1} (Fisher (1934)). Since second derivatives measure curvature, I(p) is a measure of the expected curvature of the likelihood function at the true parameter value. The importance of the expected information in the context of minimum variance estimation is provided by the following result, which is known as the Cramér-Rao theorem (1946):

If T(X) is an unbiased estimator of p then

$$Var(T(X)) > I^{-1}(p) \qquad (3.76)$$

Thus, within the class of unbiased estimators, no estimator can have a variance which is smaller than the reciprocal of the expected information. So, as required, we obtain a bound on the maximum precision that can be attained within the class of unbiased estimators. Note that the bound only applies to unbiased estimators. It is always possible to obtain an estimator with lower variance by giving up on the property of unbiasedness. (For example, the estimator T(X)=100 has zero variance for any problem).

The result is important for both theoretical and practical reasons; it formulates in a precise way the limits that can be achieved in statistical inference due to random variation in the population. Though the Cramér-Rao bound puts a lower limit on the variance of unbiased estimators, the bound may not be achievable.

So far we have used the likelihood function only to determine an estimate of an unknown parameter θ. We have noted several times, however, that values of θ with relatively high likelihood are more plausible than those with low likelihood, and that it should be possible to exploit this knowledge to construct a credible range for θ. This is achieved formally by looking at the sampling properties of the likelihood function: quantifying the variation in $L(\theta)$ from sample to sample.

In most situations exact calculation is very difficult and we will be forced to use (asymptotic) approximations that assume large sample sizes. In the next Sub-section 3.7.2 we present some methods which are useful in deriving the sampling properties of the likelihood function. It will be shown that these methods can be used to derive the sampling distributions of the ML and LM estimators of the exponential distribution (Sec. 3.7.3).

3.7.2 Methods for deriving the sampling distribution of estimators

Mood et al. (1974) present methods for deriving the sample distribution of estimators. In some cases it is possible to establish the exact sampling distribution of an estimator and use this as the basis for confidence interval estimation. More generally this is not possible but approximations can be obtained. The following relations can be used in deriving the exact sampling distributions:

If X and Y are random variables with PDF's f and g respectively. Let

$$Z=X+Y,\ \ U=X-Y,\ V=XY \text{ and } W=X/Y$$

then the PDF's of Z, U, V and W are, respectively, given by:

$$f_Z(z) = \int f(x)g(z-x)\,dx$$

$$f_U(u) = \int f(u+y)g(y)\,dy \qquad (3.77)$$

$$f_V(v) = \int f(x)g(v/x)\,|x|^{-1}\,dx$$

$$f_W(w) = \int f(xw)g(x)\,|x|\,dx$$

The proof of these formulae is given in Mood et al. (1974). In textbooks on statistics the following relation is proven:

$$E(XY) = E(X)E(Y) \qquad (3.78)$$

$$Var\left(\sum_{i=1}^{n} a_i X_i\right) = \sum_{i=1}^{n} a_i^2 Var(X_i) + 2\sum_{i=1}^{n}\sum_{j=i+1}^{n} a_i a_j Cov(X_i, X_j)$$

Furthermore it is possible to derive the following property for the product of random variables:

$$Var(V) = Var(X)Var(Y) + E^2(X)Var(Y) + E^2(Y)Var(X) \qquad (3.79)$$

If exact calculations are not possible, the following approximation rules can be used (using Taylor's formula):

$$g(X) = g(m_X) + (X - m_X)\frac{dg(x)}{dx}_{|x=m_X} + \frac{(X-m_X)^2}{2}\frac{d^2g(x)}{dx^2}_{|x=m_X} + \ldots \qquad (3.80)$$

From this, we can derive:

$$E(g(X)) \approx g(E(X)) \qquad (3.81)$$

$$Var(g(X)) \approx Var(X)\,[g'(m_X)]^2 \qquad (3.82)$$

If the coefficient of variation of X is less than c, the error involved in these approximations is less than c^2. In particular, the following useful approximations can be used:

$$E(\sqrt{X}) \approx \sqrt{E(X)}, \qquad Var(\sqrt{X}) \approx \frac{Var(X)}{4E(X)}$$
$$E(X^{-1}) \approx \frac{1}{E(X)}, \qquad Var(X^{-1}) \approx \frac{Var(X)}{E^4(X)} \tag{3.83}$$

From Eqn. (3.83) the approximation $CV\left(\frac{1}{X}\right) = \frac{\sigma\left(\frac{1}{X}\right)}{E\left(\frac{1}{X}\right)} \approx \frac{\frac{\sigma(X)}{E^2(X)}}{\frac{1}{E(X)}} = \frac{\sigma(X)}{E(X)} = CV(X)$ follows directly. This approximation was shown to be very useful in Sec. 2.6.

The second equation in (3.83) is useful in deriving the sampling distribution of the standard devation. It is well known that Var(s^2) = $2\sigma^4/N$ in which σ is the population standard deviation of the normal distributed X_i and all X_i's are i.i.d. Therefore:

$$\sigma(s) = \sqrt{\frac{\frac{2\sigma^4}{N}}{4\sigma^2}} = \frac{\sigma}{\sqrt{2N}} \tag{3.84}$$

Exact calculations for the estimations of the 2-parameter Exponential distribution will be given in the next sub-section.

3.7.3 Exact sampling distributions for the Exponential distribution

The techniques from the previous section will shown to be succesful in deriving the exact sampling distributions for the parameter estimations of the MML and MLM of the exponential distribution.

The expressions of the MML and MLM for the location and scale parameter of the exponential distribution are given in the Appendix. Suppose that $X_1, X_2, ..., X_n \sim$ Exp(ξ,α) i.i.d. This is the usual exponential model parameterized in such a way that ξ+α is the population mean. The sampling distribution of $m_X = \frac{1}{n}\sum_{i=1}^{n} X_i$ is the sum of n exponential distributions divided by n. According to Johnson et al. (1994), a summation of n independent exponential distributions, each with scale parameter α is gamma distributed with parameters (n, α), and a constant k times a gamma distribution with parameters (α,β) is also gamma distributed with parameters (α,β/k). Therefore m_X is ξ + gamma distributed with parameters (n,αn).

Furthermore $P(\min(X_1, X_2, ..., X_n) < x) = 1 - P(\min(X_1, X_2, ..., X_n) > x) =$

$= 1 - P^n(X_i > x) = 1 - (1 - P(X_i < x))^n$. So the minimum of n exponential distributions, each with scale parameter α, is also exponentially distributed with scale parameter α/n. Finally, Sukhatme (1937) showed that $2n/\alpha(m_X - \min((X_1, X_2, ..., X_n))$ has a Chi-square distribution with 2(n-1) degrees of freedom.

With the above considerations and with Eqn. (3.78) we can derive that the following properties for the sampling distribution of the MLEs:

$$
\begin{aligned}
&E(\alpha_{ML}) = \alpha(1-1/n) \\
&Var(\alpha_{ML}) = \alpha^2(n^{-1} - n^{-2}) \\
&E(\xi_{ML}) = \xi + \alpha/n \\
&Var(\xi_{ML}) = \alpha^2/n^2
\end{aligned}
\tag{3.85}
$$

In order to derive the sampling distribution for the MLM, we use the following properties of the order statistics of the standard exponential distribution (Johnson et al., 1994):

$$
\begin{aligned}
&E(X_{(i)}) = \sum_{j=1}^{i} \frac{1}{n-j+1} \\
&Var(X_{(i)}) = \sum_{j=1}^{i} \frac{1}{(n-j+1)^2} \\
&Cov(X_{(i)}, X_{(k)}) = \sum_{j=1}^{i} \frac{1}{(n-j+1)^2} = Var(X_{(i)})
\end{aligned}
\tag{3.86}
$$

The following summations have been derived:

$$
\begin{aligned}
&\sum_{j=1}^{n} \sum_{k=1}^{j} \frac{1}{n-k+1} = n \\
&\sum_{j=2}^{n} \sum_{k=1}^{j} \frac{j-1}{n-k+1} = c(n) \\
&\sum_{j=2}^{n} \frac{2j-1-n}{n(n-1)} = 1 + \frac{1}{n}
\end{aligned}
\tag{3.87}
$$

The function c(n) can be derived to $c(n) = \Psi(n)\frac{(n-1)n}{2} - \sum_{j=1}^{n-1}(n-j)\Psi(j)$, in which Ψ is the Psi-function $\Psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$, having the property $\sum_{i=k}^{n} \frac{1}{i} = \Psi(n) - \Psi(k)$ (Abromowitz and Stegun, 1965). The function c(n) behaves as n^2 for large values of n. We write $c(n)=O(n^2)$.

For n=2 the L-Moments estimator of α reads $\alpha_{LM} = x_{2:2} - x_{1:2}$. The difference $x_{2:2} - x_{1:2}$ has an exponential distribution with parameter α. Consequently $E(\alpha_{LM}) = \alpha$

(unbiased) and Var(α_{LM}) = α^2 for n=2. The Eqn. (3.87) and the asymptotic covariance matrix of the sample L-moments of Hosking (1986) are necessary to derive the following sampling properties of MLM:

$$E(\alpha_{LM}) = \alpha(4n^{-1}(n-1)^{-1}c(n)-2)$$
$$Var(\alpha_{LM}) = 4\alpha^2 / 3n + O(n^{-2})$$
$$E(\xi_{LM}) = \xi + \alpha(4n^{-1}(n-1)^{-1}c(n)-3) \qquad (3.88)$$
$$Var(\xi_{LM}) = \alpha^2 /3n + O(n^{-2})$$

Note that the variance of α_{LM} is slightly larger than the variance of α_{ML}. The variance of ξ_{LM} decreases with $O(n^{-1})$ instead of $O(n^{-2})$ in case of the maximum likelihood method. Furthermore, the expressions 3- $4n^{-1}(n-1)^{-1}c(n)$ and n^{-1} can be plotted in Figure 3.3 for a graphical comparison of the biases:

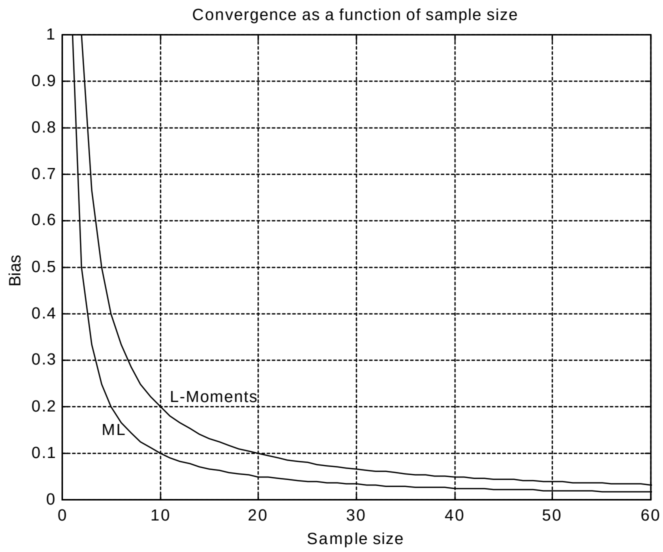


Figure 3.3: Expressions 3 - $4n^{-1}(n-1)^{-1}c(n)$ and n^{-1}

Notice that the ML-method has a slightly better performance for the parameter estimation of the exponential distribution than the L-Moments method; its bias and RMSE is lower than the MLM-equivalents.

3.7.4 Performance based on relative bias and RMSE of the estimators

With Monte Carlo simulation studies, datasets can be generated from a beforehand known probability distribution function (and known p-quantile). Different parameter estimation methods can be applied on these datasets and compared with respect to their estimates of the p-quantiles. The estimation method with the smallest bias and/or variance is then considered to be the best method for that particular distribution function. In order to familiarize the reader with the concept, in the following we start with a simulation analysis of six estimation methods for the scale parameter of a one-parameter exponential distribution $F(x)=1-e^{-\lambda x}$. In Figure 3.4 we see the performances of the six methods for one simulation of 20 values the exponential distribution with scale parameter λ=1.5. The different methods cause a very wide range of frequency lines. The 100 year event (or the 0.99 quantile) of the theoretical distribution is 3.07m.

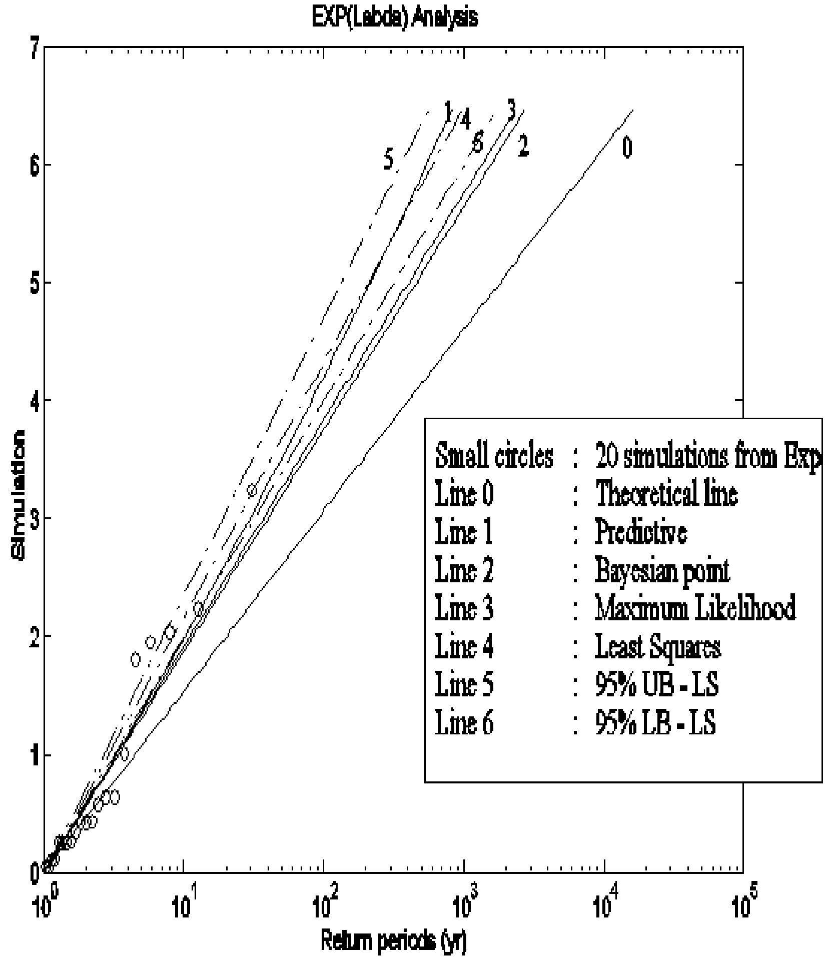


Figure 3.4: Performance of six methods

The above simulation analysis is repeated 200 times and from each simulation we store per estimation method the results of the 1/100 year prediction. This excersise is also repeated for other sample sizes. Apart from n=20 values we look at n=3, 6, 9,..., 60 values. In the next Figure 3.5 we have plotted the mean and standard deviations of each estimation method.

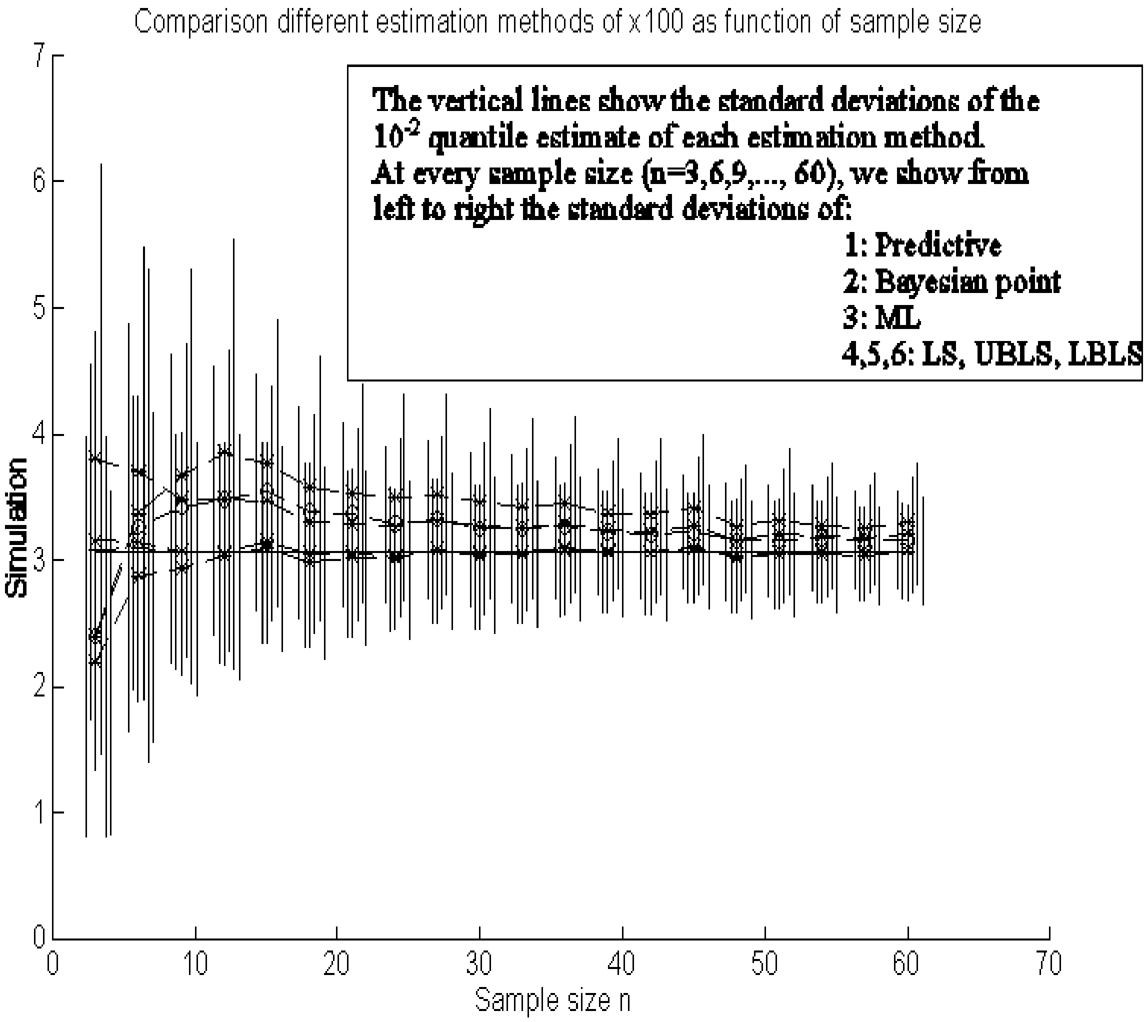


Figure 3.5: Performance of six estimation methods as function of sample size (source: Van Gelder and Vrijling (1997a))

In general it is impossible to say which estimation method is the most appropriate method for a particular model and dataset. This depends on the size of the sample, the type of the distribution, the choice of the parameters of the distribution, the inhomogeneity that is embedded in the data, and of course the choice of the criterion. However, a Monte Carlo simulation is the suitable method to examine the performance for a certain choice of the above mentioned dependencies.

Lots of simulation work has been performed to judge the performance of the estimation methods based on the relative bias and RMSE of the distribution parameters. An overview of this work is given in the following Table 3.1:

Table 3.1: Literature review (only first authors are shown)

	MOM	LM (or PWM)	ML	LS	Bayes	ME
GUM	Takara, 1989 Yamaguchi, 97 Carter, 1983 Landwehr, 79	Takara, 1989 Yamaguchi, 97 Landwehr, 79 Guo, 1991	Takara, 1989 Yamaguchi, 97 Carter, 1983 Corsini, 1995	Takara, 1989 Yamaguchi, 97 Carter, 1983	Coles, 1996	Takara, 1989
WEIB	Yamaguchi, 97 Abernethy, 83	Yamaguchi, 97	Yamaguchi, 97 Smith, 1987	Yamaguchi, 97	Smith, 1987	Singh, 1990
GPA	Moharran, 93 Hosking 1987 Castillo 1997	Moharran, 93 Hosking 1987 Castillo 1997	Moharran, 93 Hosking 1987 Castillo 1997	Moharram, 93	Coles, 1996	Singh, 1997
GEV	Takara, 1989 Yamaguchi, 97 Sank., 1999	Takara, 1989 Yamaguchi, 97 Lu, 1992b Wang. 1998	Takara, 1989 Yamaguchi, 97 Hosking, 1985	Yamaguchi, 97	Fill, 1998 Coles, 1996 Preumont, 88	Singh, 1992 Jowitt, 1979 Lind, 1991
LN	Stedinger,1980 Hoshi, 1986 Yamaguchi, 97 Goda, 1992	Takara,1990 Sank., 1999 Yamaguchi, 97 Takeuchi 1988	Stedinger,1980 Takara, 1990 Yamaguchi, 97 Takeuchi 1988	Takara, 1990 Lechner, 1991	Corbyn 1988 Lye, 1988	Singh, 1987
GAM	Hoshi, 1986 Bobee, 1991	Rasmussen, 94 Wu, 1991 Durrans, 92b	Hoshi, 1986 Hu, 1987 Stacy, 1965	Ashkar, 1998	vNoortwijk, 99 Ribeiro 1993	Singh 1985

Maximum likelihood estimation of the generalized Pareto distribution (GPA) has previously been considered in the literature, but Hosking et al. (1987) show that unless the sample size is 500 or more, estimators derived by the method of moments or the method of probability-weighted moments are more reliable. They also use simulations to assess the accuracy of confidence intervals for the parameters and quantiles of the generalized Pareto distribution.

Various estimation methods for the three-parameter case of Generalized Pareto distribution are given in Moharram et al. (1993), including an alternative method based on least squares (LS). Modified formulae for computing estimators are provided. The performances of these methods are compared by using Monte Carlo simulation. It is found that the LS method has generally a lower root mean squared error (RMSE) than that obtained using other methods. The LS method also performs best in terms of BIAS when the shape parameter is greater than zero, while the probability weighted moments (PWM) method performs best when the shape parameter less than zero.

In Castillo and Hadi (1997), it is shown that when the shape parameter of the GPA is greater than 1, the maximum likelihood estimates do not exist, and when the shape parameter is between 1/2 and 1, they may have problems. Furthermore, for shape parameters less than or equal to -1/2, second and higher moments do not exist, and hence both the method-of-moments (MOM) and the probability-weighted moments (PWM) estimates do not exist. Another and perhaps more serious problem with the MOM and PWM methods is that they can produce nonsensible estimates (i.e., estimates inconsistent with the observed data). In Castillo and Hadi (1997), a simulation study is carried out to evaluate the performance of the parameter estimation methods and to compare them with other methods suggested in the literature. The simulation results indicate that no method is uniformly best for all the parameter values.

In Fill and Stedinger (1995), it was shown that for realistic generalized extreme value (GEV) distributions and short records, a simple index-flood quantile estimator performs better than two-parameter (2P) GEV quantile estimators with probability weighted moment (PWM) estimation using a regional shape parameter and at-site mean and L-coefficient of variation (L-CV), and full three-parameter at-site GEV/PWM quantile estimators. However, as regional heterogeneity or record lengths increase, the 2P-estimator quickly dominates. Fill and Stedinger (1995) generalizes the index flood procedure by employing regression with physiographic information to refine a normalized T-year flood estimator. A linear empirical Bayes estimator uses the normalized quantile regression estimator to define a prior distribution which is employed with the normalized 2P-quantile estimator. Monte Carlo simulations indicate that this empirical Bayes estimator does essentially as well as or better than the simpler normalized quantile regression estimator at sites with short records, and performs as well as or better than the 2P-estimator at sites with longer records or smaller L-CV.

In Wang (1998), approximate goodness-of-fit tests of fitted generalized extreme value (GEV) distributions using LH moments are formulated on the basis of comparison of sample LH kurtosis estimates and theoretical LH kurtosis values of the fitted distributions. Their tests are different from those that have been derived for testing the GEV distributions of which parameter values are known a priori. The tests are intended to answer the following questions: Does a fitted GEV distribution describe adequately a given data series? If not, can the GEV distribution function describe adequately the larger events in that data series for use for high quantile estimation? If so, what degree of emphasis on the larger events is needed in order that the GEV distribution becomes acceptable? The use of the GEV distribution in conjunction with the LH moment estimation method and the formulated tests should alleviate the need for finding the "correct" distribution. The tests are evaluated by Monte Carlo simulations using generated samples of both GEV and Wakeby distributions.

Takeuchi and Tsuchiya (1988) derive PWM solutions for Normal and 3-parameter Lognormal distributions. Their paper presents their relative accuracy in comparison with other parameter estimation procedures such as Moment, Maximum-likelihood, Quantile and Sextile methods through Monte Carlo simulation experiments. Simulation results revealed that PWM estimates of quantiles are unbiased for the Normal distribution and less biased than those of the Moment method for Lognormal distribution with a large coefficient of skewness. It was also revealed that the RMSE of PWM estimates of quantiles is as small as that of the Moment method for the Normal distribution but larger for the Lognormal distribution.

In Lechner (1991) three common estimators for the parameters of the lognormal distribution are evaluated. Correction factors which eliminate essentially all the bias, and formulas for the standard deviations of the estimators, are presented. It is reported that the Persson-Rootzen estimators are about as good as the maximum-likelihood estimators, without the penalty of requiring iterative optimization. Also, the estimators resulting from (least squares) fitting a line to the plot of log lifetimes on normal (Gaussian) probability paper are reasonably good. Formulas are given for obtaining these latter estimators without actually plotting the points. Lechner (1991) simulated 5k to 30k samples (more samples for smaller N for each case) and calculated the following: the means, standard deviations, and third moments of each estimator; correlations between the two members of each pair; comparisons between the estimators; and simple corrections to improve the performance of the estimators.

In Corbyn (1988), methods are developed for the determination of the posterior distribution of the first moment of the lognormal distribution with exponential and other prior distributions. Bayesian methods of statistical inference are compared with the more generally used method of inference based on confidence limits. The general problem of Bayesian estimation of the mean of a correlated random variable is discussed.

Sankarasubramanian et al. (1999) deals with fitting of regression equations for the sampling properties, variance of L-standard deviation, and bias and variance of L-skewness, based on Monte-Carlo simulation results, for generalised Normal (Lognormal-3) and Pearson-3 distributions. These fitted equations will be useful in formulating goodness-of-fit test statistics in regional frequency analysis. The second part of their paper presents a comparison of the sampling properties between L-moments and conventional product moments for generalised Normal, generalised Extreme Value, generalised Pareto and Pearson-3 distributions, in a relative form. The comparison reveals that the bias in L-skewness is found to be insignificant up to a skewness of about 1.0, even for small samples. In case of higher skewness, for a reasonable sample size of 30, L-skewness is found to be nearly unbiased. However, the conventional skewness is found to be significantly biased, even for a low skewness of 0.5 and a reasonable sample size of 30. The overall performance evaluation in terms of "Relative-RMSE in third moment ratio" reveals that conventional moments are preferable at lower skewness, particularly for smaller samples, while L-moments are preferable at higher skewness, for all sample sizes.

Corsini et al. (1995) analyze the Maximum likelihood (ML) algorithms and Cramer-Rao (CR) bounds for the location and scale parameters of the Gumbel distribution are discussed. First they consider the case in which the scale parameter is known, obtaining the estimator of the location parameter by solving the likelihood equation and then evaluating its performance. They also consider the case where both the location parameter and the scale parameter are unknown and need to be estimated simultaneously from the reference samples. For this case, performance is analyzed by means of Monte-Carlo simulation and compared with the asymptotic CR bound.

Also results of a Monte Carlo study are presented in Guo and Cunnane (1991) comparing different simulation procedures and assessing the value of historical floods for at-site flood frequency analysis on the assumption of a Gumbel distribution.

In Wu et al. (1991), a new procedure, the method of lower-bound (MLB), is proposed for determining the design quantile Xp. Their basic concept is first to determine an estimate of the location parameter using the probability weighted moment (PWM) method, and then to transform the variable X from the original (X) space to a new (Y) space. The variable Y is considered to have a two parameter gamma distribution. In Y-space, the two parameters are estimated by the PWM or an autocovariance method, then transformed from the Y-space to the X-space. Results from the Monte Carlo experiments show that the MLB estimates are less biased than comparable moment estimates and maximum likelihood estimates, and more efficient than those of PWM for design quantiles x_p.

In Hu (1987a), the determination of confidence intervals for design floods using the Pearson Type III distribution with a known skewness is analyzed. Tables for the confidence factors based on moment and curve-fitting estimates were developed by Monte Carlo simulation technique and were used to construct the confidence intervals for frequency curves. The performance of methods using tables presented herein, the method of B values based on the curve-fitting method are evaluated.

In Naghavi and Yu (1996), it is shown that the quantile prediction accuracy of the log-Pearson type III (LP3) distribution depends largely on the accuracy of the parameter-estimation method used. The performance of a parameter-estimation method, on the other hand, depends on both the individual population chosen from the LP3 family and the sample size. In this study Monte Carlo experiments were conducted to evaluate four parameter-estimation methods that are frequently used in hydrological analysis. The four methods tested are the method of indirect moments (MMI), the method of mixed moments (MIX), the method of direct moments (MMD), and a modification of MMI using optimization techniques (MMO). A quantile ratio index (QRI) was devised to identify the limits (sample size and LP3 population subset) within which each of these methods will perform best. This study suggested that when QRI is less than or equivalent to 1.14, MMI or MMO should be used for sample size N less than equivalent to 30, MIX for 30 less than N less than 100, and any of the four methods for N greater than equivalent to 100. When QRI greater than 1.14, MMO is recommended for N less than equivalent to 30, MIX for 30 less than N less than 100, and MIX, MMO, or MMI for N greater than equivalent to 100.

In Pilon and Adamowski (1993), maximum likelihood and censored sample theory are applied for flood frequency analysis purposes to the log Pearson Type III (LP3) distribution. The logarithmic likelihood functions are developed and solved in terms of fully specified floods, historical information, and parameters to be estimated. The asymptotic standard error of estimate of the T-year flood is obtained using the general equation for the variance of estimate of a function. The variances and covariances of the parameters are obtained through inversion of Fisher's information matrix. Monte Carlo studies to verify the accuracy of the derived asymptotic expression for the standard errors of the 10, 50, 100, and 500 year floods, indicate that these are accurate for both Type I and Type II censored samples, while the bias is less than 2.5%. Subsequently, the Type II censored data were subjected to a random, multiplicative error. Results indicate that historical information contributes greatly to the accuracy of estimation of the quantities even when the error of its measurement becomes excessive. (Type I: The threshold is fixed and the number of censored values is a random variable. Type II: The number of censored values is fixed and the threshold is a random variable).

In Lye et al. (1988), the three-parameter lognormal distribution is studied with Bayesian estimates of the parameters and of the T-year quantile and their posterior variances of the estimates are obtained by using Lindley's Bayesian approximation procedure. These estimates are compared to estimates obtained by the method of maximum likelihood. In all cases the posterior variances of the Bayes estimates of the T-year flood events are less than the corresponding variances of their maximum likelihood counterparts.

In Lye et al. (1993), ML estimators were compared with Bayesian estimators for the reliability functions of the extreme value distributions.

In Yamaguchi (1997), Monte Carlo simulations have been performed to determine a preferable parameter estimation method for each of eight distribution functions. It was also shown that a jackknife method is beneficial to correct the bias and RMSE irrespective of the estimation method used.

In Takara and Stedinger (1994) the use of two-parameter distributions is recommended from the viewpoint of quantile estimation accuracy for datasets having sample skewness greater than 0.38 and less than 1.8. They found that the quantile lower bound estimators are likely to provide more accurate quantile estimators than other procedures.

Singh and Guo (1997) and Singh and Singh (1985, 1987) showed that the Method of Entropy yielded parameter estimates for the Generalized Pareto, the Gamma and the lognormal distributions which were comparable or better within certain ranges of sample size and coefficient of variation in comparison with MOM, PWM and ML.

3.7.5 Performance based on relative bias and RMSE of the quantile of inhomogeneous data

The work of the previous section is performed under the assumption of homogeneous data. However, the performance of the methods under inhomogeneous data is also an important issue of study, which has not received much attention yet in literature (Hall, 1992). The concept of homogeneity will be subject of Chapter 4. For the time being, it is enough to know that homogeneity is related with the fact that data comes from one and the same distribution function.

There are several ways to generate inhomogeneous data. One can draw data from distribution 1 with certain probability and from distribution 2 with complementary probability and combine the data. However, in this section the approach will be followed in which the parameters of the distribution function are considered uncertain. The parameters are drawn from a Normal distribution with a fixed coefficient of variation (for instance 5% or 20%; this is called the inhomogeneity factor). The realisations are substituted in the distribution function and a realisation from this distribution function is drawn (Van Gelder, 1999a).

Consider the exponential distribution with two parameters ξ (location) and α (scale). Data can be generated with $x_i = \xi - \alpha \log(u)$ in which u is a uniformly distributed random variable between 0 and 1. The parameters ξ and α themselves are also considered as random variables (normally distributed). In this way an inhomogeneous dataset is generated.

The performance of the estimation methods will be judged on two measures: the relative bias of the quantile estimation and the root mean squared error (RMSE) of the quantile estimation.

Table 3.2: Estimates of mean and standard deviation of (α, ξ, F(0.99), F(0.999), and F(0.9999)) for different homogeneties and estimation methods

Inhomogeneity 0%

	α_{mean}	α_{std}	ξ_{mean}	ξ_{std}	$F(0.99)_{mean}$	$F(0.99)_{std}$
ML	0.4462	0.1527	1.0476	0.0498	3.1025	0.7045
LM	0.4968	0.1878	0.9970	0.0995	3.2847	0.8174
CM	0.4596	0.1899	1.0342	0.1216	3.1508	0.8145
LS	0.6554	0.2784	0.9121	0.1659	3.9305	1.1596

Inhomogeneity 0%

	α_{mean}	α_{std}	ξ_{mean}	ξ_{std}	$F(0.999)_{mean}$	$F(0.999)_{std}$
ML	0.4516	0.1528	1.0490	0.0479	4.1686	1.0531
LM	0.4996	0.1919	1.0010	0.1034	4.4522	1.2690
CM	0.4623	0.2014	1.0383	0.1322	4.2319	1.3122
LS	0.6612	0.3015	0.9139	0.1885	5.4811	1.9318

Inhomogeneity 0%

	α_{mean}	α_{std}	ξ_{mean}	ξ_{std}	$F(0.9999)_{mean}$	$F(0.9999)_{std}$
ML	0.4547	0.1502	1.0502	0.0488	5.2383	1.3838
LM	0.5060	0.1899	0.9989	0.1045	5.6594	1.6930
CM	0.4667	0.1926	1.0382	0.1250	5.3363	1.7048
LS	0.6647	0.2822	0.9150	0.1696	7.0369	2.4682

Inhomogeneity 5%

	α_{mean}	α_{std}	ξ_{mean}	ξ_{std}	$F(0.99)_{mean}$	$F(0.99)_{std}$
ML	0.4642	0.1518	1.0384	0.0691	3.1761	0.6974
LM	0.5049	0.1883	0.9977	0.1039	3.3230	0.8191
CM	0.4658	0.1928	1.0368	0.1265	3.1819	0.8248
LS	0.6646	0.2858	0.9128	0.1745	3.9734	1.1865

Inhomogeneity 5%

	α_{mean}	α_{std}	ξ_{mean}	ξ_{std}	$F(0.999)_{mean}$	$F(0.999)_{std}$
ML	0.4665	0.1523	1.0300	0.0641	4.2526	1.0456
LM	0.5055	0.1885	0.9910	0.1029	4.4830	1.2486
CM	0.4676	0.1964	1.0289	0.1304	4.2592	1.2824
LS	0.6688	0.2941	0.9030	0.1839	5.5227	1.8869

Inhomogeneity 5%

	α_{mean}	α_{std}	ξ_{mean}	ξ_{std}	$F(0.9999)_{mean}$	$F(0.9999)_{std}$
ML	0.4644	0.1517	1.0369	0.0703	5.3138	1.3904
LM	0.5015	0.1873	0.9998	0.1066	5.6185	1.6711
CM	0.4621	0.1935	1.0392	0.1289	5.2954	1.7110
LS	0.6593	0.2873	0.9162	0.1776	6.9884	2.5090

Inhomogeneity 20%

	α_{mean}	α_{std}	ξ_{mean}	ξ_{std}	$F(0.99)_{mean}$	$F(0.99)_{std}$
ML	0.6044	0.1876	0.9054	0.1504	3.6889	0.7986
LM	0.5737	0.1971	0.9361	0.1377	3.5780	0.8435
CM	0.5210	0.2008	0.9887	0.1488	3.3883	0.8511
LS	0.7456	0.2951	0.8481	0.1933	4.2818	1.2193

Inhomogeneity 50%

	α_{mean}	α_{std}	ξ_{mean}	ξ_{std}	$F(0.99)_{mean}$	$F(0.99)_{std}$
ML	1.0242	0.3201	0.4802	0.3230	5.1966	1.2737
LM	0.8594	0.2706	0.6450	0.2496	4.6025	1.1353
CM	0.7640	0.2774	0.7404	0.2551	4.2586	1.1577
LS	1.1057	0.4096	0.5231	0.3135	5.6150	1.6723

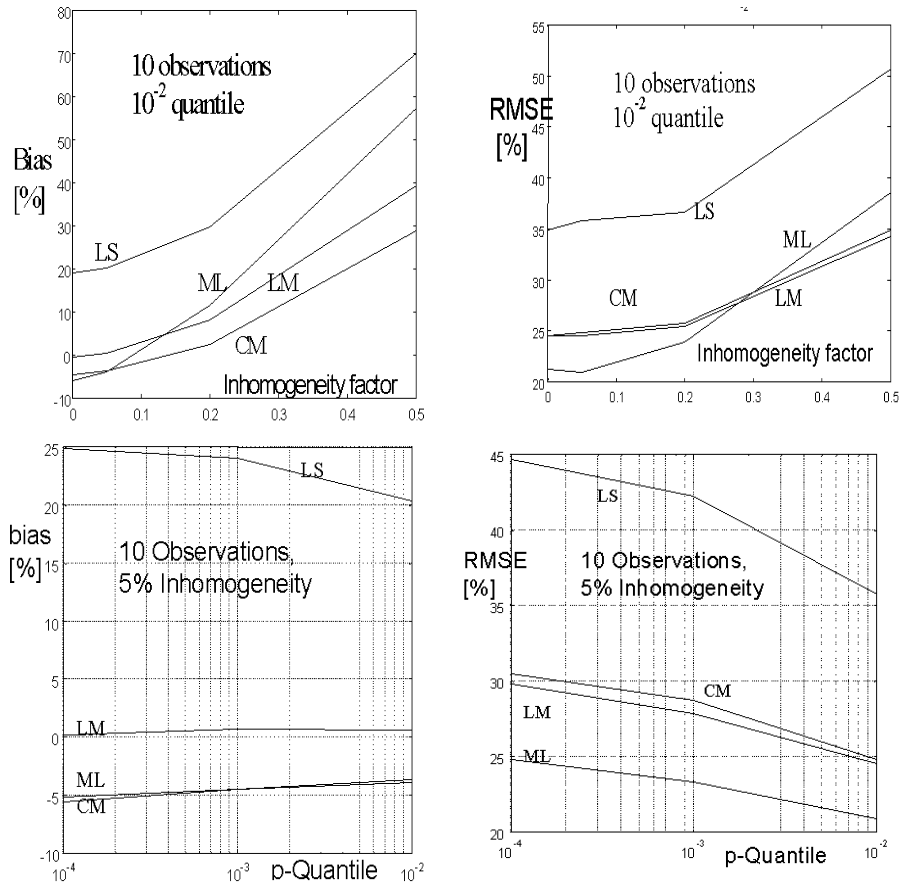
Various sample sizes are investigated varying between 10 and 100. The Monte Carlo simulation is repeated 2000 times in order to ensure convergence in the results.

The results for one typical simulation (sample size = 10) are given in Table 3.2. Each quantity (α, ξ, F(0.99), F(0.999), and F(0.9999)) will be summarized in the Table by its mean value (indicated by the subscript mean) and standard deviation (subscript std). Note that the following mean values were given as input parameters in the simulations: $\alpha = 0.5$, $\xi = 1$. For this choice the quantiles are F(0.99) = 3.30, F(0.999) = 4.45, and F(0.9999)=5.60.

Note that the relative bias in the quantiles with the L-moments method is the lowest of the four methods (CM=MOM) in the above simulation. Also the RMSE is quite low. The performance of the four methods given 5% inhomogeneous data and 10 datapoints is presented in Figures 3.6 and 3.7 showing the bias and RMSE of the 10^{-2} , 10^{-3} and 10^{-4} quantiles. LM give the smallest bias of about 1% over the whole range of p-quantile levels. The smallest RMSE is obtained with the ML method. The LS-method give the highest bias and RMSE (although the estimations are overestimations of the quantiles; i.e. safe values for engineers, whereas MOM and ML are underestimations). The performance of the 4 methods given inhomogeneous data and 10 datapoints is presented in Figures 3.8 and 3.9 showing the bias and RMSE of the 10^{-2} quantile given 0%, 5%, 20% and 50% inhomogeneity. Note that the bias in the quantiles is the smallest with the LM-method up to about 10% inhomogeneity. Above 10%, the MOM give the smallest bias. The RMSE of LM and MOM are approximately the same and are lower than the RMSE of ML for data with an inhomogeneity factor of 30% and higher.

The L-moment method shows its power in case of inhomogeneous datasets. The quantile estimation with the L-moment method gives the smallest bias and RMSE compared with other methods (in case of inhomogeneous data from an exponential distribution and for all sample sizes). For homogeneous datasets it is better to use other estimation techniques such as the Maximum Likelihood for example. In practical situations however, it is unknown if the dataset can be considered homogeneous.

Ashkar et al. (1993) examined other types of robust estimators based on trimming the data. Seki and Yokoyama (1996) used bootstrap techniques for robust parameter estimation. Konecny (1994) and Bernier (1994) consider the robustness of estimation methods and models in a general framework and Marshall and Olkin (1997) introduce an extra distribution parameter to achieve more robustness.



Figures 3.6-3.9: Comparisons of four estimation methods as a function of inhomogeneity level and quantile.

3.7.6 Performance based on over- and underdesign

In Van Gelder (1996b) it was suggested to measure the performance of statistical estimation methods with respect to over- and underdesign. In fact there is a strong relation with the performance based on relative bias and RMSE from the previous section. The relative bias and RMSE can give a first indication how much the under- or overdesign is, however, in case of very skewed distributions of the quantile, this indication might give a false impression of the amount of under- or overdesign. Under- or overestimation of the p-quantiles have an important meaning in civil engineering practice as well. Underestimation may give rise to unsafe structures whereas overestimation may lead to conservatism or too expensive structures. Therefore it is very useful to study the probabilities of under- and overdesign of a certain estimation method.

As a typical result from Van Gelder (1996b), in this section, we will in particularly concentrate on the under- and overestimation of the p-quantile of an Exponential and Gumbel distribution with a ML- and LS-parameter estimation method. Different sample sizes are considered (n=10, 30 and 100) for the same quantile of interest x_{100} such that $P(x>x_{100} \mid \text{Data}) = 1/100$. Typical results are shown in the next Tables 3.3 and 3.4.

Data from Gumbel	n	ML	LS
Fitted by Gumbel	10	0.59	0.34
	30	0.56	0.36
	100	0.53	0.38
Fitted by Exponential	10	0.18	0.19
	30	0.01	0.12
	100	0.00	0.05

Table 3.3: Probabilities of underdesign p_u

Data from Exponential	n	ML	LS
Fitted by Gumbel	10	0.85	0.50
	30	0.96	0.55
	100	0.99	0.67
Fitted by Exponential	10	0.59	0.37
	30	0.54	0.37
	100	0.52	0.37

Table 3.4: Probabilities of underdesign p_u

The probabilities of overdesign follow from the relation p_o=1-p_u. From the Tables 3.3 and 3.4, it follows that the least squares method usually gives lower probabilities of underdesign than the maximum likelihood method. That's why a least squares method is so popular under engineers. If we define assymetric loss-functions, in which we can model the risk aversion of a designer towards underdesign, we can determine optimal choices for distribution type and estimation method. For example if we penalize underdesign with a factor 4 more than overdesign we get the following Table 3.5.

n	Gumbel		Exponential	
	f*	EM*	f*	EM*
10	Exp	ML	Exp	LS
30	G	LS	Exp	LS
100	G	LS	Exp	ML

Table 3.5: Optimal choices for distribution type f* and estimation method EM*

From this table, we indeed notice a preference for the least squares method, except for large sample sizes from an exponential distribution where a ML-method is prefered and for small sample sizes from a Gumbel distribution which are better modeled by an exponential distribution with a ML-method for risk-averse engineers.

Another well known estimation method applied in civil engineering practice is the Method of Moments (MOM). In Section 3.9 a proof is shown that the p-quantile estimates by a MOM are always smaller (and therefore less risk averse) than the p-quantile estimates by a LS-method.

The notion of under and overdesign will be treated in more detail in Chapter 5.

3.7.7 Discussion

In this section, an overview and references were given of parameter estimation techniques that are well known in civil engineering practice. With Monte Carlo simulation studies, these estimation techniques can easily be compared. Table 3.1 gave an overview of all the simulation work that has been performed for the pairs (f,EM): distribution and estimation method. With the references given in this table it is in principle possible to determine the optimal (w.r.t. minimum bias and RMSE) estimation method given a certain distribution function. However, the optimal choice for a pair (f,EM) can change very quickly if the main assumption of i.i.d. data is violated (this was shown in Sec. 3.7.5), or when the criteria for the optimal pair is changed (performance measured in terms of under- and overdesign), as was shown in Sec. 3.7.6. Under- and overdesign are important measures for the engineer. Based on simulations from an Exponential distribution (in Sec. 3.7.4) and some mathematical proofs (in Sec. 3.9.1) an ordening in risk aversion of the different estimation techniques can be made. The Maximum Likelihood, Bayesian point estimation (mean of posterior distributions) and Method of Moments parameter estimation techniques give a relatively higher proportion of underdesign than the Bayesian predictive (integration over the posterior distribution) and Least Squares techniques. Asymetric loss criteria can be used to model the risk aversion of the engineer in mathematical terms. The optimal choice of the probability distribution function and the parameter estimation method can then be determined by minimizing the assymetric loss. In Van Gelder (1996b), this idea has been worked out for more types of loss functions and includes parameter and model uncertainty. In Sec. 5.5 and 5.6 we come back to this issue.

Most of the in Section 3.7 given probability models have been implemented in computer programs. Kuczera, (1995) made a program called FLIKE in which the Bayesian analysis of GEV, LN and GPA models are included. Perreault et al. (1994) and Perron et al. (1994) developed the very powerful and user-friendly AJUSTE software.

3.8 Weight factor estimation

In this section the statistical estimation methods for distribution type selection will be investigated. In many areas of civil engineering, the question arises which probability distribution should be used to model the load and resistance. Instead of choosing one particular probability distribution, it is also possible to consider various probability distributions and to attach weights to these distributions according to how good the fits are. The probability distribution for which the bias and/or standard deviation of its predictions is large should be given less weight relative to those distributions that exhibit less bias and/or scatter. Weight factors for probability distributions can be determined with different methods. In Kass and Raftery (1995) a Bayesian method is suggested to derive these weight factors. Their method was successfully applied in an economics case study by De Vos (1995) and a biometrics case study by Volinsky et al. (1996). In Tang (1980), a linear regression method was suggested to derive the weight factors. His method was successfully applied in a sea level case study by Van Gelder et al. (1997a) and by Perricchi and Rodriguez-Iturbe (1983) in hydrology. In this section, the two proposed methods will be reviewed briefly. Since the method of Tang is defined for comparing two probability distributions only, it appeared to have some disadvantages. In Van Gelder et al. (1999d), however, his method is extended to n probability distributions ($n \geq 2$) (see also Sub-section 3.9.6). Furthermore, in Van Gelder et al. (1999d), the methods are applied to estimating extreme discharges of the Oder. A weight factor estimation, based on L-moments is also introduced in this Section. The ideas will be described in Section 3.8.3 and tested with large-scale Monte Carlo simulations.

3.8.1 Bayes factors

In this section, the two methods for determining the weight factors of probability distributions are described.

Consider a data set D and two possible probability models (or hypotheses) H_1 and H_2. In the traditional approach we would determine a test statistic T and compute its p-value according to model H_1. If the test statistic of the data results in a smaller value than the p-value, then we would reject H_1. This traditional way of model testing has a lot of disadvantages. It can only be applied when two models are nested, one within the other. Furthermore it can only offer evidence *against* hypothesis H_1 under small p-values; we cannot accept H_1 under large p-values. The p-value offers only an interpretation as a long-term probability in a long repetition of the same experiment. In the Bayesian approach these disadvantages don't exist.

In the Bayesian approach, we apply Bayes theorem to the data that each of the models is supposed to predict and compute the posterior probability that a certain model is correct. There is no limit to the number of models that may be simultaneously considered, nor does any model need to be nested within any of the others.

Given prior probabilities $p(H_1)$ and $p(H_2) = 1\text{-}p(H_1)$, the data produces posterior probabilities $p(H_1|D)$ and $p(H_2|D) = 1\text{-}p(H_1|D)$. The quantity commonly used to summarise these results is the Bayes factor:

$$B=[p(H_1|D)/\mathrm{p}(H_2|D)]/[p(H_1)/p(H_2)], \qquad (3.89)$$

which can be reduced by Bayes theorem to:

$$B=p(D|H_1)/p(D|H_2). \qquad (3.90)$$

So, the Bayes factor is precisely the probability of H_1 in favour of H_2 given solely by the data and its prior beliefs.

By using non-informative improper priors, $p(D|H_\mathrm{i}) = \int \mathrm{L}(D|H_\mathrm{i},8)\mathrm{p}(H_\mathrm{i}(8))\mathrm{d}8$ will not exist as explained in Bernardo and Smith (1994). Several ideas have been suggested to repair this problem:

1. Subdivide the data into two sets $D=(D_1,\ D_2)$ and use D_1 to generate prior information for D_2. This is an idea of Berger and Pericchi (1996) who also give suggestions how to split up D. (See also Gunasekara and Cunnane (1992)).
2. Apply the Schwarz Criterion (Schwarz, 1978):

$$-2\log B \approx -2\log\left[\frac{p(D\,|\,H_1,\hat{l}_1)}{p(D\,|\,H_2,\hat{l}_2)}\right] - (r_2 - r_1)\log n\,, \tag{3.91}$$

where $\hat{l}_i$ is the maximum likelihood estimator under H_i, r_i is the number of parameters in model H_i, and n is the number of observations. In this criterion, the term $(r_2 - r_1)\log(n)$ acts as a penalty term which corrects for differences in size between the models (compare the AIC (Mutua, 1994)). Although the Schwarz criterion is independent of the prior density, it may be viewed as a useful approximation to $-2\log B$.

In order to test hypotheses using Bayes factors, Kass and Raftery (1995) suggested the following guidelines:

$2\log B$	B	Evidence against H_1
0 to 2	1 to 3	Not worth more than a bare mention
2 to 5	3 to 12	Positive
5 to 10	12 to 150	Strong
>10	>150	Decisive

Table 3.6: Guidelines testing hypotheses using Bayes factors.

Suppose $(H_1,H_2,...,H_n)$ is our collection of candidate models, and γ is our quantity of interest. Given a set of prior model probabilities $\{p(H_1),p(H_2),...,p(H_n)\}$, the posterior distribution of γ is given by

$$p(\gamma\,|\,D) = \sum_{i=1}^{n} p(\gamma\,|\,H_i,D)\,p(H_i,D)\,, \tag{3.92}$$

where $p(\gamma|H_i, D)$ is the posterior for γ under the i-th model, and $p(H_i|D)$ is the posterior probability of this model. Averaging over models can result in a better fit than using any model individually. Despite this advantage there may arise a problem in the fact that calculation procedures can be time consuming. This problem does not exist when the number of models is small (say, n=2-5). FORM-Approximations and Markov-Chain Monte-Carlo methods are developed for efficient calculation by Geyskens and Der Kiureghian (1996) and Carlin and Louis (1996) respectively.

Any approach that selects a single model and then makes inference conditionally on that model ignores the uncertainty involved in the model selection, which can be a big part in the overall uncertainty. This difficulty can be avoided if one adopts a Bayesian approach and calculates the posterior probabilities of all the competing models, following directly from the Bayes factors. A composite inference can then be made that takes account of model uncertainty in a simple way.

3.8.2 Tang Weights

Tang (1980) proposed a linear regression model whereby discrepancy between observed data and the predicted probability model, as well as uncertainty of extrapolation from observations can be incorporated into hydrologic risk assessment. The method is based on a Bayesian linear regression analysis of the observed data (after transformation) plotted on probability paper. Given the discrepancies between data and model predictions in terms of expectations and variances of a design value, Tang's method involves combining these expectations and variances over the different probability models.

Suppose $E(Y_1)$ and $\mathrm{Var}(Y_1)$ denote the mean and variance of the design value predicted using one model, whereas $E(Y_2)$ and $\mathrm{Var}(Y_2)$ denote the mean and variance of the design value predicted using another independent model. According to Tang, an overall estimate of the expectation based on the combined information of two independent models can be determined using Bayes theorem as

$$E(X_2) = \frac{\mathrm{Var}(Y_2)E(Y_1) + \mathrm{Var}(Y_1)E(Y_2)}{\mathrm{Var}(Y_2) + \mathrm{Var}(Y_1)} \tag{3.93}$$

and an overall estimate of the variance as

$$\mathrm{Var}(X_2) = \frac{\mathrm{Var}(Y_2)\mathrm{Var}(Y_1)}{\mathrm{Var}(Y_2) + \mathrm{Var}(Y_1)}\,. \tag{3.94}$$

Observe that the combined estimate $E(X_2)$ will approach $E(Y_1)$ if $\mathrm{Var}(Y_1)$ is extremely small relative to $\mathrm{Var}(Y_2)$. In other words, if the first model provides an excellent fit while the second model provides some scatter, then the information of the second model could be neglected.

Using Bayes theorem, Eqs. (3.93) and (3.94) can be determined as follows. Let us consider a random sample from a normal distribution with an unknown value of the mean and a specified value of the variance. Suppose that the prior distribution of the unknown mean is a normal distribution, then the posterior distribution is also a normal distribution with parameters similar to Eqs. (3.93) and (3.94). As a matter of fact, the first and second model can be interpreted as the prior and observed information, respectively. For example, the posterior mean is an average of the prior mean and the sample mean, weighted inversely by the respective variances. For details, see Ang and Tang (1990, Chapter 8).

Although Tang presented the formulae for comparing design-value estimates on the basis of two probability models only, they can be easily generalised to n probability models where $n \geq 2$. If Eqs. (3.93) and (3.94) hold for all possible pairs of n independent probability models, then both the expectation $E(X_n)$ and the variance $\mathrm{Var}(X_n)$ can be derived using mathematical induction. This new result is given in Van Gelder et al. (1999d) and also in Sec. 3.9.5.

3.8.3 Application of Bayes factors and Tang weights to river discharges

In Van Gelder et al. (1999c, 1999d), case studies were performed on the model selection of the extreme discharges of the rivers Maas (Berger, 1992) and Oder. Differences were found between the methods based on Bayes factors and on Tang weights. In order to test the performance of both methods, Monte Carlo simulations had been performed. The results are presented next.

Monte Carlo simulations have been performed to determine the posterior model probabilities as a function of the number of samples, and the distribution type from which the simulations were generated (Rayleigh, exponential, Gumbel and lognormal). In all cases, diffuse prior model probabilities have been used, $P(H_1)=P(H_2)=P(H_3)=P(H_4)=1/4$, and uniform parameter priors have been used. The following integrals were calculated numerically:

$$
\begin{aligned}
&P(D|H_1)=\int L_{\mathrm{exp}}(D|\lambda)p(\lambda)d\lambda,\\
&P(D|H_2)=\int\int L_{\mathrm{gumb}}(D|\delta,\lambda)p(\delta,\lambda)d\delta d\lambda,\\
&P(D|H_3)=\int L_{\mathrm{ray}}(D|\delta)p(\delta)d\delta,\\
&P(D|H_4)=\int\int L_{\mathrm{ln}}(D|\delta,\lambda)p(\delta,\lambda)d\delta d\lambda.
\end{aligned}
\qquad (3.95)
$$

with L_{exp}, L_{gumb}, L_{ray}, L_{ln} the exponential, Gumbel, Rayleigh and lognormal likelihoods, respectively. According to Kass and Raftery (1995), the weight factors are the posterior probabilities that the model H_i is correct:

$$P(H_i|D) = \frac{P(D|H_i)P(H_i)}{\sum_{j=1}^{4} p(D|H_j)\,p(H_j)} \tag{3.96}$$

The weight factors according to Tang's method have also been determined. In order to exclude statistical variability, the determination of the weight factors has been performed 1000 times and the mean values are presented in Tables 3.7. In each cell the value on the l.h.s. gives the weight factor according to the Bayes factors; the value on the r.h.s. according to Tang's method.

Tables 3.7. Simulation results (l.h.s. Bayes; r.h.s. Tang)

Simulation from exponential distr. (CV 100%; fixed value)

	n=10		*n*=20		*n*=50	
Rayleigh	0	0.35	0	0.30	0	0.22
Exponential	0.99	0.23	0.99	0.32	0.99	0.44
Gumbel	0.01	0.24	0.01	0.26	0.01	0.25
Lognormal	0	0.17	0	0.13	0	0.07

Simulation from Rayleigh distribution (CV48%; fixed value)

	n=10		*n*=20		*n*=50	
Rayleigh	0.43	0.50	0.72	0.53	0.88	0.56
Exponential	0.48	0.18	0.23	0.14	0	0.10
Gumbel	0.09	0.29	0.05	0.32	0.12	0.33
Lognormal	0	0.02	0	0.01	0	0.01

Simulation from Gumbel distribution (CV 20%)

	n=10		*n*=20		*n*=50	
Rayleigh	0	0.40	0	0.38	0	0.36
Exponential	0.95	0.17	0.83	0.17	0	0.15
Gumbel	0.04	0.25	0.15	0.29	0.93	0.34
Lognormal	0.01	0.17	0.02	0.15	0.07	0.15

Simulation from lognormal distr. (CV 20%)

	n=10		*n*=20		*n*=50	
Rayleigh	0	0.44	0	0.44	0	0.09
Exponential	0.57	0.12	0.80	0.12	0	0.43
Gumbel	0.34	0.26	0.18	0.29	0.77	0.31
Lognormal	0.07	0.17	0.02	0.16	0.23	0.16

The weight factors according to both methods can distinguish competing models, even when the length of record is quite short. However, the Bayes factors perform better than Tang's method. As the sample size increases up to *n*=50, the true model comes out with a model probability close to 100% in the case of Bayes factors, whereas Tang's method has probabilities around 50%. For the lognormal distribution both methods perform quite bad. One has to have sample sizes of the order 100-200 before the lognormal distribution is "recognised".

Also notice that for Gumbel generated data, the incorrect 1-parameter exponential model had higher posterior probabilities for low sample sizes. This is a bit surprising, since the 2-parameter Gumbel distribution even has one parameter more than the exponential distribution. In Figure 3.10, this behaviour has been analysed in more detail with Bayes factors for small sample sizes. Note the high standard deviation of the weight factors.

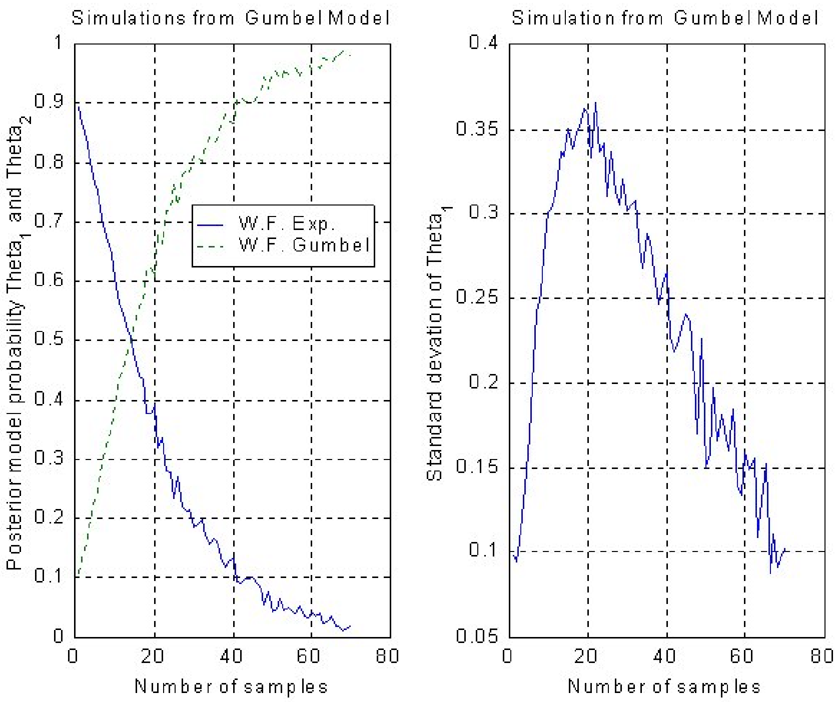


Figure 3.10: Weight factors

Probability exceedance curves are used frequently in many areas of safety and reliability. In this section, the question how to select a certain probability distribution has been tackled by using Bayes factors and Tang's method. Both methods were reviewed, extended, and investigated with Monte Carlo experiments. Bayes factors appear to perform better than Tang's method and are therefore suggested to be used in distribution selection.

3.8.4 L-Kurtosis Weights

In this section it will be shown that apart from the Bayes factors and Tang weights also the sample L-Kurtosis can give an indication for the weight factor. For that purpose, various simulation experiments were designed to illustrate the effect of distribution shape and tail weight on the accuracy of quantile estimation. In the first simulation experiment, samples were generated from the generalized Pareto distribution (parent) and five 3-parameter distributions (GLO, GEV, GNO, GPA, and PE3; the abbreviations are given on page 206) were fitted by matching the first three L-moments. Figure 3.11 shows the steps of the simulation experiment.

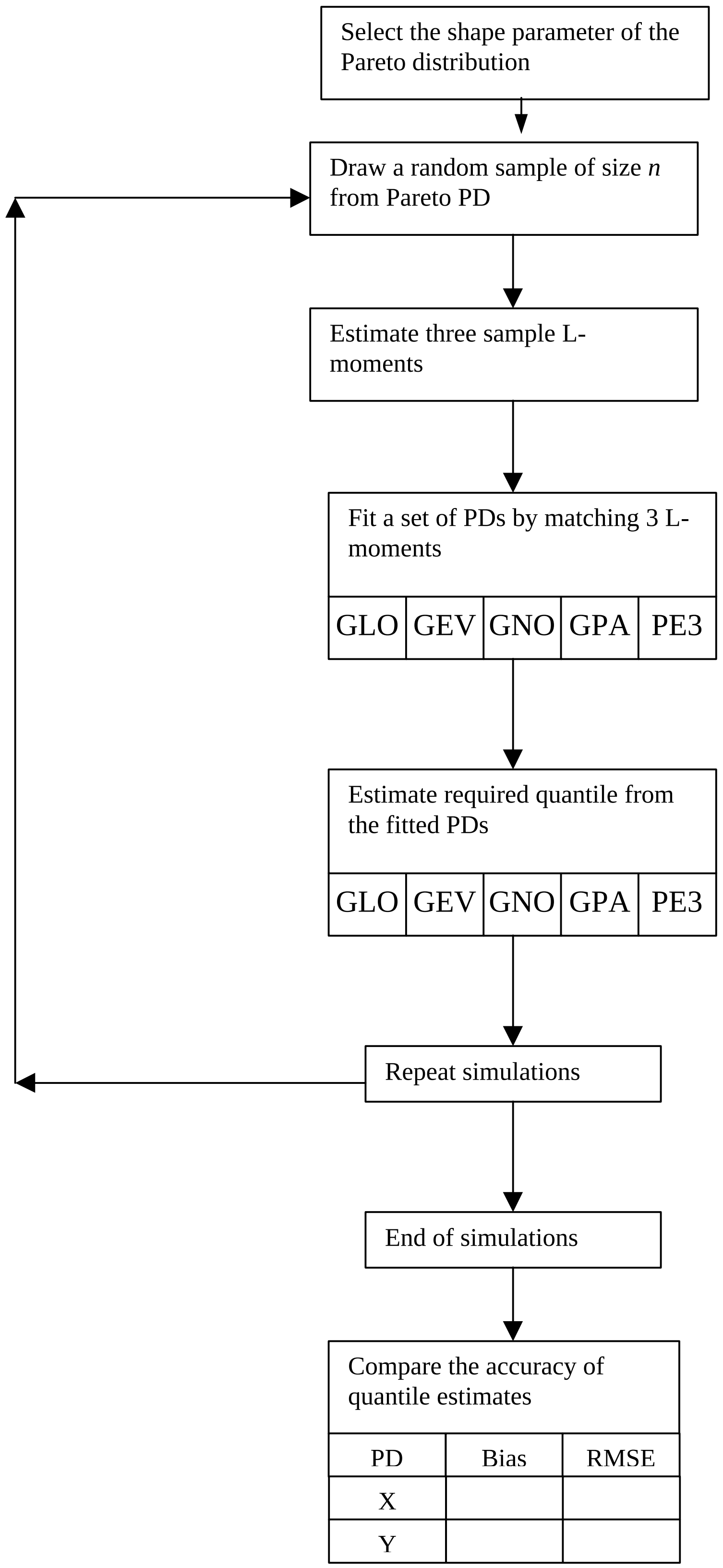


Figure 3.11: Steps in the simulation experiment

Consider the first case in which a sample size 30 was used to estimate the quantile for a POE (probability of exceedance) $p=10^{-3}$. The shape parameter, *k*, of the parent Pareto distribution was varied from –0.4 to +0.1. Note that the tail weight of the Pareto distribution as defined by Maes (1995) is equal to –*k*, such that higher the tail weight, the longer and heavier the distribution tail (Smith and Weissman, 1987). In Figure 3.12, the normalized bias of the quantile estimated from the five candidate PDF's is plotted against the shape parameter.

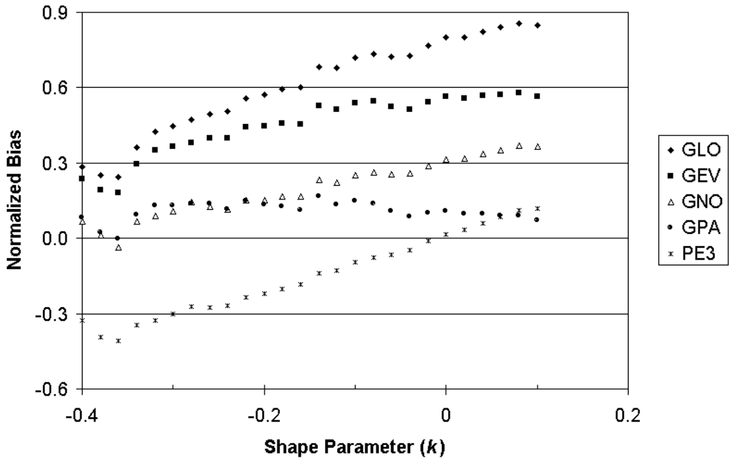


Figure 3.12: Normalized bias against the shape parameter of the GPA (POE=10^{-3}, n=30)

The normalized bias is minimum (< 10%) for the fitted GPA distribution, whereas it varies significantly (±80%) for other PDFs. It is interesting that the bias of GNO quantile estimates is fairly close to that of GPA for $k < 0$. The efficiency of the quantile estimate can be seen from Figure 3.13 where the normalized RMSE is plotted against the shape parameter. The RMSE of PE3 estimates is minimum, though the associated bias is quite large as seen from the previous Figure. Among the remaining four PDFs, GPA and GNO estimates have smaller RMSE.

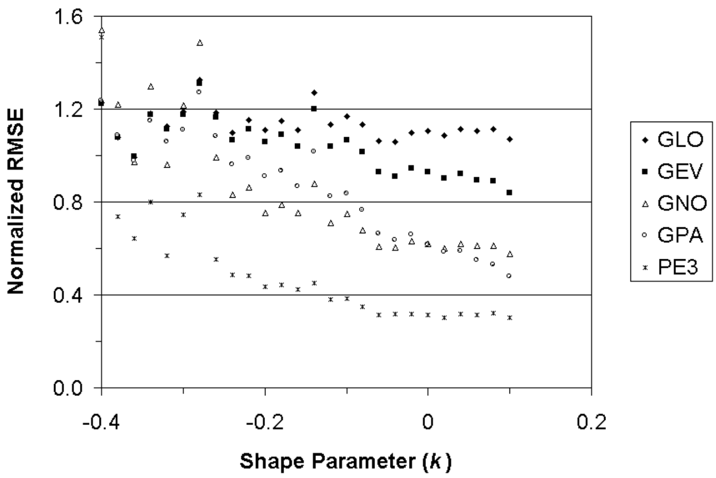


Figure 3.13: Normalized RMSE against the shape parameter (POE=10^{-3}, n=30)

However, in case of estimates of a lower quantile (probability of exceedence of 10^{-2}) the bias decreases significantly for all PDFs, as shown in Figure 3.14. In fact, for $k <$ -0.2, all PDFs result in fairly small bias (<10%). As far the efficiency is considered, Figure 3.15 suggests that the RMSE is fairly insensitive to the distribution type. With few exceptions, the RMSE for all quantile estimates lies in a narrow band.

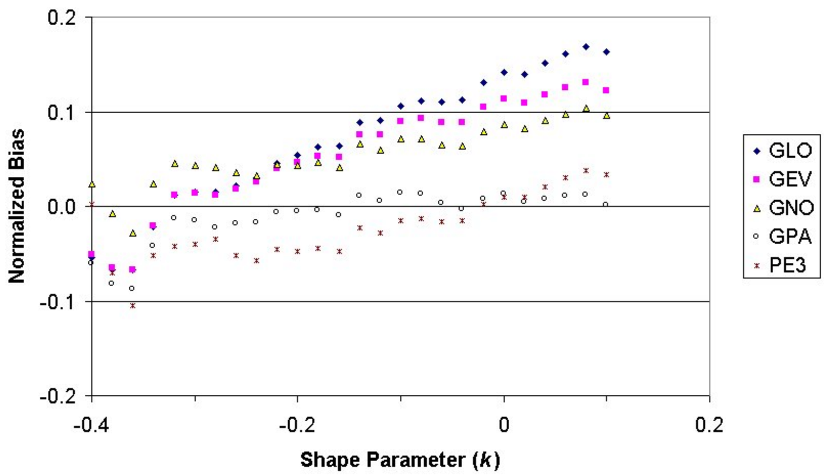


Figure 3.14: Bias of Pareto quantile estimates (POE=10^{-2}, n = 30)

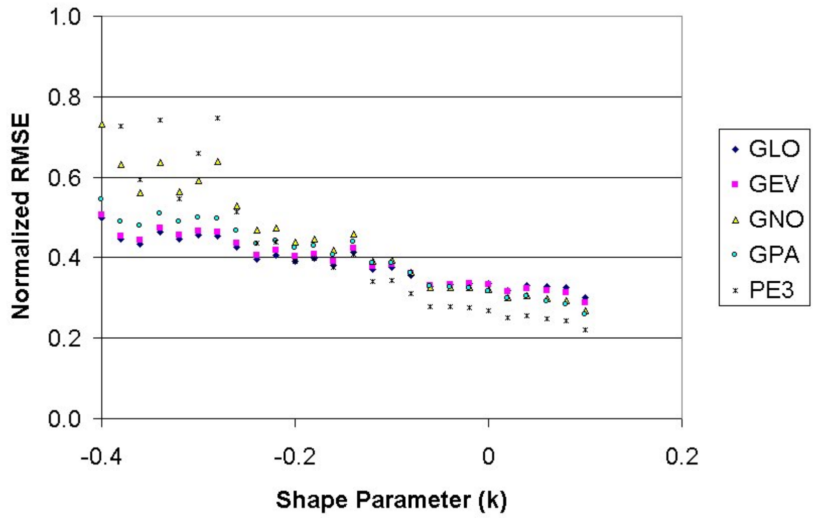


Figure 3.15: RMSE of Pareto quantile estimates (POE=10^{-2}, n = 30)

To analyze the effect of sample size on bias and efficiency, random samples of various sizes ranging from 50 to 500 were generated from GPA distribution with k = –0.4, and quantile values for 10^{-3} were estimated as before. The GPA quantile appears to be almost unbiased, followed by the GNO quantile estimates. It is interesting that the bias is almost insensitive to the sample size (for large sample sizes). The efficiency, however, improves with an increase in the sample size, as shown in Figure 3.16. In large samples, n > 100, the normalized RMSE appears to be insensitive to the distribution type. The RMSE still remains around 30% for the large sample sizes in accordance with Hosking and Wallis (1987). The quantile estimates for 10^{-2} exhibit fairly small bias as well as reduced RMSE . In fact, all five PDFs are able to provide fairly accurate estimates of low quantiles (results not shown here).

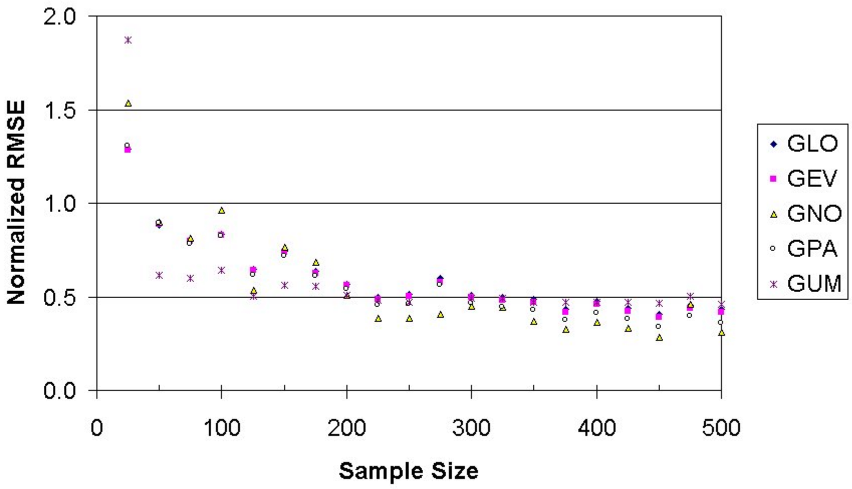


Figure 3.16: RMSE of Pareto quantile estimates for various sample sizes (prob. of exceedence 10^{-3}, k=–0.4)

The bias of an extreme quantile (with probability of exceedance < 10^{-3}) estimate is more sensitive (than the RMSE) to the distribution type fitted to the sample data. Therefore, the knowledge of the correct distribution type has relevance to the minimization of bias. Although the distribution type has major influence on the quantiles estimates, the dependence is not unique, i.e., more than one distribution can provide reliable estimates with low bias and high efficiency depending on the degree of (tail) equivalence between the population and the fitted distribution. In this respect, the importance of identifying exactly the parent distribution from small sample is limited, rather any distribution reasonably close to the parent can serve the purpose. It is proposed that L-kurtosis is such a measure that can quantify approximately the degree of closeness of the sample data to a candidate distribution (Hosking 1992).

The second simulation experiment was designed to evaluate systematically the effectiveness of the L-Kurtosis (L-K) criterion against the minimum divergence (M-D) criterion (Eqn. (3.62)) for estimating extreme quantiles. Consider a sample drawn from a known distribution (e.g., kappa distribution), and a set of several candidate distributions are fitted to the data based on some criterion, e.g., matching L-moments. The probability distribution closest to the parent can be identified by computing divergence or probabilistic distance of each of the candidate distributions from the parent. Obviously, the distribution with the least divergence would be closest to the parent PDF.

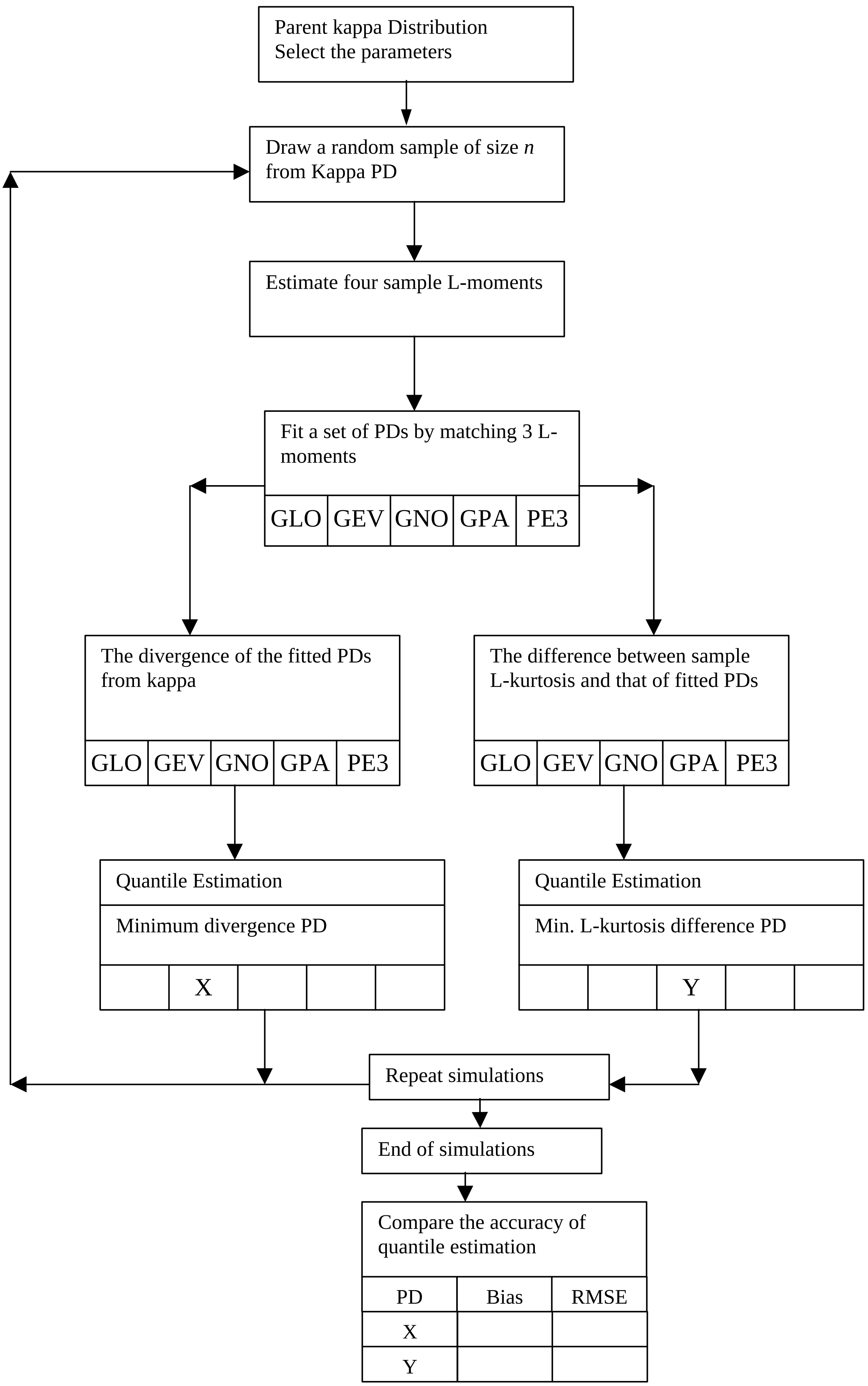


Figure 3.17: Overview of the simulation experiment

The steps involved in simulation experiment, shown in Figure 3.17, are briefly described as follows. A random sample was drawn from a four-parameter kappa distribution with preselected parameters. A set of three parameter PDFs were fitted to sample data by matching the first three sample L-moments. The divergence of the fitted PDFs from the parent was computed, and the difference of L-kurtosis of fitted PDFs from that of the sample was evaluated. Using L-K and M-D criteria representative PDFs were chosen to compute the required quantile. The process was repeated several times to estimate the bias and RMSE of quantile values. This simulation was repeated for various sample sizes, probability of exceedence values and parameters of the parent kappa distribution. Numerical computations utilized several FORTRAN routines developed by Hosking (1997).

To illustrate the application of two distribution selection criteria, random samples of size 30 were drawn from the kappa distribution with τ_2 =0.125, τ_3 = 0.266, and τ_4 = 0.182. Five 3-parameter distributions were fitted to each sample, and the best fit PDF was selected from both L-K and M-D criteria. Relative frequencies of the selected distribution in simulation, which consisted of 10,000 samples, are shown in Figure 3.18 along with a table reporting the bias and RMSE of the quantile estimate (probability of exceedence 10^{-3}). It is interesting that bias and RMSE resulting from the L-K and M-D criteria are almost identical, though the best fit distributions selected from these criteria are relatively different. This observation confirms the main conclusion of the first simulation experiment of this section regarding the non-uniqueness of the type of the best fit distribution to the sample data.

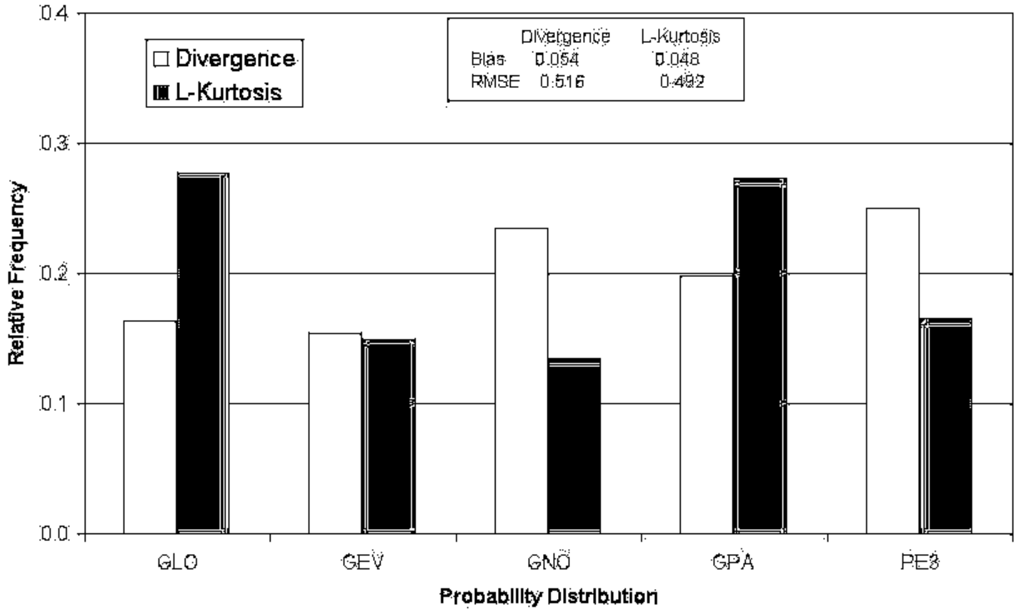


Figure 3.18: Relative frequency of probability distributions selected from L-K and M-D criteria (prob. of exceedence 10^{-2})

Now consider estimation of a quantile corresponding to a probability of exceedence of 10^{-3} from a sample of size n = 30. The shape parameters (h and k) of the kappa distribution were varied to cover a wide range of L-skewness and L-kurtosis values. For each case, the quantile bias and RMSE were estimated by the two criteria using 1000 samples in the simulation experiment.

In Figure 3.19, the variation of quantile bias resulted from the two methods is plotted against the L-kurtosis of the parent kappa distribution. The quantile bias from the two criteria is in close agreement for $\tau_4 < 0.5$. It is noteworthy that the degree of underestimation of quantile increases with increasing values of τ_4. The efficiency of both methods is almost identical for $\tau_4 < 0.5$ as shown in Figure 3.20. However, for $\tau_4 > 0.5$ the RMSE associated with L-K criterion is fairly large as compared to M-D criterion. In case of a larger sample size, n = 100, both methods result in almost the same bias and RMSE in quantile estimates (Pandey et al., 1999).

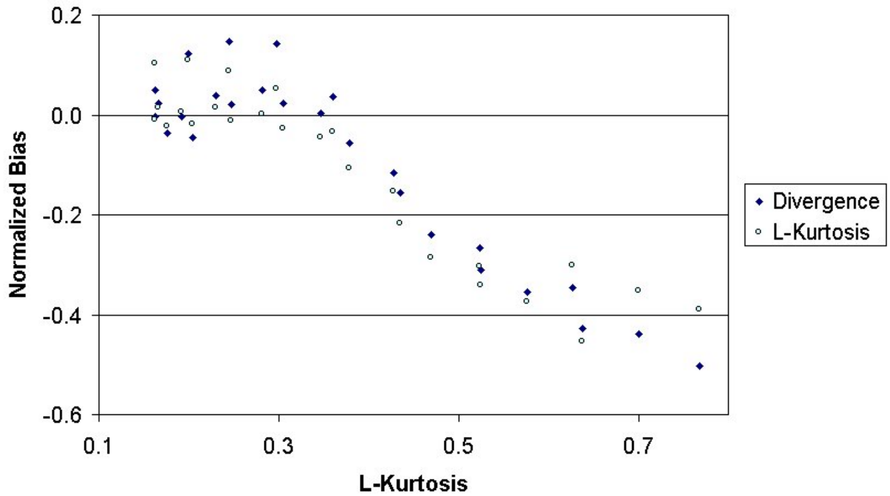


Figure 3.19: Bias of kappa quantile estimates (prob. of exc. 10^{-3}, n = 30)

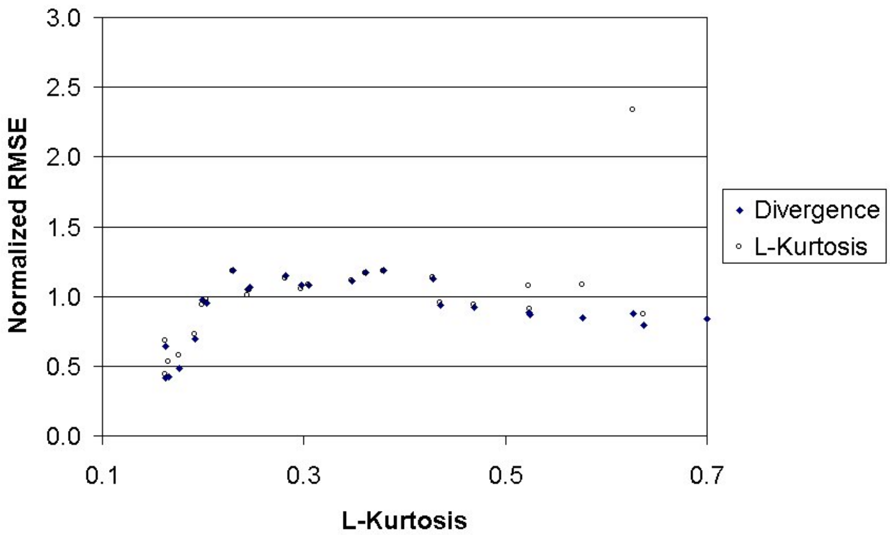


Figure 3.20: RMSE of kappa quantile estimates (prob. of exc. 10^{-3}, n = 30)

Since the probability weighted moments of higher order (> 2) can be reliably estimated from small samples, their use in distribution fitting and quantile estimation has distinct advantages. PWM's are directly related to the expectations of order statistics, as discussed in Section 3.3. L-moments are the linear combinations of PWM's, and in a simple sense they are analogous to ordinary moments. The L-kurtosis, which is the fourth order L-moment normalized by that of the second order, is considered in the literature as an effective measure of distribution shape and tail behaviour.

In modern information theory, the divergence is developed as a mathematically comprehensive measures of probabilistic distance, which is extensively used in signal analysis and pattern recognition. The central idea of this section was to compare the effectiveness of L-kurtosis in distribution fitting against benchmark results obtained from the use of divergence measure.

The impact of distribution shape on the accuracy of extreme quantile estimates was discussed. Numerical results indicate that the bias of quantile estimates is more sensitive than RMSE to the distribution shape. It also highlighted that there can be more than one distribution with similar tails that can provide accurate quantile estimates. In other words, the importance of identifying exactly the parent distribution from small sample is somewhat limited, rather any distribution reasonably close to the parent can serve the purpose. It is accepted that L-kurtosis is such a measure that can quantify approximately the degree of closeness of the sample data to a candidate distribution as proposed by Hosking (1992). However, this conclusion is subject to limitations of present investigation, and it would benefit from further validation.

Results of simulation experiments described in Section 5 indicate that quantile estimates obtained from the L-kurtosis criterion are in fairly close agreement with those obtained from the minimum divergence criterion. In this respect, it can be concluded that L-kurtosis is a reliable indicator of distribution shape and its use in quantile estimation is very effective. Remarkable simplicity of computation makes the L-Kurtosis criterion an attractive tool for distribution fitting. It can also be concluded that information theoretic concepts can deliver benchmark results to judge the performance of the method of L-moments.

In Van Gelder et al. (1999b), a few case studies are described on the L-Kurtosis model selection criterion.

3.9 Theoretical Considerations

3.9.1 Analytical comparisons between LS and MOM

Theorem 3.9.1: The LS-estimate of the scale parameter of the Exponential and Gumbel probability distributions is always larger than its MoM-estimate.

Proof : Assume that we have N observations in sorted order given by x_1, x_2, ..., x_N. We can linearize the probability distribution F(x) under consideration (with fixed scale parameter B and location parameter A). So we can find a function g such that:

$$g(F(x)) = (x-A)/B. \tag{3.97}$$

For the Exponential distribution g is given by $g_E(\zeta) = -\ln(1-\zeta)$ and for the Gumbel distribution, we have $g_G(\zeta) = -\ln(-\ln(\zeta))$.

Define the vector y by $y = i/(N+1)$, the plot position of the i^{th} observation (i=1,...,N) and the vector $y^* = g(y)$.

From linear regression theory we have $B_{LS} = \sigma_x/\rho\sigma_{y^*}$ in which $\rho = \mathrm{cov}(x,y^*)/\sigma_x\sigma_{y^*}$.
For the Exponential distribution we have $B_{MoM} = \sigma_x$.

We will proof that $B_{MoM}<B_{LS}$, or equivalently:

$\rho\sigma_{y^*}<1$, or equivalently (because $-1<\rho<1$):

$$\sigma_{y^*}<1. \tag{3.98}$$

Note that $y_i^* = g(y_i) = -\ln(1-i/(N+1)) = -\ln(z)$, in which z can be considered as a uniform distribution between δ and $1-\delta$ with $\delta = 1/(N+1)$.

$$\begin{aligned} &\text{So } E(y^*) = \int_\delta^{1-\delta} -\ln(z)dz = -\ln(1-\delta)+1+\ln(1-\delta)\delta-2\delta+\ln(\delta)\delta \uparrow 1 \ (\delta\to 0), \\ &\text{and } \mathrm{Var}(y^*) = \int_\delta^{1-\delta} \ln^2(z)dz - E^2(y^*) \uparrow 1 \ (\delta\to 0). \end{aligned} \tag{3.99}$$

So $\sigma_{y^*}<1$ (for all N).

For the Gumbel distribution we have $B_{MoM} = \sigma_x \sqrt{6}/\pi$ and

$E(y^*) = \int_{\delta}^{1-\delta} -\ln(-\ln(z))dz = (\delta-1)\ln(-\ln(1-\delta)) + \delta\ln(-\ln(\delta)) + Ei(1,-\ln(1-\delta))$
$+ Ei(1-\ln(\delta)) \uparrow \gamma \ \ (\delta \rightarrow 0)$ (3.100)

In which Ei are the exponential integrals (Abramowitz, and Stegun, 1965):

$$Ei(n,x) = \int_1^{\infty} e^{-xt} t^{-n} dt \quad (3.101)$$

where n is a non-negative integer, and defined for Re(x)>0. The 1-argument exponential integral is a Cauchy Principal Value integral, defined only for real arguments x, as follows:

$$Ei(x) = -\int_{-\infty}^{x} e^t t^{-1} dt \quad (3.102)$$

The relation between Eqn. (3.101) and (3.102) follows from: Ei(x) = -Ei(1,-x) for x<0. Furthermore, the variance is convergent as: $Var(y^*) \uparrow \pi^2/6 \ (\delta \rightarrow 0)$, so

$\sigma_{y^*} < \pi/\sqrt{6}$ (for all N) (3.103)

3.9.2 Theoretical limit distributions for maxima

Assume that X_1, X_2, ..., X_n are independent and identically distributed random variables coming from a parent distribution with a cumulative distribution function F(x) and probability distribution function f(x). Define $H_n=\max(X_1, X_2, ..., X_n)$ then H_n has a CDF given by:

$H_n(x)=P(\max(X_1, X_2, ..., X_n)<x)=F^n(x)$. (3.104)

Notice that the percentiles of H_n move to the right with increasing n, approaching the upper and lower end points if they are bounded, or going to ∞ of they are unbounded. When n goes to infinity, we have

$\lim_{n\rightarrow\infty} H_n(x)=1$ if F(x)=1
$\lim_{n\rightarrow\infty} H_n(x)=0$ if F(x)<1 (3.105)

that is, the limit distribution degenerates to a Dirac function. To avoid this degeneracy, we transform the random variable X by means of constants a_n and b_n such that

$$\lim_{n\to\infty} H_n(a_n+b_n x) = \lim_{n\to\infty} F^n(a_n+b_n x) = H(x) \qquad (3.106)$$

where H(x) is a nondegenerated CDF.

For instance:
$F_{X_{max}}(x)=(1-e^{-\lambda x})^n \to 0$ $(n\to\infty, \forall x)$. However if x also goes to infinity with $a_n+b_n x$ with $a_n=\log(n)/\lambda$ and $b_n=1/\lambda$ then:
$F_{X_{max}}(x) \approx F_{X_{max}}(a_n+b_n x) = F_{X_{max}}(\log(n)/\lambda+x/\lambda) = (1-e^{-\lambda(\log(n)/\lambda+x/\lambda)})^n = (1-e^{-x}/n)^n$
$\to \exp(-e^{-x})$ $(n\to\infty)$.
So X_{max} converges to Gumbel.
$F_{X_{max}}(a_n+b_n x)$ is Gumbel distributed with parameters (0,1) for $n\to\infty$.
So X_{max} is Gumbel distributed with parameters (a_n,b_n) for $n\to\infty$.
So: $E(X_{max})\to\log(n)/\lambda+0.57772/\lambda$ $(n\to\infty)$ and $\sigma(X_{max})\to 1.2825/\lambda$ $(n\to\infty)$.

Assume the sea levels in Hook of Holland exponentially distributed with parameter λ. In Figure 3.21 the ten highest sea levels at Hoek van Holland (>180cm) are depicted. In the same Figure the distribution of X_{max} (being $(1-e^{-\lambda x})^n$) and the limit distribution (being Gumbel with mean 281cm and standard deviation 45cm) are given.

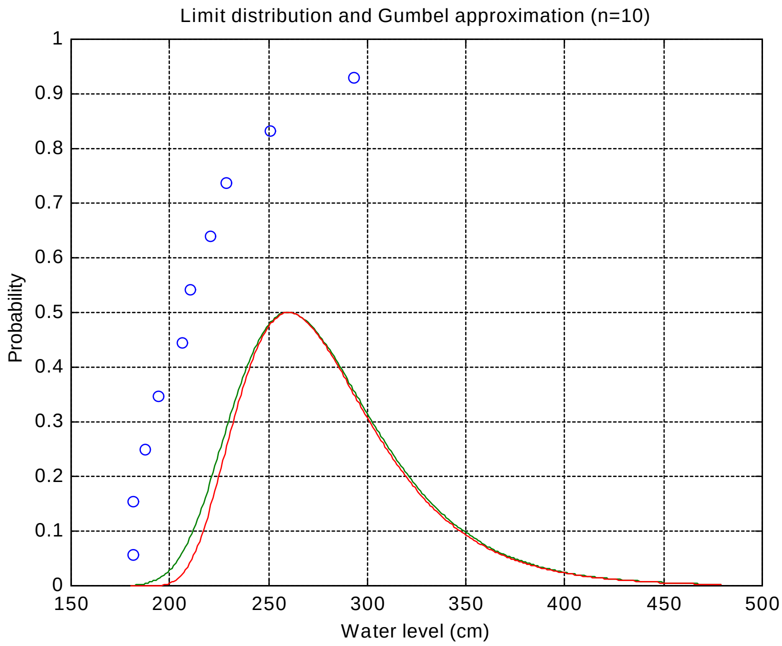


Figure 3.21. Extreme values with n=10 (parent is Exponential)

When F(x) satisfies the limit $\lim_{n\to\infty}F^n(a_n+b_nx)=H(x)$, we say that F(x) belongs to the domain of attraction of H(x). The surprising result is that there are only 3 possible CDF's for H(x) (Gnedenko, 1943). They are given by:

Frechet:	$H(x)=\exp(-x^{-c})$,	$x\geq 0,\ c>0$	
Weibull for max.:	$H(x)=\exp(-(-x)^{-c})$,	$x\leq 0,\ c<0$	(3.107)
Gumbel:	$H(x)=\exp(-\exp(-x))$,	$-\infty<x<\infty$	

The practical importance of this result is that, when we are dealing with extremes, and n is large enough, the infinite many degrees of freedom we have with the distribution function for these extremes, are reduced to 3 parametric families. In other words, no matter what F(x) is the exact parent distribution, one of the above given 3 distributions can be used for approximating the extremes of F(x). The problem is now, which distribution (or domain of attraction) is associated with our CDF F(x)? With the following result from Castillo, (1988), this question can be solved:

A necessary and sufficient condition for F(x) to belong to the domain of attraction H(x) is that:

$$\lim_{\epsilon\to 0} (F^{-1}(1-\epsilon)-F^{-1}(1-2\epsilon))(F^{-1}(1-2\epsilon)-F^{-1}(1-4\epsilon))^{-1}=2^c \qquad (3.108)$$

with the following correspondence:

Table 3.8: Model choice

c>0	H is Frechet
c=0	H is Gumbel
c<0	H is Weibull for max.

Sometimes physical considerations can be used to reduce the three limiting distibutions to two. Parent distributions with a finite endpoint (like wave heights in shallow water) cannot lie in a Frechet-type domain of attraction (because they have their domain for $x\to\infty$). Moreover, if we consider that the Gumbel distribution can be approximated as closely as desired by Weibull for maxima or Frechet, we conclude that the limit distribution can be selected solely from physical considerations. *If we are dealing with random variables limited in the tail to the right, then a Weibull for maxima distribution is the limiting distribution.*

From the assumption that we only have a set of extreme observations where the parent distribution F(x) is unknown, we would like to determine the domain of attraction. An estimator for the c-parameter of Eqn. (3.108) can be found with the Pickands' method. Pickands (1975) shows that this c-parameter is the same as the one in a Generalized Pareto distribution given by:

$$GPA(x;a,c)=1-(1+cx/a)^{-1/c} \tag{3.109}$$

Fitting its 2 parameters on the data gives us automatically the domain of attraction. A curvature observation of the data plotted on Gumbel probability paper can also be used:

Table 3.9: Model choice

Convex curve	H is Weibull
Linear curve	H is Gumbel
Concave curve	H is Frechet

If we have concluded for a Weibull for maxima domain of attraction:

$$H(x)=\exp(-(-x)^{-c}), \qquad x \le 0,\ c<0 \tag{3.110}$$

Then, with $\lim_{n\to\infty} F^n(a_n+b_n x)=H(x)$, we can derive:

$$F(x)=H^{1/n}((x-a_n)/b_n)=\exp(-((\lambda-x)/\delta)^{\beta}),\ x \le \lambda,\ \beta>0 \tag{3.111}$$

the general form of a Weibull for maxima distribution in which λ, δ and β are the unknown parameters, to be fitted to the data. λ is the location parameter, δ is the scale parameter and β is the shape parameter. Note that the maximum value that the random variable can adopt is given by λ.

Caution: there is also another type of Weibull distribution known in practice:

$$F(x)=1-\exp(-((x-\lambda)/\delta)^{\beta}),\ x \ge \lambda,\ \beta>0 \tag{3.112}$$

This is the type which was suggested by Weibull (1961). This is in fact the limiting distribution for minima, but is nowadays used a lot for fitting maximum data as well. It is also commonly used as a lifetime distribution and in corrosion engineering (Scarf and Laycock, 1996).

3.9.3. Lindley's argument and the central limit theorem

Lindley (1983) has shown that the elicitation and modulation of expert judgement concerning quantities of interest (such as model constants) often involves the use of the normal distribution.

The central limit theorem explains why we might see so many times a normal distribution in practice: a stochastic variable that is influenced by a large number of independent processes will be approximately normally distributed.

Roughly, the central limit theorem says that the sum of a number of (independent) samples taken from any distribution is approximately normally distributed. As we add more terms the approximation becomes better. This does not only apply to the sum but also to the average (which makes sense if one knows that if X has a normal distribution then it follows that also aX has a normal distribution). In mathematical terms: Given a set X_1, X_2, ..., X_n of i.i.d. random variables. Suppose each X_i has mean $E(X_i) = \mu$ and variance $var(X_i) = \sigma^2 < \infty$. Define $Y_n = \Sigma X_i$
Then:

$$\lim_{n\to\infty} \frac{Y_n - nm}{s\sqrt{n}} \text{ has a standard normal N(0,1) distribution.} \tag{3.113}$$

The central limit theorem for the sum of random variables can easily be applied on the product of random variables by noting that log Π = Σ log, and therefore:
The product of n i.i.d. random variables converges to a lognormal distribution.

3.9.4 Relationship between PDF's

Also relations between PDF's can be derived. The following relations were found:

Table 3.10: Relationships between PDFs

X	**Y=exp(X)**
Normal	Shifted Log Normal
Gumbel	Frechet
Exponential	Pareto

The proof for the last two relationships is given by:

$$F_Y(y) = P(Y \le y) = P(e^X \le y) = P(X \le \log(y)) =$$
$$F_X(\log y) = e^{-e^{-\frac{\log y - x}{a}}} = e^{-e^{-\frac{\log y}{a}} e^{\frac{x}{a}}} = e^{-e^{\frac{x}{a}} y^{-\frac{1}{a}}} = e^{-(e^{-x}y)^{-\frac{1}{a}}} = \tag{3.114}$$
$$= e^{-(y/e^x)^{-\frac{1}{a}}}$$

This is a Frechet distribution with location 0, scale e^{ξ} and shape 1/α.

$$F_Y(y) = P(Y \le y) = P(e^X \le y) = P(X \le \log(y)) =$$
$$F_X(\log y) = 1 - e^{-\frac{\log y - x}{a}} = 1 - e^{-\frac{\log y}{a}} e^{\frac{x}{a}} = 1 - e^{\frac{x}{a}} y^{-\frac{1}{a}} = \tag{3.115}$$
$$= 1 - \left(\frac{e^x}{y}\right)^{1/a}$$

This is a Pareto distribution with scale e^{ξ} and shape 1/α. Also the following relationship could be proven:
If X is Weibull then Y=Log(X-ζ) is Gumbel for minima distributed:

$$F_Y(y) = P(Y \le y) = P(\log(X - V) \le y) = P(X \le e^y + V) =$$
$$= 1 - \exp\left(-\frac{e^y}{b}\right)^d = 1 - \exp\left(-\frac{e^y}{e^{\log b}}\right)^d = 1 - \exp\left(-e^{y - \log b}\right)^d = \tag{3.116}$$
$$= 1 - e^{-e^{\frac{y - \log b}{d^{-1}}}}$$

This is a Gumbel distribution for minima.

Finally it is possible to examine the tail behaviour of distributions. The following relations were found (properties of the Halphen distributions are described in Perreault et al., 1999a-1999b):

Table 3.11: Tail behaviour as a function of the return interval T

X	**Tail**
LogNormal	exp(√lnT)
Gamma	lnT
Normal	√lnT
Halphen	T^{ν}

3.9.5 Generalization of the Tang weights to n probability models

Let

$E(X_1) = E(Y_1)$, $\mathrm{Var}(X_1) = \mathrm{Var}(Y_1)$, as well as

$$E(X_n) = \frac{\mathrm{Var}(Y_n)E(X_{n-1}) + \mathrm{Var}(X_{n-1})E(Y_n)}{\mathrm{Var}(Y_n) + \mathrm{Var}(X_{n-1})},\ \mathrm{Var}(X_n) = \frac{\mathrm{Var}(Y_n)\mathrm{Var}(X_{n-1})}{\mathrm{Var}(Y_n) + \mathrm{Var}(X_{n-1})},$$

for $n = 2,3,\ldots$,

then:

$$E(X_n) = \frac{\sum_{i=1}^{n}\left(\prod_{\substack{j=1\\ j\neq i}}^{n}\mathrm{Var}(Y_j)\right)E(Y_i)}{\sum_{i=1}^{n}\left(\prod_{\substack{j=1\\ j\neq i}}^{n}\mathrm{Var}(Y_j)\right)},\ \mathrm{Var}(X_n) = \frac{\prod_{j=1}^{n}\mathrm{Var}(Y_j)}{\sum_{i=1}^{n}\left(\prod_{\substack{j=1\\ j\neq i}}^{n}\mathrm{Var}(Y_j)\right)},\ \text{for } n = 2,3,\ldots \qquad (3.117)$$

The proof follows by mathematical induction. As the basis of induction,

$$E(X_2) = \frac{\mathrm{Var}(Y_2)E(X_1) + \mathrm{Var}(X_1)E(Y_2)}{\mathrm{Var}(Y_2) + \mathrm{Var}(X_1)} = \frac{\mathrm{Var}(Y_2)E(Y_1) + \mathrm{Var}(Y_1)E(Y_2)}{\mathrm{Var}(Y_2) + \mathrm{Var}(Y_1)}$$

and

$$\mathrm{Var}(X_2) = \frac{\mathrm{Var}(Y_2)\mathrm{Var}(X_1)}{\mathrm{Var}(Y_2) + \mathrm{Var}(X_1)} = \frac{\mathrm{Var}(Y_1)\mathrm{Var}(Y_2)}{\mathrm{Var}(Y_2) + \mathrm{Var}(Y_1)}.$$

Using the induction hypothesis, it follows that

$$E(X_n) = \frac{\mathrm{Var}(Y_n)E(X_{n-1}) + \mathrm{Var}(X_{n-1})E(Y_n)}{\mathrm{Var}(Y_n) + \mathrm{Var}(X_{n-1})} =$$

$$= \frac{\mathrm{Var}(Y_n)\sum_{i=1}^{n-1}\left(\prod_{\substack{j=1\\ j\neq i}}^{n-1}\mathrm{Var}(Y_j)\right)E(Y_i) + E(Y_n)\prod_{j=1}^{n-1}\mathrm{Var}(Y_j)}{\mathrm{Var}(Y_n)\sum_{i=1}^{n-1}\left(\prod_{\substack{j=1\\ j\neq i}}^{n-1}\mathrm{Var}(Y_j)\right) + \prod_{j=1}^{n-1}\mathrm{Var}(Y_j)} = \frac{\sum_{i=1}^{n}\left(\prod_{\substack{j=1\\ j\neq i}}^{n}\mathrm{Var}(Y_j)\right)E(Y_i)}{\sum_{i=1}^{n}\left(\prod_{\substack{j=1\\ j\neq i}}^{n}\mathrm{Var}(Y_j)\right)}$$

and

$$\mathrm{Var}(X_n) = \frac{\mathrm{Var}(Y_n)\mathrm{Var}(X_{n-1})}{\mathrm{Var}(Y_n) + \mathrm{Var}(X_{n-1})} = \frac{\mathrm{Var}(Y_n)\prod_{j=1}^{n-1}\mathrm{Var}(Y_j)}{\mathrm{Var}(Y_n)\sum_{i=1}^{n-1}\left(\prod_{\substack{j=1\\ j\neq i}}^{n-1}\mathrm{Var}(Y_j)\right) + \prod_{j=1}^{n-1}\mathrm{Var}(Y_j)} =$$

$$= \frac{\prod_{j=1}^{n}\mathrm{Var}(Y_j)}{\sum_{i=1}^{n}\left(\prod_{\substack{j=1\\ j\neq i}}^{n}\mathrm{Var}(Y_j)\right)}$$

for $n = 2,3,...$

3.10 Discussion

The choice of statistical estimation methods for probability distribution functions is one of the most challenging problems within civil engineering, and one that is filled with many controversies. However, it is a topic with great practical importance and it needs to be dealt with (see also Lambert et al. (1994), Haimes et al. (1994), and Seiler and Alvarez (1996)). Attempts to develop new methods have been extremely abundant (see also Bardsley (1994), Capehart et al. (1998), and Chow and Watt (1990)). This is the reason for the large number of references in this thesis. Chapter 3 focused on the most important statistical estimation methods that are in circulation under civil engineers. The question which estimation method can best be used is impossible to answer. This depends on too many factors. The definition of what is considered best is one of those factors. However, Table 3.1 gives an overview of the most important journal papers which investigate the performance of a certain pair (f, EM) w.r.t. minimum bias and RMSE of a quantile. Depending on various conditions, sometimes a classification in the performance of the estimation methods can be made. Furthermore, methods for deriving sampling distributions have been described in this chapter and have been applied to the Exponential distribution. The problem of weight factor estimation has been investigated with various methods. Bayes factors appear to perform very well and weight factors based on L-Kurtosis show to be in fairly close agreement with those obtained from the minimum divergence criterion.

It is recommended in this chapter (as well as in Mendel and Chick (1993)) to use theoretical considerations as much as possible in the distribution selection. Sections 3.9.2 to 3.9.4 give some material for that purpose. Chick et al. (1995, 1996) proposed a physics-based approach to determine the PDFs of extreme river discharges. In their papers, a new model for predicting the frequency of extreme river levels is proposed which encapsulates physical knowledge about river dynamics, including formulae which describe river discharge. The model accounts for the river dynamics at a given location by modeling both how water gets into the river (via upstream tributaries) and how water leaves (discharge modeled by Chézy's equation). Although the simplified physical model makes several rough approximations (using memoryless properties and Chézy's equation for approximating discharge), insights were gained in the effects of Chezy's equation parameters on the shape of the curves relating the river level and flood return frequency can be shown with Chick's approach. These shapes do not always conform to the curves found for traditional models. In particular, the relation is not necessarily linear on log paper, as with the Exponential model. It was shown that an increase in the power parameter of Chezy's equation led to a non-linear relation on log paper. As the power increased, the slope of the curve relating flood volume and the frequency of extreme floods decreased. This may be true for more complicated systems as well. It was concluded by Chick et al. (1995, 1996) that flood protection designs based on drawing straight lines on log paper would be conservative for extremely rare floods.

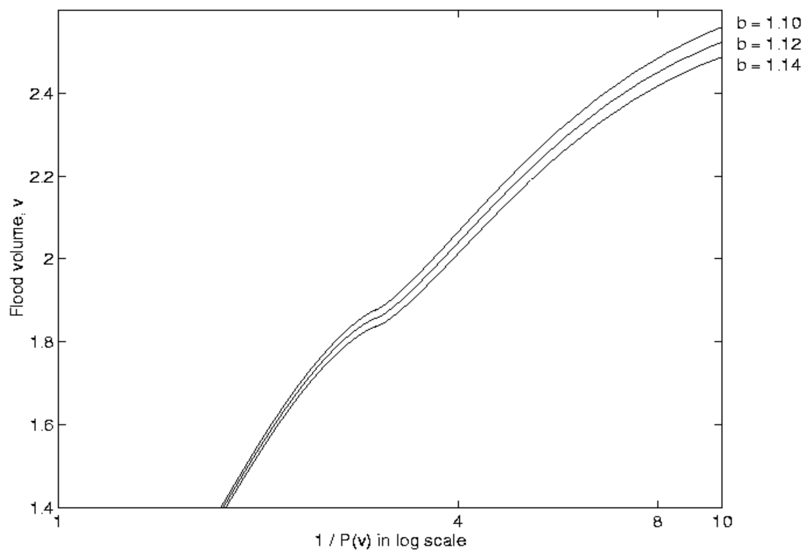


Figure 3.22: Effect of changes in the power parameter of Chezy's equation on the flood frequency curve (source: Chick et al., 1996)

Chapter 4

Homogeneity

An IBM electronic computer speeds though thousands of intricate computations so quickly that on many complex problems, it’s just like having 150 extra engineers.
- IBM Ad in National Geographic in 1952.

4.1 Introduction

In this chapter, the problem of (in)homogeneous data in civil engineering applications is studied. Statistical analysis of civil engineering data such as flood data, wind data, wave height data, etc., is essentially a problem of information scarcity. Records are usually too short to ensure reliable estimates of low-exceedance probability quantiles in many practical problems. Determination of quantiles is needed for the design, construction and operation of hydraulic structures, insurance studies and protection of populated areas. To perform a frequency analysis of data from civil engineering practice, the data first has to be tested for homogeneity. Non-homogeneous data may lead to wrong quantiles. A homogeneity test must be able to separate data sets that do not come from the same distribution. In this chapter statistical and physics-based homogeneity tests will be presented.

Homogeneity, or similarity or uniformity, of data sets is important for the comparison of data sets. If data are collected at a similar location, using similar methodology, at a similar time each year, such data can be compared over the years for changes. If, however, the data are collected using different methodologies from one year to the next, comparisons of these data would be difficult. Inconsistencies are introduced as a result of different methodologies, standards, and collection techniques. In many cases these inconsistencies are a result of improvements made to collection and/or analytical techniques. For example, improvements to water sampling techniques made over the years have allowed scientists to detect traces of chemicals that could not be measured in the past. These inconsistencies must be assessed by all users and, if significant, must be taken into account to eliminate bias or false conclusions.

Standards and guidelines are often established to help alleviate some of the compatibility and homogeneity issues described above. Standards and guidelines can be established for the way information is collected, stored, updated, and retrieved. Standards can be helpful if everyone agrees to adopt the same standard or, failing that, if differences between standards are adequately documented so that users are aware of the limitations.

As said before, statistical analysis of civil engineering data is a problem of information scarcity. Usually records are very short. Datasets with 100 years of water levels may seem much, but when one is interested in the 10^{-4} quantile, 100 years of data is very little. Apart from the data scarcity problem, there is also a data homogeneity (or rather data inhomogeneity) problem. A basic assumption is that data is coming from one and the same process. Or in mathematical terms: the data are realisations from one and the same probability distribution function. Statistical procedures are available to check the homogeneity of a dataset. A short overview of these procedures will be given in this chapter. However their weakness will appear quickly, since these procedures are not very powerful to reject inhomogeneous- or accept homogeneous data. A Monte Carlo experiment will illustrate this. Other methods to judge the homogeneity of a dataset may be more physics-based. It will be shown that this way is more powerfull than a statistical procedure. A case study will be presented in order to show the physical-based judgement of a coastal engineering dataset. Other physics-based techniques will be reviewed in this chapter. The physics-based homogeneity considerations can be seen as some sort of multivariate consideration. This will be shown in Section 4.4. In the area of multivariate analysis a special kind of outlier detection has been developed in the last decade (Rousseeuw and Leroy, 1987). It will be shown in this chapter that these detection methods for multivariate datasets can also be very well used for a homogeneity analysis of datasets. The outlier detection methods will be briefly described in Section 4.5 and it will be explained how they can be used in a multivariate-based homogeneity consideration. Its performance will again be analyzed with Monte Carlo simulations in Section 4.6 and the techniques will be applied in Section 4.7 to a case study of river flows in Europe. The chapter will end with a discussion.

4.2 Homogeneity

Statistical distribution functions play an important role in coastal engineering. In the design of coastal structures they are used to determine the so-called p-quantiles. As defined in Chapter 3, a p-quantile of a random variable is the value of that variable which is exceeded with a probability p. In coastal engineering p-quantiles in the order of 10^{-1} for small-scale defence structures to 10^{-4} for important coastal defence structures are commonly applied. In the Netherlands the seadikes for instance are designed with p-values of 10^{-4} per year and river dikes with p-values of 1/1250 per year. The reason for the difference is that a possible inundation from the rivers is not so disasterous as an inundation from the sea.

The importance to estimate the correct p-quantile is quite high. A too low estimate may lead to an unsafe structure, whereas a too high estimate may lead to a conservative overdesigned structure which costs unnecessarily too much. In Chapter 5 we will come back to this issue. It was shown in the previous chapter that much research had been carried out for finding the best p-quantile estimation method. Methods such as Maximum Likelihood (ML), Method of Moments (MOM), Least Squares (LS), Weighted Least Squares (WLS), Method of L-Moments (MLM), Bayesian methods, were described. Methods to select the optimal distribution function are also available. Various goodness-of-fit criteria such as χ^2 , Kolmogorov-Smirnov (KS), etc. (e.g. Kendall and Stuart, 1977) however are not suitable for that purpose since the parameters of the distributions are unknown beforehand. The methods of Chapter 3 have been suggested as alternatives. All these p-quantile estimation techniques and distribution selection methods have one important assumption in common, and that is that the data under consideration must be homogeneous. To verify the homogeneity assumption, statistical procedures have been developed. Buishand (1982) and Ondo et al. (1997) give excellent overviews of the various procedures which are available in literature.

Homogeneity analysis has been carried out a lot in the field of regional frequency analysis (RFA). RFA tries to combine data from other sites in order to improve the accuracy of the p-quantile estimate. However, combining data from different sites may only be done when the sites can be considered homogeneous. Therefore numerous papers have appeared dealing with this problem. We mention Dalrymple's test and the L-moment X10 test (Fill and Stedinger, 1995). Homogeneity tests based on L-moment ratios have received quite some attention lately (Rao and Hamed, 1994, Zrinji and Burn, 1993). Sample L-moments are less biased than traditional moment estimators. In Section 4.6 we will come back to the issue of homogeneity analysis in a regional setting (when datasets at more than one site are available).

Not only homogeneity tests have been developed in the field of RFA. Also literature is available from the behavioural sciences such as sociology and psychology. A well-known statistical test is for example the Mann-Whitney test (Harnett, 1970). The Mann-Whitney test is a non-parametric homogeneity test which can test the nul-hypothesis that two independent datasets are coming from the same distribution. The performance of this test can be studied with help of Monte Carlo simulations. A dataset 1 with a given size can be simulated from a given distribution function. Also a dataset 2 can be generated (from a possibly different distribution) and the two datasets can be compared with eachother from a homogeneity point of view. Under the null-hypothesis that the datasets come from the same distribution, the test statistic should be normally distributed with mean 0 and standard deviation 1. If the

Mann-Whitney test is analyzed in this way, it will appear that the test is not so powerfull for small sample sizes. Only for large sample sizes in the order of 100 or more, the test may reject or accept the null-hypothesis with high confidence. Also the recently developed L-Moment techniques have difficulties to judge the homogeneity of a single dataset. A Monte Carlo experiment was designed for that purpose. A sample of size 40 is generated from a certain extreme value distribution (the maximum of 10 Gumbel distributions: we denote it as Gumbel^{10}). From this sample the ordinary moments and the L-moments are calculated. Another sample of size 40 is generated from a quite different extreme value distribution (the maximum of 10 normal distributions: Normal^{10}) with the same mean and standard deviation however. Its ordinary and L-moments are calculated. The experiment is repeated 100 times and the moments and L-moments values of each sample are depicted in Figure 4.1. Note that as well as the ordinary moments diagram as the L-moments diagram show a non-distinguishable behaviour between the two extreme value distributions. In other words: it is impossible to separate the two distribution functions with the traditional and newly developed L-moment techniques.

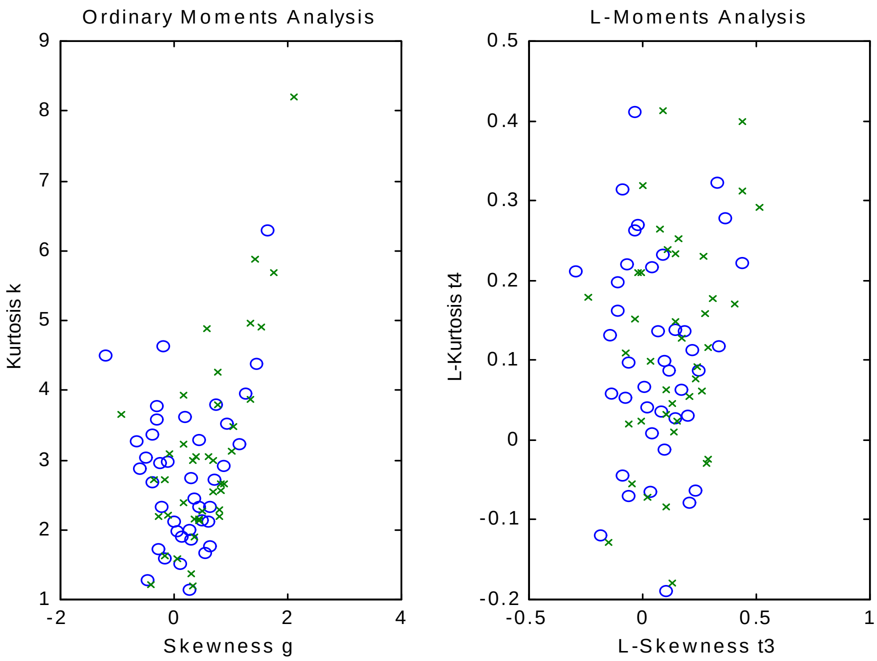


Figure 4.1. Homogeneity analysis with L-moments techniques;
Data from Gumbel^{10} are indicated with x; data from Normal^{10} with o.

The before mentioned homogeneity tests do not include any physical arguments to judge the data. In the following section we propose procedures in order to examine the homogeneity of a data set on the basis of physical arguments.

4.3 Physics-based homogeneity considerations

Rather than a statistical analysis of the data, the data is examined on the basis of its physical origin. Physical arguments to judge the homogeneity of a data set are for instance:

1. Type of spectrum (swell, wind, single or double peaked wave height spectrum);
2. Season, calendar period of the data set (sea level data in the winter or summer);
3. Physical characteristics of the phenomena (breaking, non-breaking waves).

Combining statistical tests with physical arguments leads to more homogeneous data than only applying the statistical tests. This will be shown in a case of wave height and peak period data measured in the Bay of Bengal near the city of Madras.

In Van Gelder and Vrijling (1998c), a case study was performed with registrations of the wave rider campaigns in the Bay of Bengal near the city of Madras during the south-west monsoon from mid-April to mid-August 1993. A set of peaks over threshold values was available with significant wave heights. The set consists of 144 values and the threshold is given by 39cm. A statistical data analysis gives the following Figure 4.2.

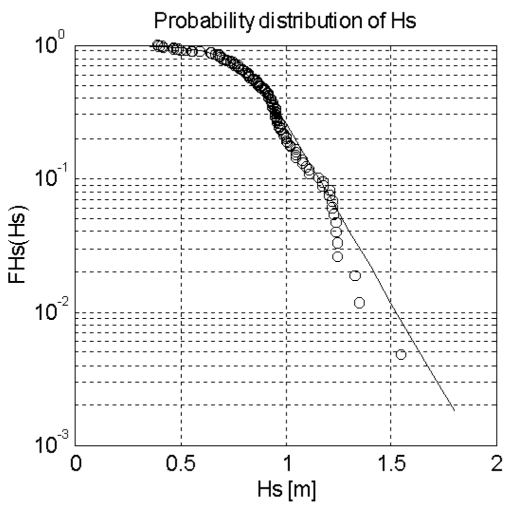


Figure 4.2. Wave height data with optimal fit (Normal).

A statistical homogeneity check on the data set with 144 values as described in the previous section does not lead to a rejection of the data set. Also a visual inspection of Figure 4.2 does not suspect inhomogeneous data.

An investigation on the origins of wave generation in the Bay of Bengal however leads to:

1. Locally generated waves; these waves are generated in the Bay of Bengal either by the north-east or south-west monsoon or by hurricanes.
2. Swell; these waves are generated in the Roaring Forties, south of Cape Town. They reach the southern point of India from the south-west in approximately three days.

From the given data set with 144 significant wave heights, also the corresponding peak periods were available. A visual inspection of the data set with the peak periods immediately leads to the conclusion of inhomogeneity (Figure 4.3). The waves can be separated into two classes. The first class contains waves with periods of about 5 sec. The second class contains waves of about 12 sec.

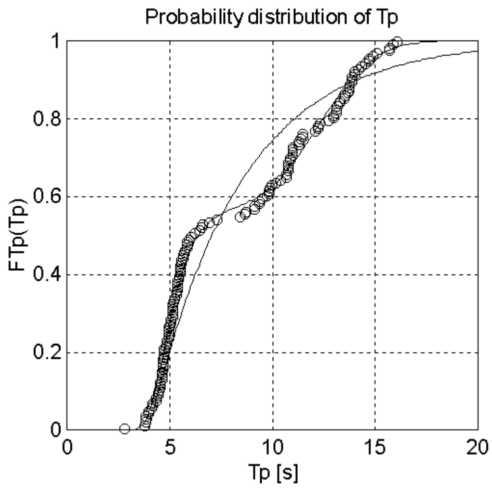


Figure 4.3. Peak periods data with Exponential inhomogeneous fit and combined homogeneous fit.

Trying to model the probability distribution of the peak periods by one single distribution function leads to inacceptable deviations, as can be seen from Figure 4.3. Therefore the inhomogeneous data set had to be split up into two sub sets. Each sub set is modeled by its own distribution and the probability model of the total data set is obtained by combining both sub models (Figure 4.3).

Returning to the original goal: a probability distribution for the significant wave height, the following result is obtained (Figure 4.4). The significant wave height with exceedance probability of 10^{-3} is 1.55m in stead of 1.90m; a difference of about 30%. This might have lead to an overdesigned coastal structure. However, the multivariate analysis of the data (including wave period in the analysis together with wave heights) had shown the inhomogeneous behaviour of the first tuple.

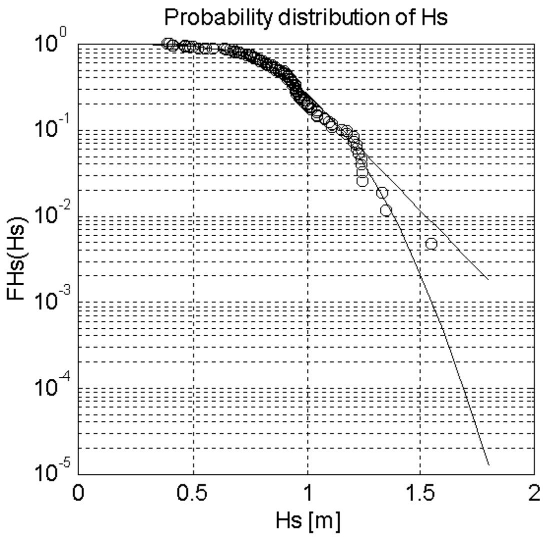


Figure 4.4. Comparison of in-/homogeneous fit.

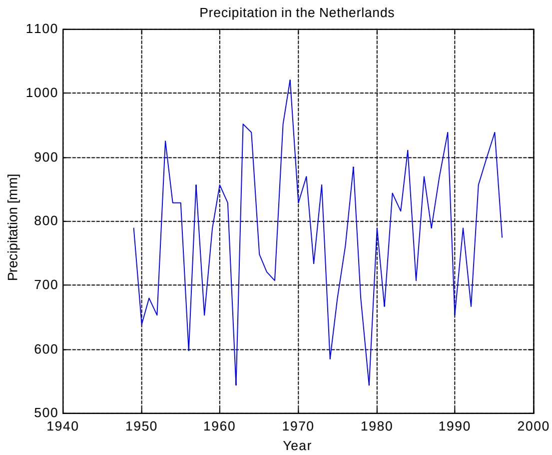
Coastal engineering data from Northern European seas differ from the data presented above. Monsoons and hurricanes are not present in Northern Europe. However, strong winds can occur during the winter months October, November, December, January and February. These winds may yield extreme wave heights and water levels along the coasts. The analysis of such datasets also requires homogeneity studies. One of the first extensive homogeneity studies of the water levels along the Dutch coast was performed by the Delta Committee (1960). They homogenize the dataset of water levels by looking at the trajectory of the depression that caused the high water level. Extreme water levels can only occur when the depression follows a trajectory through a certain area. From the meteorological archive the area could be confined to:

At 10° W between 51° N and 62° N;
at 0° W between 52° N and 61° N;
at 7° E between 52° N and 61° N.

The cause of an extreme water level is found in the trajectory of the depression combined with the behaviour of the body of water in the North Sea basin. Therefore the statistics of the water level data is confined to those water levels that were caused by the dangerous trajectories. Such a physically-based homogeneity selection method can also be succesfully applied at other locations.

4.4 Multivariate-based homogeneity considerations

What was observed from the case study of Section 4.3 was that the homogeneity of a univariate dataset can be judged by extending the univariate set to a multivariate dataset and by analyzing the other tuples of the dataset. This principle can be applied to all kind of studies and appears to be very powerful. Consider for example the total yearly precipitation in the Netherlands (Figure 4.5a).



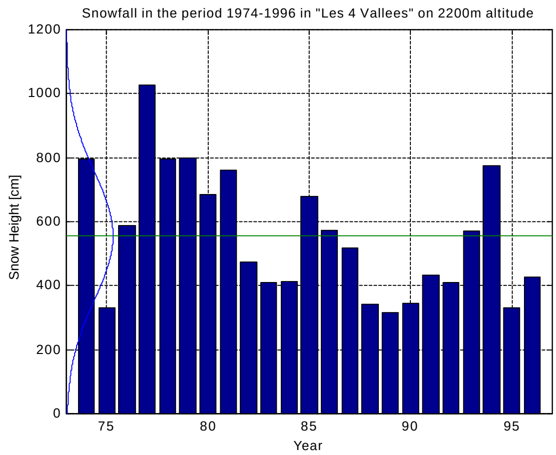


Figure 4.5a. Total yearly precipitation with high variation coefficient
Figure 4.5b. Total yearly snowfall in Les Quatres Vallees in Switzerland (mean = 557cm, std=199cm)

Although the total yearly precipitation might be expected to be a stable quantity, a coefficient of variation of 15% is observed (see also Revfeim, 1992). The univariate dataset of precipitation at each year can be extended to a multivariate dataset with (precipitation, dominant wind direction). Due to the geographic location of the Netherlands, winds from the west are very humid and bring a lot of rain; winds from the east however are very dry. Therefore two different processes can be distinguished and the precipitation dataset should be splitted into at least two subsets (see also Waylen and Woo (1982) and Rossi et al. (1986)).

Other examples were mentioned by Vrijling on the snow fall data of Switzerland (Figure 4.5b), the river discharge data of the river Maas at the location Lith by Van Gelder et al. (1999g) (Figure 4.5c, coefficient of variation of 21% of the annual average discharge, 24% for the annual maxima, and even 49% for the annual minima discharges), and in Buishand (1989a).

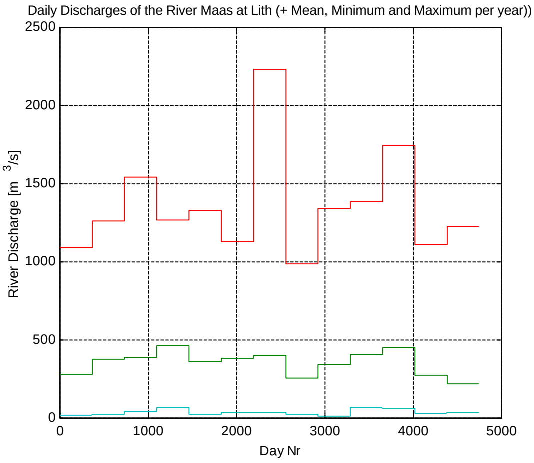


Figure 4.5c. Discharge at Lith (River Maas)

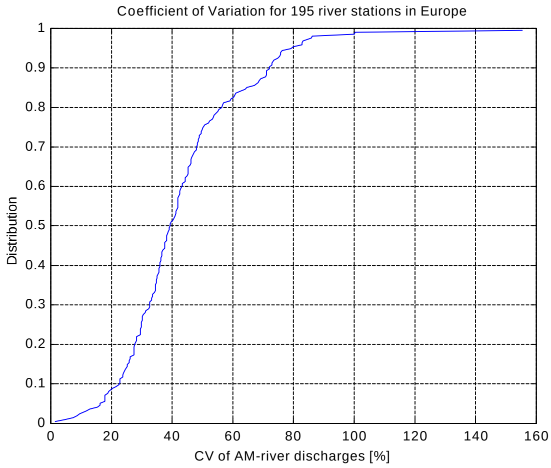


Figure 4.5d. Distribution of CV

River discharges show in general a large coefficient of variation, as can be seen from Figure 4.5d. The mean CV of the 213 discharge sets from GRDC is almost 43% and the CV of the CV itself is still 20%.

4.5 Discordance-based homogeneity considerations

In the previous section, it was shown that the homogeneity of univariate datasets should be judged by extending the univariate set to a multivariate set. The judgement of the homogeneity of the multivariate set is often much easier than the judgement of the homogeneity of the univariate set. However there may be difficulties as seen in the following illustrative example.

The following data (depicted in Figure 4.6), consisting of the body weights (in kilograms) and brain weights (in grams) of N=28 animals, are reported by Jerison and can be found also in Rousseeuw and Leroy (1987). Instead of the original data, this example uses the logarithms of the measurements of the two variables. In the lower right region there are 3 dinosaurs (observations 6, 16, and 25) with a small brain and heavy body, and in the upper left area we find the human and the rhesus monkey (observations 14 and 17) with a relative high brain weight. The classical ellipsoid is blown up by these outliers, and contains all animals except the largest dinosaur. The dataset without the largest dinosaur is therefore considered homogeneous. However, this would be an incorrect conclusion. The ellipsoid based on, for instance, the MVE (minimum volume ellipsoid) is much narrower and does not include the outliers. This method may therefore be considered to perform better than the classical method in the judgement of the homogeneity of data.

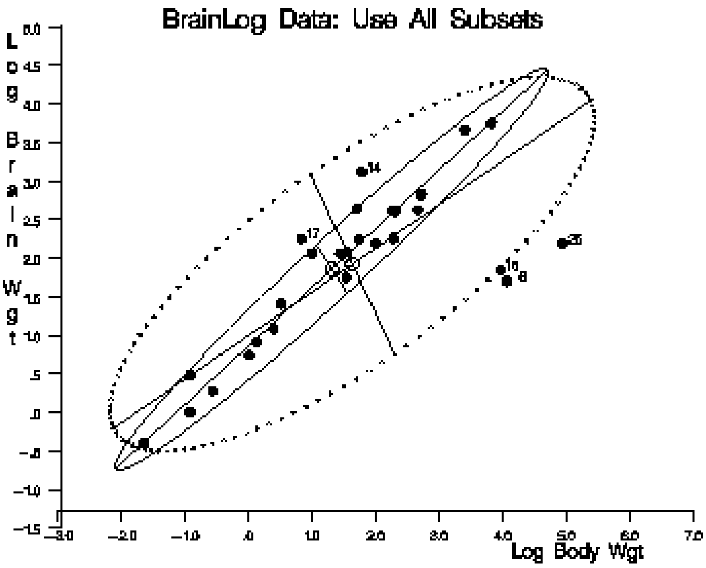


Figure 4.6: Example of inhomogeneous data and its detection methods from Rousseeuw and Leroy (1987)

The above described outlier detection methods will be used to derive so-called discordance-based homogeneity considerations.

A *discordancy measure* should be able to identify those sites that are grossly discordant with the group as a whole. Discordancy can be measured in terms of the L-moments of the sites' data. For example see Figure 4.7 (taken from Van Gelder et al., 1999g). The L-CV, L-skewness and L-kurtosis values are depicted for 34 sites (with river discharge measurements) in Southern France. Discordant sites can be considered those sites which are far from the center of the cloud; i.e. those that lie outside the outermost ellipse based on the MVE. The small squares in Figure 4.7 show the discordant sites; the small triangles fall inside the (3-dimensional) ellipse. More background on the example will be given in Section 4.7.

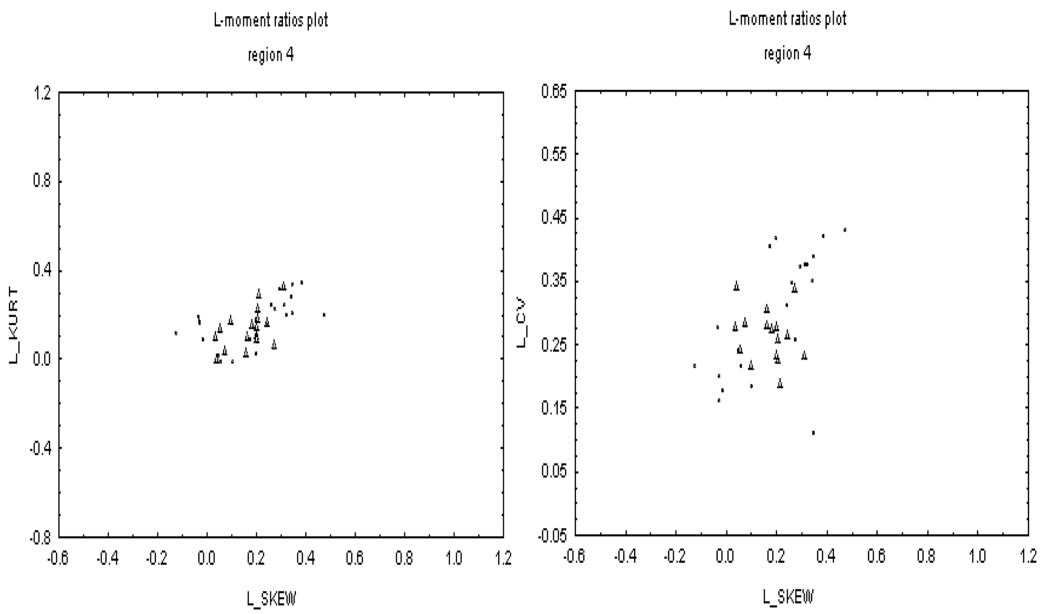


Figure 4.7: L-Moments diagrams of 34 sites in Southern France
(region 4 in Van Gelder et al. (1999g))

One can think of the three L-moment ratios as defining a three-dimensional space in which each observation can be plotted. Also, one can plot a point representing the means for all variables. This "mean point" in the three-dimensional space is called the centroid. The Mahalanobis distance (Mahalanobis, 1946) is the distance of a case from the centroid in the three-dimensional space. This measure provides an indication of whether or not an observation is an outlier. The classical Mahalanobis distance is defined as:

$$MD_i^2 = (x_i - T(X))^T C(X)^{-1}(x_i - T(X)), \tag{4.1}$$

where T(X) and C(X) are the usual mean and covariance estimates:

$$T(X) = \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$
$$C(X) = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})(x_i - \bar{x})^T \tag{4.2}$$

and $x_i = (x_{i1}, x_{i2},\dots ,x_{ip})^T$ for i=1,...,n is the i^{th} row of data matrix X, n is the number of observations, and p is the dimension of the space (p=3 if we consider the three L-Moment ratios). Points whose MD_i^2s are large are flagged. It is well known, however that the sample mean and covariance in a multivariate dataset are extremely sensitive to outliers. Rousseeuw and Leroy (1987) propose to replace the classical mean and covariance in the expression of the Mahalonobis distance by their high breakdown point (BP) robust analogs (MVE: Minimum Volume Ellipsoid or MCD: Miminum Covariance Determinant). The breakdown point is the smallest percentage of contaminated data that can cause the estimator to take arbitrarily large aberrant values. The breakdown point of the classical estimates based on the method of Maximum Likelihood, method of Moments, method of Least Squares is 0.

The MCD estimator for location T(X) is defined as the mean of the k points of X, where k is equal to [(n+p+1)/2], for which the determinant of the covariance matrix is minimal. Vandev and Neykov (1993) showed that the breakdown point of the MCD is equal to (n-k)/n if k is within the bounds (n+p+1)/2 $\leq$ k $\leq$ n-p-1 and n $\geq$ 3(p+1). Note that the number of observations should be at least three times larger than the dimensionality + 1. If k = (n+p+1)/2, then the BP is equal to ½ assymptotically.

For more information about efficient algorithms for calculating the MCD and other robust covariance estimates with high breakdown points, see Rocke and Woodruff (1996). To improve the efficiency of the estimates Rousseeuw and Van Zomeren (1990) advised to perform one step improvements for the location and scatter:

$$T_1(X) = \frac{\Sigma w_i x_i}{\Sigma w_i}$$
$$C_1(X) = \sum_{i=1}^{n} w_i (x_i - T_1(X))(x_i - T_1(X))^T / \Sigma w_i \tag{4.3}$$

where

$$w_i = \begin{cases} 1 & if \quad (x_i - T(X))^T C(X)^{-1}(x_i - T(X)) \leq c \\ 0 & otherwise \end{cases} \tag{4.4}$$

and T(X) and C(X) are given in Eqn. (4.2) and the cut-off value c might be taken equal to $c^2_{p,0.975}$. The observations with zero weights can be interpreted as discordant.

Other kinds of weights which are based on one-step M-estimates, taking T(X) and C(X) as initial values, are given by the following Equations:

$$\text{Re}\,weighting \quad (REW)$$
$$w_i(t) = \begin{cases} 1 & if \quad t \leq c \\ 0 & if \quad t > c \end{cases} \quad with \quad c = \sqrt{c^2(p, 0.975)}$$

$$Huber's \quad Weights \quad (HUB)$$
$$w_i(t) = \begin{cases} 1 & if \quad t \leq c \\ c/t & if \quad t > c \end{cases} \quad with \quad c = \{\sqrt{(2p-1)} + e\}/\sqrt{2} \quad and \quad e = 2.25 \tag{4.5}$$

$$Hampel's \quad Weights \quad (HAM)$$
$$w_i(t) = \begin{cases} 1 & if \quad t \leq c \\ \frac{c}{t}\exp\left\{-\frac{1}{2}(t-c)^2/b^2\right\} & if \quad t > c \end{cases} \quad \begin{matrix} with & c = \{\sqrt{(2p-1)} + e\}/\sqrt{2} \\ and & e = 2.25, \quad b = 1.25 \end{matrix}$$

In this chapter we shall denote them by: RD_i(MCD), RD_i(REW), RD_i(HUB), RD_i(HAM), RD_i(T-BW) – which are the robust distances (analogs of the Mahalanobis distance) based on the Minimum Covariance Determinant (MCD) estimator of the multivariate location and scatter and some of its one-step improvements based on Huber's weights (HUB), Hampel weights (HAM), Constrained M-estimates (T-BW) (see, Rousseeuw and Van Zomeren (1990), Rocke and Woodroff (1996), Todorov et al. (1992), and Campbell (1980)). For the robust distances $\chi^2(3,0.975) = 9.36$ is the threshold for which a site is declared as discordant.

Hosking and Wallis (1997) suggest to use the Wilks measure in the discordancy analysis (denoted by D_i). However the Wilks measure is equal to the Mahalonobis distance up to a fixed constant ($MD_i=3(n-1)D_i/n$), and therefore not robust against discordant sites as it is also based on the sample mean and covariance matrix (which are themselves affected by discordant sites). The above mentioned alternatives based on robust estimates of multivariate location and scatter, are suggested in this chapter as alternatives.

Apart from a discordancy measure, also a heterogeneity- and goodness-of-fit measure can be defined. The measures of Hosking and Wallis (1993, 1997) are adopted for this purpose.

A *heterogeneity measure* should be able to estimate the degree of heterogeneity in a group of sites and to assess whether the sites might reasonably be treated as a homogeneous region. A formal definition of the heterogeneity measure is given in Hosking and Wallis (1993, 1997). Basically the heterogeneity measure compares the between-site variations in sample L moments for the group of sites with what would be expected for a homogeneous region. What “would be expected” is evaluated through Monte Carlo simulation from the four parameter Kappa distribution. Hosking and Wallis (1993, 1997) suggested that regions can be classified as "acceptably homogeneous" if $H<1$, "possibly heterogeneous" if $1\leq H<2$, and "definitely heterogeneous" if $H\geq 2$. (The H(2) and H(3) statistics based on V2 and V3 respectively lack power to discriminate between homogeneous and heterogeneous regions according Hosking and Wallis (1997)).

A *goodness of fit measure* should be able to test whether a given distribution fits the data acceptably closely. A formal definition of the goodness of fit measure is given in Hosking and Wallis (1993, 1997). Basically the goodness of fit measure compares the differences between the theoretical L-moment ratio for the given distribution with the regional average of sample L-Moments. The fit is declared to be adequate if the goodness of fit measure Z is sufficiently close to zero, a reasonable criterion being $|Z| < 1.64$.

In Van Gelder and Neykov (1998), a method has been suggested to judge the homogeneity of a single data set, by using a discordancy measure based on the robust distances as described above. The method consists of splitting the data set into sub-parts and to determine of each sub-part its L-moments. If the L-moments of a certain sub-set are too far from the center of the cloud (in robust distance sense), then that part of the data set should be considered inhomogeneous with the rest of the data set. This approach had been applied to the datasets of extreme water levels along the Dutch coast by splitting the time series in three parts of about 30 years each. The homogeneity of the data sets over time was automatically judged in this way.

In the next section, the performance of the discordance-based homogeneity considerations will be assessed with large-scale Monte Carlo simulations.

4.6 Performance of discordance-based homogeneity considerations

The performance of D_i as a discordancy measure and RD_i as a robust discordancy measure were assessed in a series of Monte Carlo simulation experiments.

To aid the presentation: Let Q_{ij}, j=1,...,n_i, be observed data at N sites of a region, with sample size n_i at site i, and let $Q_i(F)$, 0<F<1, be the quantile function of the distribution at site i. A region of N sites is called homogeneous if $Q_i(F)=\mu_i q(F)$, i=1,...,N, where μ_i is the site-dependent scale factor and q(F) is the quantile of the regional frequency distribution.

For each of a number of artificial regions, 10,000 replications were made of data from the region, and the accuracy of quantile estimates and the value of the heterogeneity measure H were calculated. The regions were specified by the number of sites in the region, the record lengths at each site, and the frequency distribution at each site. The frequency distribution at each site was given by a GEV-distribution with a certain L-moment ratio of τ and τ_3 (the L-CV and 3rd/2nd L-moment respectively). The at-site mean was set to 1 (without loss of generality).

Two types of region were used in the simulations: homogeneous and "bimodal". With bimodal it is meant that one part of the sites have one distribution and the other part have another distribution. Bimodal 30% means that :

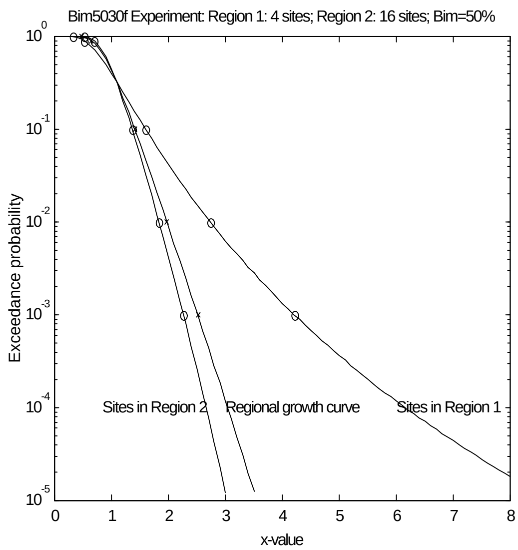
$$(\tau_{region\text{-}1} - \tau_{region\text{-}2})/(\tau_{region\text{-}1} + \tau_{region\text{-}2})/4 = 0.30 \qquad (4.6)$$

For instance, $\tau_{region\text{-}1}$ = 0.26 and $\tau_{region\text{-}2}$ = 0.14 give a bimodality of 30%. These regions should test the (dis)ability of RDs to detect discordant sites (and/or heterogeneity) when there is a sharp difference between the frequency distributions at two subsets of sites. The base region has N sites and every site has n_i=30 observations and L-moment ratios of τ=0.25 and τ_3=0.25. Variations in the regions include changing N to 2, 4, 5, 7, 10, 14, 16, 20 or 24, keeping n_i at 30, and changing the L-moment ratios τ to 0.22, 023, 025 and 0.18, 0.17, 0.15 with appropriate changes in τ_3. Both homogeneous and bimodal variants of these regions were simulated. Bimodality is simulated at levels from 20%, 30% to 50%.

In two regions with an equal number of sites no algorithm can identify which one contains discordant sites. Therefore simulations are also performed which are based on a different number of sites in both regions, i.e. 7+14=21 sites, 10+20=30 sites, 4+16=20 sites, 6+24=30 sites respectively.

First let us concentrate on one particular experiment called bim5030f. Two regions of sites are generated 10,000 times. The 1st region contains 4 sites and the 2nd region contains 16 sites. Each site contains 30 GEV-distributed observations with L-moment ratios τ and τ_3 equal to 0.25 in the 1st region and 0.15 in the 2nd region respectively. The region type bimodality is therefore 50%.

In Figure 4.8a, the exceedance probability curves are shown for both regions together with the regional growth curve. In Figure 4.8b, the L-Moment diagram for one simulation is shown, together with the classical discordancy measure and the robust measure.



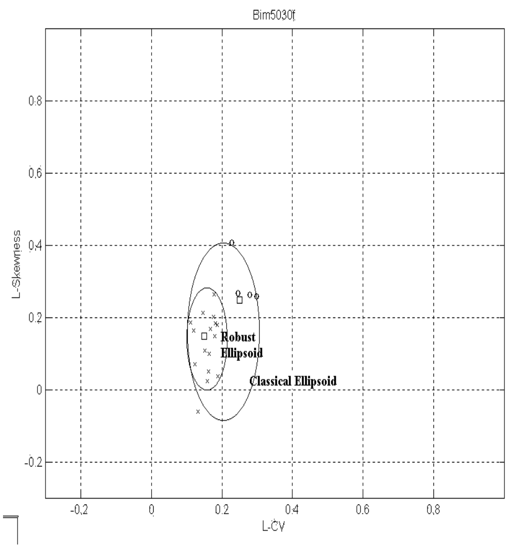


Figure 4.8a: Exceedance probability curves for both regions and
Figure 4.8b: L-Moment diagram; the small circles are realisations from Region 1, crosses from Region 2, the squares give the theoretical averages.

The following value for the heterogeneity measures was obtained: H = 6.26. Furthermore, by the classical measure of discordance the sites of the 1st region (4 sites) are identified as 7% in average as discordant whereas by the robust distances based on MCD and its one-step improvements the percentages in average of being discordant are: 86% , 86%, 53%, 76% and 70% .

The value H=6.27 (using the classical measure of heterogeneity) indicates that the region is definitely heterogeneous, so that there should be discordant sites. However, this is in contrast with the 7% value (obtained by the classical measure of discordancy).

The appearance of heterogeneity is due to the presence of a small number of atypical sites in the region. The probability to identify the atypical-surrogate sites is much greater by using the robust distances than by using the classical discordancy measure. As seen above, the results based on the robust distances do not contradict the one based on H statistics, whereas the results based on the classical measure of discordance do contradict the results based on H statistics.

From experiment bim5030f, the main conclusion is that robust distances are useful tools in the identification of homogeneous regions. However, this conclusion also has to be verified in other experiments. Therefore the following experiments have been set up (Table 4.1).

Table 4.1: Summary of the simulation experiments.

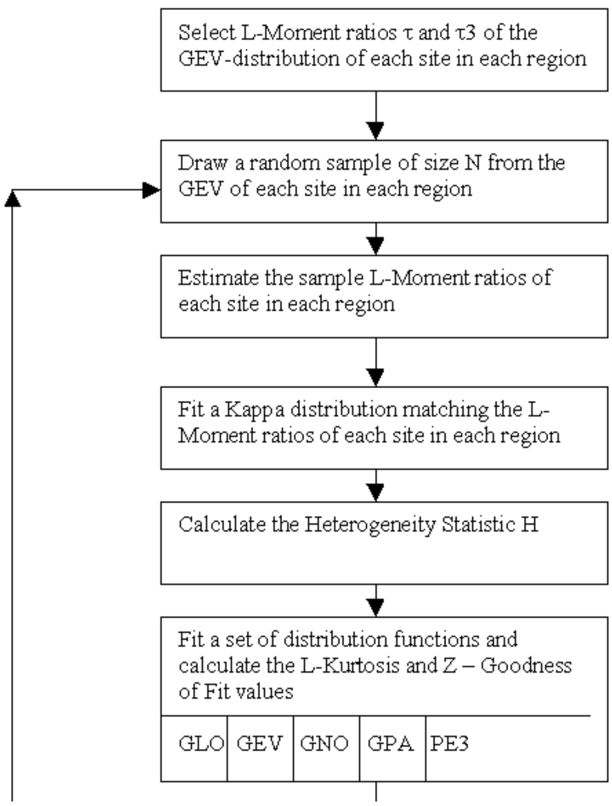
Experiment	# Sites Region 1	# Sites Region 2	# Obs / Region	τ Region 1	τ Region 2	τ_3 Region 1	τ_3 Region 2	Bimodality
2030a	2	2	30	0.22	0.18	0.22	0.18	20%
2030b	5	5	30	0.22	0.18	0.22	0.18	20%
2030c	10	10	30	0.22	0.18	0.22	0.18	20%
2030d	7	14	30	0.22	0.18	0.22	0.18	20%
2030e	10	20	30	0.22	0.18	0.22	0.18	20%
2030f	4	16	30	0.22	0.18	0.22	0.18	20%
2030g	6	24	30	0.22	0.18	0.22	0.18	20%
3030a	2	2	30	0.23	0.17	0.23	0.17	30%
3030b	5	5	30	0.23	0.17	0.23	0.17	30%
3030c	10	10	30	0.23	0.17	0.23	0.17	30%
3030d	7	14	30	0.23	0.17	0.23	0.17	30%
3030e	10	20	30	0.23	0.17	0.23	0.17	30%

3030f	4	16	30	0.23	0.17	0.23	0.17	30%
3030g	6	24	30	0.23	0.17	0.23	0.17	30%
5030a	2	2	30	0.25	0.15	0.25	0.15	50%
5030b	5	5	30	0.25	0.15	0.25	0.15	50%
5030c	10	10	30	0.25	0.15	0.25	0.15	50%
5030d	7	14	30	0.25	0.15	0.25	0.15	50%
5030e	10	20	30	0.25	0.15	0.25	0.15	50%
5030f	4	16	30	0.25	0.15	0.25	0.15	50%
5030g	6	24	30	0.25	0.15	0.25	0.15	50%

Note that in case of a small number of sites or an equal number of sites in regions the RD analysis is meaningless. In the 1^{st} case this is because the number of sites is less than 12 as pointed out in Vandev and Neykov (1993) whereas in the 2^{nd} case this is because in 2 groups with an equal number of sites there exists no algorithm that can identify which one contains discordant sites.

The more interesting results could be found in the *d, *e, *f and *g experiments, which are based on different number of sites in both groups, i.e., 7+14=21 sites, 10+20=30 sites, 4+16=20 sites, 6+24=30 sites respectively.

For each experiment the following steps are followed (Fig. 4.8):



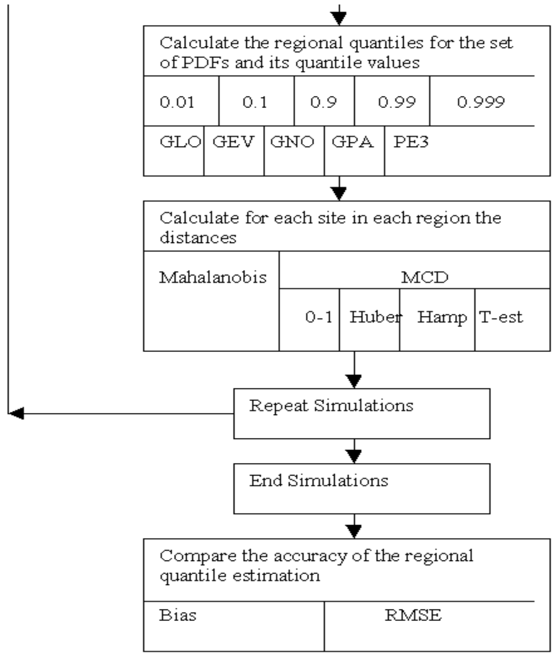


Figure 4.8: Flow diagram of the steps for each simulation experiment

The complete simulation results are reported in Van Gelder and Neykov (1999). In the next Table 4.2 a summary is given of the results of the Mahalanobis and robust distances.

Table 4.2: Summary of the results.

Experiment	# Sites Reg 1	# Sites Reg 2	Bimodality	H	Mahalanobis	MCD	MCD 0-1	MCD Huber	MCD Hamp	MCD T-est
2030a	2	2	20%	0.72	0 – 0	0 - 0	0 – 0	0 - 0	0 - 0	0 – 0
2030b	5	5	20%	1.09	0 – 0	29 - 26	29 - 26	12 - 10	22 - 19	28 - 25
2030c	10	10	20%	1.53	1 – 1	31 - 26	29 - 25	8 - 6	16 – 13	10 – 8
2030f	4	16	20%	1.13	3 – 1	42 - 25	41 - 24	15 - 5	26 - 12	19 – 8
2030g	6	24	20%	1.41	5 – 1	38 - 21	35 - 18	13 - 4	21 – 8	14 – 4
3030a	2	2	30%	1.50	0 – 0	0 - 0	0 – 0	0 - 0	0 - 0	0 – 0
3030b	5	5	30%	2.17	0 – 0	29 - 26	29 - 26	12 - 10	22 - 19	28 - 25
3030c	10	10	30%	3.02	1 – 1	31 - 25	30 - 24	8 - 5	17 - 13	11 – 8
3030f	4	16	30%	2.49	5 – 1	56 – 22	55 – 21	24 – 4	39 – 11	30 – 7
3030g	6	24	30%	3.00	7 – 1	54 - 18	51 - 15	21 - 3	33 - 6	24 – 3
5030a	2	2	50%	3.28	0 – 0	0 - 0	0 – 0	0 - 0	0 - 0	0 – 0
5030b	5	5	50%	4.81	0 – 0	29 - 26	29 - 26	12 - 10	22 - 19	28 - 25
5030c	10	10	50%	6.69	1 – 1	31 - 25	30 - 24	7 - 5	16 – 13	10 – 8
5030d	7	14	50%	7.27	2 – 1	72 - 15	71 - 15	25 - 3	55 - 8	41 – 6
5030e	10	20	50%	8.67	3 – 1	74 - 9	73 - 9	21 - 2	52 - 4	24 – 3
5030f	4	16	50%	6.26	7 – 0	86 - 16	86 - 15	53 – 3	76 – 7	70 – 5
5030g	6	24	50%	7.70	9 – 1	88 - 12	88 - 11	51 – 2	76 - 4	67 – 2

The notation used in Table 4.2 will be explained from the last row of the table. The classical measure of discordance (Mahalanobis) identifies the sites of the 1st region (6 sites) with 9% in average as discordant and the sites of the 2nd region (24 sites) with 1% in average as discordant. This is denoted in the table as 9 – 1. By the robust distances based on MCD and its one-step improvements the percentages of a site in the 1st region being identified as discordant are: 88% , 88%, 51%, 76% and 67% . The sites in the 2nd region are identified as discordant with the percentages 12%, 11%, 2%, 4% and 2%.

The group of the smaller number of sites could be considered as group of discordant sites whereas the other group with larger number of sites as regular one. It is seen that the ratio of the group with the smaller number of sites which is considered as the "discordant group of sites" is larger than the second ratio corresponding to the "regular group of sites".

That difference is not so large if the difference between τ and τ_3 respectively in both regions is not so large as well. When this difference is increased then the corresponding ratios become more distinct as well (see for instance the results with 50% bimodality). In all these cases the H statistic is greater than 1 and according to Hosking and Wallis (1997) the region should be accepted as heterogeneous. The robust distances identified these discordant sites whereas the classical measure of discordance could not identify them.

Notice that in the experiments with many discordant sites the first estimator (MCD) performs better than the one-step improvements. In all these experiments they give the highest percentages for identifying discordant sites (although the MCD by 0-1 weights approximate very closely with only a few percents difference). However in the situations without discordant sites it identifies more "regular sites" as potential outlier. Note that the generated moments are not multivariate normally distributed. Thus the results based on the classical and robust distances should be considered as a guidline rather than statistical tests.

The simulations show that 'screening the sites' by robust distances is in agreement with their H test in situation with very atypical sites whereas the classical discordance measure break down.

4.7 Use of discordance-based homogeneity in RFA

Given annual maxima stream flows at more than 200 river basins in Germany, Belgium, France, Luxemburg, The Netherlands, Switzerland, Austria, Czech Republic, Poland, Slovakia, Hungary and the UK (Figure 4.9).

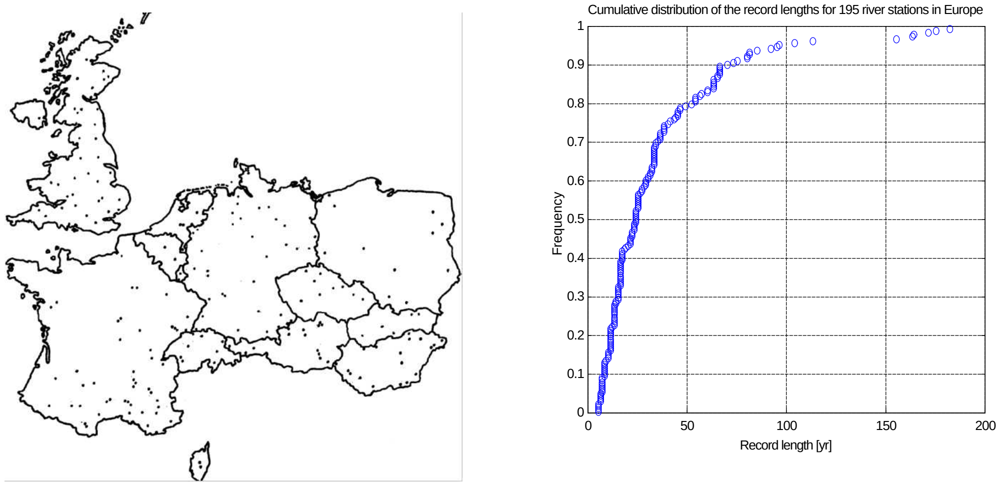


Figure 4.9: Distribution of the measurement stations over North-Western and Central Europe

Data was provided by the GRDC in Koblenz, Germany. The question can be posed how to derive homogeneous regions on the basis of statistical techniques and physics-based considerations? Regional frequency analysis may give the answer to that question, as was also shown by Demuth and Kuells (1997) in their regional analysis of droughts in southern Germany, by Pearson et al. (1991) in their RFA of Western Australian flood data, Mkhandi et al. (1996) in their flood analysis for Tanzania, Rao and Hamed (1994, 1997) in their RFA of Wabash river flood data and Upper Cauvery flood data, Vogel and Wilson (1996) in their analysis of annual maximum, mean, and minimum stream flows in the United States, and Vogel et al. (1993a, 1993b) in their flood flow frequency model selection in Australia and South-Western United States. A regional frequency analysis of North-Western and Central Europe including data from 12 countries was used as a case study by Van Gelder et al. (1999g) to apply the proposed discordance-based homogeneity considerations of Section 4.5. Some highlights of their study will be presented in this section and Section 4.8.

Regional frequency analysis involves two major steps: (1) Grouping of sites into homogeneous regions (Nathan and McMahon (1990)), and (2) Regional estimation of quantiles at the site of interest. The performance of any regional estimation method strongly depends on the grouping of sites into homogeneous regions. Geographically contiguous regions have been used for a long time in hydrology, but have been critisized for being of arbitrary character. In fact, the geographical proximity does not guarantee hydrological similarity. During the last five to ten years researchers have attempted to develop methods in which similarity between sites is defined in a multidimensional space of catchment-related characteristics or statistical characteristics.

A region can only be considered homogeneous if sufficient evidence can be established that data at different sites in the region are drawn from the same parent distribution (except for the scale parameter). L-moment ratio diagrams have become popular tools for regional distribution identification, and for testing for discordant stations. Hosking and Wallis (1993) developed several tests for use in regional studies. They gave guidelines for judging the degree of homogeneity of a group of sites, and for choosing and estimating a regional distribution. L-moment diagrams as a tool for identifying a regional distribution have been used in numerous other studies, including Chowdhury et al. (1989), Pilon and Adamowski (1992), Vogel and Fennessey (1993), and Vogel et al. (1993a,b). An alternative test for homogeneity based on estimated dimensionless 10-year floods was developed by Lu and Stedinger (1992a). Regional growth curves estimated from averaged probability weighted moments have been much used in recent studies.

Hosking and Wallis (1997) developed a unified robust approach to RFA, based on L-moments described by Hosking (1990), that involves objective and subjective techniques for defining homogeneous regions, of assigning sites to regions, identifying and fitting regional probability distributions to data and testing hypotheses about distributions. By robustness Hosking and Wallis (1997) refer to statistics that work well even if the data are contaminated or the model assumptions are slightly violated. The advantages of their approach over the conventional method of moments and the maximum likelihood method are the smaller effect of discordant sites and more reliable inference from small samples, as the L-moments are a linear combination of data (as also discussed in Sec. 3.3).

Van Gelder et al. (1999g) developed an objective tool to derive homogeneous regions of which the principles were presented in the previous section. They succesfully applied their tool to the case study of 195 river stations in Europe (the original database of over 200 river basins contained a few basins with too much missing observations). It was proposed to use first a K-means clustering as the start-up phase to group sites into regions. This method will start with k random clusters, and then move objects between those clusters with the goal to (1) minimize variability within clusters and (2) maximize variability between clusters. This is analogous to "ANOVA in reverse" (analysis of variance) in the sense that the significance test in ANOVA evaluates the between group variability against the within-group variability when computing the significance test for the hypothesis that the means in the groups are different from each other. In K-means clustering, the program tries to move objects in and out of groups (clusters) to get the most significant ANOVA results. Usually, as the result of a K-means clustering analysis, we would examine the means for each cluster on each dimension to assess how distinct our k clusters are. Ideally, we would obtain very different means for most, if not all dimensions, used in the analysis. The magnitude of the F values from the ANOVA performed on each dimension is another indication of how well the respective dimension discriminates between clusters. Hartigan and Wong (1979) and Everitt (1993) give more background information about K-means clustering techniques.

In the case-study of Van Gelder et al. (1999g), a K-means clustering of the site characteristics latitude, longitude, elevation, annual precipitation, population density and the size of basin area was performed. Latitude, longitude and size of the basin area were available from GRDC. The other three characteristics of all locations were obtained by visual inspection from a geographic Atlas (Bosatlas, 1998). The following transformations of the site characteristics were used (Smith, 1992):

Size of basin area: $Y = 3\log(X)/\sigma(\log X)$ (4.7)

Elevation: $Y=\sqrt{X} / \sigma(\sqrt{X})$ (4.8)

Latitude and Longitude: $Y=X / \sigma(X)$ (4.9)

The K-Means clustering resulted in eight clusters. The cluster with sites from Southern France contained 34 sites, however, inhomogeneous (H=6.94).

Van Gelder et al. (1999g) propose to proceed the start-up phase by determining the robust distances for the sites in each cluster and to remove those sites from the cluster which are considered discordant. This process should be iterated until convergence is reached to a homogeneous region. It was shown that the proposed tool performed very well in the case study with 195 river stations. For instance, the sites in Southern France converged in six iterations from a heterogeneous cluster of 34 sites to an extremely homogeneous region with 20 sites (H=0.32) and the GEV as the optimal regional distribution (Figure 4.10 and Table 4.3).

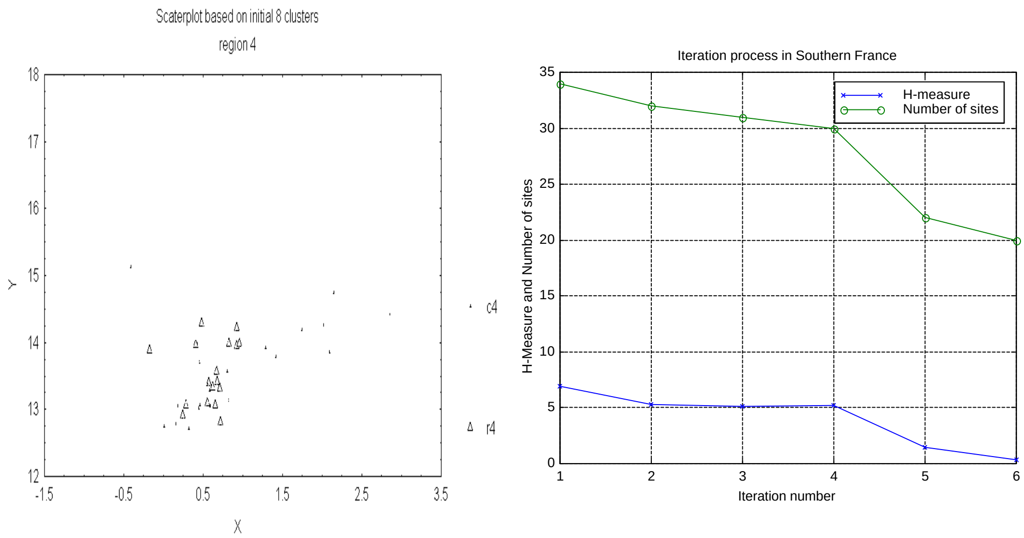


Figure 4.10: South France: XY coordinates of 34 sites (small triangles) during start-up, 20 sites (large triangles) after convergence)

#	ID	OLS	MCD	REW	HUB	HAM	T-BW
1	208	1.013	0.690	1.622	1.159	1.364	1.592
2	52	3.541	2.824	3.566	3.512	3.494	3.651
3	53	1.432	2.412	2.287	1.733	1.976	1.914
4	56	1.633	2.481	2.623	1.644	1.685	1.902
5	57	2.148	13.783*	16.494*	2.995	4.109	2.147
6	58	3.015	1.584	3.027	3.223	3.227	2.694

Table 4.3: A selection from the output of the robust distance calculation for a few sites in Southern France (97.5% cut-off value is 9.35)

Note from Table 4.3 that site 5 is considered with 97.5% certainty a discordant site, according to the MCD and REW robust distances, and should therefore be removed in the iteration process.

It is emphasized that without the use of the robust discordancy measures it would not have been possible to derive homogeneous regions so easily. If H appeared to be “significant”, i.e. $H > 1$, then it was observed that the robust distances of discordancy were also significant for some of the sites. The robust distances support the decision-making process, which is normally the most time-consuming part of the analysis. In this way the proposed iteration process of Van Gelder et al. (1999g) may give an important contribution to future RFA’s.

4.8 Discussion

Homogeneity aspects in the statistical analysis of civil engineering data have been discussed in this chapter. Apart from statistical tests to judge the homogeneity of a data set, also physical arguments have to be included in the judgement. A case study of significant wave height data has been presented to illustrate the differences between the tails of distributions when the homogeneity aspects are left out of consideration.

It was shown that the homogeneity of univariate datasets should be judged by extending the univariate set to a multivariate set. The judgement of the homogeneity of the multivariate set is often much easier than the judgement of the homogeneity of the univariate set. Robust distances should be used in the judgement of multivariate homogeneity analysis. They can also be used in the clustering of sites to homogeneous regions.

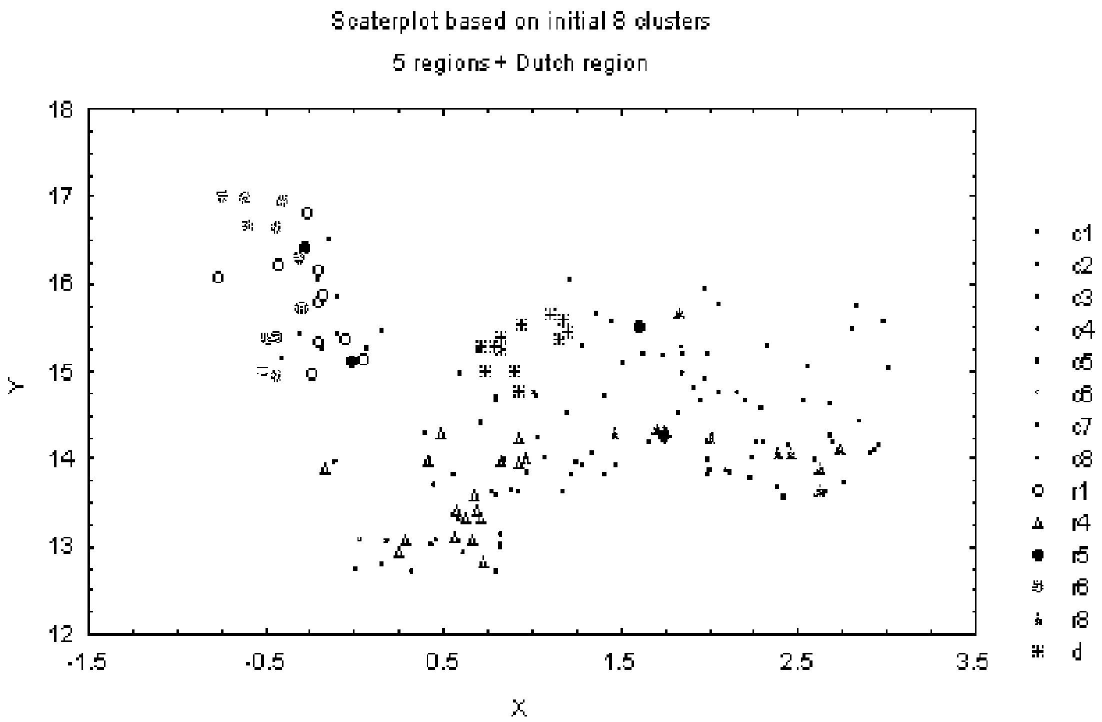


Figure 4.11: The homogeneous regions w.r.t. the frequency distribution of river discharge in North-Western and Central Europe (source: Van Gelder et al., 1999g).

The homogeneous regions with respect to the frequency distribution of river discharges in Europe were determined by Van Gelder et al. (1999g) with robust discordancy measures and physical considerations (mountaineous ridges often form the border of a homogeneous region as can be observed in Figure 4.11). The Dutch homogeneous region contains apart from the locations Borgharen and Lith along the Maas, Lobith along the Rhine, Vechterweerd along the Vecht, also the German sites Rees and Köln along the Rhine, Vlotho, Hannover-Muenden and Intschede along the Weser, Rethem along the Aller, and finally Versen along the Ems. In total 12 locations with a GEV and PE3 distribution as acceptable regional fit (H=0.33, Z=0.92 and 0.94 resp.). Attempts to include in the Dutch region also low-lying sites from Belgium, Northern Poland and South Eastern England failed to maintain the homogeneity.

Robust distances perform better in the judgement of a homogeneity analysis than the classical discordancy measures. Monte Carlo simulations have shown the validity of this statement in Sec. 4.6.

Chapter 5

Design Philosophies

Problem formulation in terms of probabilities is
typically more challeging than the calculations.
- Bertsekas.

5.1 Introduction

Probabilistic design of flood protection has become quite accepted nowadays. Already in the late 1950's a very extensive research project was initiated in the Netherlands by the Delta Committee on the probabilistic analysis of flood defences (Van Dantzig, 1956, and Delta Committee, 1960). Since then many research projects in the field of risk-based decision-making in flood protection followed of which Vrijling, (1986), and CUR, (1990), give good overviews. In the design of dikes, many uncertainties play an important role. However, the influence of uncertainties in for example construction costs, damage costs and statistical and resistance uncertainty has not been investigated in a probabilistic framework so far. This will be the subject of this chapter. It was stated in Chapter 2 already that a Bayesian or Monte Carlo approach can be used to examine the influence of uncertainties on the probabilistic design of flood defences. These will also be the applied methods of this chapter.

In this chapter two design philosophies will be distinguished (Fig. 5.1). The design of a flood protection structure on the basis of a fixed probability of failure (for example once per 100 years in average) is a very common design philosophy. Design of a flood protection structure by economic optimization, and by taking account of the failure probability as a function of the design, is another possible design philosophy

In the first design philosophy, we have to find the best possible estimate of the design loads. Therefore a certain distribution function for these loads have to be selected together with an estimation method to estimate the parameters. In Chapter 3, a criterion was suggested to penalize a too low estimate of the design load more than a too high estimate. This can be ascertained by using asymmetric loss functions. The consequences of a possible wrong fit can be quantified with this idea. Given a certain choice of the loss function and the size of the data set with the loads, three levels have been discerned in Van Gelder (1996b) to determine the optimal distribution function f* and estimation method EM*. In the first level a known distribution function including its parameters for the loads is assumed. Dependent on the loss type and sample size, the optimal pair (f*,EM*) can be determined.

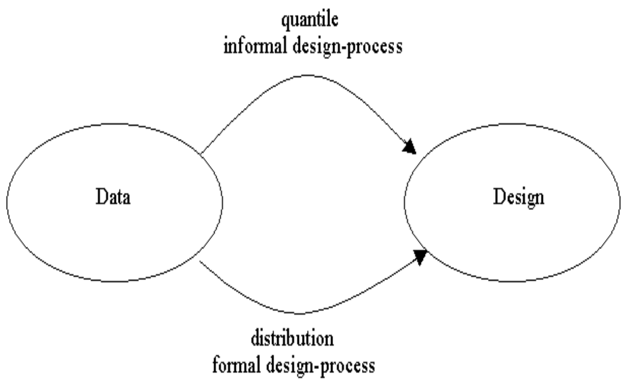


Figure 5.1: Two design philosophies

The second level assumes a known distribution function but its parameters as unknowns. Also here, the optimal pair can be determined as a function of the loss type and sample size. The parametrical (or statistical) uncertainty is treated in a Bayesian way by calculating the posterior parameter probabilities. Possible prior knowledge in the form of historical (inaccurate and subjective) data or translated data from other locations (regionalization) can be added in this Bayesian modeling approach (as opposed to the classical modeling approach). In the third level no assumption on the distribution type and its parameters of the loads is made. This approach can also be treated in a Bayesian way by using a mixture of competitive distributions with diffuse weighing parameters. Also in the third level it is possible to determine the optimal pair (f*,EM*). The determination can be performed by Monte Carlo simulations. In this chapter a typical example with a two by two case is presented for the first two approaches with an exponential and Gumbel model and with a Least Squares and Maximum Likelihood estimation method.

In the second design philosophy we take account of the whole probability distribution function rather than an estimate of only one value of the distribution function (as in the first design philosophy). The loss functions are determined via the costs of construction of the flood protection structure and the costs of possible failure. Also here the three levels as described above can be discerned. In this chapter a typical example of a two by one case has been worked out for the second level with an exponential and Gumbel model with a Maximum Likelihood estimation method.

The new elements in this chapter are the comparisons between the two design philosophies. In the first design philosophy (with a fixed probability of failure) a loss function approach is suggested for the tails of the distribution. A discrimination between under- and overdesign of the structure is made. Three levels of knowledge of the probability model are suggested (complete knowledge; limited knowledge and lack of knowledge). Furthermore, in this chapter a link between the two design philosophies is developed. One of the main results in this chapter is the appearance that the first design philosophy with a Bayes estimate of the dike height under the so-called modified linex loss function is equivalent to the second design philosophy.

The reliability concept is now widely accepted in civil engineering as a tool to find the most appropriate structural system in terms of safety and economy (Zarembka, 1974). Design engineers can determine the optimum degree of safety for a structure to satisfy the safety demands and economical concerns of owners or users. The design preferences of engineers however may change with the amount of monetary value that is involved with the structure. When the amount of damage due to a possible collapse of the structure is large, the engineer will be very risk-averse in his design. In this chapter it will be shown how risk averse behaviour can be included in a reliability-based design by using convex utility functions of monetary value. This procedure has also been applied to a design of a sea defence system in Van Gelder and Vrijling, (1997b). The chapter finishes with a discussion.

5.2 Design by given probability of exceedance

In the design of flood protection structures, we very often work with the design loads; i.e. the loads with a return period of T years. We call this way of designing: *design by given probability of exceedance* (Smith, 1993). Different choices for the assumed distribution of the loads will lead to different estimates in the T-year loads. In order to find the best distribution, we ought to specify some criteria. We would like to use a kind of criterion which is *in relation with the consequences of a possible "wrong fit"* (Slack et al., 1975). We have developed the following concept in three levels:

5.2.1 First Level

Let us assume that we know the exact distribution function F and its parameters λ for the extremes (POT or AM) of the loads. Its q-quantile can be calculated from F and we denote it with y_q. Assume that the load data is fitted by a distribution G by some kind of estimation method EM. Then the q-quantile of G can be calculated and will be denoted by y_q^*. Take in mind that y_q^* is a function of both the distribution fitted to the data and the estimation method used.

If $y_q^* > y_q$, then we are constructing the flood protection structure too high, costing an extra amount of money for the difference in height $y_q^* - y_q$ given by $a(y_q^* - y_q)$, where a is the variable cost factor.

If $y_q^* < y_q$, then we are constructing the flood protection structure too low, making an unsafer construction. We assume that there are costs involved in an unsafer construction by the amount of $b(y_q - y_q^*)$.

Hence, the overdesign loss function is given by $L_o = a\,(y_q^* - y_q)$ (5.1)

and the underdesign loss function is given by $L_u = b\,|y_q^* - y_q|$ (5.2)

The quantity $\Delta = y_q^* - y_q$ is a random variable, because y_q is a random variable and y_q^* is a function of the data. The random quantity Δ has a density given by p(d), where d is a realisation of Δ. Eqn. (5.1) and (5.2) can also be written in one expression via:

$$L(\Delta) = \begin{cases} a\Delta & \text{if} \quad \Delta \geq 0 \ \text{ or } \ y_q^* \geq y_q, \\ -b\Delta & \text{if} \quad \Delta < 0 \ \text{ or } \ y_q^* < y_q \ . \end{cases} \tag{5.3}$$

L_o is the right part of L and L_u is the left part of L. The probability of overdesign can be expressed in terms of Δ by the integral:

$$p_o = \int_0^\infty p(d)\mathrm{d}d \tag{5.4}$$

and the probability of underdesign is given by:

$$p_u = \int_{-\infty}^0 p(d)\mathrm{d}d \tag{5.5}$$

The expected overdesign loss is given by:

$$E(L_o) = \int_0^\infty L_o(d)\, p(d)\mathrm{d}d = a \int_0^\infty d\,p(d)\mathrm{d}d \tag{5.6}$$

The expected underdesign loss is given by:

$$E(L_u) = \int_{-\infty}^0 L_u(d)\, p(d)\mathrm{d}d = -\,b \int_{-\infty}^0 d\,p(d)\mathrm{d}d \tag{5.7}$$

The above expressions can be calculated with Monte Carlo simulations.

The concept proceeds as follows: Given a certain F with parameters λ for the extremes of the loads and the data set $\{x_1, x_2,..., x_n\}$ of extreme loads, the fitted distribution G and estimation method EM which minimizes the expected underdesign and overdesign losses will give us the optimal choice (= best distribution fit + best EM). This concept must lead to a decision tool with which we can decide which distribution function and EM to use for fitting the data from a certain physical process (wave heights for instance). In the first level, however, we have made the assumption that we know the distribution type and its parameters already (but no assumption on the EM). Further on, the assumption on distribution type and parameters will be weakened (second level) and even omitted (third level). The concept of the first level can be realized as follows.

With Monte Carlo sampling we simulate 600 load observations from F. Each series is going to be fitted by method EM to the same distribution function G. We calculate y_q^* and compare it with y_q. We assume a certain loss function to be representative in describing the situation of the flood protection structure; i.e. we choose a pair (a, b). Then, $E(L_o)$ and $E(L_u)$ will be numerically approximated. The distibution G and the estimation method EM which gives the lowest expected design loss $E(L_o)+E(L_u)$ is prefered to be used in the statistical analysis of the extreme loads in front of the flood protection structures.

The use of another criterion is possible here, namely the minimization of the maximum design loss rather than the expected design loss. In formula form we are looking for a G and EM for which $\max(L_o)+\max(L_u)$ is minimal.

Apart from the “2 degrees of freedom”; distribution function F and estimation method EM, we have a third degree of freedom, namely n; the number of observations of the loads. The return period T will be fixed. Finally, the loss function will be the fourth degree of freedom; we will distinguish 1 symmetric loss function and 2 asymmetric loss functions. In an asymmetric loss function we can model the fact that losses increase with underestimation error at a much faster rate than overestimation error. Overestimation of the design loads can lead to losses from needless overconstruction whereas underestimation can lead to potentially enormous costs associated with catastrophic failure of the structure. We therefore will look at the following loss function:

Asymmetric loss function I (a,b)=(4,1):

$$L_u=4(y_q^*-y_q) \text{ if } y_q^*>y_q \qquad (5.8)$$
$$L_o=-(y_q^*-y_q) \text{ if } y_q^*<y_q$$

Asymmetric loss function II (a,b)=(2,1):

$L_u=2(y_q^*-y_q)$ if $y_q^*>y_q$ (5.9)
$L_o=-(y_q^*-y_q)$ if $y_q^*<y_q$

And to compare the results, we will also look at a symmetric loss function (a,b)=(1,1) given by:

$L_u=(y_q^*-y_q)$ if $y_q^*>y_q$ (5.10)
$L_o=-(y_q^*-y_q)$ if $y_q^*<y_q$

Given F and n, the assumed distribution G that minimizes the probability of underdesign, will consequently maximize the probability of overdesign. Where is the optimum? The best fit is given by that G, which gives the lowest expected losses. So, we are not interested anymore in seeking the distribution with the best statistical fit. The optimal fit follows from a trade off between over- and underdesign losses and the designer's aversion to risk due to over- and underdesign.

Given three degrees of freedom; F, n, and type i (i=1,2,3) of loss function, we will determine the best fit f* and best estimation method EM*, defined as that choice for which the expected design loss is a minimum. It is expected that in most cases f* is the same as F; the distribution where we generate the data from.

An interesting question to ask ourselves is how much do we lose when we apply a certain combination of (f,EM) to determine the design quantile y_q ? If we assume that (f*,EM*) is the optimal pair the expected design loss EDL is given by:

EDL(f,EM)=E(L|f,EM)-E(L|f*,EM*) (5.11)

Note that EDL(f*,EM*)=0.

The relative EDL is given by:

REDL(f,EM)=EDL(f,EM)/E(L|f*,EM*) (5.12)

Using these definitions, Van Gelder (1996b) calculated Eqn. (5.11) and (5.12) for many input choices. A typical result is presented in the following Table 5.1.

LF	n	Gumbel				Exponential			
		G,ML	Exp,ML	G,LS	Exp,LS	G,ML	Exp,ML	G,LS	Exp,LS
I	10	24	0	11	9	52	5	11	0
	30	6	86	1	0	151	1	32	0
	100	45	350	0	13	381	0	78	4
II	10	0	23	12	33	32	0	13	15
	30	0	208	14	51	116	0	30	16
	100	28	582	0	66	312	0	63	19
III	10	0	87	46	100	11	0	24	43
	30	0	395	42	132	70	0	28	37
	100	10	819	0	120	220	0	44	40

Table 5.1: REDL (%) in case of two types of distribution functions, two parameter estimation methods, three types of loss functions, and three different sample sizes

Loss function type I is an asymmetric loss function with penalty factor 4 for underdesign; type II is asymetric with penalty factor 2 for underdesign; type III is symmetric.

For very asymmetric loss functions, it appears that a LS-method is prefered over the ML-method. The reason is that LS usually overestimates the p-quantile.

Furthermore, we also see that for small sample sizes and a very asymmetric loss function it happens that the data from a Gumbel distribution is better fitted (w.r.t. minimum design loss) by an exponential distribution than by a Gumbel itself. This phenomenom returned also in the model uncertainty analysis of Section 3.8.3. Furthermore, we see that for symmetric loss functions, the ML-method is prefered because they are more efficient than LS-methods; i.e. their parameters have a smaller variance. The optimal choices are summarized in Table 5.2.

Optimal choices for (f,EM)

Type of LF	n	Gumbel		Exponential	
		f*	EM*	f*	EM*
I	10	Exp	ML	Exp	LS
	30	G	LS	Exp	LS
	100	G	LS	Exp	ML
II	10	G	ML	Exp	ML
	30	G	ML	Exp	ML
	100	G	LS	Exp	ML
III	10	G	ML	Exp	ML
	30	G	ML	Exp	ML
	100	G	LS	Exp	ML

Table 5.2

5.2.2 Second Level

Let us assume that we only know the distribution type F for the extremes of the loads, but that we are uncertain about its parameters λ (i.e. they are given by the non-informative priors belonging to the distribution type F). We also have available a dataset $\{x_1,x_2,...,x_n\}$. We now want to find the distribution type G and estimation method EM, which give the lowest expected design losses. From F and the posterior distribution of the parameters, the uncertainty can be integrated out to find the q-quantile; denoted by y_q. We minimize the expected design losses over (G,EM) in the same way as was done in the first level using Monte Carlo simulation. The second level has been applied to the wave data of Pozzallo in Van Gelder (1996b). It appears that the optimal choice (f*,EM*) remains the same, in comparison with the first level. In other words: statistical uncertainty has no influence on the selection of the optimal pair (f*, EM*). The first and second level finally leads us to the third level.

5.2.3 Third Level

Let us assume that the extremes of the loads come from an uncertain probability model with uncertain parameters. We also have available a dataset $\{x_1,x_2,...,x_n\}$. What is the distribution type G and estimation method EM, which give the lowest expected design losses. This level can be treated by using a mixture of distribution functions with diffuse weights. The data should determine the optimal estimates for the weights. In Chapter 3, the determination of weights has been investigated and the approach of Bayes factors is proposed to be applied in the analysis for the Third Level. We refer to the large-scale simulations that have been performed in Section 3.8.

5.3 Design by reliability-based economic optimization

Reliability-based optimal structural design in the field of water resources has been applied by numerous authors in the past of which we mention Van Dantzig (1956), Bernier (1987a,b), Duckstein and Parent (1994), and Stedinger (1997). In the field of structural engineering much work has been done by Ghiocel and Lungu (1975), Grigoriu et al. (1979), Siddall (1983), Augusti et al. (1984), Frangopol (1985), Kanda and Ahmed (1997) and Moses (1998).

The principle of a reliability-based optimal structural design can be explained with the following illustrative example (Van Dantzig, 1956). Assume that an existing structure has a height of H_0. The structure will be heightened to an optimal height H

(Figure 5.2). There are costs involved with this heightening, which are a function of X where $X = H - H_0$. The total cost of heightening the structure with length X is assumed to be linear by:

$$I=c_f+c_vX \tag{5.13}$$

where c_f is the initial fixed cost and c_v the subsequent variable cost of heigtening the structure per meter.

Assume F is the maximum load distribution function per year with known parameters. If the load is higher than the resistance, we assume that there will be a loss c (independent of the value of the load; i.e. a step loss function (of course it is possible to make this loss function more general)). The expected loss over the unbounded design time of the structure is given by:

$$D=(1-F(H_0+X))\alpha(1-\alpha)^{-1}c \tag{5.14}$$

where α is the discount factor with $\alpha=[1+(r/100)]^{-1}$ and r is the discount rate (Van Noortwijk, 1996). Discounting costs is based on the fact that the utility of a certain amount of money decreases in time from the standpoint of the present. The present discounted value of an amount c in year n is equal to $\alpha^n c$. Note that $\sum_{i=1}^{\infty} a^i = \frac{a}{1-a}$, if we start counting from year 1 (for $0<\alpha<1$).

Summation of both types of costs gives the objective function which has to be minimized over the design variable X in order to find the optimal design:

$$dI/dX+dD/dX=0 \tag{5.15}$$

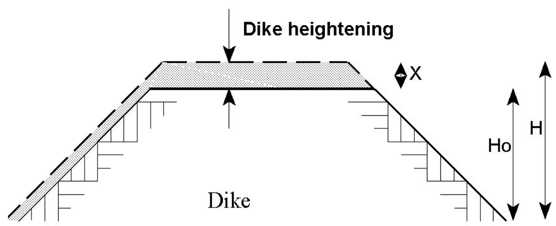


Figure 5.2 Simplied model for dike heightening

In Van Dantzig's (1956) model, the only failure mechanism of dikes that is considered is overtopping. If the applied water level h is higher than the dike height H, then inundation takes place with a total damage of c. The probability of inundation can be modeled in many different ways and is a very difficult subject as was seen in Chapter 3.

Let us assume that the true distribution for the extreme loads is given by F with non-informed prior distributions for its parameters. Let $\{x_1,x_2,...,x_n\}$ be the observations. Then we can update the prior distributions to posterior distributions and calculate the predictive function of F, denoted by PF. The above economic optimization is performed with PF rather than F and leads to an optimal design height X. If we would decide to fit the data to some other distribution G, we can calculate the predictive of G which leads to an optimal height X_{eco}^*. Then, in accordance with Eqn. (5.11), it is interesting to study the expected design loss, which is given by:

$$EDL=(PF(X)-PF(X_{eco}^*))\alpha(1-\alpha)^{-1}c+c_v(X-X_{eco}^*) \tag{5.16}$$

The distribution G giving the smallest expected design loss will be prefered to model the load data. Van Gelder (1996b) proposed a simulation scheme to determine the optimal likelihood model for the first two levels (complete knowledge and limited knowledge). The following procedure can be followed for the second level (Fig. 5.3):

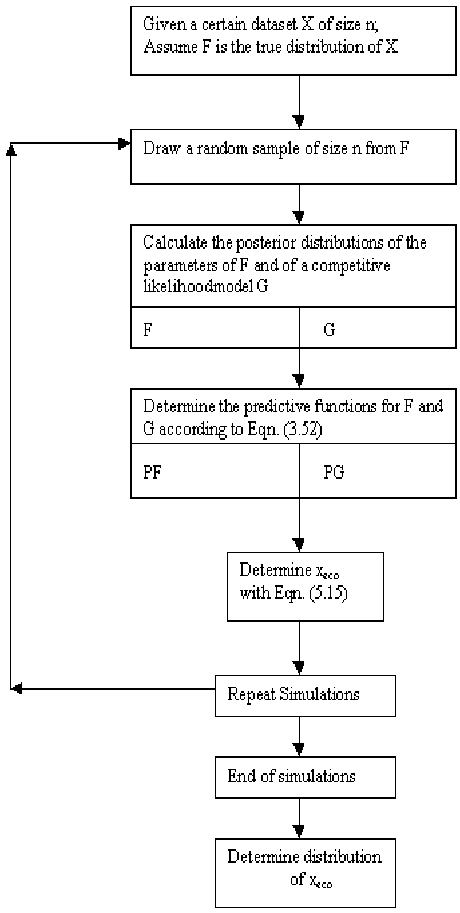


Figure 5.3: Simulation procedure

In Van Gelder (1996b) the concepts of the first and second level are applied to the Pozzallo wave data using Monte-Carlo simulations. The following two assumptions were analyzed.

- data from Pozzallo is Gumbel distributed with uncertain parameters;
- data from Pozzallo is Exponentially distributed with uncertain parameters.

Some typical results are displayed in Fig. 5.4 and Table 5.3.

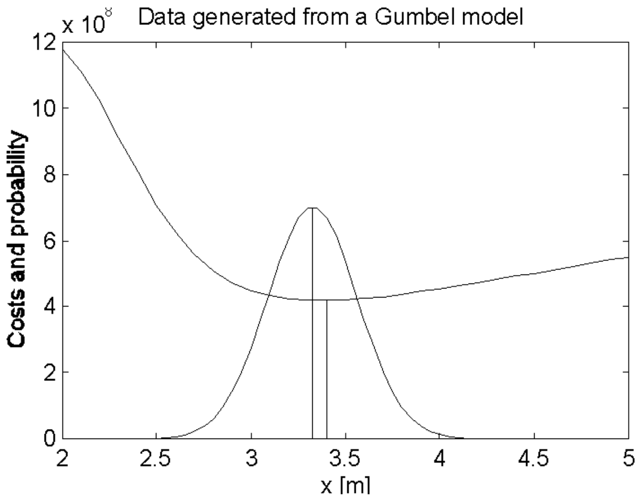


Figure 5.4: Expected losses

n	f_{xeco*}	Gumbel + p.u. L_{min}=4.175 x_{eco}=3.40m		Exponential + p.u. L_{min}=4.67 x_{eco}=3.60m	
		L=G	L=Exp	L=G	L=Exp
10	μ σ L	3.15 0.20 4.38	3.63 0.23 4.26	3.44 0.37 4.69	3.70 0.34 4.67
100	μ σ L	3.34 0.07 4.18	3.89 0.08 4.30	3.50 0.37 4.68	3.63 0.10 4.67

p.u. = parameter uncertainty

Table 5.3: The expected losses per likelihood model; shaded cells give the lowest values for the loss function

The CV of the distribution of X_{eco} , as shown in Fig. 5.4 depends on the available sample size n. When n tends to infinity, X_{eco} will be deterministic. The expected losses are obtained by integration:

$$E(L)=\int(I+D)(x)f_{X_{eco}}^{*}(x)dx \qquad (5.17)$$

and for the simulation with n=10 the expected losses are given by E(L)=4.284 10^8 \$. From this it follows that there is a relative difference of 2.6% with the theoretical minimum value of 4.175 10^8 \$. See also Goicoechea et al. (1982) for a case study.

5.4 Reliability-based optimal structural design under uncertainties and risk aversion

In this section the reliability-based structural design of flood protection structures will be examined with respect to various uncertainties and risk aversion. The effect of statistical and distribution type uncertainty in the reliability-based design should be taken into consideration, as was pointed out by, for instance, Bernier (1987a, 1987b). In Slijkhuis et al. (1997), it is described how these uncertainties can be taken into account in the reliability-based optimization of the Hook of Holland dike. The results are summarized in the next Table 5.4:

	Opt. dike height[m]	Prob. of inundation	Costs [gld]	Opt. dike height[m]	Prob. of inundation	Costs [gld]
	Exponential Model			Gumbel Model		
Without stat. uncertainty	5.88	$7.52\text{x}10^{-6}$	$157\text{x}10^{6}$	5.51	$6.43\text{x}10^{-6}$	$141\text{x}10^{6}$
With stat. uncertainty	6.00	$8.38\text{x}10^{-6}$	$164\text{x}10^{6}$	5.67	$7.24\text{x}10^{-6}$	$148\text{x}10^{6}$

Table 5.4: Influence of statistical and model uncertainties on the reliability-based optimal dike height

The costs of heightening the dikes with $h-h_0$ metres depend on the fixed cost c_f = 1.1 10^8 and the variable cost c_v = 4.0 10^7: i.e. $c_f + c_v[h-h_0]$. If the polder is inundated, an economic value of c = 2.4 10^{10} Dutch guilders is lost. The discount factor is $a = [1+0.015]^{-1}$, compounded annually, where $0 < a < 1$.

From Table 5.4, we observe a higher optimal dike height if we include statistical uncertainty in the economic optimization procedure both for the exponential and the Gumbel model. Consequently also the probabilities of inundation increase as well as the total costs increase as uncertainty is taken into account.

Apart from statistical uncertainty there is also uncertainty in the costs of dike heightening (i.e. the parameters c_f and c_v), and in the damage costs c. The influence of the uncertainty in the parameters c_f, c_v and c was analyzed in Slijkhuis et al. (1997).

These parameters were considered as random variables with a normal distribution. Rather than optimizing μ(L), as is done in the Van Dantzig (1956) calculation, the optimization is over μ(L)+kσ(L), in which k is the risk aversion index (Vrijling and Van Gelder, 1997a). A decision based on the expected value only is called risk neutral. Risk neutrality can be modelled with a linear utility function. In case of a risk averse attitude a smaller standard deviation is prefered over a larger in case of equal expected values. In the literature this is frequently modelled by convex utility functions (Barlow et al. (1993)). To show the principle the expected utility is evaluated below for a linear U=aX and a quadratic utility function U=aX^2:

$$E(U) = \int ax f(x)dx = aE(X)$$
$$E(U) = \int ax^2 f(x)dx = a(E^2(X) + \sigma^2(X)) \tag{5.18}$$

The last equality is because E(X^2) = E^2(X) + var(X). In case of risk aversion the standard deviation starts to play a role. However the strict application of quadratic utility curves has as a disadvantage that the units may hamper the understanding by the public. Quadratic utility curves are difficult to communicate to the public and most probably also to decision makers. The rule proposed above, containing E(L) + kσ(L), is clearly risk averse, and does not have the disadvantage of squared units. Risk averse designers will tend to choose a high k (k towards 2) and invest more in a design to be sure of a safe structure.

The influence of the uncertainty in the cost and damage variables, as well as the choice of the risk aversion index on the optimal dike height is shown in Figure 5.5. Costs of dike heightening can be quite well estimated in advance. Therefore, we assign to c_v and c_f a variation coefficient of 10%. Costs of damage caused by inundation is more difficult to estimate, leading to a variation coefficient for c of 30%.

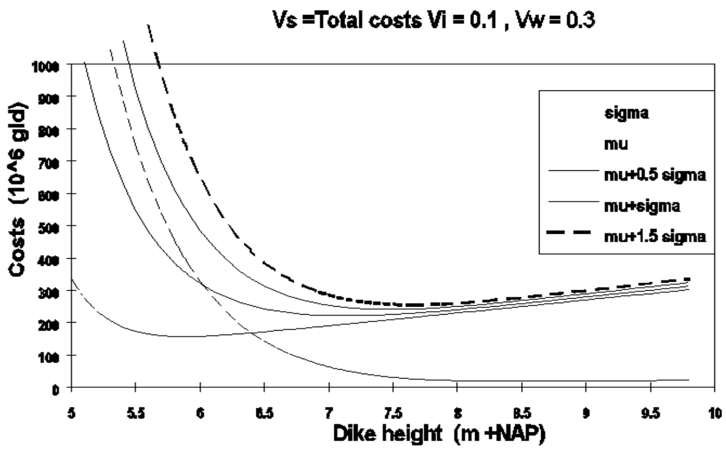


Figure 5.5: μ(L)+kσ(L) for different values of k

We observe (Table 5.5) that a higher risk aversion leads to a higher optimal dike height, which gives a safer construction (the probability of inundation decreases), but at the same time higher construction costs. For example if we add one standard deviation in the optimization procedure, the total costs increase 50% but the safety increases 2000% (probability of inundation is divided by 20) according to Van Gelder and Vrijling (1998a).

Risk avers index	Minimal Costs, [10^6gld].	Opt. dike height [m]	Prob. of inundation
k=0	157	5,88	$7{,}52.10^{-6}$
k=½	221	7,13	$1{,}18.10^{-7}$
k=1	241	7,48	$3{,}70.10^{-7}$
k=1½	255	7,65	$2{,}10.10^{-8}$

Table 5.5: Results of optimization

The influence of the value of the variation coefficient has been investigated. It appears that the CV of c has a very low influence on the optimal dike height. The reason for this can be explained if we look at Fig. 5.6 and Eqns. (5.19):

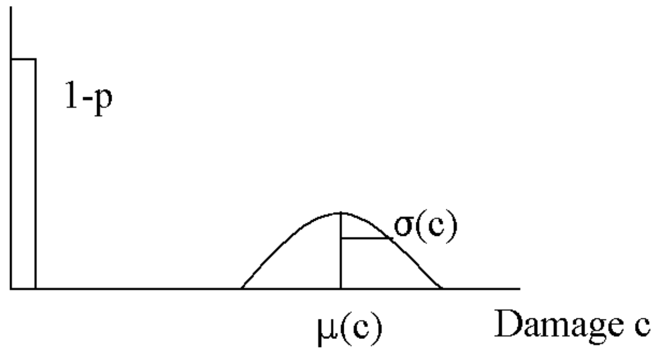


Figure 5.6: Distribution of Damage

The mean and the standard deviation of the cost function are given by:

$$m(L) = m(c_f) + m(c_v)X + \frac{pm(c)}{r}$$
$$\sigma^2(L) = \sigma^2(c_f) + \sigma^2(c_v)X^2 + \frac{p\left[\sigma^2(c) + (1-p)m^2(c)\right]}{(1+r)^2 - 1} \qquad (5.19)$$

The proof follows by treating the occurrence of a flood per year as a Bernoulli trial *Y* with occurrence probability *p* (thus representing the inherent uncertainty in time). Because the Bernoulli variabele *Y* and the random variable *C* are independent, the variance of the product *Y* and *C* can be written as (Sec. 3.7.2):

$$\mathrm{Var}(YC) = \mu^2(Y)\sigma^2(C) + \mu^2(C)\sigma^2(Y) + \sigma^2(Y)\sigma^2(C) =$$
$$= p^2\sigma^2(C) + p(1-p)\mu^2(C) + p(1-p)\sigma^2(C)$$

Furthermore, we assume independence between the costs c_v, c_f and c, and observe that:

$$\sum_{i=1}^{\infty}\left(\frac{1}{1+r}\right)^i = \frac{1}{r}$$
$$\sum_{i=1}^{\infty}\left(\frac{1}{(1+r)^2}\right)^i = \frac{1}{(1+r)^2-1}$$

The variance of the total costs contain a factor $\sigma^2(c) + (1-p)\mu^2(c)$ which can be approximated almost by the second term solely $(1-p)\mu^2(c)$ because $\mu^2(c) \gg \sigma^2(c)$. In the optimization procedure of $\mu(L)+k\sigma(L)$, there is therefore negligable influence of $\sigma(c)$. The influence of the variance of c_v is however much larger, as can be seen in Figure 5.7. The higher the variation coefficient of c_v, the lower the optimal dike height, but the higher the total costs. From a coefficient of 10% to 50%, we see an increase of about 20% in the total costs, and a decrease in optimal dike height of about 30cm.

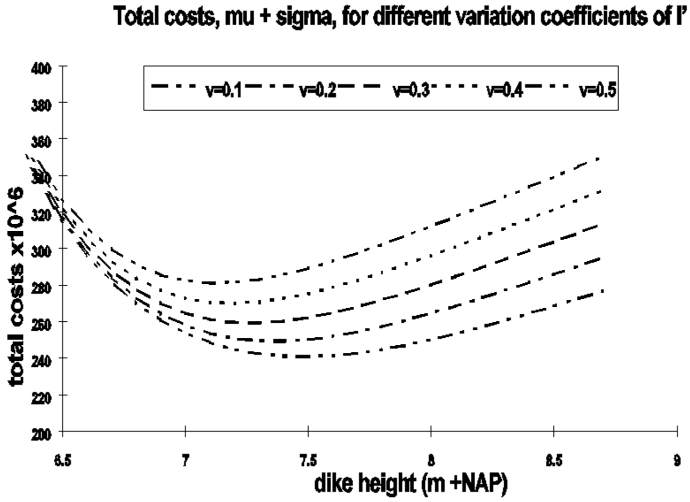


Figure 5.7: Influence of the variation coefficient of I'

Not only the minimal costs in the economic optimizations are of interest, but also the distribution of these costs are valuable to consider. We will simulate the total costs of dike heightening and damage of a dike of 5 meters over a time period of 100 years. For a certain simulation, we draw realisations from the normal distributions of c_f, c_v and c. The optimal dike heightening is determined and each year, it is simulated if an inundation occurs or not. If yes, the damage costs are discounted by the discount factor in year i (i=1..100) and given by $\frac{c}{(1+r)^i}$ (if r=0.05 and i=100, then the denominator is 131). This discount factor is also modeled by a normal distribution with a variation coefficient of 10% and compared with the case of a fixed discount factor. The simulations where inundation has occurred are plotted in the next Figure 5.8:

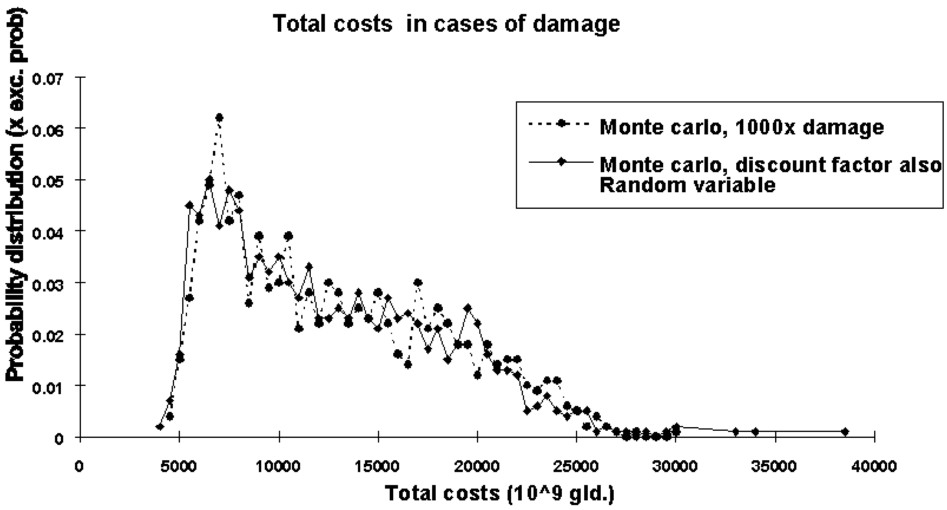


Figure 5.8: Simulation of inundation costs in 100yr.

Coming back to the issue of risk aversion. Monetary value is not always a consistent measure of utility. Consider the following example. A decision maker is asked to choose between alternatives I and II from the following pair of lotteries (see Figure 5.9).

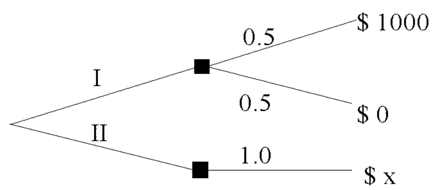


Figure 5.9: Pair of lotteries

In lottery I, the outcome will be either $1000 or $ 0 with equal likelihoods. In lottery II, the only outcome is $ 500 for sure. The expected monetary values are the same for the two lotteries. However, a decision maker might prefer lottery II, since there is a sure gain of $ 500, whereas in lottery I there is a 50% chance of gaining nothing.

The lotteries would be indifferent to the decision maker if the financial status of the decision maker is relatively good in comparison with the monetary values of the alternatives. Therefore a utility function for money should be established. If the decision maker is a large firm or a government organization, the utility function over a range of monetary values may be a straight line. If the decision maker is risk-averse, the utility function will show a convex behaviour over the range of monetary values.

Consider the following example (Fig. 5.9):

Let u(1000) = 1
and u(0) = 0 (5.20)

Suppose the decision maker chooses for x = 100, then

$$u(100) = 0.5\ u(1000) + 0.5\ u(0) = 0.5 \tag{5.21}$$

This procedure can be continued for other monetary values in order to obtain the monetary utility curve. A series of risk-averse utility functions is given by (Ang and Tang, 1990):

$$u(x) = \frac{1}{1 - e^{-g}}(1 - e^{-gx}) \tag{5.22}$$

As γ increases the utility function becomes more convex, indicating higher risk-aversion. In Fig. 5.10, the risk-averse utility functions for γ=1, 2 and 3 are depicted.

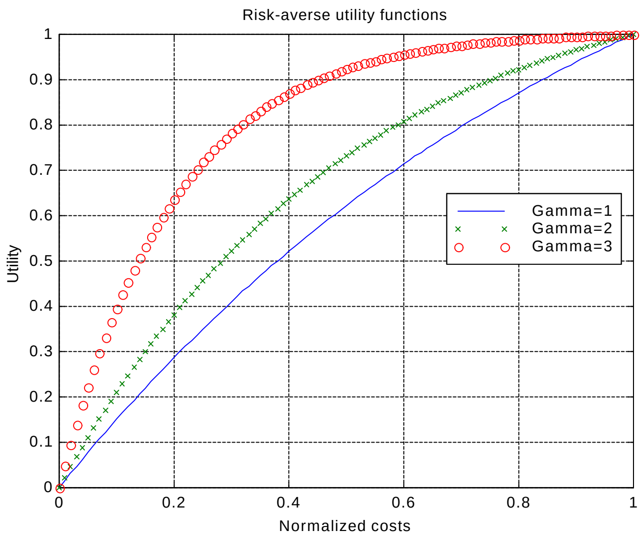


Figure 5.10: Convex utility functions

The inclusion of the concept of risk-aversion into the reliability-based optimization ends up with the following utility function which can be optimized over the decision variable h:

$$u(L(h)) = (1-e^{-\gamma})^{-1} (1-\exp(-\gamma(c_f + c_v(h-h_0) + (1-F(h))\alpha(1-\alpha)^{-1}c))) \quad (5.23)$$

This utility function has been applied by Van Gelder and Vrijling (1997b) to the Pozzallo vertical breakwater example and the following results were obtained (Figure 5.11). Without risk-aversion (i.e. risk-neutral) the reliability-based optimal design is given by h = 4m. Applying risk-aversion with different levels (low, medium, high; corresponding to γ = 1, 2, and 5), an increase in the optimal design height is found from 4.3, 4.4, to 4.8m.

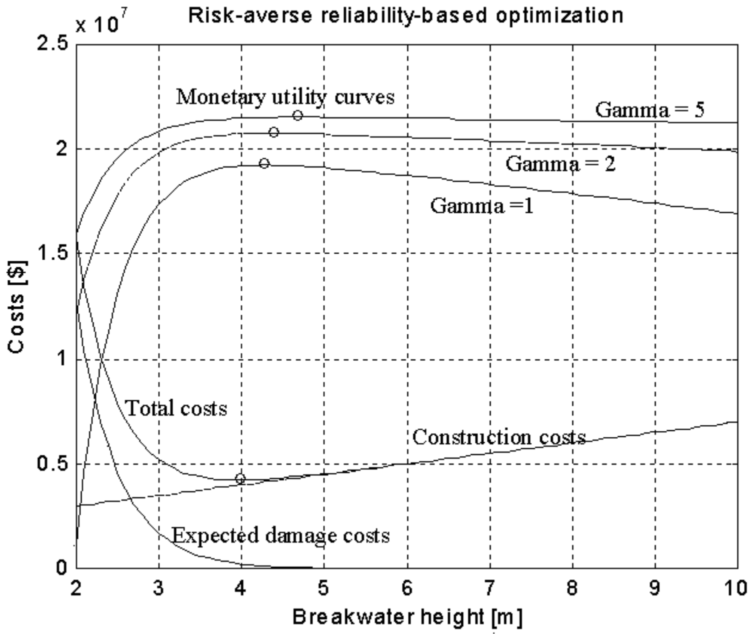


Figure 5.11. Results of the fictitious example

In Vrijling and Van Gelder (1997b) the reliability-based optimization of a structure is studied under the influence of a time-varying probability of failure over the lifetime of the structure. The total costs function can take account for time-varying probabilities of failure and is written with an extra index i:

$$L(X) = c_f + c_v X + c\sum_{i=1}^{\infty} a^i p_i(X) \quad (5.24)$$

Here is assumed that after failure at year i the failure probability at year i+1 is p_{i+1} (and not p_1). However, if we take into account that after failure the structure is repaired to the original condition (the failure probability at year i+1 is p_1), then it was derived by Van Noortwijk and Peerbolte (2000) that Eqn. (5.24) must be replaced by:

$$L(X) = c_f + c_v X + \frac{c\sum_{i=1}^{\infty} a^i p_i(X)}{1-\sum_{i=1}^{\infty} a^i p_i(X)} \quad (5.25)$$

Vrijling and Van Gelder (1997b) applied the reliability-based optimization techniques, given a decreasing hazard function, on the Pozzallo vertical breakwater and noticed that the total costs function for the vertical breakwater decreased (because of the decreasing hazard function) and that the optimal breakwater height also decreased. This behaviour was intuitively expected.

If we investigate an increasing instead of a decreasing hazard function, then we have the situation that the structure deteriorates over time. An example on the deterioration and heightening of dikes can be found in Speijker et al. (2000).

Also the uncertainties in the resistance parameters can be analyzed in the reliability-based optimization. If we assume a normally distributed resistance parameter (with mean μ and standard deviation σ), and an exponentially distributed load (with location A and scale B), then we can derive the probability of failure with the following convolution:

$$p_f(\mu,\sigma)=\int_A^\infty e^{-\frac{\eta-A}{B}}\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{\eta-\mu}{\sigma}\right)^2}d\eta=\frac{1}{\sigma\sqrt{2\pi}}\int_A^\infty e^{-\frac{\eta}{B}+\frac{A}{B}-\frac{\eta^2}{2\sigma^2}+\frac{\mu\eta}{\sigma^2}-\frac{\mu^2}{2\sigma^2}}d\eta \qquad (5.26)$$

The exponent in the above integral can be rewritten as:

$$\begin{aligned}
&-\frac{\eta}{B}+\frac{A}{B}-\frac{\eta^2}{2\sigma^2}+\frac{\mu\eta}{\sigma^2}-\frac{\mu^2}{2\sigma^2}=-\frac{1}{2\sigma^2}\left[\eta^2-\left(2\mu-\frac{2\sigma^2}{B}\right)\eta\right]+\frac{A}{B}-\frac{\mu^2}{2\sigma^2}=-\frac{1}{2\sigma^2}\left[\eta^2-2\left(\mu-\frac{\sigma^2}{B}\right)\eta+\left(\mu-\frac{\sigma^2}{B}\right)^2-\left(\mu-\frac{\sigma^2}{B}\right)^2\right]+\frac{A}{B}-\frac{\mu^2}{2\sigma^2}=\\
&=-\frac{1}{2\sigma^2}\left[\eta-\left(\mu-\frac{\sigma^2}{B}\right)\right]^2+\frac{1}{2\sigma^2}\left(\mu-\frac{\sigma^2}{B}\right)^2+\frac{A}{B}-\frac{\mu^2}{2\sigma^2}=-\frac{1}{2\sigma^2}\left[\eta-\left(\mu-\frac{\sigma^2}{B}\right)\right]^2+\frac{1}{2\sigma^2}\left(\mu^2-2\frac{\mu\sigma^2}{B}+\frac{\sigma^4}{B^2}\right)+\frac{A}{B}-\frac{\mu^2}{2\sigma^2}=\\
&=-\frac{1}{2\sigma^2}\left[\eta-\left(\mu-\frac{\sigma^2}{B}\right)\right]^2-\frac{\mu}{B}+\frac{\sigma^2}{2B^2}+\frac{A}{B}
\end{aligned} \qquad (5.27)$$

So, we have:

$$\begin{aligned}
p_f(\mu,\sigma)&=\frac{1}{\sigma\sqrt{2\pi}}\int_A^\infty e^{-\frac{\eta}{B}+\frac{A}{B}-\frac{\eta^2}{2\sigma^2}+\frac{\mu\eta}{\sigma^2}-\frac{\mu^2}{2\sigma^2}}d\eta=\frac{1}{\sigma\sqrt{2\pi}}\int_A^\infty e^{-\frac{1}{2\sigma^2}\left[\eta-\left(\mu-\frac{\sigma^2}{B}\right)\right]^2-\frac{\mu}{B}+\frac{\sigma^2}{2B^2}+\frac{A}{B}}d\eta=\\
&=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{\mu}{B}+\frac{\sigma^2}{2B^2}+\frac{A}{B}}\int_A^\infty e^{-\frac{1}{2\sigma^2}\left[\eta-\left(\mu-\frac{\sigma^2}{B}\right)\right]^2}d\eta\approx e^{-\frac{\mu}{B}+\frac{\sigma^2}{2B^2}+\frac{A}{B}}=e^{-\frac{\mu-A-\frac{\sigma^2}{2B}}{B}}
\end{aligned} \qquad (5.28)$$

The ≈ sign in Eqn. (5.28) is almost an equal sign in the case $\left(\mu-\frac{\sigma^2}{B}\right) >> A$.

The total cost function L(X) is now given by the expression with an extra parameter σ:

$$L(X) = c_f + c_v X + \alpha(1-\alpha)^{-1} c\, p_f(H_0+X,\sigma) \qquad (5.29)$$

The optimal probability of failure follows again from the equation dL(X)/dX = 0. This leads the same optimal probability of failure in the situation where there is no uncertainty in the resistance. The optimal increase in height X is the same as the optimal increase in the situation where there is no uncertainty in the resistance added with $\sigma^2/2B$.

Simple closed-form expressions for the optimum design can be obtained for several PDF's, as will be shown next. The closed-form expressions could be used to understand the significance of certain parameters in the design load determination.

If the total costs formula is given by the expression:

$$c(\lambda, h) = c_f + c_v[h - h_0] + \frac{\alpha}{1-\alpha} c(1 - F(h)) \tag{5.30}$$

Then the optimal height follows from the equation: $\frac{c_v}{c}\frac{1-\alpha}{\alpha} = f(h)$ (5.31)

If F ~ Exp, $F(h) = 1 - \exp\{-(h - x_0)/\lambda\}$ it was derived that:

$$h^* = x_0 - l \ln\left(l \cdot \frac{c_v}{c} \cdot \frac{1-a}{a} \right) \tag{5.32}$$

This corresponds to a failure probability of:

$$p_{opt} = l \frac{c_v}{c}\frac{1-a}{a} \tag{5.33}$$

Notice that the optimal failure probability is independent of the location parameter x_0. Therefore, using "exponential uncertainty modelling", the behaviour of the optimal design and probabilities of failure can be described as follows:

A larger location parameter leads to higher optimal design according to Eqn. (5.32), but has no influence on the optimal probability of failure according to Eqn. (5.33). A larger scale parameter leads to smaller optimal design (Eqn. (5.32)) and a higher probability of failure (Eqn. (5.33)).

If F ~ Gumbel, the approximation $e^{-x} \approx 1 - x + O(x^2)$ for $|x| << 1$ can be used, leading to exactly the same solution as in the Exponential case.

If F ~ Normal, $h = m + s\sqrt{-2\log\left(\frac{c_v}{c}\frac{1-a}{a} s\sqrt{2p}\right)}$ (5.34)

If F ~ LogNormal, $h = \exp\left(m - s^2 + \sqrt{s^4 - 2s^2(m + \log[s\sqrt{2p}\frac{c_v}{c}\frac{1-a}{a}])}\right)$ (5.35)

If F ~ Weibull, $h = z + b\left(\frac{1-d}{d}\right)^{1/d} lambertW^{1/d}\left(\frac{d}{1-d}\left(\frac{b}{d}\frac{c_v}{c}\frac{1-a}{a}\right)^{\frac{d}{d-1}}\right)$ (5.36)

The LambertW function satisfies (Corless et al., 1998),

$$LambertW(x) * \exp(LambertW(x)) = x . \quad (5.37)$$

The asymptotic behavior of LambertW is described by Corless et al. (1998).

The probabilities of failure are given by substituting the expressions (5.32) to (5.36) in the corresponding CCDFs.

In the Lake IJssel case study (as described by Vrijling et al. (1999)), it appeared that the various uncertainties could be very well described by exponential distributions (Figure 5.12). This was also observed in general by Cornell (1995). The probability of Z < 0 (overtopping of the Lake IJssel dikes) was calculated by a first order reliability method (FORM). Given the uncertainties in water level, wind speed, wind surge, wave height, wave steepness, wave run-up, and lake oscillations, the uncertainties in the probability of overtopping at the location Rotterdamsche Hoek were determined.

The results of the uncertainty calculations are summarized in Fig 5.12. Notice the differences between the required crest heights for the three cases: inherent (or intrinsic) uncertainty, inherent + statistical uncertainty, and inherent + statistical + model uncertainty.

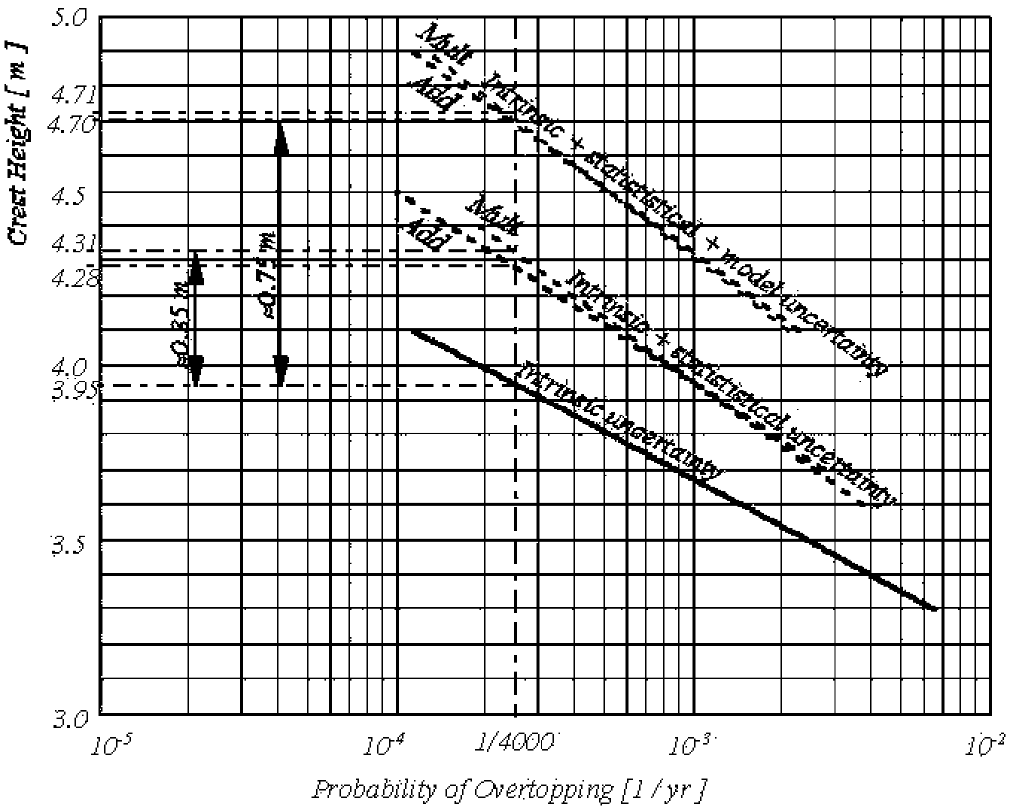


Figure 5.12: Uncertainties in the probability of overtopping at the location Rotterdamsche Hoek along Lake IJssel (Vrijling et al., 1999).

The probabilities of overtopping are given by straight lines in the semi-logarithmic Figure 5.12 and they are described by exponential distribution functions: $P(X<k) = 1-\exp(-(k-A)/B)$ with the following values (Table 5.6):

Rotterdamse Hoek							
	A in m 1	ΔA in m 2	B in m 3	ΔB in m 4	h for P = 1/4000 [1/yr] in m 5	Δh in m 6	P [1/yr] for h = 3.95 m 7
i.u.	2.303	- -	0.198	- -	NAP + 3.95	- -	1/4000
i.u. + s.u. (add.)	2.285	-0.018	0.240	0.042	NAP + 4.28	0.33	1/1000
i.u. + s.u. (mult.)	2.113	-0.190	0.267	0.069	NAP + 4.33	0.38	1/1000
i.u. + s.u. (add.) + m.u.	2.492	0.089	0.2645	0.0665	NAP + 4.71	0.76	1/250
i.u. + s.u. (mult.) + m.u.	2.357	0.054	0.2832	0.0852	NAP +4.70	0.75	1/275

Table 5.6: Location and scale parameters

The change in the optimal probability of failure (from p_{opt} to p_{opt}') which is caused by the increase in uncertainty (from B to B+ΔB) can also be expressed as follows:

$$p_{opt}' = \frac{B+\Delta B}{B} p_{opt} \tag{5.38}$$

Given an optimal probability of failure of 1/4000 yr^{-1} for Rotterdamsche Hoek, the inclusion of all uncertainties (ΔB=0.0852) leads to a new optimal probability of failure of

$$\frac{0.2645}{0.198} . \frac{1}{4000} = \frac{1}{3000} yr^{-1} \tag{5.39}$$

Instead of an economic optimal dike height of 4.70 m, a height of 4.60 m is the result.

Summarizing we can say that in this section the influence of various types of uncertainty on an economic optimal dike design was investigated. In a case study of the Hook of Holland sea dike, the statistical uncertainty is determined for the two probability distribution functions Exponential and Gumbel. The influence on the economic optimal dike height is for both models in the order of 15cm. The influences of the uncertainties in the construction costs of the dike (mobilisation cost and cost per meter dike heightening) and the uncertainty in the costs of damage due to inundation were much higher. Depending on the extent of risk aversion of the designer, the influences were in the order of 1m extra heightening. With a Monte Carlo simulation it was shown how to get an impression of the distribution of the total damage costs over a period of 100 years due to inundation. It appears that this distribution had quite a large standard deviation caused by the fact of the uncertainties in the occurrences of inundations and the uncertainty in the damage costs. Furthermore it was noted that with "exponential uncertainty modelling", a larger location parameter leads to a higher optimal design, but without influence on the optimal probability of failure. A larger scale parameter leads to a lower optimal design and a higher probability of failure. In this probabilistic dike design only one failure mechanism has been examined, namely overtopping. However, other mechanism such as sliding, piping, etc. should be analyzed as well. The number of uncertainties will then increase considerably. The framework of the uncertainty analysis however, will remain the same.

5.5 Bayesian Reliability-based Optimal Structural Design

The use of asymmetric loss functions gives us the possibility to differentiate between underestimation and overestimation of the q-quantile. As already mentioned in civil engineering applications, an underestimation error of the q-quantile generally leads to much higher losses than an overestimation error.

Although Fortin et al. (1997a) also use asymmetric loss functions for comparing statistical distributions and estimation methods, they do not apply these loss functions in a full Bayesian framework. The parameters of the probability distributions have been estimated by using three well-known methods from classical statistics: the method of maximum likelihood, the method of moments, and the method of probability-weighted moments. The Bayesian point of view only comes in when Fortin et al. (1997b) use a nonparametric Bayesian simulation methodology, called Pólya resampling, instead of the classical bootstrap to draw observations with replacement from a reference sample.

In this section three loss functions will be studied in a full Bayesian framework: (i) the asymmetric linear loss function, (ii) the asymmetric squared-error loss function, and (iii) the linex loss function. For an overview of asymmetric loss functions, see Zellner (1986) and Thompson and Basu (1993 and 1995). The linex loss function has been used in real estate assessment by Varian (1974). Basu and Ebrahimi (1991) determine an expression for the linex estimator of the survival function of a system having a Type II censored exponential lifetime. Using simulated data, Basu and Thompson (1992) and Thompson and Basu (1993) obtain linex estimates of the reliability of simple stress-strength systems. Pandey et al. (1994) study the problem of estimating the shape parameter of a Pareto distribution using a linex loss function.

The Bayes estimator of the *q*-quantile under asymmetric loss minimises the expected loss with respect to the probability distribution of an unknown parameter. In order to find the loss function that can best be applied to decision problems in civil engineering, we revisit the economic dike-height optimisation problem of Van Dantzig (1956). He assumed the annual maximum sea water levels to be exponentially distributed. It appears that Van Dantzig's economic loss function differs slightly from the linex loss function. This modified linex loss function seems to be a promising candidate for solving quantile estimation problems in other civil engineering benefit-cost analyses.

In this section an overview is given of the Bayesian estimation of quantiles by using the three above-mentioned loss functions. The economic optimisation of dike heights is subsequently addressed. We derive the relation between the economic optimisation and the loss function approach. The different methods are compared in the Hook of Holland case study.

Define the random quantity X_i to be the maximal sea water level in year i, $i = 1,\ldots,n$. We assume the random quantities $X_1,\ldots,X_n$ to be mutually independent, identically distributed, random quantities with a cumulative distribution function $\Pr\{X_i \leq x\} = F(x \mid l)$ with parameter l, $i = 1,\ldots,n$. As a function of l, the *q*-quantile of the probability distribution of *X* is defined to be

$$y_q = g(l) = F^{-1}(1-q \mid l), \tag{5.40}$$

where $g'(l) > 0$. Suppose the parameter l is unknown with a prior probability density function $p(l)$. After observing the data $\mathbf{x} = (x_1,\ldots,x_n)$, this prior density can be updated to the posterior density using Bayes' theorem:

$$p(\lambda) = \pi(\lambda \mid \mathbf{x}) \propto L(\mathbf{x} \mid \lambda)\pi(\lambda) = \prod_{i=1}^{n} f(x_i \mid \lambda)\pi(\lambda), \tag{5.41}$$

where $L(\mathbf{x} \mid \lambda)$ is the likelihood function of the observations $\mathbf{x}$ when the value of l is given.

For the purpose of flood prevention, we are interested in estimating the *q*-quantile of the probability distribution of the maximal sea water level *X* per year, denoted by $g(l^*)$. In a Bayesian framework, this can be achieved by minimising the loss due to the simple estimation error $\Delta = g(l^*) - g(l)$. Since the loss due to flooding increases with overestimation error (i.e. the real *q*-quantile is less than its estimated value: $g(l) < g(l^*)$ or $\Delta > 0$) and, at a much faster rate, with underestimation error (i.e. the real *q*-quantile is greater than its estimated value: $g(l) > g(l^*)$ or $\Delta < 0$), we focus on asymmetric loss functions (see Thompson and Basu, 1995). Beside loss functions of the simple estimation error, loss functions of the relative estimation error can also be considered.

The three most well-known asymmetric loss functions are: (i) the asymmetric linear loss function, (ii) the asymmetric squared-error loss function, and (iii) the linex loss function.

5.5.1 Asymmetric Linear Loss

The asymmetric linear loss function is defined in Section 5.2.1 by Eqn (5.3). We can best choose the estimate l^* for which the expected loss is minimal with respect to the probability distribution of l:

$$\begin{aligned} &E(L(\Delta)) = \\ &= \int_{-\infty}^{l^*} a[g(l^*) - g(l)]p(l)\,dl + \int_{l^*}^{\infty} b[g(l) - g(l^*)]p(l)\,dl = \\ &= ag(l^*)P(l^*) - a\int_{-\infty}^{l^*} g(l)p(l)\,dl - bg(l^*)[1 - P(l^*)] + b\int_{l^*}^{\infty} g(l)p(l)\,dl\,, \end{aligned} \tag{5.42}$$

where $P(l)$ is the cumulative distribution function of l. The Bayes estimator under asymmetric linear loss is the solution of the equation

$$\frac{dE(L(\Delta))}{dl^*} = g'(l^*)([a+b]P(l^*) - b) = 0, \tag{5.43}$$

which results in $l^* = P^{-1}(b/[a+b])$. Hence, the Bayes estimator l^* equals the $b/[a+b]$-quantile of the posterior distribution of l. When $a = b$, the linear loss function is symmetric and its Bayes estimator reduces to the posterior median $P^{-1}(0.5)$.

5.5.2 Asymmetric Squared-Error Loss

The asymmetric squared-error loss function is defined by

$$L(\Delta) = \begin{cases} a\Delta^2 & \text{if} \quad \Delta \geq 0 \;\text{ or }\; l \leq l^*, \\ b\Delta^2 & \text{if} \quad \Delta < 0 \;\text{ or }\; l > l^*, \end{cases} \tag{5.44}$$

where $a, b > 0$. This loss function is asymmetric for $a \neq b$ with expected value

$$E(L(\Delta)) = \int_{-\infty}^{l^*} a[g(l^*) - g(l)]^2 p(l)\,dl + \int_{l^*}^{\infty} b[g(l) - g(l^*)]^2 p(l)\,dl \,. \tag{5.45}$$

The Bayes estimator under asymmetric squared-error loss is the solution of the equation

$$\begin{aligned}\frac{dE(L(\Delta))}{dl^*} &= 2g(l^*)g'(l^*)\{[a-b]P(l^*)+b\} - \\ &\quad - 2g'(l^*)\{[a-b]\int_{-\infty}^{l^*} g(l)p(l)\,dl + b\int_{-\infty}^{\infty} g(l)p(l)\,dl\} = 0,\end{aligned} \tag{5.46}$$

which must be solved for l^* numerically. When $a = b$, the squared-error loss function is symmetric and the Bayes estimator $g(l^*)$ reduces to the posterior mean of $g(l)$.

5.5.3 Asymmetric Linex Loss

The asymmetric linex loss function is defined by

$$L(\Delta) = b[a\Delta + \exp\{-a\Delta\} - 1], \tag{5.47}$$

where $a, b > 0$. The expected loss can be written as

$$\begin{aligned}&E(L(\Delta)) = \\ &= b[\int_{-\infty}^{\infty} a[g(l^*) - g(l)]p(l)\,dl + \int_{-\infty}^{\infty} \exp\{-a[g(l^*) - g(l)]\}p(l)\,dl - 1].\end{aligned} \tag{5.48}$$

The Bayes estimator under asymmetric linex loss, l^*, is the solution of the equation

$$\frac{dE(L(\Delta))}{dl^*} = ab\,g'(l^*)[1 - \int_{-\infty}^{\infty} \exp\{-a[g(l^*) - g(l)]\}p(l)\,dl] = 0, \tag{5.49}$$

which results in

$$l^* = g^{-1}\left(\frac{\ln(\int_{-\infty}^{\infty} \exp\{ag(l)\}p(l)\,dl)}{a}\right). \tag{5.50}$$

Examples of the three loss functions are displayed in Figure 5.13: (i) the asymmetric linear loss function with $a = 5.37\ 10^7$ and $b = 1.94\ 10^7$, (ii) the asymmetric squared-error loss function with $a = 1.07\ 10^8$ and $b = 3.88\ 10^7$, and (iii) the asymmetric linex loss function with $a = 3.03$ and $b = 1.32\ 10^7$. The parameters a and b have been chosen as such that the three loss functions are equal for $\Delta = \pm 0.5$.

Note that both linear loss and squared-error loss are special cases of what Thompson and Basu (1995) called monomial-splined loss, defined, for fixed $m = 1,2,3,...$, by

$$L(\Delta) = \begin{cases} a|\Delta|^m & \text{if} \quad \Delta \geq 0, \\ b|\Delta|^m & \text{if} \quad \Delta < 0, \end{cases} \tag{5.51}$$

where $a,b > 0$. Fortin et al. (1997b) applied monomial-splined loss for $m = 1,2,3$ within a framework of classical statistics.

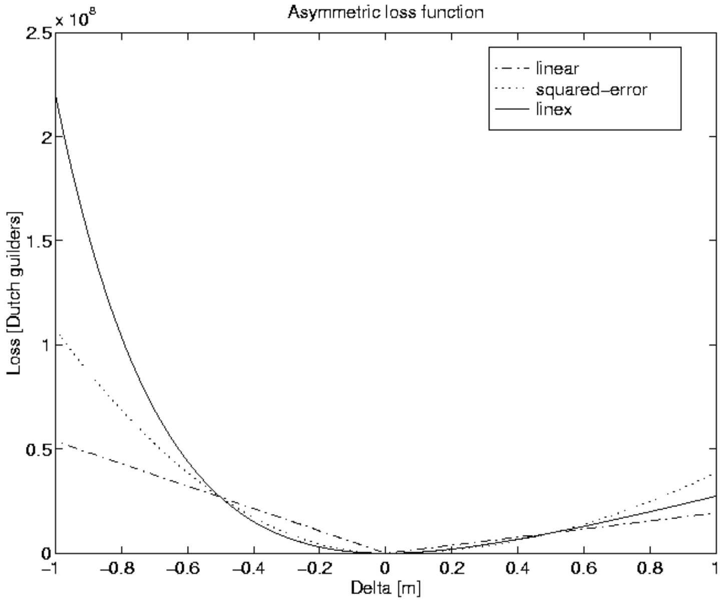


Figure 5.13. The linear, squared-error and linex loss function according to Eqs. (5.3), (5.44) and (5.47).

5.5.4 Estimation of Optimal Dike Height

Let us reconsider the benefit-cost analysis that is adapted from Van Dantzig (1956). Suppose we have to decide how high the dikes should be to prevent a polder from flooding. Let the height of the dike h be the decision variable, and let $h_0 = 3.25$

metres be the initial height of the dike at the moment the decision has to be taken. The only failure mechanism that we regard is overtopping, i.e. inundation of the polder will occur as soon as the sea water level exceeds the height of the dike. To account for the stochastic nature of the sea water level, we assume the maximal sea levels per year X_i, $i = 1,\ldots,n$, to be conditionally independent, exponentially distributed, random quantities with a known location parameter $x_0 = 1.96$ metres and an unknown scale parameter l with expected value 0.33 metres. Hence, the likelihood function is

$$l(\mathbf{x} \mid \lambda) = \prod_{i=1}^{n} f(x_i \mid \lambda) = \prod_{i=1}^{n} \frac{1}{\lambda} \exp\left\{ -\frac{x_i - x_0}{\lambda} \right\}. \tag{5.52}$$

Accordingly, the *q*-quantile of the probability distribution of *X* is

$$y_q = g(l) = x_0 - l \ln(q). \tag{5.53}$$

The cost parameters are as given in Sec. 5.4. Since the probability of inundation of the polder is $\exp\{-(h - x_0)/l\}$, the expected discounted costs due to inundation of the polder over an unbounded time-horizon can be written as

$$c(l, h) = c_f + c_v[h - h_0] + \frac{a}{1-a} c \exp\left\{ -\frac{h - x_0}{l} \right\} \tag{5.54}$$

if the decision-maker chooses dike height *h* and when the value of l is given.

The decision with minimal expected costs, i.e. the dike height for which the expected discounted costs due to inundation are minimal, is

$$h^* = x_0 - l \ln\left(l \cdot \frac{c_v}{c} \cdot \frac{1-a}{a} \right) \tag{5.32}$$

when the value of l is given. Accordingly, the inundation probability that balances the cost of investment optimally against the cost of inundation is

$$q = \exp\left\{ -\frac{h^* - x_0}{l} \right\} = l \cdot \frac{c_v}{c} \cdot \frac{1-a}{a}. \tag{5.33}$$

When the value of l is given to be 0.33 metres, the optimal inundation probability is $q = 8.25\ 10^{-6}$.

To account for the statistical uncertainty in the mean of the maximal sea water level per year, the prior density of l is assumed to be an inverted gamma distribution with scale parameter $m > 0$ and shape parameter $n > 0$:

$$\mathrm{Ig}(l \mid n, m) = [m^{v} / \Gamma(n)]\, l^{-(n+1)} \exp\{-m/l\} \tag{5.55}$$

for $l > 0$. The prior mean and variance are $E(l) = m/(n-1)$ and $\mathrm{Var}(l) = E(l)^2/(n-2)$, respectively. Hence, the larger n, the less uncertain l. On the basis of this prior density, the expected discounted costs over an unbounded horizon transform into

$$\int_0^{\infty} c(\lambda, h) p(\lambda)\, d\lambda = c_f + c_v[h - h_0] + \frac{\alpha}{1-\alpha} c \left[\frac{\mu}{\mu + h - x_0} \right]^{\nu}. \tag{5.56}$$

The dike height with minimal expected costs, while taking the uncertainty in l into account, is

$$h^{\bullet} = x_0 - m + \left[n m^{n} \cdot \frac{c}{c_n} \cdot \frac{a}{1-a} \right]^{\frac{1}{n+1}}. \tag{5.57}$$

The inundation probability that balances the cost of investment optimally against the cost of inundation, while taking the uncertainty in l into account, is

$$q^{\bullet} = \left[\frac{m}{m + h^{\bullet} - x_0} \right]^{n} = \left[\frac{m}{n} \cdot \frac{c_n}{c} \cdot \frac{1-a}{a} \right]^{\frac{n}{n+1}}. \tag{5.58}$$

When the expected value of l is 0.33 metres, the optimal inundation probability under statistical uncertainty is $q^{\bullet} = 1.02\ 10^{-5}$ for $n = 50$, $q^{\bullet} = 9.17\ 10^{-6}$ for $n = 100$, and $q^{\bullet} \to 8.25\ 10^{-6}$ as $n \to \infty$.

An advantage of the inverted gamma distribution as a prior density is that the posterior distribution of l, when the observations $x_1, \ldots, x_n$ are given, is also an inverted gamma distribution with scale parameter $m + \sum_{i=1}^{n} (x_i - x_0)$ and shape parameter $n + n$. The inverted gamma distribution is said to be a conjugate family of distributions for observations from an exponential distribution with unknown mean (scale parameter). From now on, when we use the probability density function $p(l)$, we refer to the posterior density. Note that (inverted) gamma priors have als been applied by Basu and Ebrahimi (1991), Basu and Thompson (1992), Thompson and Basu (1993), and Pandey et al. (1994).

5.6 Relation Between the two Design Philosophies

The question arises whether Van Dantzig's economic cost function and the Bayesian loss function are interrelated to each other. In this respect, we reformulate Van Dantzig's cost function in terms of Bayesian loss, i.e. we rewrite the loss function as

$$\begin{aligned} L(\Delta) &= L(g(l^*) - g(l)) = c(l, h^* + \Delta) - c(l, h^*) = \\ &= c_v \Delta + \frac{a}{1-a} c \exp\left\{-\frac{h^* - x_0}{l}\right\}\left[\exp\left\{-\frac{\Delta}{l}\right\} - 1\right]. \end{aligned} \tag{5.59}$$

There are now two possibilities for rewriting the probability of exceedence $\exp\{-(h^* - x_0)/l\}$: (i) as a constant and (ii) as a function of the unknown scale parameter l.

First, we investigate the probability of exceedence $\exp\{-(h^* - x_0)/l\}$ to be a constant, i.e. to be $q = 8.25\ 10^{-6}$:

$$L(\Delta) = c_v \Delta + \frac{a}{1-a} cq \left[\exp\left\{-\frac{\Delta}{l}\right\} - 1\right], \tag{5.60}$$

where $\Delta = g(l^*) - g(l)$ and $g(l) = x_0 - l \ln(q)$. The Bayes estimator under asymmetric loss in terms of Eqn. (5.60), l^*, is the solution of the equation

$$\frac{dE(L(\Delta))}{dl^*} = g'(l^*)\left[c_v - \frac{a}{1-a} cq \int_0^\infty \frac{1}{l} \exp\left\{-\frac{g(l^*) - g(l)}{l}\right\} p(l)\, dl\right] = 0, \tag{5.61}$$

which results in

$$g(l^*) = x_0 - l^* \ln(q) = x_0 - m + \left[nm^n \cdot \frac{c}{c_n} \cdot \frac{a}{1-a}\right]^{\frac{1}{n+1}} = h^{\bullet}. \tag{5.62}$$

Second, we consider the probability of exceedence $\exp\{-(h^* - x_0)/l\}$ to be a function of the unknown scale parameter l, by substituting the optimal dike height h^* according to Eqn. (5.32):

$$L(\Delta) = c_v \left(\Delta + l \left[\exp\left\{-\frac{\Delta}{l}\right\} - 1\right]\right), \tag{5.63}$$

where $\Delta = g(l^*) - g(l)$ and

$$g(l) = x_0 - l \ln\left(l \cdot \frac{c_v}{c} \cdot \frac{1-a}{a} \right) \tag{5.64}$$

The Bayes estimator under asymmetric loss in terms of Eqn. (5.63), l^*, is the solution of the equation

$$\frac{dE(L(\Delta))}{dl^*} = c_v g'(l^*) \left[1 - \int_0^\infty \exp\left\{ -\frac{g(l^*) - g(l)}{l} \right\} p(l)\, dl \right] = 0, \tag{5.65}$$

which results in

$$g(l^*) = x_0 - l^* \ln\left(l^* \cdot \frac{c_v}{c} \cdot \frac{1-a}{a} \right) = x_0 - m + \left[nm^n \cdot \frac{c}{c_n} \cdot \frac{a}{1-a} \right]^{\frac{1}{n+1}} = h^{\bullet} \tag{5.66}$$

5.6.1 Modified Linex Loss

We ought to notice that the two economic loss functions (5.60) and (5.63) differ slightly from the linex loss function (5.47). A difference is that both economic loss functions are not only a function of the simple estimation error Δ, but also of the relative estimation error Δ/l. In terms of l^* and l, the loss function (5.60) can be written as

$$L(\Delta) = -c_v \ln(q) \cdot (l^* - l) + \frac{a}{1-a} cq \left[\exp\left\{ \ln(q) \cdot \frac{l^* - l}{l} \right\} - 1 \right] = L(\Delta_1, \Delta_2), \tag{5.67}$$

where $\Delta_1 = l^* - l$ is the simple estimation error of l^* and $\Delta_2 = (l^* - l)/l$ is the relative estimation error of l^*. The general formulation of the modified linex loss function (5.67) is:

$$L(\Delta_1, \Delta_2) = b(a\Delta_1 + d[\exp\{-a\Delta_2\} - 1]), \tag{5.68}$$

where

$$a = -\ln(q), \quad b = c_v, \quad d = \frac{a}{1-a} \cdot \frac{c}{c_v} \cdot q. \tag{5.69}$$

Since the main aim of this section is estimating the q-quantile of a probability distribution, the most appropriate loss functions seem to be the economic loss functions (5.60) and (5.67) (in terms of $g(l)$ and l, respectively). These economic loss functions are modified linex loss functions, for which the parameters have a clear economic significance. The parameters represent the cost of investment (dike heightening) on the one hand, and the cost of flooding on the other hand. Since the modified linex loss functions are derived from estimating the mean of an exponential distribution, more research has to be undertaken to find out whether they can also be applied to estimate the statistical parameters of other probability distributions.

5.6.2 Comparative Results

On the basis of the dike heightening problem, we have compared the linear, squared-error and linex loss function with the economic loss functions. The results are summarised in Tables 5.7-8. The coefficients *a* and *b* of the linear, squared-error and linex loss function have been assessed in the following way. As suggested by the economic loss functions (5.60) and (5.63), the coefficients of the linex loss function are assumed to be $a = [E(l)]^{-1}$ and $b = c_v E(l)$. Furthermore, asymmetric linear and squared-error loss functions have been fitted to this linex loss function by, somewhat arbitrary, assuming the linear, squared-error and linex loss to be equal to each other for $\Delta = \pm 0.5$ (see Figure 5.12). Results are also presented for the symmetric linear and squared-error loss function.

Table 5.7: Bayes estimates of the scale parameter l and the dike height *h* for n = 50.

Estimation method for n = 50 observations	*a*	*b*	l^* [m]	h^* [m]
Van Dantzig without uncertainty	-	-	0.330	5.82
Van Dantzig with uncertainty	-	-	-	6.14
Bayes estimate symmetric linear loss	1	1	0.326	5.77
Bayes estimate symmetric squared-error loss	1	1	0.330	5.82
Bayes estimate asymmetric linear loss	$5.37\ 10^7$	$1.94\ 10^7$	0.356	6.13
Bayes estimate asymmetric squared-error loss	$1.07\ 10^8$	$3.88\ 10^7$	0.350	6.05
Bayes estimate linex loss	3.03	$1.32\ 10^7$	0.393	6.56
Bayes estimate modified linex loss	-	-	0.360	6.14
Bayes estimate modified linex loss	-	-	0.357	6.14

Table 5.8: Bayes estimates of the scale parameter l and the dike height h for n = 100.

Estimation method for n = 100 observations	a	b	l^* [m]	h^* [m]
Van Dantzig without uncertainty	-	-	0.330	5.82
Van Dantzig with uncertainty	-	-	-	5.98
Bayes estimate symmetric linear loss	1	1	0.328	5.80
Bayes estimate symmetric squared-error loss	1	1	0.330	5.82
Bayes estimate asymmetric linear loss	$5.37\ 10^7$	$1.94\ 10^7$	0.349	6.05
Bayes estimate asymmetric squared-error loss	$1.07\ 10^8$	$3.88\ 10^7$	0.344	5.98
Bayes estimate linex loss	3.03	$1.32\ 10^7$	0.354	6.10
Bayes estimate modified linex loss	-	-	0.345	5.98
Bayes estimate modified linex loss	-	-	0.343	5.98

From Tables 5.7-8, we can conclude the following. The cost-optimal dike height without taking the statistical uncertainties involved into account is, due to Eqn. (5.32), equal to 5.82 m. When asymmetric loss functions are applied, the optimal dike height is higher while taking the statistical uncertainty in l into account. The larger the uncertainty in the scale parameter l, i.e. the smaller the number of observations n, the higher the cost-optimal dike height. On the other hand, a symmetric squared-error loss function results in the same height without uncertainty (5.82 m) and a symmetric linear loss function can result in even lower heights (5.77 m and 5.80 m, respectively). As expected, the optimal dike height under uncertainty according to Eq. (5.57) equals the dike height that follow from both economic loss functions (5.60) and (5.63). Recall that the main difference between Eqn. (5.60) and Eqn. (5.63) is that the former is regarded as a function of the optimal q-quantile, whereas the latter contains the substitution for q in terms of Eq. (5.33). Since the linex loss function results in a dike height much greater than the height in case of the economic loss functions, we recommend using the economic loss functions (modified linex loss functions) instead.

A Bayesian approach towards the estimation of flood quantiles has been suggested. Bayes estimators of the optimal dike height under symmetric and asymmetric loss have been investigated when the annual maximum sea water levels are exponentially distributed with unknown mean. Three types of loss functions have been considered: (i) linear loss, (ii) squared-error loss, and (iii) linex loss. In order to properly account for the statistical uncertainty in the mean, a modified linex loss function can best be applied. This new modified linex loss function is derived from the economic dike heigthening problem of Van Dantzig. The Bayes estimate of the dike height under modified linex loss is equivalent to the optimal dike height for which the economic loss is minimal. The modified linex loss function seems to be a promising candidate to solve quantile estimation problems in other civil engineering benefit-cost analyses. Moreover, unlike in most Bayesian literature, the parameters of the modified linex loss function have a clear economic significance. They represent the cost of investment (dike heightening) on the one hand, and the cost of flooding on the other hand. The advantage of using a Bayesian loss function approach over a Van Dantzig approach is that the former approach is more closely related to the current design practice of hydraulic structures with fixed quantiles. The difference between under- and overdesign is more visible in the Bayesian loss function approach than in the Van Dantzig approach. The next step would be to repeat the type of work done in this section on a larger scale in order to estimate the statistical parameters of other probability distributions.

5.7 Discussion

In this chapter we have described the reliability-based optimal structural design of civil structures with respect to a certain failure mechanism. In this chapter and in papers, such as Van Gelder et al., (1997a) and Slijkhuis et al., (1997), it was shown that a reliability-based design should take account of statistical- and probabilistic model uncertainties. A Bayesian framework was developed for that purpose. Furthermore, in this chapter the reliability-based design was included with the concept of risk aversion by using utility functions of monetary value. Different levels of risk aversion can be applied in the optimal design. A higher risk aversion leads to a more conservative design of the structure. The exact level of risk aversion, however, is difficult to determine. It does not only depend on the economic status of the decision maker, as was illustrated in this chapter, but also on political and ethical issues. Especially when human lives are involved with failures of structures, the techniques in this chapter will be very difficult to apply. The recent research on the field of acceptable risks (e.g. Fischhoff (1981), Vrijling et al. (1995,1996), Stallen et al. (1996), and Vrijling and Van Gelder (1997a) looks promising to be used for these cases.

Two different design philosophies for civil structures, namely design by fixed probability of failure, and design by economic optimization, were shown to be equivalent by use of the introduced modified Linex loss function.

One of the difficulties in applying a reliability-based optimization lies in the lack of supporting data that can be used in the associated probability-based decision models. Also the uncertainty in the economic parameters of structural systems can cause difficulties in the optimization procedures. Both difficulties have been approached in the recent work of Slijkhuis et al. (1997) and Van Gelder et al. (1997a). It was noted that with "exponential uncertainty modelling", a larger location parameter leads to higher optimal design, but without influence on the optimal probability of failure. On the other hand, a larger scale parameter leads to a smaller optimal design and a higher probability of failure. A proposal to deal with time-variant probabilities of failure in a reliability-based optimization was suggested by Vrijling and Van Gelder (1997b). In Ang and Tang (1990) it was already noted that design preferences of engineers may change with the amount of monetary value that is involved with the structure. When the amount of damage due to a possible collapse of the structure is huge, the engineer will be very risk-averse in his behaviour. In this chapter the concept of risk averseness was included in the reliability-based design. The use of convex utility functions of monetary value was shown to be appropriate in modeling a risk-averse behaviour of the engineer. The procedure was applied to a design of a vertical breakwater in Van Gelder and Vrijling (1997b). Vertical breakwaters have become quite popular as a sea defence system for large harbours. The design of vertical breakwaters by a reliability-based optimization was shown to be succesful by Voortman et al. (1999). The amount of structural and economic damage due to collapse of a vertical breakwater is enormous. The inclusion of risk-aversion is therefore very important in the structural design.

In Roos et al. (1997) a systems approach has been described to determine the optimal safety level of connecting water barriers in a sea-lake environment. Connecting water barriers are large dams to lock a sea arm or a river branch from the influence of the sea or another river. These dams do not directly protect land or people from flooding, but indirectly reduce the water levels in the enclosed water body. Because of the influence on the water body behind the dam, the water-connecting barrier is an essential part in the flood protecting system. It was shown by Roos et al. (1997) that the safety level of the water connecting barrier can be analyzed in a systems approach with an economic optimization criterion. It is based on a bivariate economic minimization of the expected total costs. A case study of the Afsluitdijk in the Netherlands could succesfully be performed in this systems approach.

In Vrijling and Van Gelder (1995), Van Noortwijk (1996b), and Van Noortwijk and Van Gelder (1996) the design philosophies for optimal maintenance and inspection schemes on hydraulic structures, and berm breakwaters in particular, have been considered. As in the design philosophy for the economic optimization of civil structures, presented in this chapter, the design philosophies for maintenance as proposed by Van Noortwijk (1996) also take account for uncertainties. For the particular case of the berm breakwater, Van Noortwijk and Van Gelder (1996) take account of the (large) uncertainties in the limiting average rate of occurence of breaches in the armour layer and of the uncertainties in the limiting average rate of longshore rock transport (given a breach has occurred). The stochastic process of rock displacement was modelled by a modified generalized gamma process by which they were able to determine analytically under the presence of mentioned uncertainties the optimal inspection/maintenance interval. Vrijling and Van Gelder (1995) use FORM in their determination of the optimal interval for berm breakwaters and were in the same time able to take also account for uncertainties in the wave climate and the armour layer (size – and relative mass density of the armour units).

For centuries the dikes in the Netherlands have been protected against wave attack by revetments constructed from pitched blocks. Based on experience a size was selected and if a severe storm damaged the revetment heavier blocks were applied. Recent research has however shown that the design storm surge would cause severe damage to the protection of the dike. A new design philosophy is developed in Hussaarts et al. (1999) that assesses the function of the revetment in the entire dike and optimizes the thickness probabilistically based on the principles which have been presented in this chapter.

Part 2: Applications

Chapter 6

Sea floods frequency analysis with historical information

... and see what we can learn from our history.
- Roberta Finkelstein.

6.1 Introduction

One of the first authors who recognized the need of including historical data in a frequency analysis were Condie and Lee (1982). With a logarithmic likelihood model they performed Monte Carlo simulations and treated a few case studies of river flows. The maximum likelihood method could succesfully be applied. Cohn and Stedinger (1987) and Jin and Stedinger (1989) showed the substantial improvements in the precision of flood-quantile estimates from the inclusion of less precise historical information. Accurate historical records of the largest floods may be nearly as valuable as a systematic gage record which covered the entire historical period. Cohn and Stedinger (1987) used a maximum likelihood framework for the analysis. Jin and Stedinger (1989) used the generalized maximum likelihood estimation and the method of probability weighted moments as a framework. Hosking and Wallis (1986) observed that the more parameters a distribution has, the greater is the value of the historical data. Guo and Cunnane (1991) gave a critical appraisal that some of the methods of incorporation of historical floods and palaeological information into flood frequency analysis may perform poorly, especially when the wrong type of distribution function is assumed. Recently, Cohn et al. (1997) developed an algorithm (based on expected moments) which can utilize three types of at-site flood information: systematic stream gauge record; information about the magnitude of historical floods; and knowledge of the number of years in the historical period when no large flood occurred. Their method appears to be very efficient and much easier to implement than for instance MLE.

Apart from frequentistic techniques to include historical information in a frequency analysis, Van Gelder (1996a) showed that Bayesian methods suit very well for this purpose as well. He used historical information to estimate the prior distributions of the parameters of the likelihood model and the instrumental data to derive the posterior distributions.

Surprisingly, the use of historical data in a frequency analysis has been limited to applications within river floods. However, within other areas, such as earthquake engineering, sea floods, etc., the methods can be applied as well. Van Gelder and Lungu (1997a, 1999) developed a new statistical model for the Gutenberg-Richter magnitude of eartquakes occurring in the Vrancea fault line in Romania. Mean return periods of Vrancea earthquakes were usually calculated with statistical models of earthquake data from the last century (Marza et al, 1991). However, a very extensive historical research had been undertaken by Radu et al. (1970, 1986, 1995) to investigate the occurrences of earthquakes in Romania in the period 984-1900. This information had not been included in the statistical models for Gutenberg-Richter magnitudes versus occurrence frequency so far. In Van Gelder and Lungu's paper new calculations were presented to include this historical data in the current models by use of a Bayesian modelling approach. A generalized extreme value likelihood model served well for their purposes. Combining the historical data with the instrumental data caused the once per 475 years design earthquake to decrease 3% to 5% in magnitude. The exact decrease however was difficult to determine because it depended on the accuracy of the historical data. The Bayesian framework gave a good tool to analyze the effect of accuracy of the historical data by one single fine-tuning weighing parameter.

In this chapter another application will be presented: the use of historical information on sea floods in a frequency analysis. Also here, a Bayesian modelling approach is adopted. The Netherlands is a low-lying country which has to protect itself against flooding from the sea and its rivers. Reliable flood defenses are essential for the safety of the country. The sea dikes are designed to withstand floods with a height of once every 10,000 years. This height is used to be calculated using statistics on sea levels measured along the Dutch coast since 1880. In this chapter calculations will be presented taking account of known sea floods in the period before 1880.

The statistical work on sea floods frequency analysis in the Netherlands started in the fifties and was under direction of professor D. Van Dantzig (Deltacommittee, 1960). The several recommendations of the committee for a new statistical approach of dike design were adopted by the government and most of the Dutch sea dikes have since been adapted to meet the new standard. Although the statistical analysis was the decisive argument for determining the new standard, a serious effort has been made to study the problem of determining an appropriate height for the sea dikes from an economic and also from a physical point of view. The economic analysis compared the cost of building dikes of a certain height with the expected cost of a possible inundation given that specific height. In the mathematical-physical analysis the effects of a wind field on a rectangular water basin was studied.

Now, more than 30 years later, both the number of observations of sea levels and the statistical methodology have grown considerably. This has led to a new investigation (Dillingh et al. 1993, De Haan 1990). They investigated the problem solely from a statistical point of view of determining a level of the sea dikes such that the probability that there is a flood in a given year equals p, where the number p is to be determined by the politics ranging between 10^{-3} and 10^{-4} (depending on the importance of the area under subject). For the data they used the complete set of high tide water levels recorded along the Dutch coast since the end of the 19th century. Although this set consists of many observations (over 100,000 per location), the problem of determing the dike level is a problem of extrapolation: estimating a water level that can occur once every 10,000 years out of a data set of a small 100 years.

After the 1953 flood, historical data about floods in previous centuries have been studied systematically (Gottschalk 1977, Jonkers 1989). Their reports give a good overview about the severeness of the floods that took place between 1500 and 1850. Up till now, this information has not been used in the calculation of the dike levels. In this paper Bayesian statistics is shown to be useful to take account of historical data in the extrapolation problem. It will appear that in this Bayesian framework it is possible to include the global warming effect on earth and its result on the sea level rise as well.

The chapter is organised as follows: the flood historical data and the assumptions on the sea level rise will be discussed in Sec.6.2. The measurements of sea level data since the end of 1800 will be discussed in Sec.6.3. We make a distinction there in POT and AM data (Rasmussen and Rosbjerg, 1989). In Sec.6.4, we will look at a Bayesian model for the AM data, followed by a Bayesian model for the POT data in Sec.6.5. Finally in Sec.6.6, we will draw the conclusions.

6.2 Flood historical data

Recently historical research has been undertaken to retrieve information about floods in the period 1500-1850 (Gottschalk 1977, Jonkers 1989). In diaries, signs of flood levels on old churches etc. a collection of old data could be gathered. The old data on flood levels however have to be taken as an indication, not as accurate data points. Different sources on the same sea flood give contradictive reports about the actual height which occurred. That’s why in this paper the floods will be classified into 4 classes: Class A for very severe floods; B for heavy floods; C for less heavy floods and D for small floods. Every class will be connected with a water level and uncertainty level, based on the historical research data and based on the sea level rise data. Class A floods are assumed to be realisations from a normally distributed random variable of flood levels with a mean of 390cm and a standard deviation of 10cm, denoted as N(390,10). B - floods are realisations from N(360,10); C - floods are in N(330,10) and D - floods from N(300,10). The following floods in the period 1500-1850 are mentioned including its classification:

1570 - A	1775 - D
1672 - D	1776 - C
1686 - D	1806 - B
1715 - D	1808 - B
1717 - D	1825 - B

It is argued that the sea level rise is, in contrary of popular belief, not a new phenomenon but a known fact from geological observations (Vrijling 1994). About 8,000 BC, the northern edge of the North Sea was dry and connected the British Isles with the continent. As the present depth of this area is equal on average to MSL-35m, a sea-level rise of approximately 35m must have occurred in the last 10,000 years. This means a rise of 35cm per 100 year.

6.3 Instrumental data

Since 1888 the sea levels at Hook of Holland are recorded intensively. If we perform a linear regression analysis on the year maxima data of Hook of Holland, we obtain as the estimated regression line:

$$y=\alpha+\beta x=226.7+(\text{year}-1888)*0.2017\text{ cm.} \tag{6.1}$$

This line is dotted in Figure 6.1.

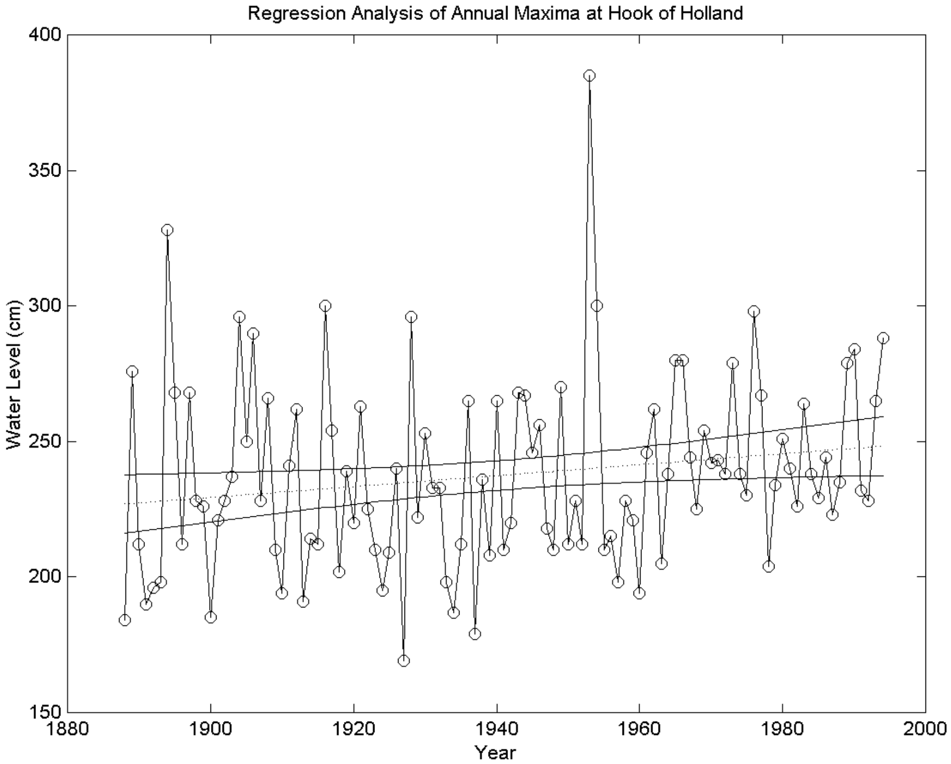


Figure 6.1: Regression Analysis of AM-data at Hook of Holland

We observe a sea level rise of 20 cm in 100 years. From the 90% confidence limits, given in the Figure by the 2 solid lines, we notice that the hypothesis $\beta=0$ would also be accepted. However, in the Delta committee, the sea level rise for the Netherlands during the last centuries is determined at 20cm per 100 year, including the correction for the sinking of the soft soil of the polders (Deltacommittee 1960). We will apply a sea-level rise correction on both the flood historical data as well as the sea level data since 1888.

Since 1888 accurate recordings of the sea levels at Hook of Holland are availabe. For the purpose of determining the level of the sea dikes such that the probability that there is a flood in a given year equals p=1/10,000 yr^{-1} (denoted as $x_{10,000}$ and called the10^{-4} quantile), we filter the recordings to 2 data sets. The AM-data set, which consists of all the annual maxima of sea levels and the POT-data set, which consists of all the peaks of water levels over a certain threshold. The AM-data set has the data of the period 1888 - 1995; in total n=108 points. The number of data points of the POT-data set depends on the level of the threshold (see Sec.6.5). Both the AM and POT data sets are corrected for the sea-level rise influence (of 20 cm per century).

6.4 Model Annual Maxima Data

Assume that the maximum sea level in a period of half a day is modelled by a stochastic variable X with (unknown) probability distribution F_X. Divide the year in 730 sections of 1/2 days. Let X_j be the maximum sea level in Section j (j=1..730). For Hook of Holland the Delta Committee (1960) showed that the X_j's are independent; i.e. the period of half a day is large enough to make storm 1 independent of storm 2. Let us now look at $X=\max_{j=1..730} X_j$. From extreme value theory, it follows that X must tend towards an extreme value distribution (Castillo (1988), and Tawn (1992)). So X is a Frechet, Weibull or Gumbel distribution, dependent on the parent distribution of X_j. From a graphical analysis (plotting the AM-data on Gumbel paper and examining convexity, concavity or linearity), we suggest a Gumbel distribution of the AM sea levels at Hook of Holland (see Figure 6.2).

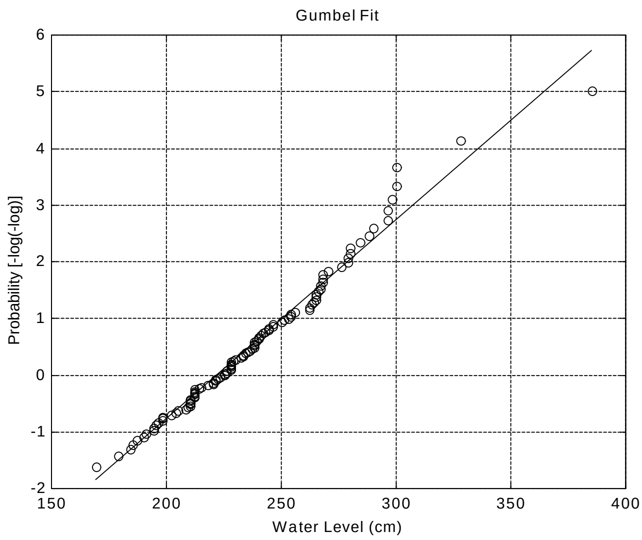


Figure 6.2: Graphical analysis of AM-data

6.4.1 Frequentistic analysis

With classical methods the calculation of the 10^{-4} quantile and its uncertainty will be presented. From Sec. 6.4, we assume the AM data to be a Gumbel distribution with cumulative distribution function $F(x|\lambda,\delta)=\exp(-\exp(-(x-\lambda)/\delta))$. The parameters λ and δ are estimated with a maximum likelihood method. This means (see Chapter 3) that the estimators for λ and δ will approximately (for large n) be normally distributed. Their means are asymptotically equal to the true parameters (i.e. are asymptotically unbiased). The asymptotic confidence interval at significance level α is given by Johnson et al. (1995):

$$\left(\frac{\lambda_{ML}-\lambda}{\delta}\right)^2 - 0.84556\left(\frac{\lambda_{ML}-\lambda}{\delta}\right)\left(\frac{\delta_{ML}-\delta}{\delta}\right) + 1.82367\left(\frac{\delta_{ML}-\delta}{\delta}\right)^2 \leq -\frac{2}{n}\log\alpha \qquad (6.2)$$

These are ellipses in the (λ,δ) domain.
Under the assumption of a Gumbel likelihood model we have:

$$E(x_{10,000}) = \lambda_{ML} - \delta_{ML}\log(-\log(1-p)) \qquad (6.3)$$

$$Var(x_{10,000}) = \frac{\delta^2}{n}\left\{1+\frac{6}{\pi^2}\{1-\gamma-\log(-\log p)\}^2\right\}$$

Here is $p=10^{-4}$ the probability that there is a flood in a given year, q=log(-log(1-p))=9.2103 and γ=0.5772.

With the AM-data we get $E(x_{10,000})=483.6$ cm and $var(x_{10,000})=393.7$ or $\sigma(x_{10,000})=19.8$ cm. The calculation of this 10^{-4} quantile and its uncertainty is based solely on the sea level data from the period since 1888. The data that we have from the period of 1500-1850 has not been used in this frequentistic analysis. That is why we perform a Bayesian analysis next.

6.4.2 Bayesian analysis

We continue the sea level analysis with a Bayesian approach in which the unknown parameters λ and δ are treated as random variabels. As argued in 6.4 we assume the likelihood model to be given by Gumbel:

$$f(x_i|\lambda,\delta)=(1/\delta)^n\exp(-(\Sigma x_i-\lambda)/\delta)\exp(-\Sigma\exp(-(x_i-\lambda)/\delta)) \qquad (6.4)$$

in which x_i (i=1..108) are the AM data.

We start the Bayesian analysis with vague (or uniform) prior distributions:

$$p(\lambda,\delta)=1/(\lambda_{max}-\lambda_{min}) \times 1/(\delta_{max}-\delta_{min}) \qquad (6.5)$$

i.e. two independent uniform distributed variabels between the wide bounderies $\lambda_{min}=100$, $\lambda_{max}=300$, $\delta_{min}=10$, $\delta_{max}=40$. The posterior distribution is then given with Bayes theorem by:

$$p(\lambda,\delta|x)=Cp(\lambda,\delta)\,(1/\delta)^n\exp(-(\Sigma x_i-\lambda)/\delta)\exp(-\Sigma\exp(-(x_i-\lambda)/\delta)) \qquad (6.6)$$

in which C is the normalisation constant such that p(λ,δ|x) integrates to 1.

The posterior predictive distribution of X can be calculated from:

$$P(X<x_B)=\int\int F(x_B|\lambda,\delta)p(\lambda,\delta|x)d\lambda d\delta \qquad (6.7)$$

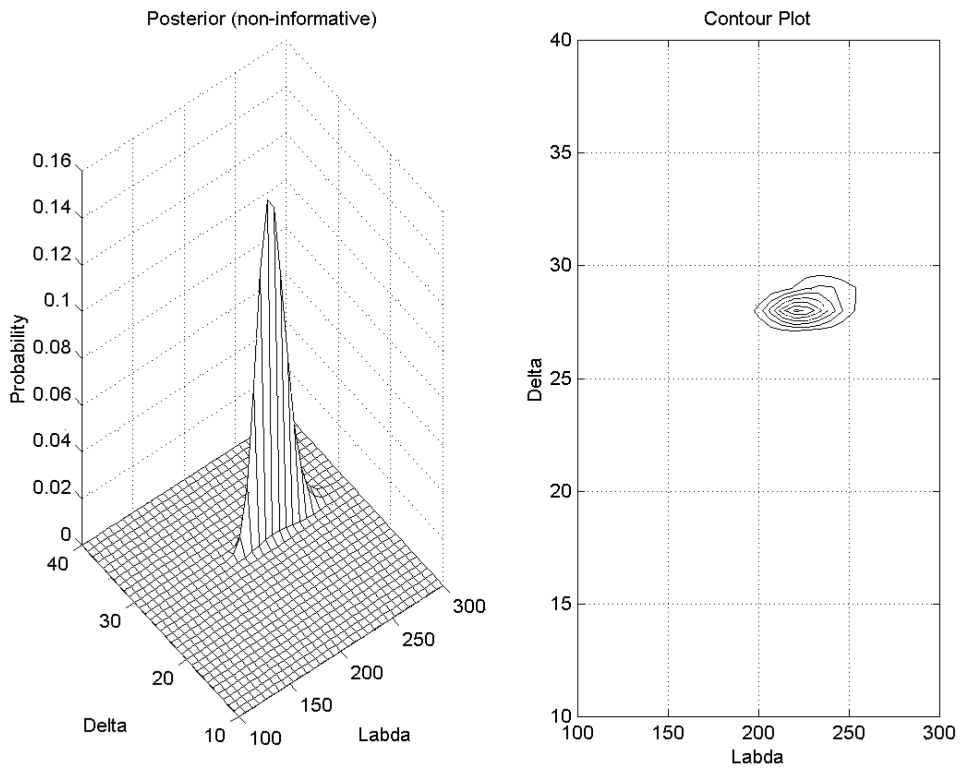


Figure 6.3: Posterior analysis

From the posterior predictive, $x_{10,000}$ follows from $P(X<x_{10,000})=1 - 10^{-4}$.
Figure 6.3 gives the results of this non-informative analysis. We obtain almost the same estimators for the parameters λ and δ as in the frequentistic method.
$E(x_{10,000})$=490.1cm and $\sigma(x_{10,000})$=21.7cm.

We now continue the Bayesian analysis with informative prior distributions. Bear in mind that we have historical flood data to our access. We will use this information to get prior information on the δ and λ parameters.

Let X_1, X_2, ..., X_{350} represent the annual maxima in year 1501, 1502, ... 1850. The r-th order statistic of this series is given by the r-th member in the ordered sequence $X_{1:350}$, $X_{2:350}$, ..., $X_{350:350}$, which is a rearranging of X_1, X_2, ..., X_{350} in increasing order. If we assume that the annual maxima of the water level at Hook of Holland is modelled by a Gumbel distribution then the distribution of the r-th order statistic is given by:

$$f_{X_{r:350}}(x)=350!/(r-1)!/(350-r)!F^{r-1}(x)[1-F(x)]^{350-r}f(x) \quad (6.8)$$

in which f(x) and F(x) are the Gumbel PDF and CDF respectively. We are interested in the joint probability distribution of $X_{341:350}$, $X_{342:350}$, ..., $X_{350:350}$, given by:

$$f_{X341:350,\ X342:350,...,\ X350:350}(x_1,x_2,...,x_{10})= \quad (6.10)$$
$$350!\Pi_{i=1}^{10}f(x_i)\Pi_{j=1}^{11}\ [F(x_j)-F(x_{j-1})]^{r(j)-r(j-1)-1}/(r(j)-r(j-1)-1)!$$

in which r(0)=0, r(1)=341, r(2)=342, ... r(10)=350, r(11)=351, $x_0=-\infty$, $x_1 = x_{341:350}$,..., $x_{10} = x_{350:350}$ and $x_{11}=\infty$.

On basis of the classification of the 10 historical observations, flood levels x_1, x_2,..., x_{10} are simulated from the normal distributions. With a maximum likelihood method (or Bayesian method with non-informative prior for λ and δ) the parameters λ and δ are estimated. We repeat this procedure until both parameters have reached its steady state. After 1000 simulations of x_1, x_2,..., x_{10} this is the case. If we make a plot of λ against δ (Fig. 6.4), we observe a high correlation between the two parameters (correlation coefficient = -0.9939).

We have $E(\lambda)=220.3$, $\sigma(\lambda)=28.4$, $E(\delta)=36.18$, $\sigma(\delta)=6.23$ and $E(x_{10,000})=553.6$cm; based on 10 extreme floods in a period of 350 years. We model both parameters by the bivariate normal distribution:

$$f_{\Lambda,\Delta}(\lambda,\delta) = (2\pi\sigma_\Lambda\sigma_\Delta(1-\rho^2)^{-½})^{-1}\ \exp\{-2(1-\rho^2)^{-1} \quad (6.11)$$
$$[((\lambda-\mu(\Lambda))/\sigma(\Lambda))^2 - 2\rho\ (\lambda-\mu(\Lambda))$$
$$(\delta-\mu(\Delta))/\sigma(\Lambda)\sigma(\Delta) + ((\delta-\mu(\Delta))/\sigma(\Delta))^2\ ]\}$$
$$\text{for } -\infty<\lambda<\infty,\ -\infty<\delta<\infty$$

With the informative prior as bivariate normal, we obtain the following results for the posterior: $E(\lambda)= 225.5$, $var(\lambda)= 8.7$, $E(\delta)= 34.70$, $var(\delta)= 0.76$, $\rho(\lambda,\delta)=-0.6385$ and $E(x_{10,000})=545.1$cm.

Furthermore, using $Var(x_{10,000})=var(\lambda+q\delta)=var(\lambda)+q^2var(\delta)+2q\rho(\lambda,\delta)\sigma_\lambda\sigma_\delta$ we find $\sigma(x_{10,000})=6.56$cm. Here is $p=10^{-4}$ the probability that there is a flood in a given year, q=log(-log(1-p))=9.2103. Note that the standard deviation of the quantile has become much smaller than the standard deviation that was found in the frequentistic analysis of the instrumental dataset ($\sigma(x_{10,000})=19.8$ cm). The weight which is given to the historical data is (due to Eqn. (6.10)) rather high. This is also clear from the role of the sea level data since 1888 on the 10^{-4} quantile which has little effect because the quantile decreased from 553.6 to 545.1cm only. It may indicate that the historical flood data and the observed sea levels do not form a homogeneous dataset.

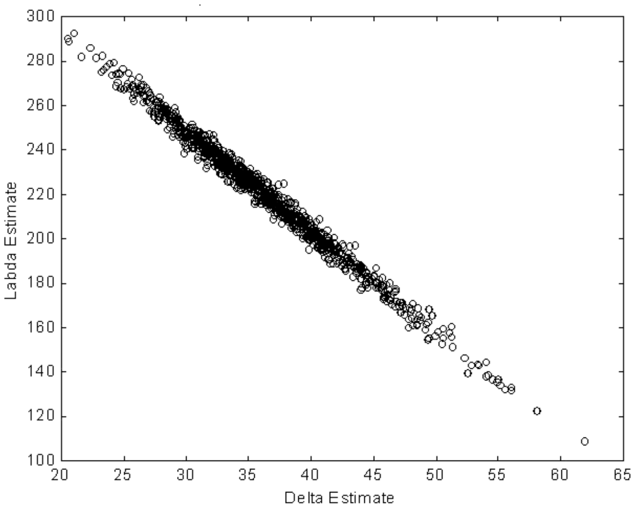


Figure 6.4: Dependency between distribution parameters

6.5 Model Peaks Over Threshold Data

6.5.1 The threshold

POT data can be obtained by selecting peaks of sea levels above a certain level (Buishand, 1989b). In this section some sort of stability criterion is suggested to determine the threshold. First of all the POT data is modeled by an exponential distribution $\lambda e^{-\lambda x}$ (other methods are described in Davison and Smith (1990) and Ekanayake and Cruise (1993)). Let x_j (j=1..n) denote the peak sea-levels above threshold T. Then we can calculate the 10^{-4} quantile as a function of T under the assumption of the exponential model.

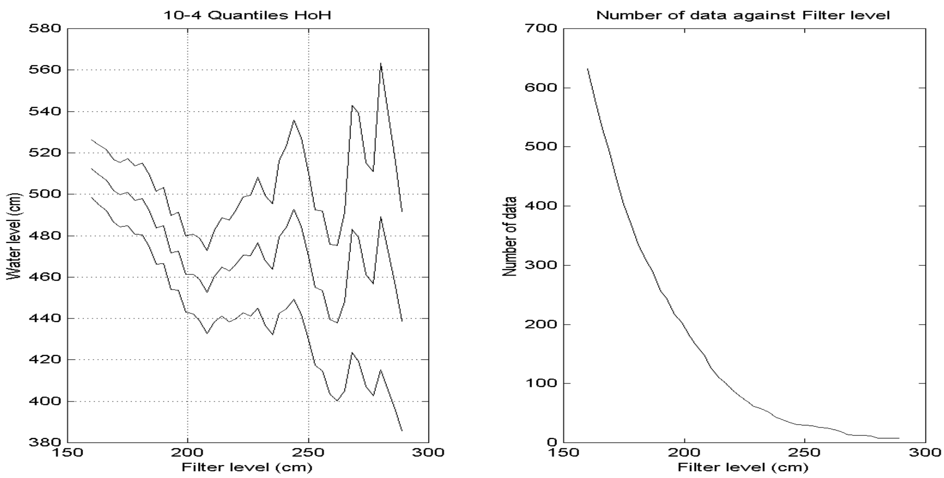


Figure 6.5: Threshold analysis

Figure 6.5 gives the results (including its standard deviation interval). At the value of the threshold level for which the 10^{-4} quantile remains more or less stable, we choose T. For the Hook of Holland data, our choice is T=200cm. The number of peaks over T for this choice is 195.

6.5.2 Frequentistic analysis

In contrast with the Gumbel model for the AM data, many analytic solutions can be found for the exponential model for the POT data. The ML-estimator for λ is given by $\lambda=n/(\Sigma_{i=1..n}\, x_i)$. The ML-estimator for the 10^{-4} quantile $x_{10,000,ML}$ follows from:

$$P(x>x_{10,000,ML})=\mu\exp(-\lambda x_{10,000,ML})=1/10.000 \text{ or} \qquad (6.12)$$
$$x_{10,000,ML}=\log(10.000\mu)*\Sigma_{i=1..n}\, x_i\,/n$$

In which $\mu=195/108=1.8$ is the average number of peaks in a year. The distribution of $x_{10,000,ML}$ can also be calculated analytically: From $x_{10,000,ML}=\log(10,000\mu)*\Sigma_{i=1..n}x_i/n$, and the fact that a summation of n exponential distributed stochasts, each with parameter λ is gamma distributed with parameters (n, λ), and that a constant k times a gamma distributed stochast with parameters (α,β) is also gamma distributed with parameters $(\alpha_1, \beta_1)=(\alpha,\beta/k)$, we have that $x_{10,000,ML}$ is gamma distributed with parameters $(n, n\lambda/\log10000\mu)$. So:

$$E(x_{10,000,ML})=(\log10000\mu)/\lambda \qquad (6.13)$$
$$Var(x_{10,000,ML})=(\log^2 10000\mu)/(\lambda^2 n).$$

6.5.3 Bayesian analysis

In contrast with the Gumbel model for the AM-data for which analytical solutions are very difficult to obtain, we can derive the Bayesian formulae of the exponential model very easily. The likelihood model is assumed to be given by an exponential model:

$$f(x|\lambda)=\lambda^n\exp(-\lambda\Sigma x_i) \qquad (6.14)$$

in which x_i (i=1..195) are the POT data. With a non-informative prior ($p(\lambda)=1/\lambda$), we obtain as posterior:

$$p(\lambda|x)=\lambda^{n-1}\exp(-\lambda\Sigma x_i)/\int_{\lambda=0..\infty} \lambda^{n-1}\exp(-\lambda\Sigma x_i)d\lambda \qquad (6.15)$$

This is a gamma distribution with parameters $(n,\Sigma_{i=1..n}x_i)$. The predictive distribution follows from:

$$P(x>x_{10,000,B})=\int_{\lambda=0..\infty}(1-F(x_{10,000,B}|\lambda))p(\lambda|x)d\lambda=(\Sigma_{i=1..n}x_i \,/\, (x_{10,000,B} + \Sigma_{i=1..n}x_i))^n \quad (6.16)$$

The Bayes estimator for the 10^{-4} quantile $(P(x>x_{10,000,B})=10^{-4})$ can be derived from Eqn. (6.16) and is given by:

$$x_{10,000,B}=(10.000^{1/n}-1)*\Sigma_{i=1..n}x_i \quad (6.17)$$

Note that for $n\to\infty$, $x_{10,000,B}\to x_{10,000,ML}$; because $n(10.000^{1/n}-1)\to\log 10.000$ $(n\to\infty)$.
To derive the posterior distribution for $x_{10,000,B}$, we observe that $\lambda\sim Ga(n,\Sigma_{i=1..n}x_i)$, so $1/\lambda\sim Ig(n,\Sigma_{i=1..n}x_i)$ in which Ig is the inverted gamma distribution. Furthermore, if $X\sim Ig(\alpha,\beta)$ then $kX\sim Ig(\alpha,k\beta)$. With

$$x_{10,000,B} = (1/\lambda)\log(10.000\mu) \quad (6.18)$$

it follows that:

$$x_{10,000,B} \sim Ig(n,\log(10.000\mu)\Sigma_{i=1..n}x_i) \quad (6.19)$$

We conclude:

$$E(x_{10,000,B})=n(10.000^{1/n}-1)(\Sigma_{i=1..n}x_i)/(n-1) \quad (6.20)$$
$$var(x_{10,000,B})=(n/(n-1))^2\,(10.000^{1/n}-1)(\Sigma_{i=1..n}x_i)^2/(n-2)$$

Figure 6.6 gives us the results of the ML-analysis and the non-informative Bayesian analysis. From the 195 data points, it is estimated that $\lambda=0.0375$. The 10^{-4} quantile is calculated in the frequentistic way to be $E(x_{10,000,ML})=461.6$ cm and $\sigma(x_{10,000,ML})=18.7$ cm. The non-informative Bayesian analysis almost gave the same results $E(x_{10,000,B})=462.9$ and $\sigma(x_{10,000,B})=19.1$cm.

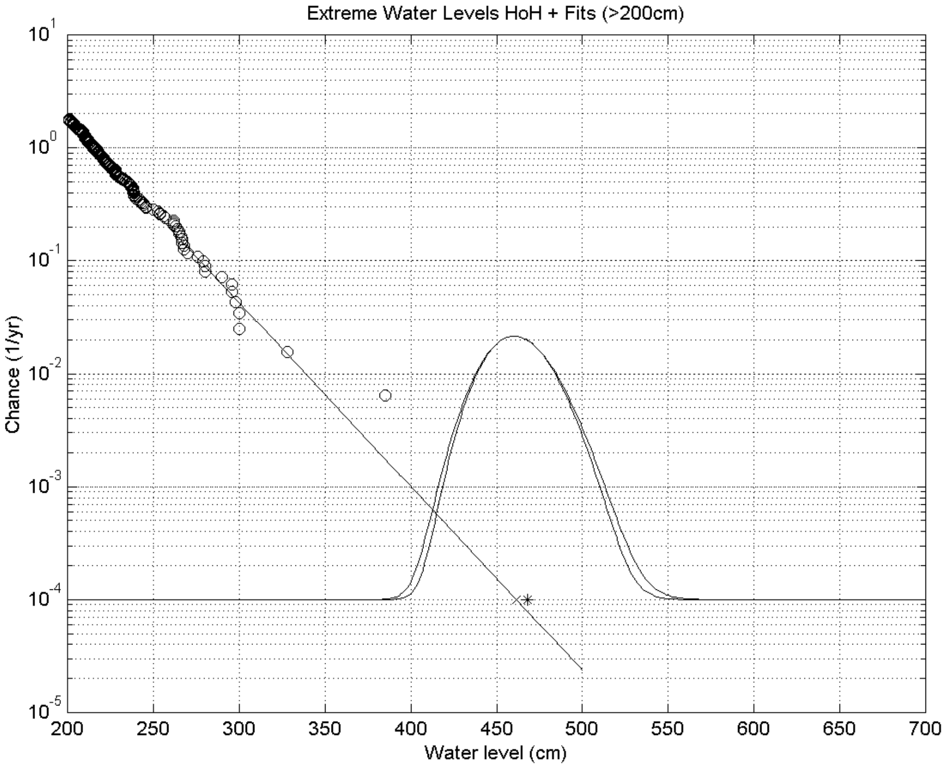


Figure 6.6: The exponential model

We continue the Bayesian analysis with informative prior distributions. In approximately 100 years, we have about 200 POT data (above 200cm). In the period 1500-1850, we therefore assume 700 POT data (in a sensitivity analysis performed later, it appeared that this amount of 700 was not very sensitive for the results). We have 10 POT data available during this period. In fact they are the 10 largest POT data, so we can use order statistics to estimate the most likely λ value if we assume an exponential parent distribution. We repeat this procedure 100 times and use the λ-estimates to fit a gamma distribution (which is conjugate with an exponential likelihood model). In Figure 6.7 the fit is shown. The prior distribution becomes $p(\lambda)=Ga(\lambda|\alpha,\beta)$, with $\alpha=183$ and $\beta=6392$. The conjugate posterior is also gamma distributed:

$$p(\lambda|x)=Ga(\lambda|\alpha+n,\beta+\Sigma_{i=1..n}x_i\ ),\text{ where } n=195 \text{ and } \Sigma_{i=1..n}x_i=5204. \quad (6.21)$$

In Figure 6.7, the prior distribution (right function), the posterior distribution with non-informative prior $Ga(\lambda|n,\Sigma_{i=1..n}x_i)$ (left function), and the posterior distribution with informative prior (middle function), which is the normalized product of both other functions, are shown. The conjugate posterior predictive is given by:

$$P(x<x_B) = \int_{\lambda=0..\infty} F(x_B|\lambda)p(\lambda|x)d\lambda \ = \int_{x=0..xB} Gg(x|\alpha+n,\beta+\Sigma_{i=1..n}x_i,1)dx \qquad (6.22)$$

in which Gg stands for the Gamma-gamma distribution:

$$Gg(x|a,b,n) = \frac{b^a}{\Gamma(a)}\frac{\Gamma(a+n)}{\Gamma(n)}\frac{x^{n-1}}{(b+x)^{a+n}} \qquad (6.23)$$

The distribution is generated by the mixture:

$$Gg(x|a,b,n) = \int_0^\infty Ga(x|n,l)Ga(l|a,b)dl \ . \qquad (6.24)$$

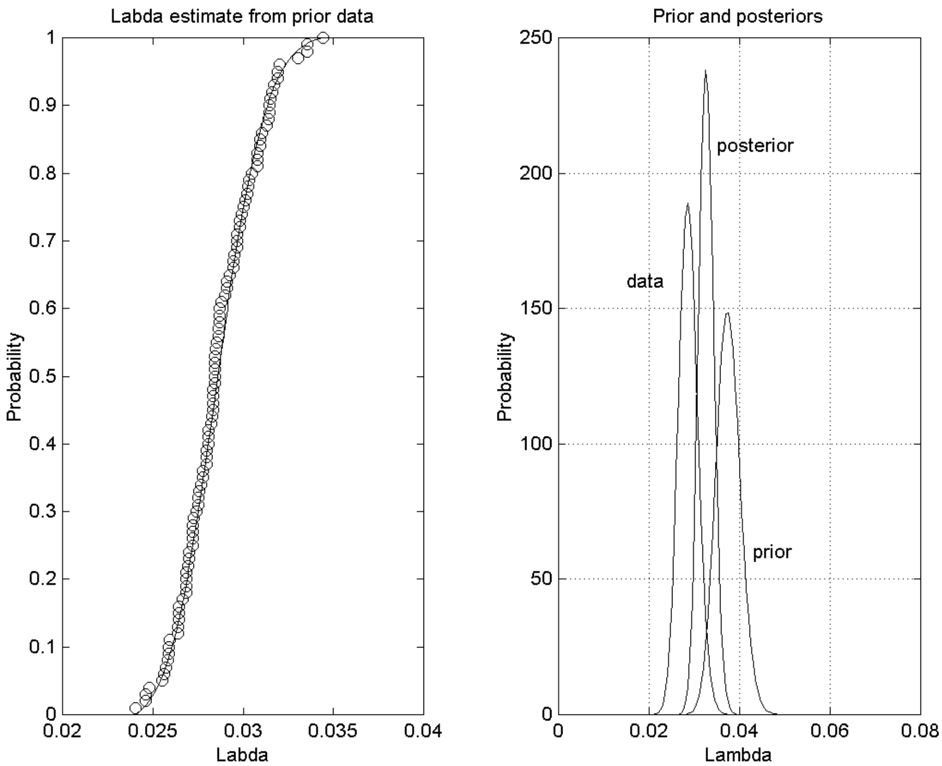


Figure 6.7: Bayesian analysis of the Labda parameter

We determine $x_{10,000}$ for which $P(x<x_{10,000})=1-10^{-4}$. We obtain 482.4cm. So notice that the influence of the amount of sea level data since 1888 on the 10^{-4} quantile is larger than in the Gumbel model for the AM data, because this time the quantile decreases from 521.4 to 482.4cm. See Figure 6.8 for the process of updating the prior beliefs with new data.

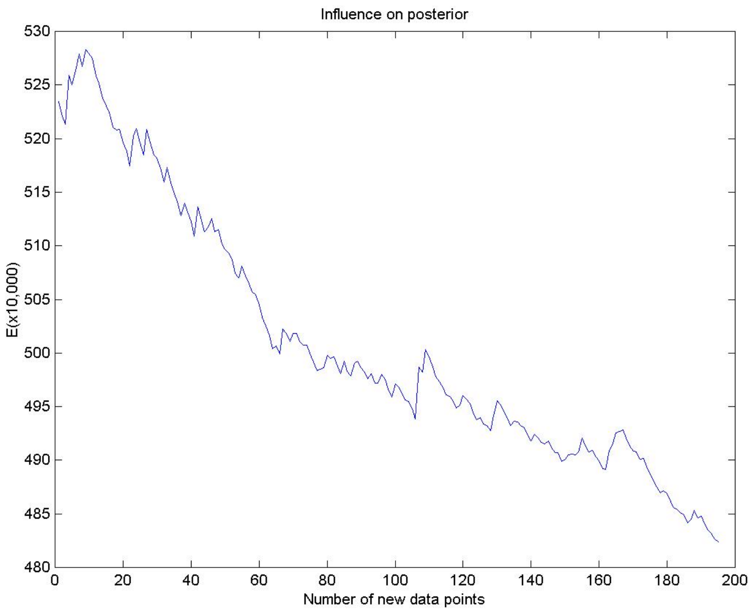


Figure 6.8: The updating process

6.6 Conclusions

In this chapter, we have examined the problem of determining the sea level with a return period of 10,000 years. The known existing statistical models for this problem use the accurate sea level measurements of the period from 1888 up till now. However, flood historical data has recently been published about the period 1500 - 1850. In order to take account of this information into the current statistical model a Bayesian approach has been used in this paper. An analysis has been made with a Gumbel model for the Annual Maxima data. The 10^{-4} quantile of the sea level increases significantly if the historical flood data is taken into account; from 490 to 545cm.

A second analysis has been made with an exponential model for the Peaks Over Threshold data. In this case the 10^{-4} quantile increases from 462 to 482cm. The particular choice for an AM model or a POT model and the choice for the distribution type (Gumbel, exponential, etc.) remains unanswered in this case study. The choice for a Bayesian framework, however, shows to be very useful when taking account of historical data in the analysis of extreme water levels.

The recent work of Buisman and Van Engelen (1995, 1996, and 1998) contains much historical information about the weather in North-Western Europe (especially the Netherlands, Belgium and the North-Western low-lying part of Germany) of three large time periods: until 1300, from 1300-1450, and from 1450-1575. These valuable collections of work are very suitable to be used in the frequency analysis of all weather related quantities.

Chapter 7

Wave heights frequency analysis with regional information

By far the mightiest of the forces arrayed against the
harbour barrier is the sea wave. This mysterious product
of wind and water is endowed with tremendous disruptive power.
It acts with all the magnificent impulse of a huge battering ram,
while, at the same time, it is equipped with the point of a
pick and the edge of a wedge. It is, in fact, one of the most complex,
the most volatile, the most pertinacious and the
most uncomprehensible of natural forces.

\- Brysson Cunningham, 1908.

7.1 Introduction

Some of the important elements to be considered by the coastal engineer when designing erosion control, scour protection, foundation, environmental and marine structures includes the determination of the significant and maximum wave height at the location of interest. Sea waves caused by the local wind, are often superimposed on swell moving in from a distance. Interaction between the two can cause unpredictably high waves and dangers for mariners. Extreme wave heights have an adverse effect on shipping and other marine operations. Once the significant wave height rises above 7 metres most vessels need to reduce speed to limit damage to cargo or structure.

Extreme wave heights have severe consequences for coastal structures. During the North Atlantic Halloween storm in 1991, the area of peak winds was several hundred miles in diameter, with buoy measurements of significant wave height in the 15 meter range and maximum wave height in the 30 meter range. How frequently the events of extreme wave heights may be expected to occur is of great importance. Design of civil engineering structures and insurance risk calculations, for instance, rely on knowledge of the occurrence frequency of these extreme events. Estimation of these frequencies is, however, difficult because extreme events are by definition rare and data records are often short. In other words: the uncertainties related to the distribution analysis of wave heights are high.

In Chapter 2 the general outlines of an uncertainty analysis were presented. It was shown that in most practical cases data is far too limited to give reliable estimates for the distribution function of wave heights, water levels, etc. The statistical uncertainty, i.e. the uncertainty of the parameter estimates due to the limited number of observations, has to be assessed. In that case besides the inherent uncertainty, the statistical uncertainty must be considered. For the statistical analysis of wave heights this was also noted by Earle and Baer (1982), Guedes Soares (1988), Bernier (1993), and Guedes Soares and Henriques (1996). The effect of the uncertainties in the wave heights on the design of coastal structures was investigated by Le Mehaute and Wang (1985), and the effect on the design of vertical breakwaters was reported by Van Gelder (1996b and 1997b).

For the short-term scales (a few hours), Longuet-Higgens (1952) already showed that the Rayleigh distribution is the most appropriate to describe the distribution of the wave heights. Without using a spectral description, various parameter estimation methods are available to determine the free parameter of the Rayleigh distribution. In Chapter 3, these parameter estimation methods were compared with eachother w.r.t. relative bias and root mean squared error of the p-quantile (p<<1). In Van Gelder and Vrijling (1999) the performance of the parameter estimation methods for the Rayleigh distribution has been investigated. Green (1994) adapted the Rayleigh distribution to take account for wave breaking in shallow waters. Rodriguez et al. (1999) study the uncertainty of various sea state parameters, under which the significant wave height, resulting from the methods of spectral estimation. The use of the spectral estimator $4\sqrt{m_0}$ in which m_0 is the 0^{th} order spectral moment $\int_0^\infty S(f)df$ appears to be a robust estimator, in the sense that it does not differ much with the choice of the estimation method of *S(f)*. Three different spectral estimation methods, Blackman-Tukey, Fast Fourier Transform and Maximum Entropy give very similar estimation results (Rodriguez et al., 1999).

Wave height distributions on long term scales (years) have been intensively investigated by numerous authors. Battjes (1970) noticed that symmetric distributions, such as the normal distribution, were not suitable to describe the long-term distribution for the wave heights. Skewed distributions, such as the Gumbel and Weibull distribution, fitted much better. To the same conclusions came Teng et al. (1993) and Teng and Palao (1996), who analyzed wave buoy data at the Pacific and Atlantic Oceans, Goda and Kobune (1990) with wave data from the Sea of Japan and the East China Sea, Rossouw (1988) with wave data from the Indian Ocean, Van Vledder et al. (1993) with wave data from the Norwegian coast, and Burcharth and Liu (1994) with data from the Meditarenean Sea. Mathiesen et al. (1994) and Goda et al. (1993) developed in an IAHR Working Group a recommended practice for extreme wave analysis. The 3-parameter Weibull distribution was advised by them.

Ferreira and Guedes Soares (1999) investigate the long-term distribution with the Beta and Gamma models. They show that these models are very flexible to cover the three types (Sec. 3.9.2) of tail behaviour.

Guedes Soares and Ferreira (1995) propose a model to describe the long-term wave height distribution, which is achieved by averaging past distribution functions, or, in case a priori distribution functions for the model parameters are given, by conditioning over their possible values. In this way they take account for the long-term time-varying character of the significant wave height data. A similar approach is being adopted in this chapter, but to take account for the space-varying character of the significant wave height data.

In this chapter, the regional frequency approach (RFA) will be applied in order to investigate the distribution function for the wave heights on long-term scale. The main idea behind the RFA was to 'trade space for time' (Chapter 4). It does so by using data from several sites, which are judged to have frequency distributions similar to the site of interest, in estimating event frequencies at that site. Analysis of sea levels (such as mean levels, tides, extremes and currents) and wave heights is currently restricted to data from a relatively sparse network of coastal gauges (containing long records of up to 100 years) and a few offshore gauges (containing short records of up to 25 years). Recent increases in theoretical models have enabled significant improvements in predicting occurrence frequencies of the extremes. For instance the so-called max-stable extreme value models (Coles, 1993) offer a suitable basis for analysing extremal characteristics of environmental processes with a spatial dimension. However, these models will require a considerable extension and development in form to handle (i) the high spatial dimensionality of the problem, (ii) knowledge of covariates and directionality, (iii) trends and seasonality of marginal and spatial (such as changes in occurrence of storm types) extreme values. Therefore, in this paper the aim was to use extreme value models based on other methods. A simple regional frequency approach based on L-moments, including as much of the available data as possible, is proposed to analyze the extreme value distributions of the coastal data.

Surprisingly, the regional frequency approach is almost solely applied within the area of hydrology. However, within other areas, such as coastal engineering, the approach can be applied satisfactorily as well. Van Gelder et al. (1999e) showed how hydraulic boundary conditions along coast lines, such as storm surge levels and wave heights, can be determined with a regional frequency approach and applied the results to a case study of the Dutch Petten Sea Dike.

In Van Gelder et al. (1999f), annual maximum sea level data along the North Sea coasts obtained from locations along the German coast (Leichtweiss-Institut für

Wasserbau), the Belgian coast (Afdeling Waterwegen Kust-Hydrografie, Oostende), the French coast (French naval hydrographic and oceanographic service, Brest), the Dutch and English coasts (Rijkswaterstaat RIKZ/ITB, Den Haag), and the Danish coast (Kystinspektoratet, Lemvig) were analyzed in a regional framework. Tides were not included in the analysis. The sea surface height variations due to tides are on the shallow North Sea, even as the wind-driven set-up quite large. Tidal heights should be added to the wind-driven heights at any given time to provide the total height. The sea levels along the coast lines are influenced by the depth of the continental shelf. There are many more factors that can cause very local effects along the coast. For example, the local coastline shape and bathymetry can produce large deviations from the sea levels across the shelf. The winds at the coast will also play an important role. Local coastal winds may have large deviations on the sea levels. From the available sea level measurement stations, the Northern stations were selected by the regionalized frequency algorithm to form a moderately homogeneous region for the analysis of extreme sea levels. Also from a physical point of view these sites gave the best resemblance. The southern stations along the French and English coasts appeared to be discordant in comparison with the Northern stations. For the Northern region the Generalized Logistic Distribution came out as the optimal choice for the regional frequency distribution of the extreme sea levels. This distribution function had its origin in the biological sciences, but is presently also used within hydrological fields by other authors (Hosking and Wallis, 1997).

This chapter presents a case study of extreme wave height prediction on the Dutch North Sea. First, data has been collected as peaks over threshold data sets of wave heights from nine North Sea locations along the Dutch coast. It appeared that the locations except for one can be very well considered as a homogeneous region and that the optimal distribution function to describe the occurrence frequency of extreme wave heights is given by the Generalized Pareto distribution.

In this chapter the available data sets will be described first. Then the results of the RFA will be presented, followed by the conclusions.

7.2 The data sets

Much information on wave heights has been described by "The WASA Group" (1998). In WASA it was found that the storm and wave climate has roughened in recent decades, but that the present intensity of the storm and wave climate seems comparable with that at the beginning of the 20th century. The WASA project furthermore analysed and used the output from a high-resolution (T106 spectral truncation) climate change scenario experiment, mimicking global warming due to increase greenhouse gas concentrations. It was found that storm and extreme wave activity was slightly increased in the Bay of Biscay and in the North Sea in a warmer climate, while this activity was slightly weakened at several other places. The experimental set-up of the climate model simulations on which these results were based has been described by Beersma et al. (1997).

Wave heights are measured at many locations in the world seas. Only in the North Sea there are hundreds of stations. They are indicated by the small triangles in Figure 7.1. From the following measurement stations on the Dutch North Sea, data was made available by the Dutch Ministry of Transport, Public Works and Water Management (RIKZ):

Station Name	Location in Northern Width and Eastern Length.	Acronym
Eierlandse gat (Wadden)	N531637.0, E0043942.0	ELD
Europlatform	N515955.0, E0031635.0	EUR
K13 Alpha	N531304.0, E0031313.0	K13
Lichteiland, Goeree	N515533.0, E0034011.0	LEG
Meetpost Noordwijk	N521626.0, E0041746.0	MPN
Munitiestortplaats IJm	N523330.0, E0040330.0	YM6
Scheur West	N512332.0, E0030257.0	SCW
Schiermonnikoog (Wadden)	N533544.0, E0061000.0	SON
Schouwenbank	N514445.0, E0031818.0	SWB

Table 7.1: Locations of wave height measurements

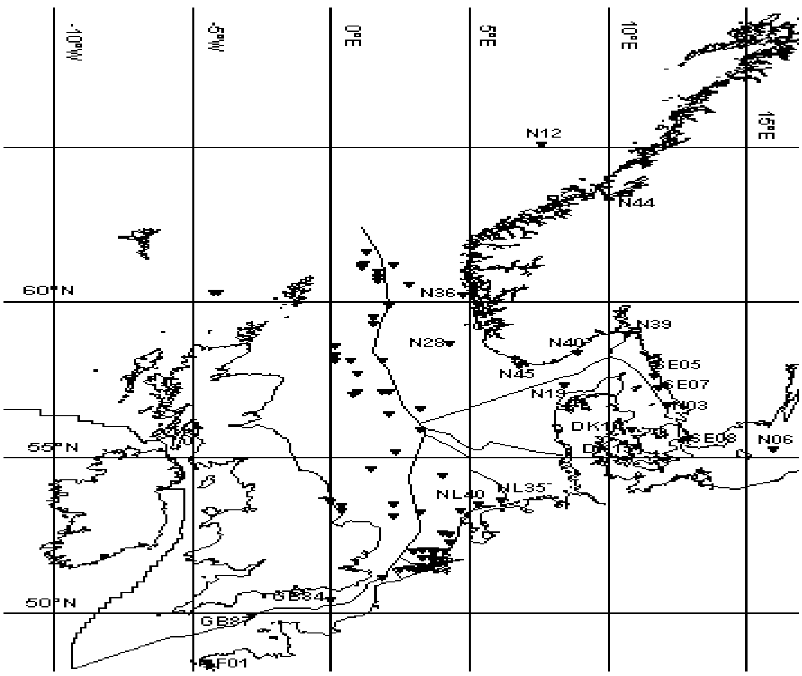


Figure 7.1: Measurement Stations at the North Sea

The following information is available at the nine sites:

- date (yyyymmdd)
- time (hhmm : UT)
- wave height H_{m0} (m)

The data are peaks over tresholds. They are made i.i.d. by using a filter of 48 hours. Each site contains between 20 to 70 peaks over the period 1979-1993. As stressed by Castillo and Sarabia (1992), and Ferreira and Guedes Soares (1998) peaks over thresholds provide a modern and soundly based input for extrapolation problems. A regionalized frequency analysis of the peaks over threshold data of the nine stations will be performed next.

7.3 RFA of wave heights

A RFA of the extreme wave heights along the Dutch coast will be performed using POT (Peaks over Threshold) data at various sites along the Dutch coast. The objective is to identify regional probability distributions for the extreme wave heights.

For each site the following characteristics were calculated:

SITE	N	ACRON	L-CV	L-SKEW	L-KURT	D(I)	MD(I)
1	23	SCW	.0720	.1191	-.0316	1.96	5.21
2	24	MPN	.0427	-.0510	.0725	2.43	6.47*
3	47	SWB	.0485	.2892	.1423	1.35	3.61
4	33	LEG	.0538	.1924	.0467	.90	2.39
5	52	ELD	.0630	.2246	.1082	.11	.29
6	67	EUR	.0606	.2393	.1477	.45	1.19
7	72	K13	.0613	.2756	.1139	.17	.44
8	58	SON	.0692	.2635	.1509	1.23	3.29
9	61	YM6	.0673	.2876	.0780	.42	1.11

Table 7.2: Characteristics of the nine sites
(N: Number of peaks, D(I): Wilks measure, MD(I): Mahalanobis distance)

The weighted means (based on the record-lengths) of L-CV, L-Skewness and L-Kurtosis are given by: 0.0609, 0.2330 and 0.1064 respectively. The number of sites should be at least 3 times larger than the dimensionality +1 in order to use the robust discordancy measures from Chapter 4. Because the dimensionality is 3 (L-CV, L-Skewness and L-Kurtosis), the number of sites should be at least 10. However, if we neglect this requirement for the moment then the station MPN should be considered as discordant according to the Mahalanobis distance (3D(i)=6.47; the 90% critical value is 6.25).

The L-moment diagrams for the data are shown in Figure 7.2. The figure also shows the unusual sites (outside the inner (1-sigma) or outer (2-sigma) ellipse) whose data need a closer examination; i.e. those sites whose L-moments are notably different from those of the other sites in the data set.

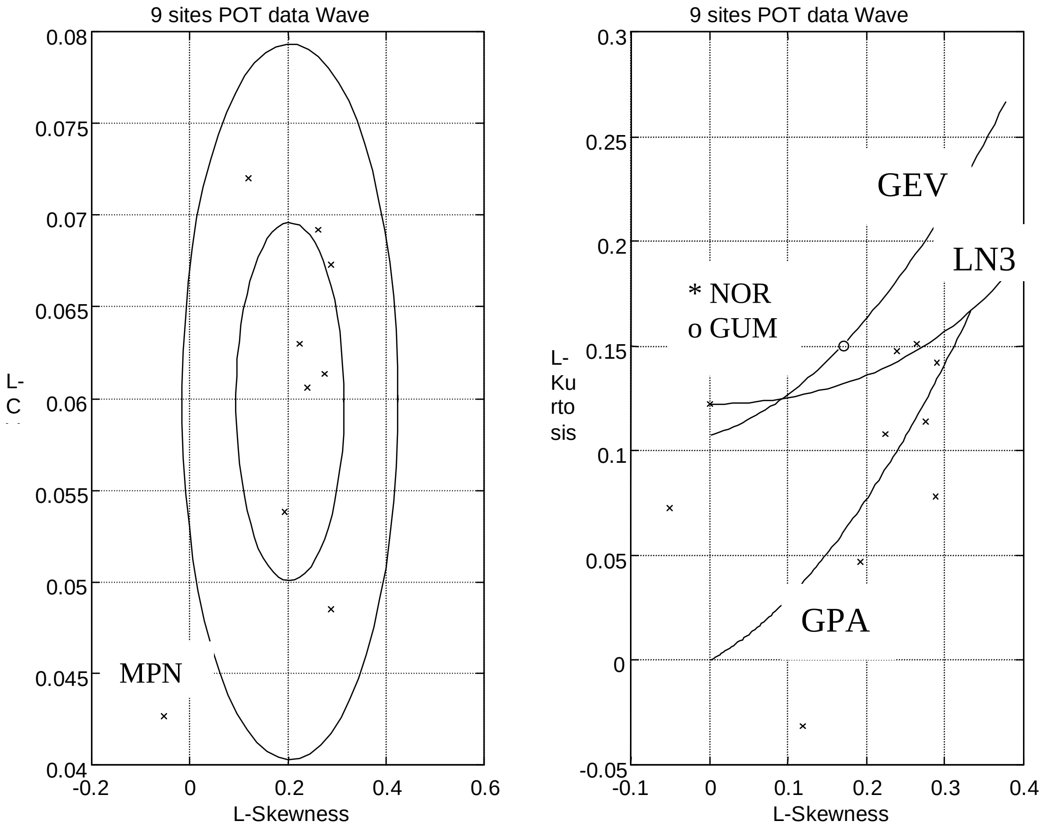


Figure 7.2: L-Moment diagrams Wave Heights Data Sets

Site MPN is the left-lower cross in Figure 7.2 (left)). It is far outside the 2-sigma ellipse. Together with the rather high Mahalanobis distance of MPN, it was decided to exclude MPN from the RFA. Thus the RFA was performed for the remaining 8 platforms. In general, a RFA consists of the following three steps:

i) the Wilks discordancy measure for outliers
ii) the heterogeneity measures
iii) the goodness-of-fit measures

From step i), it followed that MPN was a discordant site. From step ii), the following output was generated:

```
OBSERVED     S.D. OF GROUP L-CV          =   .0078
```

```
SIM. MEAN OF S.D. OF GROUP L-CV          =   .0062
SIM. S.D. OF S.D. OF GROUP L-CV          =   .0014
STANDARDIZED TEST VALUE H(1)             =  1.13  *

OBSERVED AVE.  OF L-CV / L-SKEW DISTANCE =   .0518
SIM. MEAN OF AVE. L-CV / L-SKEW DISTANCE =   .0469
SIM. S.D. OF AVE. L-CV / L-SKEW DISTANCE =   .0122
STANDARDIZED TEST VALUE H(2)             =   .41

OBSERVED AVE.  OF L-SKEW/L-KURT DISTANCE =   .0681
SIM. MEAN OF AVE. L-SKEW/L-KURT DISTANCE =   .0644
SIM. S.D. OF AVE. L-SKEW/L-KURT DISTANCE =   .0136
STANDARDIZED TEST VALUE H(3)             =   .27
```

Table 7.3: Heterogeneity measures (number of simulations 500)

These results indicate that the remaining 8 sites of the data set are acceptably homogeneous according to the H(1) statistics (Sec. 4.5). The H(2) and H(3) statistics based on V2 and V3 respectively lack power to discriminate between homogeneous and heterogeneous regions according to Hosking and Wallis (1997).

In step iii), five three-parameter distributions (generalized logistic, generalized extreme value, generalized Pareto, lognormal (LN3), Pearson type III) were fitted to the region. Table 7.4 shows that the generalized Pareto is selected as the optimal distribution function, according to the Hosking and Wallis (1997) goodness-of-fit measure. The following goodness of fit measures were obtained:

```
GEN. LOGISTIC         L-KURTOSIS=  .212   Z VALUE=  6.21
GEN. EXTREME VALUE    L-KURTOSIS=  .178   Z VALUE=  4.23
GEN. NORMAL           L-KURTOSIS=  .165   Z VALUE=  3.48
PEARSON TYPE III      L-KURTOSIS=  .141   Z VALUE=  2.10
GEN. PARETO           L-KURTOSIS=  .097   Z VALUE=  -.52 *
```

Table 7.4: Goodness of fit measures (number of simulations 500)

In the data preparation phase (Sec. 7.2), peaks over thresholds had been selected. Exceedances over high thresholds are often modelled by fitting a generalized Pareto distribution (GPD) to this part of the data (Castillo and Hadi, 1997). This is particularly useful in presence of heavy tails. One difficult aspect is the selection of the threshold, above which the GPD assumption is valid. This is often done by repeated choice of the threshold below which data are not considered. As shown by Haug et al. (2000), tail estimation with the Generalised Pareto Distribution is also possible without threshold selection. A mixture model, where one term of the mixture is the GPD, and the other is a light-tailed density distribution may be a solution. The two components are put together by means of a continuous weight function that, in some way, takes the role of automatic threshold selection. The full data set is used for inference. Maximum likelihood provides estimates with approximate standard deviations for all parameters of the model, including those present in the weight function.

7.4 Regional parameters and quantiles versus at-site parameters and quantiles

The regional frequency algorithm gives the following output (Tables 7.5 and 7.6):

```
REGIONAL PARAMETER ESTIMATES FOR DISTRIBUTIONS ACCEPTED AT THE 90% LEVEL
 GEN. PARETO           .863   .170   .244
 WAKEBY                .861   .045  1.574   .141  -.164

 REGIONAL QUANTILE ESTIMATES
.010   .020   .050   .100   .200   .500   .900   .950   .990   .999
 GEN. PARETO
.865   .867   .872   .881   .900   .972  1.163  1.225  1.333  1.431
 WAKEBY
.863   .865   .871   .880   .901   .973  1.160  1.224  1.346  1.473

REGIONAL AVERAGE
L-MOMENT RATIOS   1.0000    .0619    .2496    .1083    .0572
```

Table 7.5: Regional parameter, **-** quantile estimates and regional averages

Apart from the parameter estimates for the GPA, also the parameter estimates for the 5-parameter Wakeby distribution are given in Table 7.5. The differences between GPA and Wakeby remain very small. From the regional quantile estimates, the quantile estimates per site can be determined by multiplying the regional quantile estimates with the at-site average values. The results are shown in Table 7.6.

SITE NUMBER	QUANTILES									
	.0100	.1000	.5000	.8000	.9000	.9500	.9800	.9900	.9990	.9999
1 SCW (μ=23/15)	338.53	344.56	379.33	425.96	455.95	482.03	511.37	530.24	577.00	606.42
2 SWB (μ=47/15)	390.24	397.20	437.28	491.03	525.60	555.66	589.49	611.24	665.14	699.06
3 LEG (μ=33/15)	447.66	455.64	501.62	563.28	602.93	637.42	676.23	701.18	763.01	801.91
4 ELD (μ=52/15)	488.24	496.94	547.09	614.34	657.59	695.20	737.53	764.74	832.17	874.61
5 EUR (μ=67/15)	414.71	422.10	464.69	521.82	558.55	590.50	626.45	649.57	706.84	742.89
6 K13 (μ=72/15)	477.07	485.57	534.57	600.28	642.54	679.30	720.65	747.24	813.13	854.60

7 SON	(μ=58/15)								
469.60	477.97	526.20	590.89	632.48	668.66	709.37	735.55	800.40	841.22
8 YM6	(μ=61/15)								
452.66	460.73	507.22	569.57	609.67	644.54	683.78	709.01	771.53	810.88

Table 7.6: Regional quantile estimation per site

The platform wave data is in centimetres. The RFA-based estimations can now be compared with the at-site estimations (Tables 7.7 and 7.8).

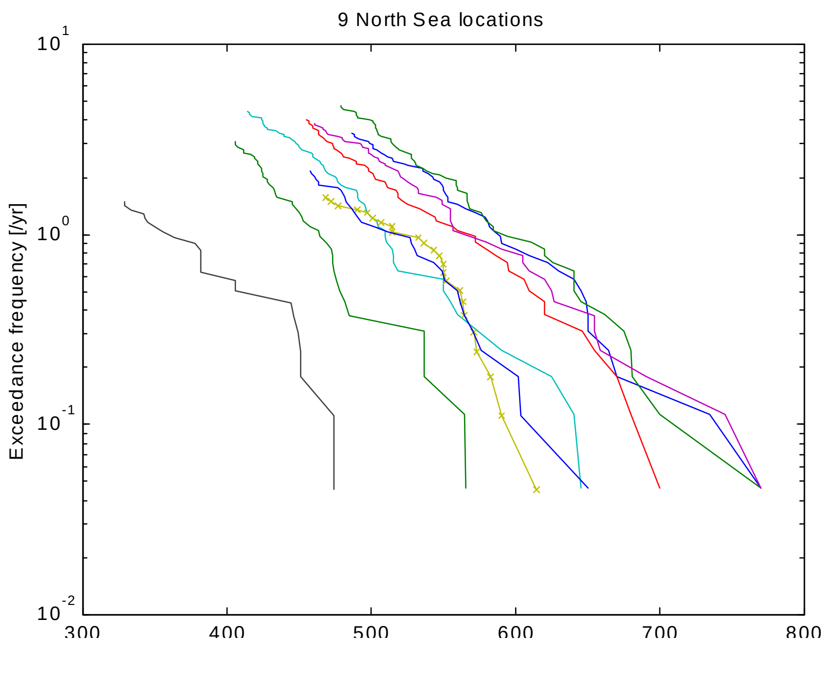
	location	scale	shape
1 SCW	318.7178	114.1414	.5744
2 SWB	405.0237	50.6725	.1027
3 LEG	451.8168	88.7547	.3547
4 ELD	483.6249	102.0713	.2663
5 EUR	414.5443	79.4354	.2276
6 K13	479.1845	81.9333	.1358
7 SON	461.3438	94.8344	.1658
8 YM6	448.9252	82.0869	.1064

Table 7.7: At-site parameter estimations of the GPA based on L-Moments

SITE NUMBER				AT-SITE QUANTILES					
.0100	.1000	.5000	.8000	.9000	.9500	.9800	.9900	.9990	.9999
1 SCW	(μ=23/15)								
319.86	330.39	383.98	438.60	464.49	481.88	496.43	503.33	513.68	516.44
2 SWB	(μ=47/15)								
405.53	410.33	438.93	480.20	508.94	535.70	568.29	590.98	655.74	706.88
3 LEG	(μ=33/15)								
452.71	461.00	506.36	560.66	591.47	615.58	639.57	653.19	680.46	692.51
4 ELD	(μ=52/15)								
484.65	494.23	548.23	617.23	659.32	694.31	731.68	754.48	806.02	833.93
5 EUR	(μ=67/15)								
415.34	422.81	465.48	521.59	556.91	587.07	620.30	641.21	691.13	720.69
6 K13	(μ=72/15)								
480.01	487.76	533.38	597.63	641.19	680.83	727.82	759.68	846.35	909.74
7 SON	(μ=58/15)								
462.30	471.25	523.44	595.30	642.86	685.25	734.30	766.76	851.34	909.08
8 YM6	(μ=61/15)								
449.75	457.53	503.78	570.34	616.56	659.48	711.58	747.75	850.42	930.77

Table 7.8: At-site quantile estimations of the GPA based on L-Moments

Note the differences between the regional and at-site estimates which can be in the order of 100 cm for the 10^{-4} quantile. The results from the tables can also be presented



by the following figures.

Figure 7.3: The frequency exceedance curves for the nine locations (MPN with crosses)

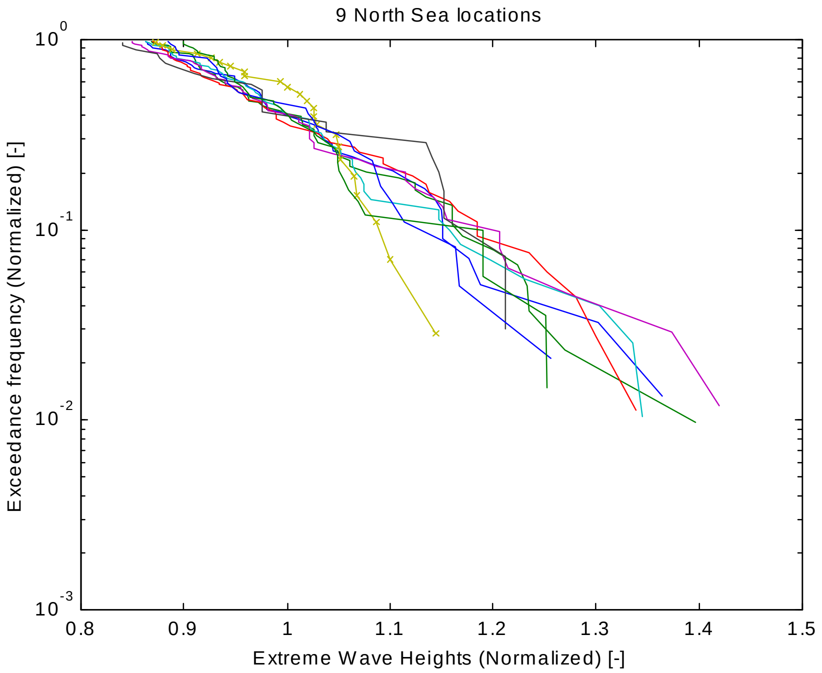


Figure 7.4: The frequency exceedance curves for the nine locations (normalized);
Site MPN with crosses

From figure 7.3 it is difficult to notice the discordant behaviour of site MPN. However, after normalization (figure 7.4), the discordancy becomes clear. From figure 7.5 it can be seen that indeed the GPA gives a satisfiable regional fit. It has a curvature downwards, indicating that there should be some maximum credible normalized extreme wave height.

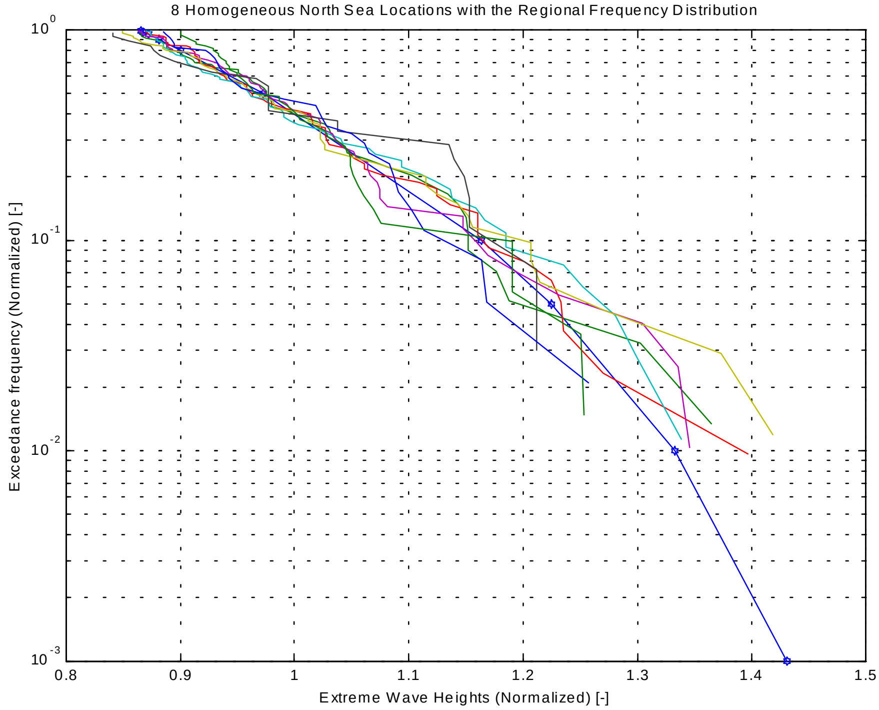


Figure 7.5: The regional frequency distribution: Generalized Pareto.

7.5 Conclusions

A Generalized Pareto Distribution appears to be the optimal regional fit for the extreme wave heights at the North Sea. Differences between the at-site quantile estimates and the regional quantile estimates can be quite high (up to 1m for the extreme extrapolations). It is better to rely on the regional quantile estimates for decision making, as is shown in Hosking and Wallis, 1997. For further investigation, it would be good to extend the data base with measurements from other stations in the neighbourhood (Van Gelder et al., 1999a). Nowadays, due to the Internet, it is relative easy to obtain wave height measurement at almost every desired location in the world. For instance, the National Data Buoy Center at http://www.ndbc.noaa.gov/index, the Coastal Data Information Program at http://cdip.ucsd.edu/ and the Prototype Measurement and Analysis Branch of the WES at http://sandbar.wes.army.mil/ provide many data sets from buoy measurements. Satelite observations of wave heights with the TOPEX/Poseidon Satellite are available from

http://podaac.jpl.nasa.gov/topex/

At this http address animation files of significant wave heights of every three days of the last 5 years at every location at the world seas and oceans are free downloadable.

Adding more sites to the existing data base of 9 stations on the Dutch North Sea may result in more accurate predictions of the quantiles. In that case also the alternative more advanced method (based on the robust distances as presented in Chapter 4) can be used in the discordancy analysis of the sites.

Finally, intersite correlations are certainly present in this case study. Hosking and Wallis (1988), however, mentioned that their RFA-procedure is quite robust against intersite correlations in case of low heterogeneity measures. In the present case study, a heterogeneity measure of 1.13 was obtained, which indicates a low value of heterogeneity, and therefore intersite correlations will have no major disturbance on the estimates.

The regional frequency analysis RFA in this chapter deals with peaks over threshold data. In this approach there are two problems that do not occur in an RFA of annual maxima:

(i) The L-CV or the coefficient of variation of the peaks generally changes with the height of the threshold (see also Buishand, 1990). The choice of the threshold for the various sites thus strongly determines how far the region is homogeneous.

(ii) The peaks have a well-defined lower bound, namely the height of the threshold. For instance, for the Generalized Pareto distribution GPD one should set the location parameter ξ equal to the height of the threshold, or equivalently, one should fit a two-parameter distribution to the excesses. There is a similar parameter constraint for the other distributions in Table 7.4. Since the mean of the peaks generally varies over the region, there remains in fact only one parameter that may be constant over the region. If the peaks in the peaks over threshold model follow a GPD, then it is possible to re-express the mean number of threshold exceedances and the GPD parameters in terms of the parameters of the associated GEV distribution for the annual maxima (Coles, 1994). This reparametrization offers more possibilities for regionalization (e.g. the fact that two parameters may be constant over the region).

Chapter 8

Conclusions and Recommendations

Non scholae, sed vitae discimus.
- Seneca.

In this thesis statistical methods for the risk-based design of civil structures have been tested, compared, and extended. For that purpose, the thesis has been divided into four main topics:
- the structuring and analysis of various kinds of uncertainties,
- the treatment of the statistical estimation methods,
- the analysis of the homogeneity aspects of data,
- the investigation of design philosophies of civil structures.

At the end of each chapter a detailed discussion is provided about the conclusions of that chapter. Here, more general conclusions will be presented.

Uncertainties in decision and risk analysis can primarily be divided in two categories: uncertainties that stem from variability in observable populations and therefore represent randomness in samples (inherent uncertainty), and uncertainties that come from basic lack of knowledge of fundamental phenomena (epistemic uncertainty). Inherent uncertainties represent randomness or the variations in nature. For example, even with a long history of data, one cannot predict the maximum water level that will occur in, for instance, the coming year at the North Sea. It is not possible to reduce inherent uncertainties. Epistemic uncertainties are caused by lack of knowledge of all the causes and effects in physical systems, or by lack of sufficient data. It might be possible to obtain the type of the probability distribution, or the exact model of a physical system, when enough research could and would be done. Epistemic uncertainties may change as knowledge increases. In this thesis the inherent uncertainty and epistemic uncertainty is subdivided into five types of uncertainty: inherent uncertainty in time and in space; parameter uncertainty and distribution type uncertainty; and model uncertainty. Looking forward to the objectives of this thesis, also uncertainties with respect to the design of flood protection structures are considered. Uncertainties such as construction cost uncertainties, damage cost uncertainties and financial uncertainties are examples of these uncertainties and are classified as model uncertainties. In Chapter 2 the treatment of the various uncertainties have been illustrated by a hypothetical example of a flood protection structure. A method to analyse the effect of inherent uncertainty in time and space on the probability model with respect to the failure probability and the failure rate has been given. The method is a very practical and simple method to get insight in the problems concerned with the lack of information in hydraulic engineering models. It shows that the more uncertainty is expected to be reduced in the future, the higher the mean and the larger the standard deviation of the distribution of the reliability index will be. The method also works for complex limit-state functions with a large number of variables. Furthermore it was shown that the treatment of the uncertainty in the resistance is not so important, because of the insensitivity of the failure probability to the resistance uncertainty. On the other hand the uncertainty in the loads should be treated carefully. Almost always these uncertainties lead to a larger failure probability for a given design.

The estimation of quantiles from available records has been an area of extensive research during the last decades. However, the large number of distributions and estimation methods that have been proposed in the literature has led to a state of confusion, and a gap exists between theory and practice. In order to bridge that gap, this thesis surveys, tests, compares and extends current methodologies, and recommends a number of statistical distributions and estimation procedures. Every statistical estimation method is attended by some degree of uncertainty due to factors which were investigated in this thesis. A best estimate may be defined as one that produces an unbiased and minimum-variance estimate in the objective function, defined as the function that is critical in the application. In this thesis it is shown that if the object function is the net expected cost for which underdesign and overdesign are each appropriately penalised (with the use of the in this thesis introduced modified Linex loss functions), minimising net expected cost will automatically provide an optimal design. A Bayesian framework is developed for that purpose.

This thesis may be a step towards the clarification of the correct quantile estimation. The proposal to use Monte Carlo simulations added with analytical derivations gives the broad lines of a possible comparison strategy. Various Monte Carlo simulations to judge the performance of a certain combination of estimation method and probability distribution have been performed in this thesis. It is however impossible to conclude which method performs in general the best. Such a conclusion depends on too many factors. One of the important factors is the amount of inhomogeneity that is contained in the data. In this respect the use of the so-called L-Moment estimators appears to be very promising. Its robustness against contaminated data and its possibility to define model selection criteria can be considered as advantageous. These issues were covered in detail by Chapter 3.

In areas where datasets are generated by more than one distinct process (e.g. snow-melt and rainfall generated peaks), the data should be considered to be drawn from subpopulations with different statistical characteristics. In the thesis a method is proposed for the recognition and incorporation of the distinct characteristics of the generating processes. When the datasets are produced by two independent processes characterised by functions F1 and F2, an overall distribution F can be derived. The approach is based on simple multivariate techniques and it is applied to a case study of wave height analysis.

The homogeneity aspect becomes important in the so-called regional frequency analysis of data records. Inference from individual data records can be extremely misleading, even for large samples. One is often tempted to trust information available from a record rather than to exploit regional average statistics of those records. This thesis documents that regional average statistics usually contain much more information about the variability and persistence of the data set at a particular site than does the individual record for that site. Discordance-based homogeneity considerations have been given. It was shown that robust distances perform better in the judgement of a homogeneity analysis than the classical discordancy measures. Monte Carlo simulations have shown the validity of this statement. Splitting datasets was shown also to be successful for analysing the homogeneity of a single dataset. These were the topics of Chapter 4.

Civil engineers require nowadays more clear decision making processes than in the past and therefore cost-benefit analyses have become common to determine the safety target of a structural design. Failure cost evaluation is a key parameter for the safety target. The designer should model the trade-off mechanism between structural costs and benefit due to a reduction in the failure probability. When additional information becomes available (such as quantitative values for various parameters, in particular the failure costs), it should be used to recalculate the structural design. Bayesian methods are suitable for doing this. One of the factors that hampers application of probability theory to risk analysis and design is the difficulty in the probability distribution selection. The problem is particularly severe when high degrees of reliability are demanded and information about the distribution is very scarce, for example, in the selection of tail probability laws for load and resistance variables. The origin of the difficulties lies in the statistical inference methods. The problem is, by its nature, one of decision making and should be resolved through methods of decision theory. If this is done, then a number of interesting facts emerge. The optimum design of a structure corresponds to the point at which the cost of the structure and the cost of exceeding the design value (causing failure of the structure) is minimal. Ideally, the optimising process should be carried out for each design variable of the structure and first attempts for doing this are shown to be successful. However, if such an optimisation process, for practical reasons, would not be possible, then design value frequencies are traditionally being used (giving generally satisfactory results). In this thesis it was shown that it is recommendable to apply in such case asymmetric loss functions. The design philosophy of economic optimisation can therefore be translated into the philosophy to design the civil structure allowing a fixed probability of failure with the use of the correct loss function.

Simple closed-form expressions for the optimum design are obtained for several probability distributions. The closed-form expressions could be used to understand the significance of certain parameters in the design load determination. The question how in a reliability-based design one should include explicitly statistical parameter and model uncertainties has been tackled using Bayesian methods. Also the concept of risk aversion was included in this framework. The linear combination $E(X) + k\sigma(X)$ is proposed as a simple risk-averse measure of risk-based decision making. Chapter 5 could be considered as the chapter with the most important results of this thesis.

In Chapters 6 and 7, two case studies were presented. The first case study described the use of (inaccurate) historical data in the estimation of flood quantiles along the Dutch coast. Bayesian techniques appeared to be very useful for that purpose. The second case study described the use of regional data in the estimation of wave height quantiles on the densely built North Sea. The L-Moment estimation methods appeared to be successful for that purpose.

Finally, an overview is given of the most commonly used probability distributions and its estimation methods in the Appendix. Such a compact overview has not been published before in the literature and it may give a quick reference for every engineer who works with various distribution functions and estimation methods. Also the comprehensive list of references gives an up-to-date list of all the important papers that have been published in the area of risk-based design of civil structures.

Index of notation and abbreviations

Appendix

References

Author Index

Index of Notation and Abbreviations

Symbol	*Description*
$I(\theta)$	Fisher Information Matrix
$\xi, \zeta, \mu, \lambda, A$	Location parameters of PDFs
$\alpha, \sigma, \beta, \delta, k, B$	Scale parameters of PDFs
a, h, k, δ	Shape parameters of PDFs
θ, λ	General parameters of PDFs
n	Sample size
$X_{(r:n)}$	r-th order statistic
$F_{(r)}$	CDF of r-th order statistic
X	Random variable
x	Realisation of X
f_X	PDF of X
F_X	CDF of X
μ	Mean
μ_r	r-th central moment
σ	Standard deviation
$\beta_1^{½}$	Skewness
β_2	Kurtosis
α_r, β_r	r-th PWM
τ	L-CV
τ_r	r-th L-Moment ratio of a PDF
λ_r	r-th L-Moment of a PDF
Φ	CDF of the standard Normal distribution
ϕ	PDF of the standard Normal distribution
l_r	r-th L-Moment of a sample
t	L-CV of a sample
t_r	r-th L-Moment ratio of a sample
Γ	Gamma function
b_r	r-th PWM of a sample
$H(f)$	Entropy of PDF f
$I(f,g)$	Cross entropy of PDF f from prior distribution g
L	Loss function
RI	Recurrence interval
D_i	Wilks measure
$\chi_{\{x>0\}}X$	= 0 ($x \leq 0$) and 1 otherwise
T	Threshold level

$J(f,g)$	Divergence between PDFs f and g
Z	Reliability function $Z = R - S$
R	Resistance or Strength variables
S	Stress or Load variables
P_f	Probability of failure $P(Z<0)$
β	Reliability index $\beta = \Phi^{-1}(P_f)$
h	(Conditional) failure rate $h = f / (1-F)$
ρ	Correlation coefficient
h	required crest height
K	crest height (m)
M_p	lake level (m)
W	wind speed (m/s)
Δ	wind surge (m)
Osc	oscillations
H_s	significant wave height (m)
$z_{2\%}$	2%- wave runup (m)
$s_{op} * 100$	wave steepness × 100
T_p	peak wave period (s)
ξ	surf similarity parameter
c_f	fixed cost
c_v	variable cost
c	cost of failure
r	discount rate per unit time
α	discount factor per unit time $\alpha = [1 + r]^{-1}$
k	risk aversion index
γ	monetary risk aversion index
u	utility
$\pi(\Theta)$	prior distribution
$f(\Theta \| x)$	posterior distribution
$L(X \| \Theta)$	Likelihood function
B	Bayes factor
Z	Goodness of fit measure
H	Heterogeneity measure

ADD	Additive
AM	Annual Maxima
ANOVA	Analysis of Variance
BP	Breakdown Point
CBA	Cost Benefit Analysis
CCDF	Complementary Cumulative Distribution Function
CDF	Cumulative Distribution Function
CM	Classical Moments (=MOM)
COV	Covariance
CR	Cramer Rao
CV	Coefficient of Variation
ECO	Economic
EDL	Expected Design Loss
EM	Estimation Method
EXP	Exponential Distribution
FFA	Flood Frequency Analysis
FORM	First Order Reliability Method
GAM	Gamma Distribution
GEV	Generalized Extreme Value Distribution
GLO	Generalized Logistic Distribution
GNO	Generalized Log Normal Distribution
GPA	Generalized Pareto Distribution
GRDC	Global Runoff Data Centre
GUM	Gumbel Distribution
HAM	Hampel's Weights
HUB	Huber's Weights
i.i.d.	Independent and Identically Distributed
IU	Intrinsic Uncertainty
KS	Kolmogorov Smirnov
LB	Lower bound
L-CV	L-Moment Coefficient of Variation
LINEX	Linear Exponential
L-K	L-Kurtosis
LM	L-Moments
LN	Log Normal Distribution
LP3	Log Pearson Type III Distribution
LS	Least Squares
MAX	Maximum
MB	Method of Bayes
MC	Monte Carlo
MCD	Minimum Covariance Determinant
MD	Mahalanobis Distance
M-D	Minimum Divergence
ME	Method of Entropy
ML	Maximum Likelihood
MLE	ML-Estimators
MLM	Method of L-Moments
MLS	Method of Least Squares
MML	Method of Maximum Likelihood
MOM	Method of Moments
MPWM	Method of Probability-Weighted Moments
MSE	Mean Squared Error
MU	Model Uncertainty
MULT	Multiplicative
MVE	Minimum Volume Ellipsoid
NAP	Amsterdam Ordnance Datum
NPP	Nuclear Power Plant
OLAS	Ocean Land Atmosphere Studies
PDF	Probability Distribution Function
PE3	Pearson Type III Distribution
PGA	Peak Ground Accelaration
PIANC	Permanant International Navigation Commission
POE	Probability of Exceedance
POT	Peaks over Threshold
RAY	Rayleigh
RD	Robust Distance
REDL	Relative Expected Design Loss
REW	Reweighing
RFA	Regional Frequency Analysis
RIKZ	Dutch National Institute for Coastal and Marine Management
RIZA	Dutch Institute for Inland Water Management and Waste Water Treatment
RMSE	Root Mean Squared Error
SHA	Seismic Hazard Assessment
SKE	Standard Kernel Estimator
SLS	Serviceability Limit State
SU	Statistical Uncertainty
T-BW	Constrained M-Estimates
UB	Upper bound
ULS	Ultimate Limit State
UTCB	Technical University of Civil Engineering, Bucharest
VAR	Variance
WEIB	Weibull Distribution
WF	Weight Factor

Appendix

Definitions of probability distribution functions and parameter estimation methods

However big floods get, there will always be a bigger one coming;
So says one theory of extremes, and experience suggests it is true.
- President's Water Comm.

In this appendix the following PDF's and their estimation methods will be given:

Uniform (Bernoulli, 1667-1748)

Exponential (LaPlace, 1749-1827)

Rayleigh (Rayleigh, 1880)

Gumbel (Gumbel, 1958)

Weibull (Weibull, 1961)

Frechet (Frechet, 1937)

Generalized Extreme Value (Von Mises, 1923)

Normal (De Moivre, 1733)

(Generalized) Pareto (Pareto, 1887)

(Generalized) Lognormal (Gauss, 1777-1855)

Pearson III (or Gamma) (Pearson, 1949)

Generalized Logistic (Verhulst, 1838)

Kappa (Hosking, 1994)

Wakeby (Houghton, 1978)

Distribution	Uniform	
Expressions	$f(x) = 1/(\beta - \alpha)$ $F(x) = (x - \alpha)/(\beta - \alpha)$ $x(F) = \alpha + (\beta - \alpha)F$	
Range	$\alpha < x < \beta$	
Central Moments	$\mu = (\alpha+\beta)/2$ $\sigma = 1/6\sqrt{3}\ (\beta-\alpha)$ $\beta_1 = 0$ $\beta_2 = 3.24$	$\alpha = \mu - \sqrt{3}\sigma$ $\beta = \mu + \sqrt{3}\ \sigma$
L-Moments	$\lambda_1 = \frac{1}{2}(\alpha + \beta)$ $\lambda_2 = \frac{1}{6}(\beta - \alpha)$ $\tau_3 = 0$ $\tau_4 = 0$	$\alpha = \lambda_1 - 3\lambda_2$ $\beta = \lambda_1 + 3\lambda_2$
Log-Likelihood	$\log L = -n \log(\beta - \alpha)$	$\alpha = \min(x_1, x_2, ..., x_n)$ $\beta = \max(x_1, x_2, ..., x_n)$
Linear-Regression Transformation	$F(x) = (\beta-\alpha)^{-1}x - \alpha(\beta-\alpha)^{-1}$	$\alpha = (s_X^2/s_{XY})\ m_Y - m_X$ $\beta = (m_Y + 1)\ (s_X^2/s_{XY}) - m_X$ $Y = -\log(1- i/n+1)$
Non-informative prior	For convenience we write $f(x)= 1/\lambda$ for $0 < x < \lambda$ $f(\lambda)=1/\lambda$	
Non-informative posterior	$f(\lambda\|x)=PA(\lambda\|\ n, \max_{i=1..n}x_i)$	
Conjugate prior	$f(\lambda)=PA(\lambda\|\alpha,\beta)$	
Conjugate posterior	$f(\lambda\|x)=PA(\lambda\|\alpha+n,\max(\beta,\max_{i=1..n}x_i))$	
Predictive function	$f(y\|x) = (\alpha+n)/(\alpha+n+1)/\max(\beta,\max_{i=1..n}x_i)$ if $y<\max(\beta,\max_{i=1..n}x_i)$ $= 1/(\alpha+n+1)\ PA(y\|\ \alpha,\max(\beta,\max_{i=1..n}x_i))$ if $y>\max(\beta,\max_{i=1..n}x_i)$	
Entropy	$E =\log(\beta-\alpha)$	

PA is the Pareto distribution $PA(x|a,k)=ak^a x^{-(a+1)}$

Distribution	Exponential	
Expressions	$f(x)=\alpha^{-1}\exp\{-(x-\xi)/\alpha\}$ $F(x)=1-\exp\{-(x-\xi)/\alpha\}$ $x(F)=\xi-\alpha\cdot\log(1-F)$	
Range	$\alpha > 0$, $-\infty < \xi < \infty$, $\xi < x$	
Central Moments	$\mu = \xi + \alpha$ $\sigma = \alpha$ $\sqrt{\beta_1} = 2$ $\beta_2 = 9$	$\alpha = \sigma$ $\xi = \mu - \alpha$
L-Moments	$\lambda_1 = \xi + \alpha$ $\lambda_2 = \frac{1}{2}\alpha$ $\tau_3 = \frac{1}{3}$ $\tau_4 = \frac{1}{6}$	$\alpha = 2\lambda_2$ $\xi = \lambda_1 - \alpha$
Log-Likelihood	$\log L = -n \log \alpha - \alpha^{-1} \Sigma(x_i - \xi)$	$\alpha = m_X - \xi$ $\xi = \min(x_1, x_2, ..., x_n)$
Linear-Regression Transformation	$-\log(1- F) = - \xi\alpha^{-1} + \alpha^{-1}x$	$\alpha = (s_{XY} / s_X^2)^{-1}$ $\xi = -\alpha\ (m_Y - \alpha\ m_X)$ $Y = -\log(1- i/n+1)$
Non-informative prior	For convenience we write $f(x)=\lambda\exp(-\lambda(x-\xi))$ $f(\lambda) \propto \lambda^{-1}$	
Non-informative posterior	$f(\lambda\|x) = Ga(\lambda\|n,\Sigma x_i)$	
Conjugate prior	$f(\lambda)=Ga(\lambda\|\alpha,\beta)$	
Conjugate posterior	$f(\lambda\|x)=Ga(\lambda\|\alpha+n,\beta+\Sigma_{i=1..n}x_i)$	
Predictive function	$f(y\|x) = Gg(y\|\alpha+n,\beta+\Sigma_{i=1..n}x_i,1)$	
Entropy	$E = \log(e\alpha)$	

- Conjugate priors on location parameters do not exist. Only censoring to the first k order statistics in a sample of size n may lead to explicit Bayesian expressions.
- The Bayesian expressions for the exponential distribution were derived in Chapter 6.

Distribution	Rayleigh
Expressions	$f(x) = 2\frac{x}{\beta}\exp[-\{x/\beta\}^2]$ $F(x) = 1 - \exp[-\{x/\beta\}^2],$ $x(F) = \beta\sqrt{-\log(1-F)}$
Range	$0 \le x < \infty$ β>0
Central Moments	μ = ½β√π σ = β-π/4 β_1 = 0.3969 β_2 = 3.26
L-Moments	$\lambda_1 = -\frac{1}{2}\sqrt{\pi}$ $\lambda_2 = \beta\left(\frac{1}{2}\sqrt{\pi} - \sqrt{\frac{1}{8}\pi}\right)$ $\tau_3 = 2(1-3^{-1/2})/(1-2^{-1/2}) - 3$ $\tau_4 = \{5(1-4^{-1/2}) - 10(1-3^{-1/2}) + 6(1-2^{-1/2})\}/(1-2^{-1/2})$
Log-Likelihood	$\log L = -n\log\frac{\beta}{2} + \sum \log x_i - \beta^{-2}\sum x_i^2$ $\beta = \frac{2}{\sqrt{n}}\sqrt{\sum x_i^2}$
Linear-Regression Transformation	log(-log(1-F)) = 2log(x) - 2logβ or $(-\log(1-F))^{1/2} = x/\beta$
Non-informative prior	$f(\beta) \propto \beta^{-1}$
Non-informative posterior	$f(\beta\|x) \propto \beta^{-n}\exp(-\beta^{-2}\Sigma x_i^2)$
Conjugate prior	$f(\beta) \propto \beta^{-d}\exp(-\beta^{-2}b)$
Conjugate posterior	$f(\beta\|x) \propto \beta^{-n-d}\exp(-\beta^{-2}(t+b))$ with $t = \Sigma x_i^2$
Predictive function	$f(y\|x) \propto 2^{n+d}\, y\frac{\Gamma\left(\frac{n+d}{2}\right)}{\left(y^2+b+\sum x_i^2\right)^{\frac{n+d}{2}}}$
Entropy	$E = \log\left(\frac{\beta^2 e}{2}\right) - \frac{1}{n}\sum_{i=1}^{n}\log x_i$

In the derivation of f(y|x), the substitution $\beta^{-2}=\theta$ is used.

Distribution	Gumbel	
Expressions	$f(x)=\alpha^{-1}\exp\{-(x-\xi)/\alpha\}\exp[-\exp\{-(x-\xi)/\alpha\}]$ $F(x)=\exp[-\exp\{-(x-\xi)/\alpha\}]$ $x(F)=\xi-\alpha\cdot\log(-\log F)$	
Range	$\alpha > 0$, $-\infty < \xi < \infty$, $-\infty < x < \infty$	
Central Moments	$\mu = \xi + \gamma\alpha$ (γ=0.577) $\sigma = 1.283\ \alpha$ $\beta_1 = 1.3$ $\beta_2 = 5.4$	$\alpha = \sigma\ 1.283^{-1}$ $\xi = -\ \mu + \gamma\alpha$ (γ=0.577)
L-Moments	$\lambda_1 = \xi + \alpha\gamma$ $\lambda_2 = \alpha\cdot\log 2$ $\tau_3 = 0.1699 = \log(9/8)/\log 2$ $\tau_4 = 0.1504 = (16\log 2 - 10\log 3)/\log 2$	$\alpha = \lambda_2/\log 2,$ $\xi = \lambda_1 - \gamma\alpha$
Log-Likelihood	$LogL = -n\log\alpha - \Sigma(x_i-\xi)/\alpha - \Sigma\exp\{-(x_i-\xi)/\alpha\}$	
Linear-Regression Transformation	$-\log(-\log F)) = (x-\xi)/\alpha$	$\alpha = (s_{XY}/s_X^2)^{-1}$ $\xi = -\alpha\ (m_Y - \alpha\ m_X)$ $Y = -\log(-\log(i/n+1))$
Non-informative prior	$f(\xi) \propto \exp(c_i\alpha^{-1}\xi)$ with c_1=0 non-informative with c_2=0.46 least-informative	
Non-informative posterior	$f(\xi\|x) \propto \exp(c_i\alpha^{-1}\xi)\ \exp(-b\exp(\alpha^{-1}\xi))$	
Conjugate prior	$f(\xi) \propto e^{d\xi/\alpha}\exp(-be^{\xi/\alpha})$ $t = \Sigma\exp(-\alpha^{-1}x_i)$	
Conjugate posterior	$f(\xi\|x) \propto e^{(d+n)\xi/\alpha}\exp(-(b+t)e^{\xi/\alpha})$	
Predictive function	$f(y\|x) \propto e^{-y/\alpha}\dfrac{\Gamma(d+n+1)}{\left(e^{-y/\alpha}+\sum e^{-x_i/\alpha}+b\right)^{d+n+1}}$	
Entropy	$E=\dfrac{1}{\alpha}\left(\dfrac{1}{n}\sum_{i=1}^{n}x_i-\xi\right)+\log(e\alpha)$	

The derivation of the Bayesian expressions is given in Chapter 3 (pp.57-60). The least-informative prior and posterior distributions are derived by Engelund and Rackwitz (1994).

Distribution	Weibull
Expressions	$F(x)=1-\exp[-\{(x-\zeta)/\beta\}^{\delta}]$, $f(x)=\frac{\delta}{\beta}\left(\frac{x-\zeta}{\beta}\right)^{\delta-1}\exp[-\{(x-\zeta)/\beta\}^{\delta}]$, $x(F)=\zeta+\beta(-\log(1-F))^{\frac{1}{\delta}}$
Range	$\varsigma \le x < \infty$ $\beta>0, \delta>0$ $-\infty < \zeta < \infty$
Central Moments	$\mu = \varsigma + \beta\Gamma(1/\delta+1)$ $\sigma = \beta\sqrt{}\Gamma(2/\delta+1)-\Gamma2(1/\delta+1)$ β_1 = see Abernethy, 1983 β_2 = ,, ,,
L-Moments	$\lambda_1=\varsigma-\Gamma(1+1/\delta)$ $\lambda_2=\beta(1-2^{-1/\delta})\Gamma(1+1/\delta)$ $\tau_3=2(1-3^{-1/\delta})/(1-2^{-1/\delta})-3$ $\tau_4=\{5(1-4^{-1/\delta})-10(1-3^{-1/\delta})+6(1-2^{-1/\delta})\}/(1-2^{-1/\delta})$
Log-Likelihood	$\log L = n\log(\frac{\delta}{\beta})-n(1-\delta)\log\left(\frac{x-\zeta}{\beta}\right)-\Sigma\{(x_i-\zeta)/\beta\}^{\delta}$
Linear-Regression Transformation	$\log(-\log(1-F)) = \delta\log(x-\zeta) - \delta\log\beta$ or $(-\log(1-F))^{1/\delta} = (x-\zeta)/\beta$
Non-informative prior	$f(\beta) \propto \beta^{-c_i}$ with $c_1=1$ non-informative with $c_2=1+0.38\delta$ least-informative
Non-informative posterior	$f(\beta\|x) \propto \beta^{ci}\exp(-\beta^{-\delta}t)\,\delta\, t^{(ci-1)\delta}/\Gamma((c_i-1)/\delta)$ with $t = \Sigma(x_i-\varsigma)^{\delta}$
Conjugate prior	$f(\beta)\propto \beta^{-ci}\exp(-\beta^{-\delta})\,\delta/\Gamma((c_i-1)/\delta)$
Conjugate posterior	$f(\beta\|x) \propto \beta^{-n\delta-ci}\exp(-\beta^{-\delta}t)\,\delta\, t^{n+(ci-1)\delta}/\Gamma(n+(c_i-1)/\delta)$
Predictive function	$f(y\|x) \propto (n+(c_i-1)/\delta)\, n\delta/t\, ((x-\varsigma)^{\delta}/t+1)^{-n-1-(ci-1)/\delta}(x-\varsigma)^{\delta-1}$
Entropy	$E=\log\left(\frac{\beta^{\delta}e}{\delta}\right)-(\delta-1)\frac{1}{n}\sum_{i=1}^{n}\log(x_i-\varsigma)$

Special cases: $\delta=1$ leads to the Exponential distribution
$\delta=2$ to the Rayleigh distribution

If X is Weibull then $Y=\log(X-\varsigma)$ is Gumbel
If X is Exponential then $Y=X^{\delta}$ is Weibull

The Bayesian formulae are derived by Engelund and Rackwitz (1994). They can also be determined with the methods of Chapter 3 (pp. 57-60).

Distribution	Frechet
Expressions	$F(x)=\exp[-\{(x-\zeta)/\beta\}^{-\delta}]$, $f(x)=\frac{\delta}{\beta}\left(\frac{x-\zeta}{\beta}\right)^{-\delta-1}\exp[-\{(x-\zeta)/\beta\}^{-\delta}]$, $x(F)=\zeta+\beta(-\log(F))^{-\frac{1}{\delta}}$
Range	$\varsigma \le x < \infty$ β>0, δ>0 $-\infty < \zeta < \infty$
Central Moments	$\mu=\varsigma+\beta\Gamma(1-1/\delta)$ $\sigma^2=-\mu^2+\beta^2\Gamma(1-2/\delta)$
L-Moments	$\lambda_1=-\varsigma+\beta\Gamma(1-1/\delta)\}$ $\lambda_2=-\beta(1-2^{1/\delta})\Gamma(1-1/\delta)$ $\tau_3=2(1-3^{1/\delta})/(1-2^{1/\delta})-3$ $\tau_4=\{5(1-4^{1/\delta})-10(1-3^{1/\delta})+6(1-2^{1/\delta})\}/(1-2^{1/\delta})$
Log-Likelihood	$\log L=n\log(\frac{\delta}{\beta})-n(\delta+1)\log\left(\frac{x-\zeta}{\beta}\right)-\Sigma\{(x_i-\zeta)/\beta\}^{-\delta}$
Linear-Regression Transformation	$\log^{1/\delta}(F)=-\frac{x-\varsigma}{\beta}$
Non-informative prior	$f(\beta)\propto\beta^{-1}$
Non-informative posterior	$f(\beta\|x)\propto\beta^{\delta n}\exp(-t\beta^{\delta})$ with $t=\Sigma(x_i-\varsigma)^{-\delta}$
Conjugate prior	$f(\beta)\propto\beta^{\delta d}\exp(-b\beta^{\delta})$
Conjugate posterior	$f(\beta\|x)\propto\beta^{\delta(n+d)}\exp(-(b+t)\beta^{\delta})$
Predictive function	$f(y\|x)\propto\delta(y-\zeta)^{-\delta}\frac{\Gamma\left(n+3+d-\frac{1}{\delta}\right)}{\left((y-\zeta)^{-\delta}+\Sigma(x_i-\zeta)^{-\delta}+b\right)^{n+3+d-\frac{1}{\delta}}}$
Entropy	$E=\log\left(\frac{\beta^{\delta}e}{\delta}\right)-(\delta-1)\frac{1}{n}\sum_{i=1}^{n}\log(x_i-\varsigma)$

The predictive function can be derived analytically by substitution of $u=\beta^{\delta}$

Distribution	Generalized Extreme Value	
Expressions	$f(x)=\alpha^{-1}e^{-(1-k)y-e^{-y}},\ \ y=\begin{cases}-k^{-1}\log\{1-k(x-\xi)/\alpha\}, & k\neq 0\\ (x-\xi)/\alpha, & k=0\end{cases}$ $F(x)=e^{-e^{-y}}$ $x(F)=\begin{cases}\xi+\alpha\{1-(-\log F)^k\}/k & k\neq 0\\ \xi-\alpha\cdot\log(-\log F), & k=0\end{cases}$ type I: $F(x)=\exp(e^{-x}),\quad -\infty<x<\infty,$ type II: $F(x)=\exp(-x^{-\delta}),\quad 0\leq x<\infty,$ type III: $F(x)=\exp(-\lvert x\rvert^{\delta}),\quad -\infty<x\leq 0.$	
Range	$\alpha>0,\ -\infty<\xi<\infty,\ -\infty<k<\infty$ $if\ k>0,\ \ x<\xi+\frac{\alpha}{k}$ $if\ k=0,\ -\infty<x<\infty$ $f\ k<0,\ \ x>\xi+\frac{\alpha}{k}$	
L-Moments	$\lambda_1=\xi+\alpha\{1-\Gamma(1+k)\}/k$ $\lambda_2=\alpha(1-2^{-k})\Gamma(1+k)/k$ $\tau_3=2(1-3^{-k})/(1-2^{-k})-3$ $\tau_4=\{5(1-4^{-k})-10(1-3^{-k})+6(1-2^{-k})\}$ $\Gamma(x)=\int_0^\infty t^{x-1}e^{-t}dt$	$k\approx 7.8590\cdot c+2.9554\cdot c^2,$ $c=\frac{2}{3+\tau_3}-\frac{\log 2}{\log 3}$ $\alpha=\frac{\lambda_2 k}{(1-2^{-k})\Gamma(1+k)},$ $\xi=\lambda_1-\alpha\{1-\Gamma(1+k)\}/k$

Substitution scheme from GEV to Frechet:

$$F(x)=e^{-\exp\{k^{-1}\log(1-k(x-\xi)/\alpha)\}}=e^{-(1-k(x-\xi)/\alpha)^{1/k}}=$$

$$=e^{-\left(\frac{\alpha+k\xi-kx}{\alpha}\right)^{1/k}}=e^{-\left(\frac{\frac{\alpha+k\xi}{-k}+x}{-\alpha/k}\right)^{1/k}}=e^{-\left(\frac{-\varsigma+x}{\beta}\right)^{-\delta}}$$

$while\ choo\sin g:$

$$\varsigma=-\frac{\alpha}{k}-\xi$$

$$\beta=-\alpha/k$$

$$\delta=-1/k$$

$With:$

$$\xi=\beta-\varsigma$$

$$\alpha=\frac{\beta}{\delta}$$

$$k=-\frac{1}{\delta}$$

Distribution	Normal
Expressions	$f(x)=\sigma^{-1}\phi\left(\frac{x-\mu}{\sigma}\right)$ $F(x)=\Phi\left(\frac{x-\mu}{\sigma}\right)$ $\phi(x)=(2\pi)^{-1/2}\exp(-\frac{1}{2}x^2),$ $\Phi(x)=\int_{-\infty}^{x}\phi(t)dt.$
Range	$\sigma>0$, $-\infty < \mu < \infty$
Central Moments	$E(X)=\mu$ $Var(X)=\sigma^2$ $\beta_1=0$ $\beta_2=3$
L-Moments	$\lambda_1=\mu$ $\lambda_2=0.5642\sigma=\pi^{-1/2}\sigma$ $\tau_3=0$ $\tau_4=0.1226=30\pi^{-1}\arctan\sqrt{2}-9$
Log-Likelihood	$f(x\|\mu,\lambda)=\Pi\phi(x_i\|\mu,\sigma)$ $\log L=-n\log\sigma-\frac{n}{2}\log(2\pi)-\frac{1}{2}\Sigma\left(\frac{x_i-\mu}{\sigma}\right)^2$
Linear-Regression Transformation	$\Phi^{-1}F=\frac{x-\mu}{\sigma}$
Non-informative prior	$f(\sigma)=\sigma^{-1}$
Non-informative posterior	$f(\sigma\|x)\propto\left(\frac{1}{\sigma}\right)^n e^{-\frac{\Sigma(x_i-\mu)^2}{2\sigma^2}}$
Conjugate prior	$f(\sigma)\propto\left(\frac{1}{\sigma}\right)^d e^{-\frac{b}{2\sigma^2}}$ $t=\Sigma(x_i-\mu)^2$
Conjugate posterior	$f(\sigma\|x)\propto\left(\frac{1}{\sigma}\right)^{n+d} e^{-\frac{b+t}{2\sigma^2}}$
Predictive function	$f(y\|x)\propto\frac{\Gamma(n+d)}{\left((y-\mu)^2+\Sigma(x_i-\mu)^2+b\right)^2}$
Entropy	$E=\log(\sigma\sqrt{2\pi e})$

The Bayesian expressions are valid under the assumption of a known mean μ. Similar expressions are given in case of a known standard deviation σ, or in case of two unknown parameters μ and σ, in Bernardo and Smith (1994).

Distribution	Pareto
Expressions	$F(x)=1-\left(\frac{k}{x}\right)^{a}$ $x(F)=k(1-F)^{-\frac{1}{a}}$ $f(x)=ak^{a}x^{-(a+1)}$
Range	$k \le x < \infty$ a>0 k>0
Central Moments	μ = ak(a-1)$^{-1}$ σ = a^{½}k(a-1)$^{-1}$(a-2)$^{-½}$ β_1 = 2(a+1)^{2}(a-3)$^{-2}$(a-2)/a β_2 = 3(a-2)(3a^{2}+a+2)/{a(a-3)(a-4)}
L-Moments	$\lambda_1=-\frac{1}{2}\sqrt{\pi}$ $\lambda_2=\beta\left(\frac{1}{2}\sqrt{\pi}-\sqrt{\frac{1}{8}\pi}\right)$ $\tau_3=2(1-3^{-1/2})/(1-2^{-1/2})-3$ $\tau_4=\{5(1-4^{-1/2})-10(1-3^{-1/2})+6(1-2^{-1/2})\}/(1-2^{-1/2})$
Log-Likelihood	$\log L=n\log a+an\log k-(a+1)\sum\log x_i$ $a_{ML}=n\left(\sum\log\left(\frac{x_i}{k_{ML}}\right)\right)^{-1}$ $k_{ML}=\underset{i}{min}\,x_i$
Linear-Regression Transformation	log(1-F) = 2log(k) – a logx
Non-informative prior	$f(k,a)\propto 1$
Non-informative posterior	$f(k\|x)\propto k^{an}e^{-t(a+1)}$ $t=\sum\log x_i$
Conjugate prior	$f(k)\propto k^{ad}e^{-b(a+1)}$
Conjugate posterior	$f(k\|x)\propto k^{a(n+d)}e^{-(b+t)(a+1)}$
Predictive function	$f(y\|x,a)\propto a^{n+1+d}e^{-(a+1)(\log y+\Sigma\log x_i+b)}$ $f(y\|x,k)\propto\frac{\Gamma(n+2+d)}{(\log y+\Sigma\log x_i+b-(n+1+d)\log k)^{n+2+d}}$
Entropy	$E=\log\left(\frac{\beta^2 e}{2}\right)-\frac{1}{n}\sum_{i=1}^{n}\log x_i$

Distribution	Generalized Pareto	
Expressions	$f(x)=\alpha^{-1}e^{-(1-k)y}, \qquad y=\begin{cases}-k^{-1}\log\{1-k(x-\xi)/\alpha\}, & k\neq 0\\ (x-\xi)/\alpha, & k=0\end{cases}$ (so if k≠ 0 then f(x) = 1/α (1-k(x-ξ)/α) $^{-1+1/k}$) $F(x)=1-e^{-y}$ $x(F)=\begin{cases}\xi+\alpha\{1-(1-F)^k\}/k, & k\neq 0\\ \xi-\alpha\cdot\log(1-F), & k=0\end{cases}$	
Range	$\alpha>0,\ -\infty<\xi<\infty,\ -\infty<k<\infty$ $if\ k>0,\ \xi\le x\le \xi+\dfrac{\alpha}{k}$ $if\ k=0,\ \xi\le x<\infty$ $f\ k\le 0,\ x\ge\xi$	
Central Moments	$\mu=\alpha/(1+k)+\xi$ $\sigma^2=\alpha^2/\{(1+k)^2(1+2k)\}$ $\sqrt{\beta_1}=2(1-k)(1+2k)^{1/2}/(1+3k)$ $\beta_2=\dfrac{3(1+2k)(3-k+2k^2)}{(1+3k)(1+4k)}$	
L-Moments	$\lambda_1=\xi+\alpha/(1+k)$ $\lambda_2=\alpha/\{(1+k)(2+k)\}$ $\tau_3=(1-k)/(3+k)$ $\tau_4=(1-k)(2-k)/\{(3+k)(4+k)\}$ $\tau_4=\dfrac{\tau_3(1+5\tau_3)}{5+\tau_3}$	$k=(\lambda_1-\xi)/\lambda_2-2,$ $\alpha=(1+k)(\lambda_1-\xi).$ $k=(1-3\tau_3)/(1+\tau_3),$ $\alpha=(1+k)(2+k)\lambda_2,$ $\xi=\lambda_1-(2+k)\lambda_2.$

Special cases: k = 0 --> Exponential
k = 1 --> Uniform on (0,1)
k < 0 --> Pareto: $F(x) = 1 - (k/x)^a$ (k>0, a>0, x>k)
$f(x) = ak^a x^{-(a+1)}$
Relation between GPD and GEV is given in Johnson et al., 1995 (p.615).
Derivation of the predictive functions for the Pareto distribution:

$$g(k)=ak^a,\ h(x)=1,\ t(x)=\log x,\ c(a,k)=-(a+1)$$

$$\tilde{d}=d+n,\ \tilde{b}=b+\Sigma\log x_i$$

$$f(y|x,a)=\int a^{\tilde{d}+1}k^{a(\tilde{d}+1)}e^{-(a+1)(\log y+\tilde{b})}dk \propto a^{\tilde{d}+1}e^{-(a+1)(\log y+\tilde{b})}$$

$$f(y|x,k)=\int a^{\tilde{d}+1}k^{a(\tilde{d}+1)}e^{-(a+1)(\log y+\tilde{b})}da=\int a^{\tilde{d}+1}e^{-a(\log y+\tilde{b}-(\tilde{d}+1)\log k)}e^{-(\log y+\tilde{b})}da$$

Distribution	LogNormal
Expressions	$f(x)=\sigma^{-1}x^{-1}\phi\left(\frac{\log x-\mu}{\sigma}\right)$, $F(x)=\Phi[\{\log x-\mu\}/\sigma]$, $\phi(x)=(2\pi)^{-1/2}\exp(-\frac{1}{2}x^2)$, $\Phi(x)=\int_{-\infty}^{x}\phi(t)dt.$
Range	$0\le x<\infty$
Central Moments	$E(X)=e^{\mu+\frac{1}{2}\sigma^2}$ $Var(X)=e^{2\mu+\sigma^2}(e^{\sigma^2}-1)$ $\beta_1=(e^{\sigma^2}-1)(e^{\sigma^2}+2)^2$ $\beta_2=3+(w-1)(w^3+3w^2+6w+6);\; w=e^{\sigma^2}$
L-Moments	Follow from the Generalized LogNormal Distribution (p. 223)
Log-Likelihood	$\log L=-n\log\sigma-\Sigma\log x_i-\frac{n}{2}\log(2\pi)-\frac{1}{2\sigma^2}\Sigma(\log x_i-\mu)^2$
Linear-Regression Transformation	$\Phi^{-1}F=\{\log x-\mu\}/\sigma$
Non-informative prior	f(σ)=1/σ
Non-informative posterior	$f(\sigma\mid x)\propto\left(\frac{1}{\sigma}\right)^n e^{-\frac{\sum(\log x_i-\mu)^2}{2\sigma^2}}$
Conjugate prior	$f(\sigma)\propto\left(\frac{1}{\sigma}\right)^d e^{-\frac{b}{2\sigma^2}}$ $t=\Sigma\log(x_i-\mu)^2$
Conjugate posterior	$f(\sigma\|x)\propto\left(\frac{1}{\sigma}\right)^{n+d} e^{-\frac{b+\sum(\log x_i-\mu)^2}{2\sigma^2}}$
Predictive function	$f(y\|x)\propto\frac{1}{y}\frac{\Gamma\left(\frac{n+d}{2}\right)}{\left((\log y-\mu)^2+\Sigma(\log x_i-\mu)^2+b\right)^{\frac{n+d}{2}}}$
Entropy	$E=\log(\sigma\sqrt{2\pi e})+\frac{1}{n}\sum_{i=1}^{n}\log(x_i-\varsigma)$

Distribution	Generalized LogNormal	
Expressions	$f(x)=\dfrac{e^{ky-y^2/2}}{\alpha\sqrt{2\pi}}, \qquad y=\begin{cases}-k^{-1}\log\{1-k(x-\xi)/\alpha\}, & k\neq 0\\ (x-\xi)/\alpha, & k=0\end{cases}$ $F(x)=\Phi(y)$ x(F) has no explicit analytical form	
Range	$\alpha>0,\ -\infty<\xi<\infty,\ -\infty<k<\infty$ $if\ k>0,\ x<\xi+\dfrac{\alpha}{k}$ $if\ k=0,\ -\infty<x<\infty$ $f\ k<0,\ x>\xi+\dfrac{\alpha}{k}$	
L-Moments	$\lambda_1=\xi+\alpha(1-e^{k^2/2})/k$ $\lambda_2=\dfrac{\alpha}{k}e^{k^2/2}\{1-2\Phi(-k/\sqrt{2})\}$ $\tau_3\approx -k\dfrac{A_0+A_1k^2+A_2k^4+A_3k^6}{1+B_1k^2+B_2k^4+B_3k^6},$ $\tau_4\approx\tau_4^0+k^2\dfrac{C_0+C_1k^2+C_2k^4+C_3k^6}{1+D_1k^2+D_2k^4+D_3k^6}.$	$k\approx-\tau_3\dfrac{E_0+E_1\tau_3^4+E_2\tau_3^4+E_3\tau_3^6}{1+F_1\tau_3^2+F_2\tau_3^4+F_3\tau_3^6}.$ $\alpha=\dfrac{\lambda_2 k e^{-k^2/2}}{1-2\Phi(-k/\sqrt{2})},$ $\xi=\lambda_1-\dfrac{\alpha}{k}(1-e^{k^2/2}).$

If k=0, the Normal distribution arises with parameters $\xi$and α.

The Generalized LogNormal distribution is related with the standard Normal distribution Z~N(0,1) by:

$$X=\begin{cases}\xi+\alpha(1-e^{-kZ})/k, & k\neq 0\\ \xi+\alpha Z, & k=0\end{cases}$$

The Generalized LogNormal distribution can be rewritten to the expression:
$F(x)=\Phi[\{\log x-\mu\}/\sigma],$
by choosing:
$k=-\sigma, \qquad \alpha=\sigma\cdot e^{\mu}, \qquad \xi=\zeta+e^{\mu}$

Distribution	Pearson III (or Gamma distribution)
Expressions	$f(x)=\dfrac{(x-\xi)^{\alpha-1}e^{-(x-\xi)/\beta}}{\beta^{\alpha}\Gamma(\alpha)}$, $F(x)=G\left(\alpha,\dfrac{x-\xi}{\beta}\right)/\Gamma(\alpha).$ $G(\alpha,x)=\int_0^x t^{\alpha-1}e^{-1}dt$
Range	$\xi\le x<\infty$
Central Moments	$\mu=\xi+\alpha\beta$ $\sigma=\sqrt{\alpha}\beta$ $\beta_1=\dfrac{4}{\alpha}$ $\beta_2=\dfrac{3(\alpha+2)}{\alpha}$
L-Moments	$\lambda_1=\xi+\alpha\beta$ $\lambda_2=\pi^{-1/2}\beta\Gamma(\alpha+\dfrac{1}{2})/\Gamma(\alpha)$ $\tau_3=6I_{1/3}(\alpha,2\alpha)-3$ in which: $I_x(p,q)=\dfrac{\Gamma(p+q)}{\Gamma(p)\Gamma(q)}\int_0^x t^{p-1}(1-t)^{q-1}dt.$ if $\alpha\ge 1$ $\tau_3\approx\alpha^{-1/2}\dfrac{A_0+A_1\alpha^{-1}+A_2\alpha^{-2}+A_3\alpha^{-3}}{1+B_1\alpha^{-1}+B_2\alpha^{-2}}$, $\tau_4\approx\dfrac{C_0+C_1\alpha^{-1}+C_2\alpha^{-2}+C_3\alpha^{-3}}{1+D_1\alpha^{-1}+D_2\alpha-2}$; if $\alpha<1$ $\tau_3\approx\dfrac{1+E_1\alpha+E_2\alpha^2+E_3\alpha^3}{1+F_1\alpha+F_2\alpha^2+F_3\alpha^3}$, $\tau_4\approx\dfrac{1+G_1\alpha+G_2\alpha^2+G_3\alpha^3}{1+H_1\alpha+H_2\alpha^2+H_3\alpha^3}$ The coefficients are given in Hosking and Wallis (1997). Parameters: If $0<\lvert\tau_3\rvert<1/3$, let $z=3\pi\tau_3^2$, then: $\alpha\approx\dfrac{1+0.2906\cdot z}{z+0.1882\cdot z^2+0.0442\cdot z^3}$; If $1/3<\lvert\tau_3\rvert<1$, *let* $z=1-\lvert\tau_3\rvert$, *then* $\alpha\approx\dfrac{0.36067\cdot z-0.59567\cdot z^2+0.25361\cdot z^3}{1-2.78861\cdot z+2.56096\cdot z^2-0.77045\cdot z^3}$.

	$\beta=\lambda_2\pi^{1/2}\Gamma(\alpha)/\Gamma(\alpha+\frac{1}{2}),$ $\xi=\lambda_1-\beta\alpha$
Log-Likelihood	$\log L=n\log\Gamma(\alpha)-n\alpha\log\beta-\sum\frac{x_i-\xi}{\beta}+(\alpha-1)\sum\log(x_i-\xi)$
Linear-Regression Transformation	$G^{-1}(\Gamma(\alpha)F)=\frac{x-\xi}{\beta}$
Non-informative prior	$f(\beta)\propto\beta^{-1}$
Non-informative posterior	$f(\beta\|x)=\frac{t^n}{\Gamma(n)}\beta^{-(n+1)}e^{-\frac{t}{\beta}}$ $t=\sum(x_i-\xi)$
Conjugate prior	$f(\beta)=\frac{b^\alpha}{\Gamma(\alpha)}\beta^{-(\alpha+1)}e^{-\frac{b}{\beta}}$
Conjugate posterior	$f(\beta\|x)=\frac{(b+t)^\alpha}{\Gamma(\alpha+n)}\beta^{-(\alpha+n+1)}e^{-\frac{b+t}{\beta}}$
Predictive function	$f(y\|x)=(y-\xi)^{\alpha-1}\frac{\Gamma(a+n+1+\alpha)}{\Gamma(n+1+\alpha)}\left\{\begin{matrix}[(y-\xi)+\\ \sum(x_i-\xi)+b]^{(-a+n+1+\alpha)}\end{matrix}\right\}$
Entropy	$E=\log(\beta^\alpha\sqrt{\alpha})+\frac{\frac{1}{n}\sum_{i=1}^{n}x_i-\xi}{\beta}-(\alpha-1)\frac{1}{n}\sum_{i=1}^{n}\log(x_i-\xi)$

If α=1, the exponential distribution arises.
The sum of two independent exponential distributions (each with scale parameter α) leads to a Pearson III distribution with parameters (2,α).
The 2-parameter Log-Gamma distribution is defined as the distribution of the random variable Y when –logY has a Pearson III distribution with parameters (α,β). The PDF is given by: $f_Y(y)=\frac{1}{\beta^\alpha\Gamma(\alpha)}\frac{(-\log y)^{\alpha-1}}{y^{1+1/\beta}},\quad 0<y<1$. Expectation and variance of the Log-Gamma distribution are given by:

$$E(Y)=\left(\frac{\beta}{\beta+1}\right)^\alpha,\quad and\quad Var(Y)=\left(\frac{\beta}{\beta+2}\right)^\alpha-\left(\frac{\beta}{\beta+1}\right)^{2\alpha}.$$

Distribution	Generalized Logistic	
Expressions	$f(x)=\frac{\alpha^{-1}e^{-(1-k)y}}{(1+e^{-y})^2}$, $\quad y=\begin{cases}-k^{-1}\log\{1-k(x-\xi)/\alpha\}, & k\neq 0\\ (x-\xi)/\alpha, & k=0\end{cases}$ $F(x)=1/(1+e^{-y})$ $x(F)=\begin{cases}\xi+\alpha[1-\{(1-F)/F\}^k]/k, & k\neq 0\\ \xi-\alpha\cdot\log\{(1-F)/F\}, & k=0\end{cases}$	
Range	$\alpha>0,\ -\infty<\xi<\infty,\ -\infty<k<\infty$ $if\ k>0,\ \ x<\xi+\frac{\alpha}{k}$ $if\ k=0,\ -\infty<x<\infty$ $f\ k<0,\ \ x>\xi+\frac{\alpha}{k}$	
L-Moments	$\lambda_1=\xi+\alpha(1/k-\pi/\sin(k\pi))$ $\lambda_2=\alpha k\pi/\sin(k\pi)$ $\tau_3=-k$ $\tau_4=(1+5k^2)/6$	$k=-\tau_3$, $\alpha=\frac{\lambda_2\sin(k\pi)}{k\pi}$, $\xi=\lambda_1-\alpha\left(\frac{1}{k}-\frac{\pi}{\sin(k\pi)}\right)$

If k=0, the logistic distribution arises.

Distribution	Kappa
Expressions	$f(x)=\alpha^{-1}\{1-k(x-\xi)/\alpha\}^{1/k-1}\{F(x)\}^{1-h}$ $F(x)=[1-h\{1-k(x-\xi)/\alpha\}^{1/k}]^{1/h}$ $x(F)=\xi+\frac{\alpha}{k}\left\{1-\left(\frac{1-F^h}{h}\right)^k\right\}$
Range	if k>0, x<ξ+α/k if h>0, x>ξ+α(1-h^{-k})/k if h<0 and k<0, x>ξ+α/k
L-Moments	$\lambda_1=\xi+\alpha(1-g_1)/k$ $\lambda_2=\alpha(g_1-g_2)/k$ $\tau_3=(-g_1+3g_2-2g_3)/(g_1-g_2)$ $\tau_4=(-g_1+6g_2-10g_3+5g_4)/(g_1-g_2),$ $g_r=\begin{cases}\frac{r\Gamma(1+k)\Gamma(r/h)}{h^{1+k}\Gamma(1+k+r/h)}, & h>0\\ \frac{r\Gamma(1+k)\Gamma(-k-r/h)}{(-h)^{1+k}\Gamma(1-r/h)}, & h<0\end{cases}$

If h=-1, the Generalized Logistic distribution arises,
If h=0, the Generalized Extreme Value distribution arises,
If h=1, the Generalized Pareto distribution arises
Furthermore, the Kappa distribution can be written as:
$F(x)=(x/b)^{\theta}\{a+(x/b)^{a\theta}\}^{-1/a}$,with $x\geq 0,\quad a,b,\theta>0$
by taking: $\xi=b,\alpha=\frac{b}{a\theta},k=-\frac{1}{a\theta},h=-a.$
The Kappa distribution is very useful as a general distribution for use in simulating artificial data in order to assess the accuracy of statistical methods (see Chapter 3).

<table>
<tr><td>Distribution</td><td>Wakeby</td></tr>
<tr><td>Expressions</td><td>f(x) and F(x) not explicitly defined
$x(F) = \xi + \frac{\alpha}{\beta}\{1-(1-F)^{\beta}\} - \frac{\gamma}{\delta}\{1-(1-F)^{-\delta}\}$</td></tr>
<tr><td>Range</td><td>if δ>0 and γ>0, then x>ξ,
if δ<0 or γ=0, then ξ<x<ξ+α/β-γ/δ</td></tr>
<tr><td>L-Moments</td><td>$\lambda_1 = \xi + \frac{\alpha}{(1+\beta)} + \frac{\gamma}{(1-\delta)}$
$\lambda_2 = \frac{\alpha}{(1+\beta)(2+\beta)} + \frac{\gamma}{(1-\delta)(2-\delta)}$
$\lambda_3 = \frac{\alpha(1-\beta)(2-\beta)}{(1+\beta)(2+\beta)(3+\beta)(4+\beta)} + \frac{\gamma(1+\delta)}{(1-\delta)(2-\delta)(3-\delta)}$
$\lambda_4 = \frac{\alpha(1-\beta)(2-\beta)}{(1+\beta)(2+\beta)(3+\beta)(4+\beta)} + \frac{\gamma(1+\delta)(2+\delta)}{(1-\delta)(2-\delta)(3-\delta)(4-\delta)}$

Parameter estimation:
If ξ is unknown, define,
$N_1 = 3\lambda_2 - 25\lambda_3 + 32\lambda_4,$ $C_1 = 7\lambda_2 - 85\lambda_3 + 203\lambda_4 - 125\lambda_5,$
$N_2 = -3\lambda_2 + 5\lambda_3 + 8\lambda_4$ $C_2 = -7\lambda_2 + 25\lambda_3 + 7\lambda_4 - 25\lambda_5$
$N_3 = 3\lambda_2 + 5\lambda_3 + 2\lambda_4,$ $C_3 = 7\lambda_2 + 5\lambda_3 - 7\lambda_4 - 5\lambda_5.$

β and -δ are the roots of the equation:
$(N_2C_3 - N_3C_2)z^2 + |(N_1C_3 - N_3C_1)z + (N_1C_2 - N_2C_1) = 0,$
The other 3 parameters are:
$\alpha = (1+\beta)(2+\beta)(3+\beta)\{(1+\delta)\lambda_2 - (3-\delta)\lambda_3\}/\{4(\beta+\delta)\},$
$\gamma = -(1-\delta)(2-\delta)(3-\delta)\{(1-\beta)\lambda_2 - (3+\beta)\lambda_3\}/\{4(\beta+\delta)\},$
$\xi = \lambda_1 - \alpha/(1+\beta) - \gamma/(1-\delta).$

If ξ is known, define,
$N_1 = 4\lambda_1 - 11\lambda_2 + 9\lambda_3,$ $C_1 = 10\lambda_1 - 29\lambda_2 + 35\lambda_3 - 16\lambda_4,$
$N_2 = -\lambda_2 + 3\lambda_3,$ $C_2 = -\lambda_2 + 5\lambda_3 - 4\lambda_4,$
$N_3 = \lambda_2 + \lambda_3,$ $C_3 = \lambda_2 - \lambda_4.$
β and -δ are the roots of the equation:
$(N_2C_3 - N_3C_2)z^2 + |(N_1C_3 - N_3C_1)z + (N_1C_2 - N_2C_1) = 0,$
The other 2 parameters are:
$\alpha = (1+\beta)(2+\beta)\{\lambda_1 - (2-\delta)\lambda_2\}/(\beta+\delta),$
$\gamma = -(1-\delta)(2-\delta)\{\lambda_1 - (2+\beta)\lambda_2\}/(\beta+\delta).$

The following relation between τ_4 and τ_3 is useful:
$\tau_4 = \sum_{k=0}^{8} A_k \tau_3^k$ with A_k tabulated in Hosking and Wallis (1997).</td></tr>
</table>

For suitable choices of its parameters, the Wakeby distribution can mimic the shapes of many distributions (GEV, GNO, GAM, etc).

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Author Index

Curriculum Vitae

The author of this thesis, Pieter van Gelder, was born on December, 20, 1968 in Oss, the Netherlands. In 1987, he received his Atheneum-B diploma at the Maasland College in Oss. After that he studied Technical Mathematics with specialisation Applied Statistics and Systems Theory at Eindhoven University of Technology (EUT) from which he received the engineer degree (Ir.) in 1991. His graduation project was performed at the Integrated Production Control Systems Group of the software company Datex on the robust control of a chemical process under supervision of Prof. Hautus (EUT) and Prof. Backx (EUT). Parallel, he also fulfilled an additional doctoral programme in mathematical physics from 1990 to 1991.

After his study he was appointed as mathematical consultant at the Road and Hydraulic Engineering Division of the Ministry of Transport, Public Works and Water Management from 1991 to 1994 in Delft. During this period he was involved in the design of a Handbook on Hydraulic Loads on Structures, a Handbook on Risk Analysis in Civil Engineering and several other projects in the field of reliability engineering and systems safety.

From November 1994 until January 2000, Van Gelder has been working on his PhD research at the Faculty of Civil Engineering and Geosciences of Delft University of Technology (DUT) within the section of Hydraulic Engineering under the supervision of Prof. Vrijling (DUT). The PhD-research was partially funded by the Ministry of Public Works, Transport and Water Management. Apart from his PhD research on the risk-based design of civil structures, he was also involved in the MAST (MArine Science and Technology) programme of the European Commission on the PROVERBS project (Probabilistic Design of Vertical Breakwaters).

Van Gelder is a member of the Dutch Royal Institution of Engineers, the Dutch Society of Statistics, the Dutch Association of Risk Analysis and Reliability, the International Association for Bridge and Structural Engineering, the European Geophysical Society, the International Association of Hydrological Sciences, and the International Society for Bayesian Analysis.

Van Gelder will accept a position as UD (University Docent) at the section of Hydraulic Engineering, starting January 2000.

STELLINGEN

PROPOSITIONS

behorende bij het proefschrift

appended to the dissertation

Statistical Methods for the Risk-Based Design of Civil Structures

van

by

P.H.A.J.M. van Gelder

Technische Universiteit Delft

10 - 01 - 00

1. De vraag welke parameterschattingsmethode de beste prestatie heeft is, om in zijn algemeenheid te beantwoorden, onmogelijk. Monte Carlo simulaties zijn echter bij uitstek geschikt om deze vraag voor specifieke situaties te beantwoorden.

The question which parameter estimation method shows the best performance is impossible to answer in a general way. However, Monte Carlo simulations are very well-suited to answer this question for specific situations.

2. Robuuste afwijkingsmaten (robust discordancy measures) geven een belangrijke ondersteuning bij het uitvoeren van regionale frequentie-analyses.

Robust discordancy measures contribute significantly in the performance of a Regional Frequency Analysis.

3. Een optimaal ontwerp gebaseerd op economische minimalisatie kan vertaald worden naar een kwantiel-ontwerp met gebruikmaking van gemodificeerde LINEX verlies functies in geval van exponentieel verdeelde belastingen.

An optimal design based on economic minimisation can be translated into a quantile design by use of modified LINEX loss functions in case of exponentially distributed loads.

4. Het werk van historici betreffende oude natuurrampen levert een belangrijke aanvulling in een statistische analyse van instrumentele meteorologische, hydrologische en seismologische gegevens.

The work of historicians concerning old natural disasters gives important information in the statistical analysis of instrumental meteorologic, hydrologic and seismic data.

5. De beschikbaarheid van satellietgegevens van golfkarakteristieken zal de populariteit van regionale frequentie analyses alleen maar doen toenemen.

The availability of satellite data of wave characteristics will only increase the popularity of Regional Frequency Analyses.

6. Om de productiviteit c.q. kwaliteit van een onderzoeker te beoordelen aan de hand van het aantal hits in een zoekmachine op internet of een science citation index moet men rekening houden met het feit dat een groot aantal hits ten gevolge van verwijzingen naar zichzelf of binnen een kleine kring, evenals verwijzingen naar een overview- of foutief paper, deze productiviteits- c.q. kwaliteitsmaat kunnen beïnvloeden.

In the judgement of the productivity or quality of a researcher based on the number of hits of a search engine on the Internet or a science citation index, one must take into account that a large number of hits is caused by hits to the researcher himself or within a small circle, as well as hits to overview papers or papers with mistakes, and that these causes can influence the productivity or quality measures.

7. Doordat er steeds meer variabelen worden gemeten, worden er steeds vaker records verbroken met name in de meteorologie, financiële-, sport- en bouwwereld.

Because more and more quantities are measured, the number of records broken increases also, especially within meteorology, finance, sports and construction engineering.

8. Doordat de reviewrapporten van ingediende tijdschriftartikelen vaak zeer verschillende beoordelingen opleveren zou het raadzaam zijn om in dat geval ook reviewrapporten te laten reviewen.

Because the review reports on submitted journal papers often show contrary judgements, it would be wise to review the review reports themselves as well.

9. De methode ter bepaling van gewichtscoëfficiënten voor kansverdelingen slaat als een Tang (1980) op een varken. De methode d.m.v. Bayes factoren zet meer zoden aan de dijk.

The method to determine the weight factors for probability distribution functions, as proposed by Tang (1980), shows a very slow convergence and should therefore be replaced by a method based on Bayes factors.

> Tang, W.H., 1980. Bayesian frequency analysis, Journal of the Hydraulics Division, Vol. 106, No. HY7, July 1980, pp. 1203-18.

10. Door bij de bepaling van maatgevende belastingen wiskundigen in te zetten, mag de onzekerheid van menselijke denkfouten significant lager verondersteld worden dan de aanwezige inherente-, statistische- en modelonzekerheden bij de bepaling hiervan.

By employing mathematicians in the determination of the design loads, the uncertainty due to human error may be assumed to be smaller than the present inherent, statistical and model uncertainties in this determination.

11. Of deze stellingen millenniumproof zijn, was bij het drukken van dit proefschrift nog niet bekend.

Whether these propositions are Y2K compliant, was unknown at time of press of this thesis.

12. De wiskundige, meegevoerd op zijn stroom van symbolen, schijnbaar bezig met puur formele waarheden, kan hiermee nochtans resultaten van eindeloos belang voor de beschrijving van het fysisch universum bewerkstelligen.

The mathematician, carried along on his flood of symbols, dealing apparently with purely formal truths, may still reach results of endless importance for our description of the physical universe.

> Quotation by Karl Pearson (27 March 1857 - 27 April 1936, London, England). Quoted in N. Rose (ed.) Mathematical Maxims and Minims (Raleigh N C: Rome Press Inc., 1988).