Frequency-Size Statistics of Coastal Soft-Cliff Erosion

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Abstract: Predicting the retreat of a coastal soft cliff is a difficult and uncertain operation, which has both theoretical and practical significance. Recession of soft cliffs occurs through a combination of processes, including slope failures and surface erosion, that are difficult to model jointly. Deterministic models for predicting coastal retreat are hampered by the complex nature of coastal erosion, which is highly nonuniform in space and episodic in time. To overcome these limitations, stochastic approaches have been proposed. These models assume distributions for the size and the time of the recession events. In this paper we investigate the frequency-size statistics of soft-cliff erosion based on two historical datasets of coastal retreat measurements at two sites in England. We find that the two datasets exhibit a similar behavior. The frequency of the recessions decreases with increasing size of the retreat. For small retreats the decrease is slow. For medium to large retreats the decrease is rapid and follows a power law. The frequency-size statistics of soft-cliff erosion is similar to the statistics of medium to large landslide areas, which are also power-law distributed. This is a significant but not conclusive result. More data are needed to confirm this finding.

DOI: 10.1061/(ASCE)0733-950X(2005)131:1(37)

CE Database subject headings: Cliffs; Erosion; Coastal environment; Statistics.

Introduction

Soft cliffs are steep escarpments made up chiefly of soft rocks. Many examples of soft-cliff coastal retreats have been documented in the literature. Schuster and Highland (2001) showed that the typical average annual retreats in the Western Hemisphere ranged from zero (no noticeable retreat occurred) to more than 2 m and that episodic events of retreat may greatly exceed the average values. Although the large retreat events are infrequent, their impact on the morphodynamics of a cliff-beach system is significant, particularly over an engineering time scale (i.e., years to tens of years). When determining development setbacks from coastal cliffs, the effects of long-term (average) retreat rates and of episodic events must be considered.

Deterministic models for predicting coastal retreat are based on average erosion rates produced by a single (dominant) failure mechanism. For example, Komar et al. (2002) related cliff erosion to the average period of time for which the cliff toe is attacked by waves. Kamphuis (1987) related average erosion rates along the Great Lakes bluff to the rate at which the clay foreshore is undercut. Mano and Suzuki (1999), investigating the soft cliff along the Fukushima coast in Japan, proposed an empirical relationship between the mean erosion rate and the on-shore component of wave energy flux at the breaking point, the Young's modulus, and the cliff height.

The deterministic models are highly site specific, and account for limited erosion processes active on the beach-cliff system. More importantly, the deterministic approaches fail to recognize that the retreat of soft cliffs is highly nonuniform in space, and episodic in time. The average values could conceal the effect of rare, large-magnitude events, such as those produced by the El Niño events in 1982–1983 and 1997–1998 along the coast of California, when erosion occurred in the form of 1.5- to 3 m-wide blocks, which failed catastrophically (Flick 1998). Similarly, along the shores of the Great Lakes, combined bluff and dune erosion varies from nearly zero to several meters per year, depending on the annual variability of wave patterns and lake levels (Kamphuis 1987).

To overcome the limitations of deterministic models, probabilistic approaches have recently been attempted. Hall et al. (2002) proposed a model based on the rate of erosion (i.e., the time between successive failures) and the size of the erosion (i.e., the length of coastal retreat). In this model, retreat events are described as random series of erosion times and sizes. The former is assumed to be gamma distributed, and the latter is log-normal distributed.

The work described in this technical note was prompted by the recognition that the frequency-size distributions proposed by Hall et al. (2002) may not be log-normal distributed, but instead power-law distributed, like other geological processes, such as landslides (Hovius et al. 1997; Dai and Lee 2001; Stark and Hovius 2001; Guzzetti et al. 2002; Malamud et al. 2004) and earthquakes (Gutenberg and Richter 1954). To test this hypothesis we analyze the frequency-size statistics of soft-cliff erosion measurements taken at two coastal sites in England.

Processes and Mechanisms

The retreat of soft cliffs may occur suddenly, through slope failures, or gradually, through removal of sediment "grains" from the cliff face by marine, subaerial, and groundwater processes (sur-

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Note. Discussion open until June 1, 2005. Separate discussions must be submitted for individual papers. To extend the closing date by one month, a written request must be filed with the ASCE Managing Editor. The manuscript for this technical note was submitted for review and possible publication on June 24, 2004; approved on July 20, 2004. This technical note is part of the ***Journal of Waterway, Port, Coastal, and Ocean Engineering***, Vol. 131, No. 1, January 1, 2005. ©ASCE, ISSN 0733-950X/2005/1-37–42/$25.00.

face erosion). These are two extremes in a geomorphologic continuum. Generally, the existing models to predict coastal retreats address one or the other of the two processes, but not both. However, soft-cliff retreat is due to both slope failures and surface erosion.

The conservation equation of sediment-mass for a beach-cliff system at any time scale can be written as:

$$\frac{\partial y(t)}{\partial t} + \frac{\partial q(x,t)}{\partial x} = S(x,t) \tag{1}$$

where t=time; y=time-dependent beach elevation measured at any point x in the cross-shore profile; $q(x,t)$=instantaneous volumetric transport rate of the beach material per unit cliff length; and $S(x,t)$=spatially distributed source term. As short-term beach processes are dominated by time-varying phenomena, such as waves and tides, their effects average out over long periods of time, with the longer-term evolution being determined by the residual effects caused by extreme events, or the threshold conditions of the beach-cliff system. Also, for most soft cliffs, large failure events are episodic depending on the instantaneous forcing parameters and the stability conditions prior to the failure events. Therefore, at the short-time scale of hydrodynamics, the exact size and timing of large failures is essentially unpredictable. Only for a reasonably long period of time the cliff erosion is (statistically) predictable. For this reason, a lumped-modeling approach is required. To achieve this, Eq. (1) can be integrated over a prescribed intermediate time interval (e.g., 1 year to avoid seasonal effects), and over the active zone of the beach-cliff system. For simple beach erosion (no material input to the beach from the cliff), deterministic or probabilistic modeling methods have been developed by Reeve and Fleming (1997) and Dong and Chen (1999). For an eroding beach-cliff system, the main difficulty lies in the lack of data for the parameterization of material input to the beach from the cliff at suitable time scales. Furthermore, even where geotechnical characteristics of the cliff and the removal mechanism of the falling debris are known, erosion of the cliff will be less uniform spatially than the erosion of the beach. Thus, model results will be largely dependent on the time step used for averaging (aggregating) erosion processes in addition to the triggering mechanisms.

In comparison with the deterministic approach, the probabilistic approach of Hall et al. (2002) that defines the distributions of erosion time and size is promising. It does not require knowledge of the detailed hydrodynamic forcing conditions or the lithology of the cliff. By explicitly treating the erosion events as episodic and randomly distributed, the approach is consistent with the known behavior of soft cliffs, allowing for the rational use of field data.

It is however noticed that in the model proposed by Hall et al. (2002) the evidence given for the log-normal distribution used to describe the size (length) of erosion is not particularly convincing as it was entirely based on the results of wave basin tests on a model cliff made up of damp sand subject to short-term wave attack. As the sand model cliff fails more easily than a real soft cliff, the laboratory data contain an excess of small landslides, which significantly affect the frequency-size distribution of the events.

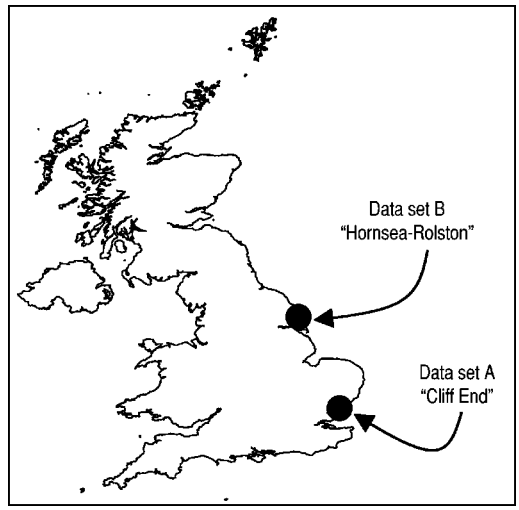

Fig. 1. Location of study areas along the eastern coast of England

An alternative way of estimating the frequency-size distribution of soft-cliff erosion is to study field measurements of coastal retreats. We attempt this by analyzing two datasets of coastal retreat measurements available for the eastern coast of England (Fig. 1).

Available Data

Dataset A was compiled by Hall et al. (2002) along a 20-m high soft cliff at Cliff End, in East Essex, on the south coast of England. The dataset lists 32 distance measurements of coastal retreat, 26 of which are larger than zero (Table 1). The measurements were obtained along eight profiles, a few hundred meters apart, in the 85-year period from 1907 to 1991. Surveys were conducted at different time intervals, from 7 to 29 years (average 21). Due to the long time between successive surveys, retreat measurements are affected by amalgamation problems, i.e., measurements may represent single or multiple events. Measurements of cliff retreat vary from 0 to 46 m, with an average of 9.9 m and a standard deviation of 11.6 m. This corresponds to an average annual retreat of 0.47 m. Inspection of Table 1 reveals that average retreat measurements varied largely in time (i.e., along the same profile) and space (i.e., at the same time, along neigh-

Table 1. Dataset A: Coastal Recession Measurements Compiled by Hall et al. (2002) at Cliff End, England.

Period	Section 1 (m)	Section 2 (m)	Section 3 (m)	Section 4 (m)	Section 5 (m)	Section 6 (m)	Section 7 (m)	Section 8 (m)
1907–1929	2.0	46.0	36.0	20.0	14.0	0.0	2.0	28.0
1929–1936	10.0	14.0	4.0	4.0	2.0	2.0	0.0	0.0
1936–1962	35.0	8.0	0.0	6.0	8.0	4.0	14.0	14.0
1962–1991	0.0	2.0	10.0	9.0	8.0	10.0	5.0	0.0
Average	11.8	17.5	12.5	9.8	8.0	4.0	5.3	10.5

Note: Data collected in the period 1907–1991 along eight profiles.

Table 2. Dataset B: Coastal Recession Measurements Compiled by Meadowcroft et al. (1999) at Hornsea-Rolston, England

Year	Section 47 (m)	Section 48 (m)	Section 49 (m)	Section 51 (m)	Section 52 (m)	Section 53 (m)
1953	1.22	0.61	0.00	0.31	3.65	0.91
1955	0.61	3.66	0.61	2.13	1.83	3.05
1957	0.00	0.91	0.00	1.22	0.00	0.91
1959	5.78	5.18	21.94	2.74	2.13	*7.60*
1960	4.26	2.13	1.52	3.66	0.61	7.01
1961	0.91	1.83	0.00	2.12	0.31	2.13
1962	2.44	10.06	0.91	5.48	7.23	4.27
1963	0.00	0.00	0.00	3.05	6.09	14.01
1964	0.31	1.22	0.00	0.31	0.31	3.66
1966	11.57	0.00	1.52	0.31	0.00	3.95
1967	0.91	0.00	6.10	0.00	5.48	0.00
1968	4.26	5.49	12.51	3.36	1.52	0.00
1969	8.53	4.88	2.43	3.96	1.83	0.31
1970	5.48	1.22	2.52	2.13	0.91	5.18
1971	0.00	0.00	0.31	3.05	0.00	0.00
1972	3.35	3.66	0.00	0.00	0.91	1.83
1973	0.30	7.01	7.32	0.00	0.00	0.00
1974	0.10	1.25	4.40	4.10	0.00	10.62
1976	5.30	7.30	1.90	2.90	7.67	6.40
1977	0.50	0.25	3.70	5.15	0.00	4.75
1978	1.10	2.00	1.50	1.55	9.10	*3.05*
1979	9.00	1.15	1.70	3.20	0.20	0.50
1980	0.10	3.80	5.70	1.10	0.30	0.20
1981	2.00	0.00	1.40	4.50	0.80	0.00
1982	6.00	5.90	0.80	4.90	4.80	0.90
1983	1.90	7.40	7.40	3.30	2.00	4.40
1984	3.20	0.20	2.30	2.60	4.60	1.40
1985	2.00	0.60	0.00	3.00	6.20	4.40
1986	5.05	8.60	*5.33*	5.20	4.40	3.40
1987	0.95	2.00	*1.81*	3.30	0.90	1.90
1988	*3.04*	8.30	4.00	1.50	0.40	1.00
1989	*2.02*	1.20	0.00	1.10	4.90	2.90
1990	5.40	0.10	5.10	2.50	5.00	5.60
Average	2.87	2.88	3.08	2.46	2.47	3.12

Note: Data for the period 1953–1990 along six profiles. Estimated data are in italic.

boring profiles), indicating the unsystematic and episodic nature of soft-cliff erosion.

Dataset B was compiled by the East Riding Council for the soft cliff at Hornsea-Rolston, along the Holderness in eastern England, which is the fastest eroding coastline in the United Kingdom, with an average erosion rate ranging from 1 to 3 m per year (Meadowcroft et al. 1999). In the area, measurements of coastal retreat were made at regular time interval (mostly once a year) from 1953 to the present time. The dataset we have used lists 198 measurements of coastal retreat distances, 168 of which are larger than zero, in the 38-year period from 1953 to 1990 (Table 2). The particular time period and six profiles (200–800 m apart) were selected because they are substantially complete, unaffected by manmade structures, and they contain a wide range of erosion sizes, including some very large events. The measured yearly cliff retreats vary from 0 to 21.94 m, with an average of 2.90 m and a standard deviation of 3.07 m, corresponding to a mean annual retreat of 2.52 m. It should be pointed out that the measurements at Hornsea-Rolston may also be affected by amalgamation. However, given the short time interval between the surveys, we argue that the problem is significantly less severe than in Dataset A. Inspection of Table 2 reveals that retreat measurements also varied significantly in time and in space.

Methods

To investigate the frequency-size statistics of coastal soft-cliff erosion at Cliff End (Dataset A) and Hornsea-Rolston (Dataset B) we obtain and analyze the noncumulative and the cumulative distributions of the available retreat measurements. The noncumulative distribution of the medium and large retreat measurements correlates with a power-law relation of the type

$$y = cL^{-\alpha} \tag{2}$$

where y=(noncumulative) frequency of retreat measurements of length L; and c and α=intercept and scaling exponent, respectively. The equivalent cumulative distribution is a power-law relation of the type

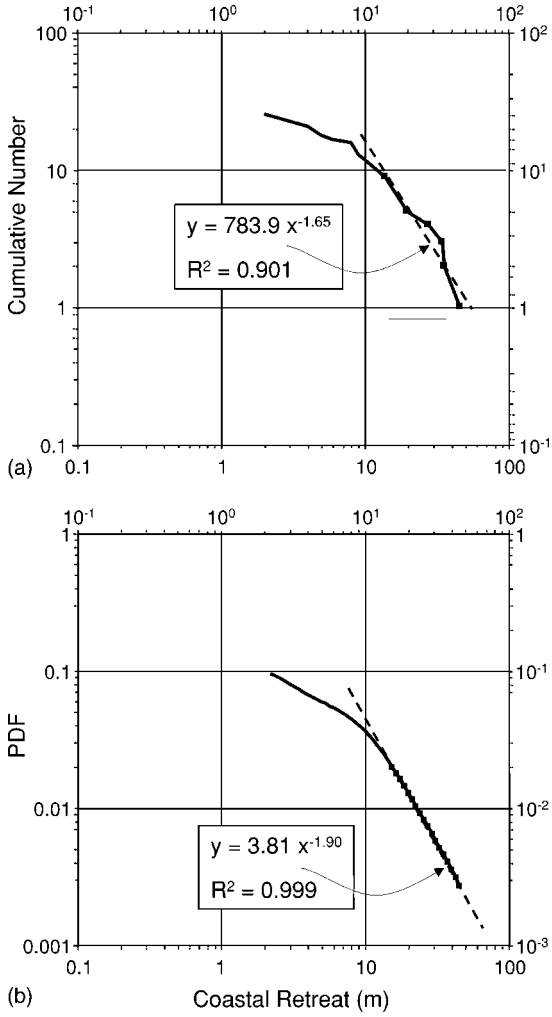

Fig. 2. Dataset A, Cliff End: (a) cumulative distribution, and (b) noncumulative distribution

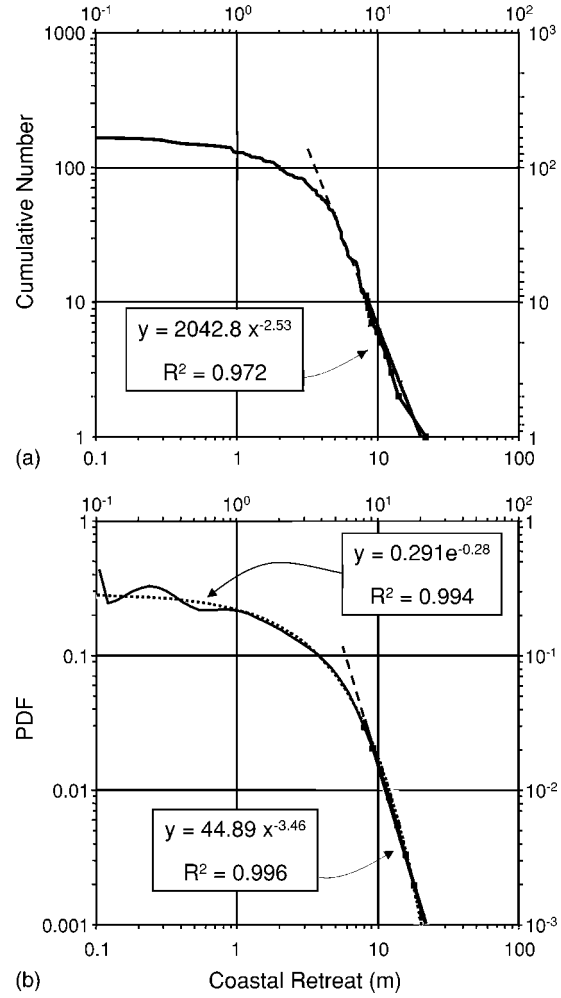

Fig. 3. Dataset B, Hornsea-Rolston: (a) cumulative distribution, and (b) noncumulative distribution

$$Y = c'L^{-\beta} = c'L^{-(\alpha-1.0)} \tag{3}$$

where Y=cumulative frequency of the retreat measurements of length $\geqslant L$; and c' and β=intercept and scaling exponent, respectively. The exponents for the cumulative and the noncumulative distributions are related. For a noncumulative power law distribution [Eq. (2)] with $a>1.0$, the corresponding cumulative distribution is a power law [Eq. (3)] with exponent $\beta=(\alpha-1.0)$.

We treat both datasets using the same methodology. We start by preparing the cumulative distribution from the measured length of coastal retreat (the raw data). We do this by (1) counting the number of events of any given size, (2) listing the events from largest to smallest, (3) counting the cumulative number of events, and (4) graphing the measured distance (x-axis) versus the cumulative number of events (y-axis), in a log-log space. Results are shown in Figs. 2(a) and 3(a) for Datasets A and B, respectively. We then fit a power law to the tail of the noncumulative distribution, where it exhibits a clear linear trend in the log-log plot.

Obtaining the noncumulative distribution from the raw data is no trivial task because of the irregularity in the data and the scarcity of large events. We adopt a kernel-density estimation technique implemented in a Bayesian scheme. Results of the kernel density estimations are shown in Figs. 2(b) and 3(b), for Datasets A and B, respectively. Again, we fit a power law to the tail of the noncumulative distribution, where it exhibits a clear linear trend in the log-log plot. The scaling exponent of the cumulative distribution, β, should be equal to $\alpha-1$, where α is the scaling exponent of the noncumulative distribution.

Results and Discussion

We start our discussion of the results from Dataset B as it is larger, taken at regular time intervals, and less affected by amalgamation problems. For this dataset the slope of the cumulative distribution ($\beta=-2.53$) corresponds to the slope of the noncumulative distribution ($\alpha=-3.46$, $\beta=\alpha-1=-(3.46-1.00)\sim-2.53$). We attribute the slight difference (2%) in the scaling exponents to the fact that we fitted different series, one for the cumulative distribution (the raw data) and one from the Bayesian estimate of the noncumulative distribution. This has probably introduced minor differences in the fits.

Fig. 3(b) reveals that the noncumulative distribution deviates from the power-law fit for retreats of less than about 8 m. This can be interpreted in different ways: the dataset may be incomplete for smaller retreat values (e.g., because of minor amalgamation problems), or it may be a length scale at which the distribution changes (i.e., a break in scale). Fig. 3(b) also shows that for values of retreat smaller than about 8 m an exponential relation (dotted line) well approximates the distribution while the

power-law model clearly breaks down for these small retreats. Beyond 8 m the exponential model slightly overestimates lengths from 10 to 20 m, and underestimates the lengths greater than about 30 m. Considering the fact that for very large values of costal retreat the exponential model consistently (and increasingly) underestimates the distribution, these values may be best approximated by a power-law relationship. Thus, one can argue that the Hornsea-Rolston data can be approximated by a hybrid model: exponential for small values of the retreat, and power-law for large values of the retreat.

The cumulative distribution of Dataset B can be fitted (at least apparently) by a power law for a larger range of values ($L>5$ m) than the corresponding noncumulative distribution ($L>8$ m). The difference is small, but significant. There are 46 measurements >5 m (25.6%), and only 11 measurements >8 m (6.5%). Thus, by fitting the cumulative distribution one can get the misleading impression that the power-law fit holds for a larger range of values than it actually does.

In comparison with Dataset B, Dataset A is more difficult to analyze and interpret, due to the very small number of measurements and to severe amalgamation problems. The tail of the noncumulative distribution obeys a power law with scaling, $\alpha=-1.90$, for $L>15$ m. This indicates that the scaling for the cumulative distribution, $\beta=\alpha-1$, should be ~ 0.9. However, by fitting the tail of the cumulative distribution we obtain a scaling exponent $\beta=1.65$, which is much larger than expected. We attribute the difference (mismatch) to the very small number of samples, and to the difficulty of correctly estimating a distribution from a very small dataset.

Despite this problem, meaningful comparison of the two datasets is still possible. Both datasets exhibit similar cumulative and noncumulative distributions. In particular, both datasets exhibit power-law scaling for large retreats. For Dataset B (Hornsea-Rolston) the scaling behavior begins at $L=8$ m, whereas for Dataset A (Cliff End) the scaling behavior starts at $L=15$ m. The difference can be attributed to more accurate (regular and high resolution) measurements in Dataset B. The scaling exponents for the two datasets are different. For Dataset B, the cumulative $[\beta=(\alpha-1)=-2.53]$ and the noncumulative ($\alpha=-3.14$) exponents are in good agreement. This is not the case for Dataset A, for which we obtained values of $\beta=(\alpha-1)=-1.65$ and of $\alpha=-1.90$, for the cumulative and noncumulative distributions, respectively. Both values do not match with what we have obtained for Dataset B. We attribute this to the limited number of measurements, and to the fact that as a result of amalgamation the dataset contains an excess of large events, which makes for a heavier tail.

It should be emphasized that the self-similar behavior holds for a limited range of values: from $L=8$ to $L=20$ m for the noncumulative distribution of Dataset B and (less clearly) from $L=15$ to $L=40$ m for the noncumulative distribution of Dataset A. This makes it difficult to argue definitively in favor of a fractal (self-similar) behavior of all sizes of coastal retreats although we believe it is more likely than a log-normal distribution, at least for the size ranges of engineering significance.

Lastly, we note that Malamud et al. (2004) have shown that the scaling exponent of the noncumulative distribution of medium to large landslides is $\alpha=-2.4$. Assuming that the area of the coastal erosion (in plan view) is $A\approx L^{1.5}$, accounting for an elongated shape of the erosion events, from Eq. (2) we obtain $A\approx L^{-3.6}$, which is in reasonable agreement with the scaling exponent obtained for Dataset B ($\alpha=-3.46$).

Conclusions

Prediction of soft-cliff erosion remains a difficult task in coastal morphodynamics. For the development of both deterministic and probabilistic models, it is essential to acquire high resolution series of coastal retreats and to obtain the frequency-size statistics of the retreats from the data.

Our finding indicates that the distribution of erosion sizes at Cliff End and at Hornsea-Rolston in England exhibits a characteristic power-law behavior, at least for retreats larger than about 8 m. This result is significant, albeit not conclusive. The fact that the power-law distributions diverge for smaller coastal retreats may indicate incompleteness (and amalgamation) in data, or it may signify that frequency-size statistics of erosion is dependent on two distributions, one being governed by rare, large-magnitude events, and the other by frequent, small-size events.

Results presented here are indicative rather than conclusive. They are limited to two sites in England that exhibit significant differences in the pattern and (average) magnitude of erosion. More data, collected at different soft-cliff sites and involving different failure mechanisms, are needed to confirm that the frequency-size statistics of soft-cliff retreats indeed obey a power law. Further investigation is also required to establish the connections between the empirical scale relationships of the distributions and the various forces and mechanisms governing soft-cliff erosion.

Acknowledgments

The writers thank Neil Mclachlan of East Riding Council for supplying the Holderness data. The research is supported, in part, by a research grant from the Leverhulme Trust (Grant No. F/00143/D).

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