Bayesian Analysis Applied to Statistical Uncertainties of Extreme Response Distributions of Offshore Wind Turbines

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ABSTRACT

Extreme response is an important design variable for wind turbines. The statistical uncertainties concerning the extreme response distribution are simulated here with data concerning physical characteristics obtained from measurements. The extreme responses are the flap moment at the blade root and the overturning moment of the support structure of an offshore wind turbine situated in the North Sea. The statistical uncertainties concern the choice of the distribution model and uncertainties concerning the distribution parameters. The uncertainties are treated with Bayesian analysis. The inclusion of the uncertainties has only marginal effect for the calculated long-term estimates of extreme responses when non-informative priors for the distribution parameters are used. The inclusion of uncertainties may have larger effect if data concerning the moments are obtained from measurements.

1. STATISTICAL UNCERTAINTIES OF EXTREME RESPONSES

The extreme response of a wind turbine can be an important design parameter. Extreme responses can be the flap moment at the blade root, bending moment at the tower foot, or extreme deflections of the blade tip etc. The extreme response is a stochastic variable, thus, it can be described statistically. In this study, the extreme responses are obtained using time domain simulations with input from real wind turbine data. The result is fitted to theoretical distribution models. The extreme response distribution can also be obtained using random peaks or random process models [10]. In this study, the extreme response distribution is obtained by fitting the maxima of the response from the simulations.

The estimate of extreme response is ridden with uncertainties. There are many types of uncertainties present in the whole process of the response estimation. In this context we only consider the statistical uncertainties that arise in fitting of the parametric models, such as the uncertainties of the distribution models and the parameters of the distribution models. The influence of the uncertainties is incorporated using Bayesian analysis.

2. BAYESIAN ANALYSIS OF THE UNCERTAINTIES

2.1. Introduction

The result of simulations has to be subjected to statistical analysis. The uncertainties that are associated with the statistical analysis are, among others, the choice of the distribution model and the parameters of the distribution. The choice of the distribution cannot be determined unambiguously in most cases, thus a subjective choice has to be made. The parameters of the distribution are determined using, for example, the Least Squares Method or the Maximum Likelihood Method. These uncertainties can be considered using the Bayesian analysis. The core of the Bayesian analysis is the Bayes theorem [8]

$$P(A|B) = \frac{P(A|B) \cdot P(A)}{P(B)} \quad [1]$$

where $P(A|B)$ denotes the conditional probability of A given B. The subjectivity of the Bayesian analysis lies in the term $P(A)$ which represents the prior probability of the parameters. The $P(A|B)$ is the so called posterior probability. The choice of the prior probability can be rather arbitrary. It is usually based on a subjective judgement about the character of the distribution (through experience, expert opinions etc.). Therefore, a Bayesian analysis is not a sound procedure in the eyes of frequentist statisticians because of this subjective element. However, in an engineering approach where the decision making process cannot always be based on an objective judgement, the Bayesian analysis is a useful tool to include statistical uncertainties.

As an example of the Bayesian analysis, we consider the extreme flap moment of the blade fitted to a Weibull distribution The Bayesian theorem can be written as

$$f''(\theta) = C \cdot \Pi f(Y_i|\theta) \cdot f'(\theta) \quad [2]$$

f'' is the posterior and f' is the prior probability density of the Weibull distribution parameters. θ represents the vector of the distribution parameters and C is a normalisation factor to be determined. $\Pi f(Y_i|\theta)$ is the so called data likelihood function and Y_i are the data obtained by simulations, e.g. maximum flap moments. The data likelihood function is also used to determine the distribution parameters,

$$\theta_{MLE} = \max \{\Pi f(Y_i|\theta)\} \quad [3]$$

θ_{MLE} is the maximum likelihood estimate (MLE) of the distribution parameters, where the likelihood function has a maximum. In the case of a 3 parameter Weibull distribution, we obtain a three dimensional probability density function of the distribution parameters and the hyper-volume of the function is normalised by the constant C to unity.

The uncertainties of the distribution parameters can be taken into account through the total probability theorem,

$$F(y) = \int\int\int F(y|\theta) \cdot f''(\theta) d\theta \quad [4]$$

$F(y|\theta)$ is in this case, the conditional distribution of the peak flap moment given a set of distribution parameters.

In the same manner that the uncertainties of the parameters are dealt with, the Bayesian analysis can be applied to the uncertainties in the choice of distribution models. Instead of the continuous density distribution of the parameters we have a prior set of weighting factors for different types of distributions.

$$f''(F_i) = C \cdot f(Y_i|F_i) \cdot f'(F_i) \quad [5]$$

F_i represents the different distribution types taken into consideration. The term $f(Y_i|F_i)$ is calculated using the following integral [7]

$$f(Y_i|F_i) = K_i = \int f(Y_i|\theta_i)\, f'(\theta) d\theta \qquad [6]$$

There is no information available that indicates a certain distribution is more likely than the others. Hence, a uniform prior is used, the prior weights of the distributions, $f'(F_i)$, are equally divided among the distributions. In this case the normalisation factor is simply $C = \frac{1}{\sum_i f'(F_i) \cdot K_i}$.

Using equation 5 the posterior weighting factors for the different distributions functions are obtained. The posterior weighting factors are taken in to account analogously, as described in equation 4 where the summation sign replaces the integration sign.

$$F(y) = \sum_i F(y|F_i) \cdot f''(F_i) \qquad [7]$$

Notice that the equation 4 takes into account all the possible variation of the distribution parameters, while equation 7 can only take a finite number of distribution functions into account. This implies also that the end result strongly depends on the selection of distribution function. Thus, one should make visualisations of the sample data in the form of P-P or Q-Q plots to determine if the distribution functions chosen are appropriate.

2.1. Uncertainties in the distribution parameters

In this section we are going to examine the uncertainties of the distribution parameters. The Bayesian analysis that is applied to treat the uncertainties of the distribution parameter is given in the previous sections. We applied the method to three different distribution functions: Gumbel, Weibull and the Normal distribution, to model the extreme flap moment obtained in 50 simulations. The choice of 50 simulations is based on a previous study that has shown that with 50 simulations the estimates of the 99 percentile of the response distribution does not change considerably, even if more simulations are added [9].

First the data likelihood function defined in equation 3 is obtained. The Weibull distribution has three parameters (see Appendix), thus the data likelihood is a function of three variables. Figure 1 shows the data likelihood for different combinations of the three parameters (that is, one parameter is fixed).

In principle one needs to integrate the data likelihood function over all the possible parameter values. However, the function decays rapidly toward zero, hence practically only a limited region need to be considered. The slight dependency between the two parameters s and k can be seen by following the maximum of the likelihood function for a given location parameter u (Figure 1, lower left). It seems rather symmetrical along a constant value of s or k but actually the symmetrical axis is slightly oblique. A larger s corresponds to larger shape factor k and vice versa. For constant k (Figure 1, lower right) and (Figure 1, upper left) there is also a slight symmetry between the two parameters.

First the posterior probability density of the distribution parameters is obtained. In this case, a uniform prior is assumed, so the posterior probability is the data likelihood function with a constant normalisation factor. Using the total probability theorem (equation 4) the distribution of the extreme flap response that includes the uncertainties of the distribution parameters is obtained. Instead of determining the probability of non-exceedance of a certain flap moment, it is also possible to determine the fractile values that are relevant for the design. This can be done by replacing the distribution function with its inverse. In this case the 100-year return period value of the extreme flap response is determined.

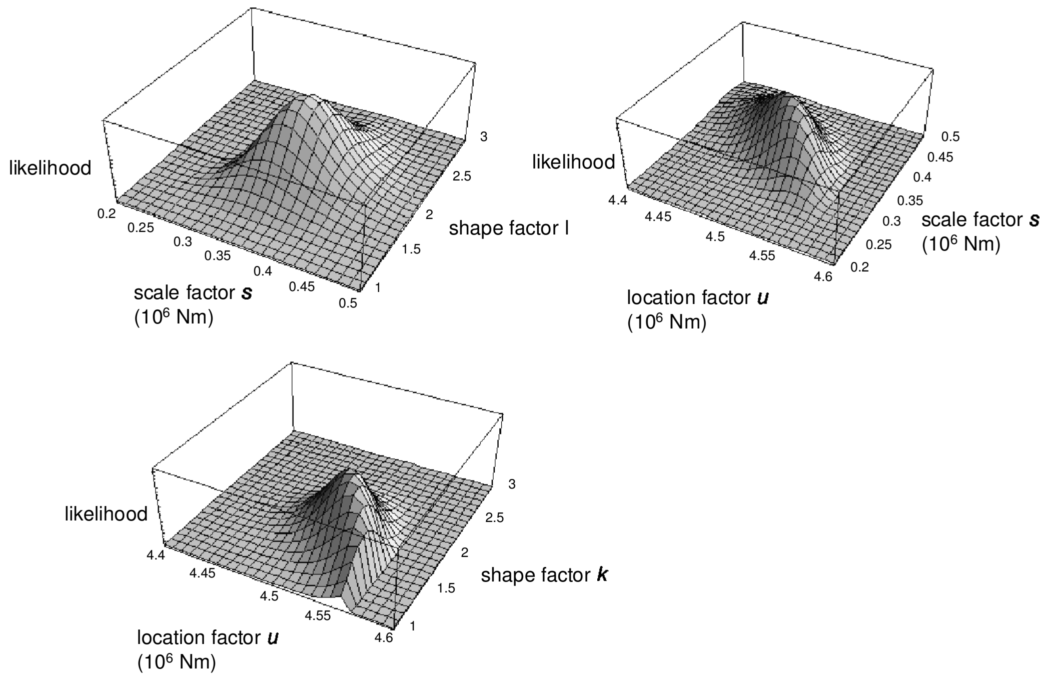

Figure 1 The data likelihood function of the extreme flap moment at 15 m/s, a 3 parameter Weibull distribution function.

To avoid the triple integration, the integral of equation 4 is split into a double and a single integral. First the data likelihood function is integrated over the s and k domain, which gives the marginal distribution of the location parameter u.

Then, the equation 4 is integrated over the domain of s and k. This yields the conditional probability (or the conditional quantile) as a function of the location parameter u. By applying the total probability theorem to the conditional quantile and the marginal distribution of the location parameter, the quantile value that includes the uncertainties of all the distribution parameters is obtained.

For the Weibull distribution, the variation of the scale parameter and shape parameter has the strongest impact on the estimate of the quantile values, because, (i), the location parameter u varies linearly with the physical variable x ($x = u + s\tilde{x}$, $\tilde{x}$ is the standard variable used in the Weibull distribution) and, (ii) the location parameter is usually determined by the lowest value of the data. The shape parameter k determines the 'gust factor' $\tilde{x}$ and the variation of the scale factor is always augmented by the 'gust factor'. The difference of the estimate is less than 1%, if instead of a variable location parameter, a constant location parameter is used (calculated with MLE)

This procedure is applied to the Gumbel and Normal distribution as in equation 4 and the quantile values that are representative for the 100-year return period value are obtained. The results are shown in the Table 1

Clearly the uncertainties concerning the distribution parameters do not have very significant effects on the long-term estimate in this case. The difference is no more than 2% for all the distributions. This can lead to the assumption that the uncertainties of the distribution parameters do not affect significantly the long-term estimate of the extreme flap response. However, one should keep in mind that the synthetic data set from simulations may present less variability than the real distribution (e.g. measurement). In that case the variability of the distribution on the long-term estimate may be more significant.

Table 1. 100-year flap moment with distribution parameter uncertainties and without

Distribution function	Bayesian estimate/10^6 Nm	Least squares estimate/10^6 Nm
Weibull	5.57	5.62
Gumbel	6.21	6.13
Normal	5.45	5.43

Usually, uncertainty would contribute to a higher estimate of the long-term response. In the Weibull case the inclusion of the distribution parameter uncertainty leads to a lower estimate instead. This can be seen in the marginal distribution of the location parameter u. (Figure 2). The probability that the location parameter is below the location parameter estimated with the least square method is much higher. Hence the long-term estimate is lower because integrating over the marginal probability density of u with the conditional quantile of the extreme response (Figure 3) yields a lower value. This has to do with the fact that the 3 parameter Weibull distribution is a limited distribution; i.e. the data likelihood for a location parameter larger than the minimum value of the sample is zero. Thus the marginal distribution of the location parameter has a right end point. However, the estimate depends on the conditional quantile as well, which in this case does not vary rapidly with the change of the location parameter u.

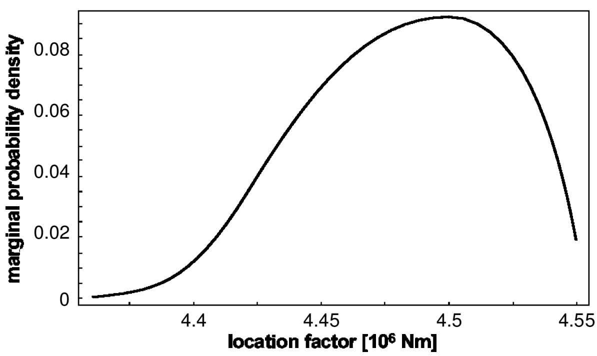

Figure 2 Marginal probability density function of the location parameter u of the Weibull distribution function.

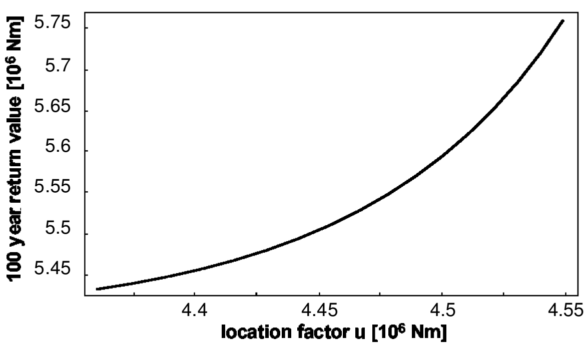

Figure 3 100 year flap moment conditioned on the location parameter u.

3. UNCERTAINTIES IN THE CHOICE OF DISTRIBUTION

The choice of the distribution model is surrounded by subjective decisions. However, there are different diagnostics to distinguish the more likely ones from the 'less likely' ones; but even these diagnostics cannot be always free of subjective elements. Before we proceed to choose the distribution function, we can describe the data. Such statistical descriptors can be the mean, median, variance, skewness etc. These will give us a general idea of how the data are distributed. As an example, we use a data set consisting of 50 maxima of extreme flap moment at the blade root, taken from 50 simulations each of 10 minutes. The different statistical descriptors are shown in Table 2.

From Table 2 it can be seen that the data have a small coefficient of variation and are positively skewed. The kurtosis is less than 3 (i.e. Normal distribution). We fit the data to 4 different functions: the Normal distribution, the Gumbel distribution, the Fréchet distribution and the Weibull distribution. The distributions are fitted with the Least Square Method. We also fitted the data to the Generalised Extreme Value (GEV) distribution [6], however, the fit yields a reverse Weibull distribution and this distribution is associated with a right end point. Considering that we will extrapolate the response distribution to a much longer period, this would be a serious limitation. Besides it is not clear where the physical limit is. For this reason we do not consider the GEV fit here.

The test for the goodness of fit, [3], is made for all the four distributions using the χ^2 test. For the present sample size of 50 data points, we choose a bin size that is about $\sigma/3$, where σ is the sample standard deviation. With a significance level of 5%, the null hypothesis cannot be rejected for all the 4 distributions. However, the bin size is critical for the χ^2 test. If we choose a larger bin size, the test result can lead to the rejection of the null hypothesis of the Normal distribution. There are also recommendations about choosing the bin size so that at least 5 samples will fall into a bin, or about collapsing the bins if the number of samples in the bin is too small. However, for small sample size, this again leads to large bin sizes, therefore one should be careful with such manipulations.

An empirical distribution test is also carried out, i.e. the Kolmogorov-Smirnov (K-S) test. This test is recommended if the sample size is small. It measures the absolute deviation of the fitted distribution from the sample distribution. Another class of test is the Empirical Distribution Function test. One of the tests is the Anderson-Darling test. It measures the deviation of the cumulative probability from the uniform distribution, giving a higher weight to the value in the tail region. If the detection of deviation in the tail region is important, the Anderson-Darling (A-D) test is to be preferred, which is the case for the estimate of extreme responses. According to the K-S and A-D tests, [3], the four distributions are acceptable models.

Figure 4 shows the sample distribution function with the different fits. The Gumbel fit and the Fréchet fit are almost identical, because the shape factor for the Fréchet distribution is in the order of 10^8 and the Fréchet distribution converges to Gumbel distribution if the shape factor $s \rightarrow \infty$. For this reason only Gumbel distribution is taken into account.

It can be seen that the Gumbel has the heaviest tail, followed by the Weibull and Normal distributions. Since a visual inspection cannot offer much help to choose a suitable distribution, we analyse the statistical descriptors of the distributions. The mean and standard deviations of the Normal distribution are identical to the sample mean and standard deviation. The rest can be calculated from the distribution functions. Table 2 shows the first four descriptors of the distribution functions.

From Table 2 it can be seen that the mean and standard deviations are well approximated by the Gumbel and Weibull distribution functions. The distribution parameters of the Normal distribution are explicitly defined by the sample mean and standard deviation. The difference between the distributions lies in the skewness and the kurtosis. The samples suggest a slight positive skewness and a kurtosis that is below the kurtosis of the Normal distribution. The Gumbel distribution has a fixed skewness and kurtosis, which are independent of the scale and location parameters. This skewness and kurtosis are much higher than for the Gaussian distribution, which therefore is expected to yield also the highest estimate. Weibull is a more flexible distribution, which can model the skewed samples without producing a too heavy upper tail because of the extra shape parameter. The Normal distribution is also acceptable considering the light tail of the sample.

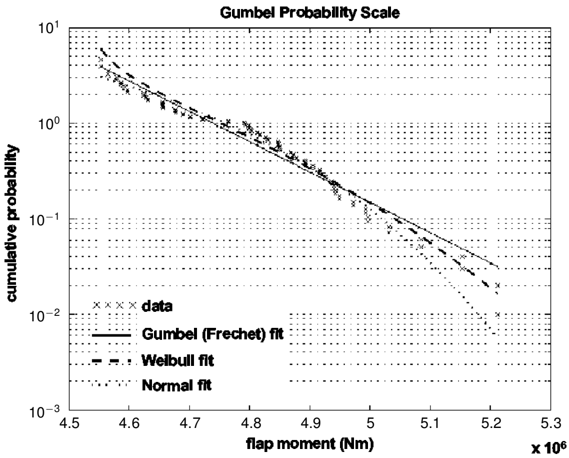

Figure 4 Simulated maxima of the flap moment fitted to different distribution functions.

Table 2. Statistical moments of the simulated flap moment and of the fitted distributions.

	Data	Weibull	Gumbel	Normal
Mean/10^6 Nm	4.81	4.82	4.81	4.81
Standard deviation/10^6 Nm	0.15	0.15	0.17	0.15
Skewness	0.22	0.69	1.13	0
Kurtosis	2.58	3.2	5.4	3

4. BAYESIAN ANALYSIS OF THE DISTRIBUTION CHOICE APPLIED TO THE MOMENTS

Using the procedure described in the previous section, we can take into account the uncertainty of the distribution choice in a semi-empirical way. Again, the 3 distributions that have been tested before are chosen, and the sample data are the extreme flap moment taken from 50 simulations with a mean wind speed of 15 m/ s because it represents the most severe load condition for this pitch regulated turbine [1].

The prior probabilities are equal for all the three distributions, i.e. 1/ 3. The posterior probabilities are calculated using the integration formula with a uniform prior $f'(\theta)$, for the distribution parameters (see equation 6).

Table 3. Participation factors and fractile values of the extreme flap moment distributions.

Distribution	Participation	99 percentile/10^6 Nm	100 year flap moment/10^6 Nm
Gumbel	14.6%	5.36	6.13
Normal	0.8%	5.28	5.61
Weibull	84.6%	5.25	5.62
Bayes	-	5.26	5.93

Table 3 shows the 99 percentiles of the distributions of the maximum flap moment for different distribution functions. It also shows the posterior probabilities of the distribution functions as percentage of the participation, i.e. the contribution in percentage to the total distribution. There is a distinct dominance of the Weibull distribution. Even if the Normal distribution does give a very close estimate to the Weibull's estimate, the contribution of the Normal distribution is insignificant. The difference in all the 99 percentile estimates is less than 4%.

The extrapolation from the short-term distribution of a random sea state to a long-term distribution is straightforward. The number of the sea states included in the analysis are considered to be independent and the extreme distribution for the period of one year can be calculated according to the equation $F_{1year} = F^N$, where F = extreme flap moment distribution and N number of independent sea states in one year. However, by carrying out the Bayesian analysis before the extrapolation, the estimate is higher than carrying out the Bayesian analysis after the extrapolation. In this case the former procedure has been chosen.

Table 3 shows also the estimates of the extreme flap moment with a return period of 100 years. The difference in the estimates increases with the extrapolation to 100-year response to about 12%. Figure 5 shows the tail behaviour of the different distributions. It can be seen that the Gumbel distribution gives the highest estimates due to the heavy tail. Weibull distribution and the Normal distribution have an overlapping tail for high fractiles.

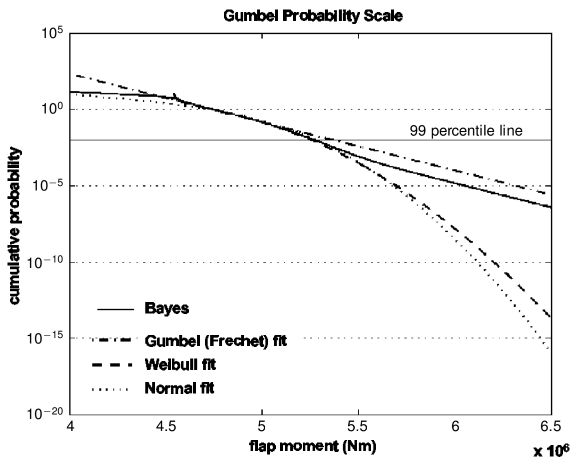

Figure 5 Cumulative probability function of the extreme flap moment at a mean wind speed of 15 m/s with different distribution models.

It can be seen that the influence of the Gumbel distribution increases with the extrapolation and that the Bayesian analysis yields effectively an average between the Gumbel estimate and the Weibull estimate. The extrapolation favours the tail of the distribution, thus even with less contribution in percentage, the absolute contribution is higher because of the heavier tail.

However, one should keep in mind that the estimate is always bounded by distributions that are included in the Bayesian analysis. For this reason three more distributions are added to the Bayesian analysis to determine the effect of including more diverse distributions. The additional distributions are the Log-Normal, 3 parameter Gamma and the Exponential distributions.

Table 4. participation factors and 100 year return values of the distribution of the flap moment using Bayesian analysis.

Distribution	Participation	100 year flap moment/10^6 Nm
Gumbel	15%	1.03
Normal	0.8%	0.93
Weibull	34.3%	1.00
Gamma	48.7%	1.01
Exponential	0%	1.08
Log-Normal	1.2%	0.94
Bayes	-	1.005

Table 4 shows the participation factors of different distributions for the flap moment. The participation factors are applied after the extrapolation. The weights of the 3 parameter Weibull and Gamma distribution are the dominant ones. The Weibull and Gamma distribution are strongly related to each other. Thus, the addition of the three extra distributions has not changed significantly the 100-year estimate of the flap moment. The Exponential distribution has the highest estimate, however, the Bayesian has given it a weighting factor of 0, despite the fact that the Exponential distribution is not rejected by the goodness of fit test. Figure 6 shows the different fitted distributions to the extreme flap moment. The data are plotted in Weibull scale so that if the data follow a Weibull distribution they will appear as a straight line. Figure 7 shows the tail region of the fitted distributions. As can be seen the 6 distributions cover the data reasonably in the probability space.

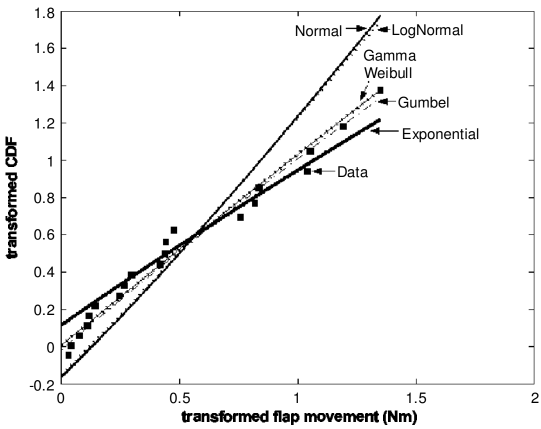

Figure 6 Fitted distributions of the extreme flap moment plotted in Weibull scale together with the original data.

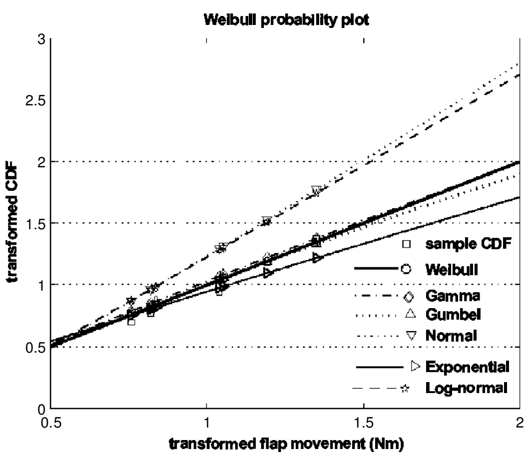

Figure 7 Tail region of the 6 fitted distributions of the extreme flap moment plotted in Weibull scale together with the original data.

Table 5 shows the estimate of the 100-year overturning moment (OTM) of the support structure using different distribution functions. The participation factors as result of the Bayesian analysis is also shown. The 100-year OTM is normalised with respect to the 3 parameter Weibull estimate. As can be seen, again the dominance of Gamma and Weibull is very strong. The 100-year OTM is dominated by the estimate of these two distributions.

Table 5. participation factors and 100 year return values of the distribution functions of the overturning moment (OTM) using Bayesian analysis.

Distribution	Participation	100 year flap moment/10^6 Nm
Gumbel	2%	1.05
Normal	0.3%	0.93
Weibull	10.4%	1.00
Gamma	86.7%	1.00
Exponential	0%	1.12
Log-Normal	0.6%	0.94
Bayes	-	0.997

5. BAYESIAN ANALYSIS OF THE DISTRIBUTION CHOICE APPLIED TO THE EXTREME DISPLACEMENTS OF THE BLADE TIP

In this section, the Bayesian analysis is applied to the extreme displacements of the rotor blade of a modern pitch controlled turbine. In the modern wind turbines, the blade tip deflection becomes more critical due to the flexible and lightweight design. The maximum blade deflection can become a design variable that determines the blade stiffness in the flapwise bending direction.

Theoretically one needs to determine the conditional distribution of the blade tip deflection (conditional on the mean wind speed). By means of convolution, the contributions from the different mean wind speeds are taken into account. In this case the maximum deflection occurs at 14 m/s. For the demonstration of the method, it is assumed that the contributions from the other mean wind speeds are insignificant. One has to keep in mind that this simplification can lead to an underestimate of the extreme deflection, if the extreme deflections from other mean wind speeds have comparable magnitudes to the deflections from the most extreme wind speed.

The extreme deflections of the blade tip at 14 m/s are obtained by time domain simulations. The deflections are fitted to different distribution functions. The selected distribution functions are: the Gumbel distribution, the 3 parameter Weibull distribution the Log-Normal distribution and the Generalised Extreme Value distribution (GEV). The Normal distribution and the Exponential distributions are rejected by the K-S and A-D tests.

The deflection data consist of N values and the data are ranked in ascending order. The cumulative probability are given by the following plotting positions (Gumbel)

$$P = \frac{r}{N+1} \qquad [8]$$

where r is the rank of the data point and N is the total number of data. Using the Least Square fit with this plotting position, a small bias is introduced when each point is given equal weight [2]. Gringorten has suggested a different plotting position [4] to remove the bias.

$$P = \frac{r - 0.44}{N + 0.22} \qquad [9]$$

The probability of the deflection data are transformed to Gumbel scale and plotted in the modified plotting position (equation (9)). If the distribution of the extreme deflection were Gumbel distributed, than the data would follow a straight line in the Gumbel scale. Figure 8 shows the deflection data with the different distribution fits. As can be seen, the data do not follow the straight line of the Gumbel fit. The upper tail of the distribution of the extreme deflection seems to have a lower exceedence probability than the Gumbel fit would suggest. The 3 parameter Weibull distribution and the GEV distribution follow more accurately the upper tail of the distribution. The Log-Normal distribution seems to underestimate the extreme deflection.

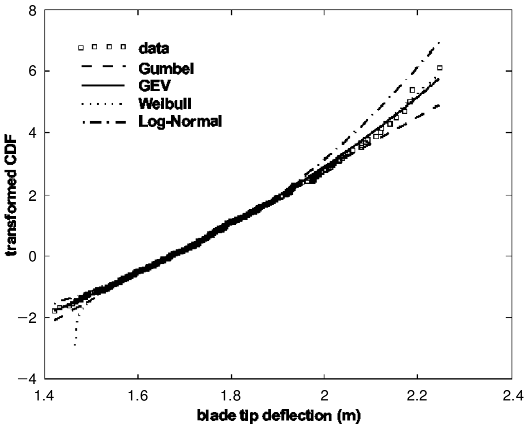

Figure 8 Maximum deflections of the blade fitted to different distribution functions, plotted in Gumbel scale together with the original data.

The GEV distribution has a right end point at 3.7 m. The fact that the extreme deflection of the blade tip is limited may seem physically plausible, but this limit has to be determined by a physical analysis rather than a statistical fit. A simplified theoretical limit of the maximum deflection of 2.53 m has been calculated [5], assuming maximum stationary lift. Instead of the right end point determined by the fitting procedure, the 2.53 m limit is used as the right end point. The prior probabilities are equal for all the four distributions, i.e. 1/4. The posterior probabilities are calculated using the integration formula (see equation 5 and 6). Table 6 shows the estimate of the 100-year deflection for different distribution functions and the Bayesian estimate. As can be seen, the contribution of the Log-Normal distribution is insignificant. The GEV has more than 3/4 of the contribution, while Gumbel and Weibull distributions share the rest. Since Gumbel and Weibull distributions are not limited in the upper tail, the Bayesian estimate of the maximum deflection with a return period of 100 year is higher than the calculated theoretical maximum.

Table 6. participation factors and 100 year extreme deflection of the blade tip using Bayesian analysis.

Distribution	Participation	100 year deflection (m)
Gumbel	8.8%	3.44
Weibull	14.1%	2.74
Log-Normal	0.1%	2.63
GEV	77.0%	2.46
Bayes	-	2.59

6. CONCLUSIONS

From a practical point of view, the analysis of the uncertainties contributes to the decision making process. The question of which distribution type to choose was made on the basis of the Bayesian analysis. The evidence of the predominance of one distribution function makes it unnecessary to combine the different distributions. In this case the Weibull distribution and

Gamma distribution have large participations in the response distribution. However, in case of the extreme blade tip displacement, the uncertainty about the theoretical maximum of the blade tip deflection is significant, thus it was necessary to take the unbounded distributions into account to compensate this uncertainty. This can be seen as a rational compromise.

One should be aware of the fact that the data are obtained using numerical simulations, which resulted in a more homogenous dataset than measurements. It may not be the case for data obtained from measurements, in such case the participation factors of the chosen distributions may be distributed very differently.

One can regard the Bayesian analysis as a useful tool that offer a rational way of dealing with limited information with uncertainties, concerning the choice of distribution models and the uncertainty on distribution parameters.

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7. APPENDIX: LIST OF DISTRIBUTION FUNCTIONS

List of Probability Density Functions (PDF)

u	Location parameter
s	scale parameter
k, γ	shape parameter
μ	mean
σ	standard deviation

- Normal $$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$
- Log-Normal $$f(x) = \frac{1}{z\sqrt{2\pi\sigma^2}} e^{-\frac{(\ln x-\mu)^2}{2\sigma^2}}$$
- Exponential $$f(x) = \frac{1}{s} e^{-\frac{x-\mu}{s}}$$
- Gamma $$f(x) = \frac{s^{-k}(x-u)^{k-1}\, e^{-\frac{x-\mu}{\sigma}}}{\Gamma(k)}$$
- Weilbull $$f(x) = \frac{k}{s}\left(\frac{x-\mu}{s}\right)^{k-1} e^{-\left(\frac{x-\mu}{s}\right)^{k}}$$
- Gumbel $$f(x) = \frac{e^{-\frac{x-\mu}{\sigma}}\, e^{-e^{\left(\frac{x-\mu}{\sigma}\right)}}}{s}$$
- GEV $$f(x) = e^{-\left(1+\gamma\frac{(x-\mu)}{\sigma}\right)^{-\frac{1}{\gamma}}} (1+\gamma\frac{(x-\mu)}{\sigma})^{-\frac{1}{\gamma}-1}, \quad 1+\gamma(x-u)/\sigma > 0,\ \gamma \pi 0.$$

List of Cumulative Distribution Functions (CDF)

- Normal $$F(x) = \Phi\left(\frac{x-\mu}{\sigma}\right)$$
- Log-Normal $$F(x) = \Phi\left(\frac{\ln x - \mu_{\ln x}}{\sigma_{\ln x}}\right)$$
- Exponential $$F(x) = 1 - e^{-\left(\frac{x-\mu}{\sigma}\right)}$$
- Gamma $$F(x) = \frac{\sigma^{-k}}{\Gamma(k)} \int_0^x (x-\mu)^{k-1} e^{-\frac{x-\mu}{s}}\, dt$$
- Weibull $$F(x) = 1 - e^{-\left(\frac{x-\mu}{s}\right)^{k}}$$
- Gumbel $$F(x) = e^{-e^{-\left(\frac{x-\mu}{s}\right)}}$$
- GEV $$F(x) = e^{-\left(1+\gamma\frac{(x-\mu)}{s}\right)^{-\frac{1}{\gamma}}}, \quad 1+\gamma(x-u)/s > 0,\ \gamma \pi 0.$$