

# Optimal inspection decisions for the block mats of the Eastern-Scheldt barrier

J.M. van Noortwijk<sup>a,\*</sup>, H.E. Klatter<sup>b</sup>

<sup>a</sup>*HKV Consultants, PO Box 2120, 8203 AC Lelystad, The Netherlands*

<sup>b</sup>*Ministry of Transport, Public Works, and Water Management, Civil Engineering Division, PO Box 20000, 3502 LA Utrecht, The Netherlands*

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## Abstract

To prevent the southwest of The Netherlands from flooding, the Eastern-Scheldt storm-surge barrier was constructed, has to be inspected and, when necessary, repaired. Therefore, one is interested in obtaining optimal rates of inspection for which the expected maintenance cost is minimal and the barrier is safe. For optimisation purposes, a maintenance model was developed for part of the sea-bed protection of the Eastern-Scheldt barrier, namely the block mats. This model enables optimal inspection decisions to be determined on the basis of the uncertainties in the process of occurrence of scour holes and, given that a scour hole has occurred, of the process of current-induced scour erosion. The stochastic processes of scour-hole initiation and scour-hole development was regarded as a Poisson process and a gamma process, respectively. Engineering knowledge was used to estimate their parameters. © 1999 Elsevier Science Ltd. All rights reserved.

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## 1. Introduction

In this article, we consider the problem of inspecting the block mats of the Eastern-Scheldt storm-surge barrier. These mats are part of the barrier's sea-bed protection and they must be inspected to detect possible scour holes that might endanger the stability of the barrier. Therefore, one is interested in obtaining optimal rates of inspection for which the expected maintenance cost is minimal and the probability of failure of the block mats is safe. For this purpose, mathematical expressions were obtained for the probability of failure of one scour hole (the probability that a scour hole is deeper than a certain failure level) and for the probability of failure of the block mats (the probability that at least one scour hole is deeper than the failure level).

A large number of articles were published on the subject of optimising maintenance through mathematical models (see e.g., [3]). Most maintenance optimisation models are based on lifetime distributions or Markovian deterioration models. As it is often hard to gather data for estimating either the parameters of a lifetime distribution or the transition probabilities of a Markov chain, few of these models

were applied (see [3]). Moreover, in the case of well-planned preventive maintenance, complete lifetimes will rarely be observed. Therefore, we propose to model maintenance on the basis of the main uncertainties involved: the rate of scour-hole initiation and the rate of scour-hole development.

The inspection problem is a result of Jorissen and De Leeuw van Weenen [8] and van Noortwijk et al. [16]. Our model differs from the model in Ref. [8] in the sense that, among other things, we regard scour erosion as being a stochastic process rather than a deterministic process. It differs from the model in Ref. [16] in the sense that we assume the rate of scour erosion to be decreasing rather than being a constant. The stochastic processes of scour-hole initiation and scour-hole development was regarded as a Poisson process and a gamma process, respectively. The rate of scour erosion was determined by using an empirical law. Moran [11] used gamma processes in his theory of water storage in dam reservoirs. In The Netherlands, gamma processes have also been used to model decision problems for optimising maintenance of dykes, beaches, and berm breakwaters (see [14, 17–18]).

The article is set out as follows. A brief description on the Eastern-Scheldt barrier is given in Section 2. In Section 3, we present the inspection model for the block mats of the barrier. Conclusions can be found in Section 4.

\* Corresponding author. Tel.: + 31-320-251-126; Fax: + 31-320-253-901.

E-mail address: j.m.van.noortwijk@hkv.nl (J.M. van Noortwijk)

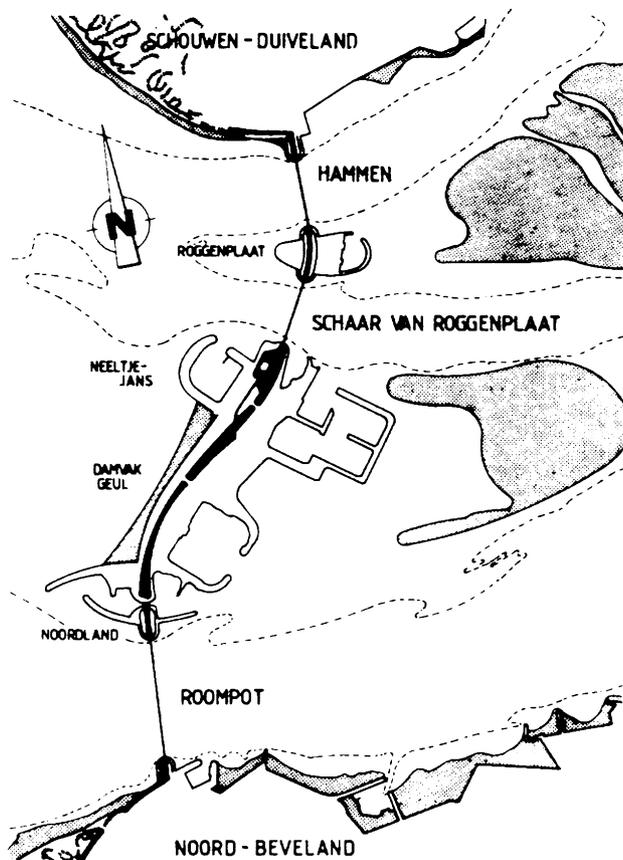


Fig. 1. The map of the Eastern-Scheldt estuary showing the two artificial dredge-improved islands ('Roggenplaat' and 'Neeltje Jans') and the three tidal channels controlled by lift-gate barriers. ('Hammen', 'Schaar', and 'Roompot').

## 2. The Eastern-Scheldt barrier

With storm-induced tides of some 4 m above average sea level, the flood of February 1, 1953, caused a severe catastrophe in Zeeland, The Netherlands. Almost 200 000 ha of polderland flooded, resulting in huge losses of life and property. In the southwest of The Netherlands, 1835 people and tens of thousands of animals drowned. To avoid future losses caused by floods like the one in 1953, the Dutch parliament adopted the so-called Delta Plan. The greater part of this plan called for raising the dykes as well as closing the main tidal estuaries and inlets by utilising a network of dams and barriers. As the Delta plan was completed, attention has shifted from building structures to maintaining structures. Hence, the use of maintenance optimisation models is of considerable interest.

This article is devoted to modelling preventive maintenance of one part of the sea-bed protection of the most expensive and most complicated structure of the Delta Works: the Eastern-Scheldt storm-surge barrier. The design of this lift-gate barrier is complex for it has to satisfy requirements in the following areas:

1. safety (flood protection during severe storm-surges when the gates are closed),
2. environment (preservation of the natural salt-water environment during normal weather and hydraulic conditions when the gates are open), and
3. transport (shipping access to the North-Sea as well as a road-connection).

The Eastern-Scheldt barrier was built in three tidal channels separated by two artificial islands (see Fig. 1). It has 62 pier-supported steel gates, each with a span of nearly 42 m and height variations between 6–12 m. To provide for the long-term stability of the barrier, the supporting concrete piers are embedded in several layers of rock and an adjoining sea-bed protection was constructed with a width varying from 550 to 650 m on either side of the centre line of the barrier. This sea-bed protection consists of asphalt mastic and block mats in the outer periphery, and graded filter mattresses under the piers (see Fig. 2). As the protection can be damaged, the remote part is monitored for the occurrence of scour holes. Given this situation, the rates of inspection and the cost of maintenance have to be optimised. For a brief summary on the technical aspects of the Eastern-Scheldt barrier, see [12,19].

## 3. Maintenance of the block mats

The block mats consist of synthetic material to which small concrete-blocks (with a height of 17 cm) are attached in a regular pattern. The purpose of this section is to obtain safe and cost-optimal rates of inspection for these mats.

### 3.1. Inspection and repair of scour holes

Owing to extreme tidal currents or ship anchorings, the block mats could be damaged in such a way that sand washes away and scour holes occur. To detect possible scour, the block mats are inspected by means of acoustic measurements. If acoustic inspection reveals a scour hole, then a detailed inspection will be carried out, followed by a repair. To confirm the statement that there is often a lack of data in practice: since the completion of the barrier in 1986, no scour holes were detected!

As preventive maintenance is based on the condition of the block mats, we are dealing with so-called condition-based preventive maintenance. Apart from condition-based repairs, it might be economic to perform (non-periodic) condition-based inspections as well. In practice, however, periodic inspections are often preferred as the necessary manpower and budget can be anticipated and scheduled well beforehand. Further, we assume that inspection of all the block mats takes negligible time, does not degrade the block mats, and entails a cost  $c_1$ . Since the completion of the barrier, inspections were carried out four times a year.

A repair is defined as placing graded rip-rap on a

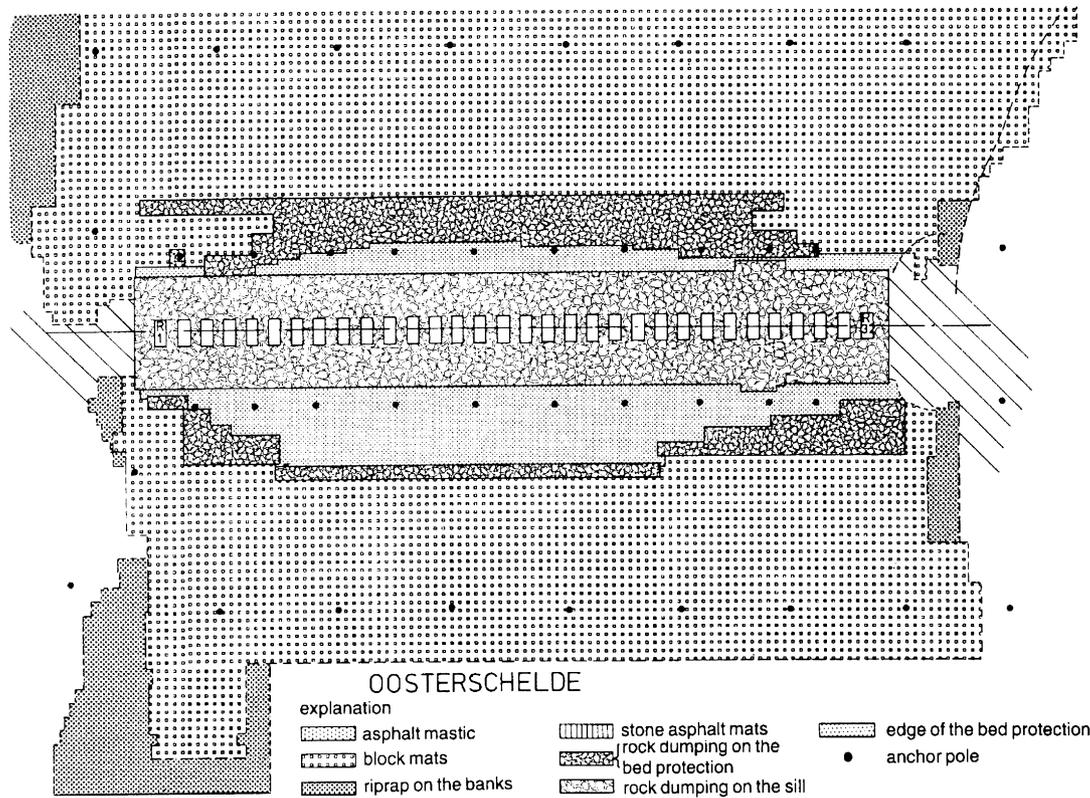


Fig. 2. View from above of the sea-bed protection of the Eastern-Scheldt barrier at the tidal channel ‘Roompot’ [12].

scour-hole surface, approximately being a hemisphere of radius  $h$ , where  $h$  is the maximum scour-hole depth (in m). The cost of repairing one scour hole can be subdivided into the fixed cost  $c_f$  (cost of mobilisation like shipping transport) and the variable cost  $c_v$  (cost per square metre rip-rap). Hence, the costs of repairing one scour hole, which is  $h$  metres deep, are

$$c(h) = c_f + 2\pi h^2 c_v. \tag{1}$$

In short, the block mats must be inspected to avoid instability of the barrier because of the following uncertain deterioration characteristics: (i) the average rate of occurrence of scour holes at all the block mats and (ii) the decreasing rate of current-induced scour erosion given that a scour hole has occurred. Because there is no direct deterioration data available, we have to rely on prior engineering judgement.

### 3.2. The stochastic process of scour-hole initiation

As suggested by van Noortwijk et al. [16], we assume the scour-hole inter-occurrence times to be exchangeable and to exhibit the ‘‘lack of memory’’ property. Exchangeability means that the order in which the scour holes occur is irrelevant. The ‘‘lack of memory’’ property means that the probability distribution of the remaining time until the occurrence of the first scour hole does not depend on the fact that a scour hole has not yet occurred since the completion

of the barrier in 1986. Let us denote the successive times between occurrences of scour holes by the infinite sequence of non-negative real-valued random quantities  $T_1, T_2, \dots$

Under the assumptions that the infinite sequence  $T_1, T_2, \dots$  is exchangeable and satisfies the ‘‘lack of memory’’ property of all  $n \in \mathbb{N}$ , we can write the joint probability density function of  $T_1, \dots, T_n$  as a mixture of conditionally independent exponentials:

$$p_{T_1, \dots, T_n}(t_1, \dots, t_n) = \int_{\lambda=0}^{\infty} \prod_{i=1}^n \frac{1}{\lambda} \exp\left\{-\frac{t_i}{\lambda}\right\} dP_{\Lambda}(\lambda) = f_n\left(\sum_{i=1}^n t_i\right), \tag{2}$$

for  $(t_1, \dots, t_n) \in \mathbb{R}_+^n$  and zero otherwise, where  $\mathbb{R}_+ = [0, \infty)$ . The infinite sequence of random quantities  $\{T_i : i \in \mathbb{N}\}$  is said to be  $l_1$ -isotropic (or  $l_1$ -norm symmetric), as its distribution can be written as a function of the  $l_1$ -norm. The random quantity  $\Lambda$ , with probability distribution  $P_{\Lambda}$ , describes the uncertainty about the limiting average inter-occurrence time of scour holes:  $\lim_{n \rightarrow \infty} [(\sum_{i=1}^n T_i)/n]$  (see e.g., [1,15]). The characterisation of Eq. (2) in terms of the ‘‘lack of memory’’ property is a result of Diaconis and Freedman [4]

For modelling the occurrences of scour holes, only the probability distribution of the average inter-occurrence time remains to be determined. To keep the mathematics of the

Table 1  
The parameters of the maintenance model for the block mats

Parameter	Description	Value	Dimension
$a$	Physical constant scour erosion process	10	m/year <sup>0.4</sup>
$b$	Physical constant scour erosion process	0.4	—
$\sigma$	Standard deviation of scour erosion	2.9	m
$\theta$	Measure of uncertainty scour erosion process	0.34	m
$\Lambda$	Average scour-hole inter-occurrence time	(0,∞)	year
$\lambda_{0.05}$	5th percentile of $Ig(\lambda \nu, \mu)$	1	year
$\lambda_{0.95}$	95th percentile of $Ig(\lambda \nu, \mu)$	10	year
$\nu$	Shape parameter of $Ig(\lambda \nu, \mu)$	2.46	—
$\mu$	Scale parameter of $Ig(\lambda \nu, \mu)$	5.46	year
$E(\Lambda)$	Expected average inter-occurrence time	4	year
$c_i$	Cost of inspection	87 500	Dfl
$c_f$	Fixed cost of repairing one scour hole	83 333	Dfl
$c_v$	Variable cost of rip-rap	1698	Dfl/m <sup>2</sup>
$c(5)$	Cost of repairing scour hole that is 5 m deep	350 000	Dfl
$c(10)$	Cost of repairing scour hole that is 10 m deep	1 150 000	Dfl
$c_F$	Cost of failure per scour hole	0	Dfl
$h$	Scour-hole depth	(0,∞)	m
$r_0$	Initial depth of a scour hole	0	m
$s$	Design depth of a scour hole	− 25	m
$y$	Scour-hole failure level ( $y = r_0 - s$ )	25	m
$P_{norm}$	Annual design norm probability of failure	0.003	—
$k$	Inspection interval length	(0,∞)	year
$k^*$	Cost-optimal inspection interval length	0.6	year
$\tilde{k}$	Largest acceptable inspection interval length	6	year

decision model tractable, we impose the property of *posterior linearity* introduced by Diaconis and Ylvisaker [5,6], i.e.,  $E(T_i|T_{i-1} = t) = c_1t + c_2$  for some constants  $c_1, c_2 > 0$  and  $i = 2, 3, \dots$ . The constants  $c_1$  and  $c_2$  represent the weights attached to the sample mean and the prior mean, respectively. Note that, because of exchangeability, before observing  $T_1$ ,  $E(T_i) = E(T_1)$  for all  $i$ . If posterior linearity holds, then the mixing distribution  $P_\Lambda$  is an inverted gamma distribution. That is, the probability density function of  $\Lambda$  is given by:

$$Ig(\lambda|\nu, \mu) = [\mu^\nu/\Gamma(\nu)]\lambda^{-(\nu+1)} \exp\{-\mu/\lambda\}I_{(0,\infty)}(\lambda), \quad (3)$$

where  $I_A(x) = 1$  for  $x \in A$  and  $I_A(x) = 0$  for  $x \notin A$ , and  $\Gamma(a) = \int_{t=0}^{\infty} t^{a-1} e^{-t} dt$  is the gamma function for  $a > 0$ . The mathematical tractability is especially useful if one wants to update the prior distribution  $Ig(\lambda|\nu, \mu)$  with actual observations  $t_1, \dots, t_n$ . In fact, using Bayes' theorem, the posterior distribution is  $Ig(\lambda|\nu + n, \mu + \sum_{i=1}^n t_i)$ . The inverted gamma distribution is said to be *conjugate* with

respect to the exponential likelihood function, as both prior and posterior distribution belong to the family of inverted gamma distributions (see e.g., [2], ch. 9). As the posterior mean can be written as a linear combination of the prior mean and the sample mean, the property of posterior linearity is satisfied. The conjugate family of inverted gamma distributions is sufficiently rich to approximate reasonably closely any prior belief. Moreover, Diaconis and Ylvisaker [6] have shown that any prior density for an exponential family parameter can be approximated arbitrarily closely by a mixture of conjugate priors.

### 3.3. The stochastic process of scour-hole development

Given that a scour hole has occurred, the question that arises is how its depth increases over time. Empirical studies show that the expected maximum scour-hole depth at time  $t$ , denoted by  $h(t)$ , behaves according to the following power law [7,9,13]:

$$h(t) = at^b \quad (4)$$

for some physical constants  $a > 0$  and  $0 < b < 1$ . This power law is based on laboratory scour observations with various bed materials, flow velocities, and geometries. Unfortunately, the uncertainty in the maximum scour-hole depth is rather large: extrapolating results from a small laboratory set-up to the large Eastern-Scheldt barrier induces many uncertainties. To account for these uncertainties, scour-hole development was regarded as a stochastic process on the basis of the earlier mentioned power law. Further, we assume the increments of erosion to be non-negative. In other words, we assume that the stochastic erosion process is non-decreasing.

In order for the stochastic process of scour erosion to be non-decreasing, we can best regard it as a gamma process (see e.g., [16]). First, recall that a random quantity  $X$  has a gamma distribution with shape parameter  $\alpha > 0$  and scale parameter  $\beta > 0$  if its probability density function is given by:

$$Ga(x|\alpha, \beta) = [\beta^\alpha/\Gamma(\alpha)]x^{\alpha-1} \exp\{-\beta x\}I_{(0,\infty)}(x).$$

Further, let  $\alpha(t)$  be a non-decreasing, left continuous, real-valued function for  $t \geq 0$ , with  $\alpha(0) \equiv 0$ . The gamma process with shape function  $\alpha(t) > 0$  and scale parameter  $\beta > 0$  is a continuous-time stochastic process  $\{Y(t): t \geq 0\}$  with the following properties:

- (i)  $Y(0) = 0$  with probability one;
- (ii)  $Y(\tau) - Y(t) \sim Ga(\alpha(\tau) - \alpha(t), \beta)$  for all  $\tau > t \geq 0$ ;
- (iii)  $Y(t)$  has independent increments.

Let  $X(t)$  denote the maximum scour-hole depth at time  $t$ ,  $t \geq 0$ , and let the probability density function of  $X(t)$ , in conformity with the physical law in Eq. (4), be given by

$$p_{X(t)}(x) = Ga(x | [at^b]/\theta, 1/\theta) \quad (5)$$

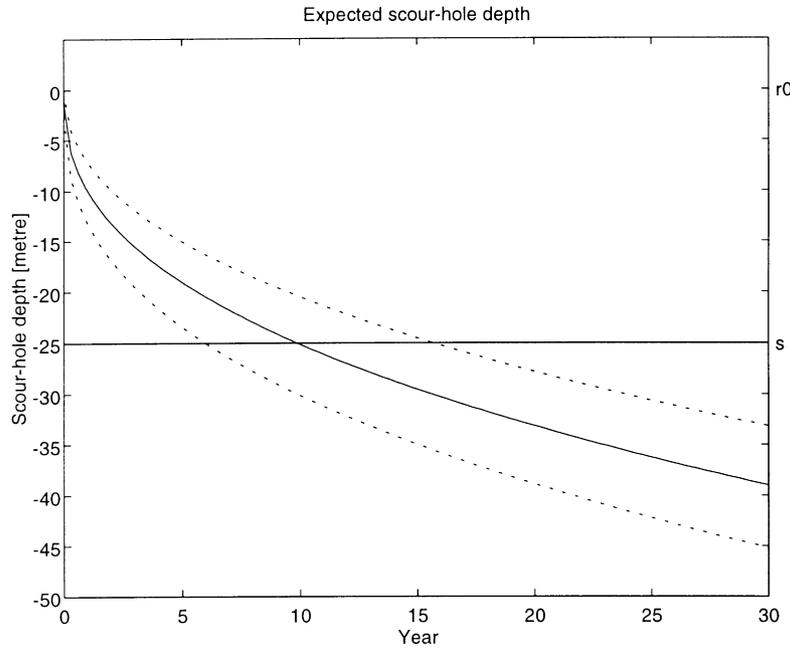


Fig. 3. The expected scour-hole depth, and its 5th and 95th percentile, as a function of time.

for  $\theta > 0$ , with

$$E(X(t)) = at^b, \quad E(X^2(t)) = (\theta + at^b)at^b, \quad (6)$$

$$\text{Var}(X(t)) = \theta at^b.$$

The parameter  $\theta$  represents the uncertainty in the scour erosion process: the larger  $\theta$ , the more uncertain the erosion process. In assessing  $\theta$ , we take account of the uncertainty in the maximum scour-hole depth in both time and space.

A scour hole is said to fail when it is deeper than a certain failure level, say  $y$ , where  $y$  is defined as the initial depth ('resistance')  $r_0$  minus the design depth at which failure occurs ('design stress')  $s$ . The values of  $y$ ,  $r_0$ , and  $s$  can be found in Table 1. Let the time at which failure occurs, i.e., at which the failure level is crossed, be denoted by the lifetime  $T$ . Owing to the gamma distributed maximum scour-hole depth in Eq. (5), the lifetime distribution can be written as:

$$F(t) = \Pr\{T \leq t\} = \Pr\{X(t) \geq y\} = \int_{x=y}^{\infty} p_{X(t)}(x) dx$$

$$= \frac{\Gamma([at^b]/\theta, y/\theta)}{\Gamma([at^b]/\theta)}, \quad (7)$$

where  $\Gamma(a, x) = \int_{t=x}^{\infty} t^{a-1} e^{-t} dt$  is the incomplete gamma function for  $x \geq 0$  and  $a > 0$ .

As we are mainly interested in the uncertainty in the lifetime of a scour hole, the uncertainty parameter  $\theta$  can best be assessed by specifying the standard deviation of the maximum scour-hole depth at the time at which the expected depth equals the failure level. When this standard deviation is denoted by  $\sigma$ , the parameter  $\theta$  follows immediately from Eq. (6): i.e.,  $\theta = \sigma^2/y$ .

As there is engineering knowledge available about the

stochastic process of scour-hole development, in terms of Eq. (4), the uncertainty in the decreasing rate of scour-hole development is smaller than the uncertainty in the average rate of occurrence of scour holes. Therefore, we may assume the former rate to be deterministic, i.e., we may assume the physical constants  $a$  and  $b$ , and the statistical parameter  $\theta$ , to be deterministic. The parameters  $a$ ,  $b$ , and  $\theta$  were determined by choosing a representative part of the block mats, evaluating the corresponding scour-hole development on the basis of empirical studies, and estimating the uncertainties involved (see Table 1).

With respect to the probability of detection, we note that scour holes can only be detected when they are deeper than about 2 m. Owing to the power law in Eq. (4), the time-period during which the expected maximum scour-hole depth develops to the detectability level is very small (to be precise: 0.018 y; see also Fig. 3). Therefore, we may assume that a scour hole is detected as soon as it occurs.

### 3.4. The maintenance decision model

Based on the two stochastic processes of the occurrence and development of scour holes, which are judged to be independent, we can formulate the maintenance optimisation model. Subsequently, we determine the expected average costs of maintenance and failure per year, and the expected probability of failure of the block mats.

Our main interest is to determine an inspection interval of length  $k$ ,  $k > 0$ , for which the expected maintenance costs are minimal and the barrier is safe, where inspections are carried out at times  $\{jk: j \in \mathbb{N}\}$ . Let  $L(\lambda, k)$  be the monetary loss when the decision-maker chooses inspection interval  $k$  and when the limiting average inter-occurrence time  $\lambda$  is

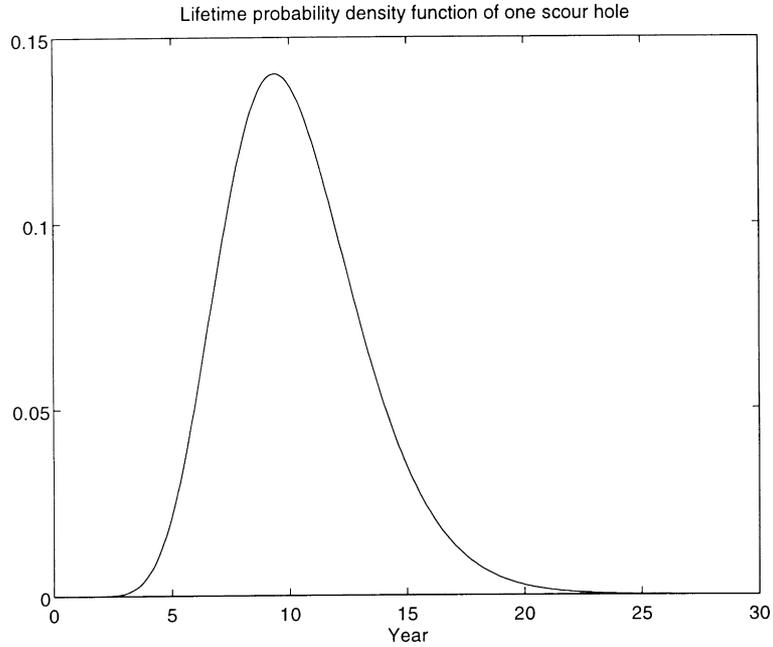


Fig. 4. The lifetime probability density function of one scour hole.

given. As a matter of fact, given the value of  $\lambda$ , the scour holes occur according to a Poisson process with arrival rate  $\lambda^{-1}$ . Note that, because of the ‘lack of memory’ property, this rate does not depend on time. The decision-maker can be best choose the inspection interval  $k^*$  whose expected loss,  $E(L(\Lambda, k^*))$ , is minimal. A decision  $k^*$  is called an *optimal decision* when  $E(L(\Lambda, k^*)) = \min_{k \in (0, \infty)} E(L(\Lambda, k))$  (see e.g., [2], ch. 8).

As the decision problem at hand is not a question of optimally balancing the initial building cost against the future maintenance cost, we may assume the loss function to be the *expected average costs per year* (for a discussion, see e.g., [17]). Moreover, because only small inspection intervals are considered, the time value of money is of little consequence to us and there is no need to use discounting. The expected average costs per year can be determined by

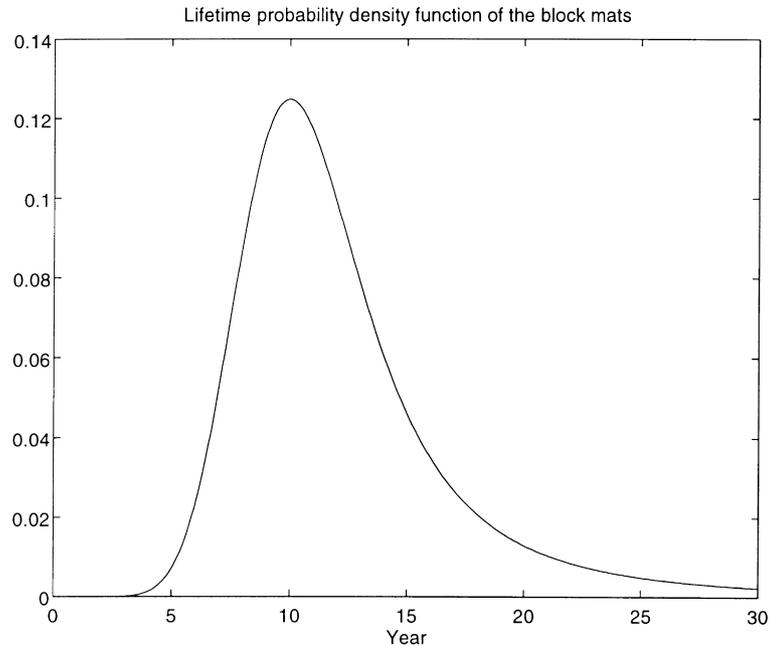


Fig. 5. The lifetime probability density function of the block mats.

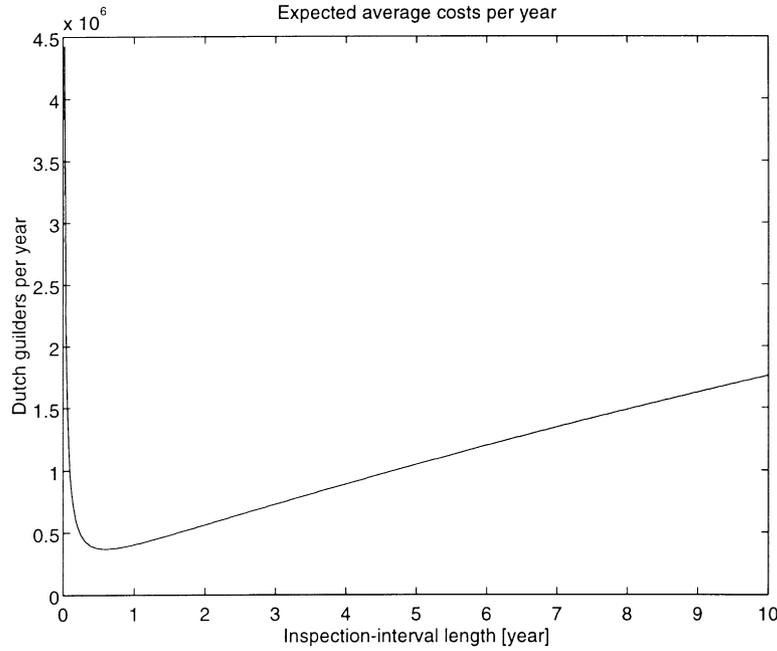


Fig. 6. The expected average costs per year for the block mats.

averaging the costs of maintenance and failure over an unbounded time-horizon. Recall that the costs of maintenance consist of the cost of inspection, which is  $c_I$  per inspection, and the cost of repair, which is given by Eq. (1) for every scour hole and depends on the observed size of the hole. In addition, we assume that every failed scour hole entails  $c_F$  cost of failure.

Using a renewal argument, we can write the expected average costs per year as follows:

$$\begin{aligned}
 L(\lambda, k) &= \frac{1}{k} \left\{ c_I + \frac{1}{\lambda} \int_{t=0}^k [c_f + 2\pi c_v E(X^2(t)) + c_F F(t)] dt \right\} \\
 &= \frac{c_I}{k} + \frac{1}{\lambda} \left[ c_f + 2\pi c_v \left\{ \frac{\theta a k^b}{b+1} + \frac{a^2 k^{2b}}{2b+1} \right\} \right. \\
 &\quad \left. + \frac{c_F \int_{t=0}^k F(t) dt}{k} \right],
 \end{aligned}
 \tag{8}$$

where  $\lambda^{-1}$  is the expected number of scour holes occurring per year. The decision-maker can now best choose the inspection interval  $k^*$  whose expected average costs per year are minimal. The expected value  $E(L(\lambda, k))$  can be easily obtained by using  $E(L(\lambda^{-1})) = \nu/\mu$ .

A disadvantage of the aforesaid maintenance model might be that the assessment of the cost of failure is very difficult. This cost does not only consist of the cost of failure caused by damaged block mats, but also of possible costs caused by instability of the barrier and, if there is a severe storm-surge, of possible costs caused by flooding. In this respect, it is important to realise that the acceptable risk of

failure of a barrier seldom results from economic optimisation, but mostly from political considerations. This problem can be solved by leaving the cost of failure out of consideration and by introducing an upper bound for the inspection interval with the following property instead: when this upperbound is crossed, the block mats are said to be unsafe in the sense that there is at least one scour hole deeper than the failure level  $y$ . The probability of this event should be smaller than a predefined design norm probability, which in itself is a function of the inspection interval length  $k$ . For example:  $1 - (1 - P_{\text{norm}})^k$ , where  $P_{\text{norm}}$  is the annual design norm failure probability of the block mats because of scouring.

In mathematical terms, the failure probability of the block mats can be expressed as follows. By assuming the scour holes to be independent and by rewriting the probability of the event ‘‘in  $(0, k]$  at least one scour hole occurs which is deeper than  $y$  metres’’ as one minus the probability of the event ‘‘in  $(0, k]$  no scour holes occur that are deeper than  $y$  metres’’, we get ([10], p. 180):

$\text{Pr}\{\text{in } (0, k] \text{ at least one hole is deeper than } y\}$

$$\begin{aligned}
 &= 1 - \int_{\lambda=0}^{\infty} \exp \left\{ - \frac{\int_{t=0}^k F(t) dt}{\lambda} \right\} \text{Ig}(\lambda | \nu, \mu) d\lambda \\
 &= 1 - \left[ \frac{\mu}{\mu + \int_{t=0}^k F(t) dt} \right]^\nu.
 \end{aligned}
 \tag{9}$$

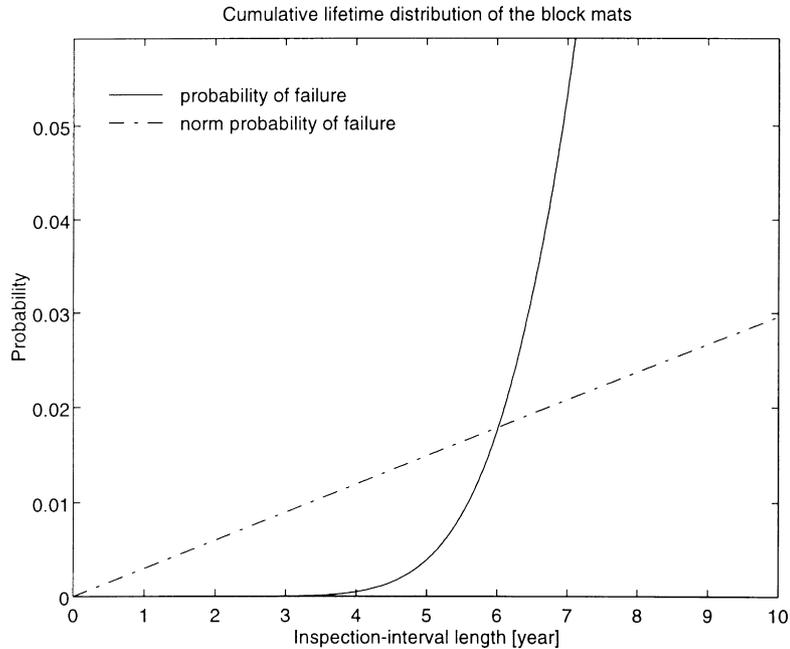


Fig. 7. The cumulative lifetime distribution of the block mats.

Therefore, an alternative decision might be an inspection interval  $\tilde{k}$  of which the expected average costs of maintenance are minimal amongst those inspection intervals of which the expected probability of failure of the block mats is safe (i.e., the probability of failure per inspection interval is smaller than a predefined design norm probability per inspection interval). We further note that the probability of failure of the block mats is influenced by the size of the area considered. The larger the area of the block mats, the smaller the expected scour-hole inter-occurrence time, and the larger the probability of failure.

For obtaining an optimal inspection interval for the block mats, we use the parameters in Table 1. The fixed cost  $c_f$  and the variable cost  $c_v$  in Eq. (1) was solved from the actual cost of repairing a scour hole of 5 m deep and the actual cost of repairing a scour hole of 10 m deep. The probabilistic characteristics of the stochastic process of scour erosion, in terms of its expected value and its 5th and 95th percentile, are shown in Fig. 3. The 5th and 95th percentile of the maximum scour-hole depth at the time at which the *expected* depth equals the failure level is 20 and 30 m, respectively. For each scour hole, the corresponding lifetime distribution is given by Eq. (7) and its probability density function is viewed in Fig. 4. The lifetime distribution of the block mats, which is based on a combination of the two stochastic processes of scour-hole initiation and scour-hole development, is given by Eq. (9) and its probability density function is displayed in Fig. 5.

The optimal decision  $k^*$  is an inspection interval of 0.6 y of which the expected average costs per year are  $3.7 \times 10^5$  Dutch guilders (see Fig. 6). A 'one at a time' sensitivity analysis has shown that  $k^*$  does not depend on the cost of failure  $c_F$ :  $k^*$  remains even and unchanged for  $c_F$  varying

from 0 to  $10^7$  Dutch guilders (Jorissen and De Leeuw van Weenen [8] suggested the cost of failure to be  $10^7$  Dutch guilders). Therefore, taking these considerations into account, the cost of failure can be neglected.

According to Fig. 7, the largest inspection interval  $\tilde{k}$  which is acceptable from the safety standpoint is 6 y. For inspection intervals that are larger than 6 y, the barrier is unsafe. The decision-maker can find an optimum balance between cost and safety using the curves in Figs. 6 and 7.

#### 4. Conclusions

In this article, we have presented a maintenance model that enables optimal inspection and repair decision to be determined for the block mats of the Eastern-Scheldt barrier. The model is based on the stochastic processes of scour-hole initiation and scour-hole development. They were regarded as a Poisson process and a gamma process, respectively. A physics-based approach was used to estimate the decreasing rate of current-induced scour-hole development and a case study has shown the usefulness of the maintenance model. Owing to the results of this case study, the manager of the Eastern-Scheldt barrier has changed the rate of inspection for the block mats from four times a year into twice a year.

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