

A sequential condition-based repair/replacement policy with non-periodic inspections for a system subject to continuous wear

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SUMMARY

This paper studies a condition-based maintenance policy for a repairable system subject to a continuous-state gradual deterioration monitored by sequential non-periodic inspections. The system can be maintained using different maintenance operations (partial repair, as good as new replacement) with different effects (on the system state), costs and durations. A parametric decision framework (multi-threshold policy) is proposed to choose sequentially the best maintenance actions and to schedule the future inspections, using the on-line monitoring information on the system deterioration level gained from the current inspection. Taking advantage of the semi-regenerative (or Markov renewal) properties of the maintained system state, we construct a stochastic model of the time behaviour of the maintained system at steady state. This stochastic model allows to evaluate several performance criteria for the maintenance policy such as the long-run system availability and the long-run expected maintenance cost. Numerical experiments illustrate the behaviour of the proposed condition-based maintenance policy. Copyright © 2003 John Wiley & Sons, Ltd.

KEY WORDS: Condition-based maintenance; inspection; replacement; repair; wear; stochastic model; Markov renewal process

1. INTRODUCTION AND PROBLEM STATEMENT

This paper considers the maintenance of a technical device or a structure (hereafter referred to as a ‘system’) subject to continuous gradual random wear and deterioration which can be modelled by a continuous-state stochastic process. Assuming that this deterioration can be monitored in practice, the maintenance decisions are based on a sequential monitoring of the system level of deterioration. The system failure occurs as soon as the deterioration level crosses a critical threshold and after a failure, the system has to wait for an inspection to be maintained. For a running system, the PM actions are performed when the measured deterioration level

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exceeds pre-set thresholds. Not surprisingly, PM policies based on the monitoring of the actual system state are usually more efficient than classical ‘static’ PM policies based on the *a priori* statistical knowledge of the system lifetime [1, 2]. CBM policies are thus particularly justified for critical systems and the collected information on the system condition can be exploited by the maintenance decision-maker in order to maximize the availability of the system and to minimize its long run expected cost. The price for this higher efficiency is an enhanced modelling work of the maintained system behaviour in order to support correctly the maintenance decisions. The objective of this paper is precisely to construct a stochastic model for the behaviour in time of a repairable system checked with sequential non-periodic inspections and maintained by a general repair/replacement. Such a stochastic model can then be used to assess and optimize the maintenance decisions.

PM policies, and especially condition-based inspection-replacement policies, have been widely studied in the literature [1–19]. Most of the existing CBM models involve discrete-state (or discretized) deterioration processes or shock models. Even if many real-world systems exhibit a continuous and gradual wear behaviour, only few CBM models deal with continuous-state deterioration processes, see for example References [9, 13–16, 20, 21] and [1, Section 3.6.6], but these models consider mainly inspection/replacement policies (or even assume a perfect and permanent knowledge of the system state), and not general CBM policies with different possible maintenance actions. In this paper, we do not limit the policy to a simple inspection/replacement one and we propose a more general CBM policy for a system subject to a continuous-state wear process. This general model allows to take into account more realistic situations, mainly through the following features:

- The possible maintenance actions are not limited to perfect inspections and replacements (or as good as new repairs). Partial (or imperfect) repairs which reduce the deterioration level of the system without returning it to its initial as good as new state are also allowed.
- The durations of the maintenance actions (inspections, repairs or replacements) are not negligible and the inactivity during maintenance contributes to an increased unavailability of the system.
- The inspection schedule is not periodic, but adaptive and dynamically constructed from the evolution of the deterioration state of the maintained system.

A unit cost is associated to each of the maintenance actions and two different cost rates per unit time are associated respectively to the operation time and to the unavailability time of the maintained system. The work presented in this paper is twofold: first, we propose a CBM decision framework taking into account the knowledge of the system state. Then we develop its associated cost model in order to assess and to optimize the performance of the resulting maintenance policy. From the methodological point of view, we show that continuous-state semi-regenerative (or Markov renewal) stochastic techniques [22, 23] provide efficient tools for maintenance modelling (more powerful than classical renewal techniques) and allow to solve analytically and numerically maintenance problems which could only be studied through Monte-Carlo simulations [24, 25].

This paper is divided into three parts. First, the assumptions on the stochastic deterioration model and the maintenance modelling framework are presented in Section 2. Section 3 is devoted to the construction of a stochastic model of the behaviour of the maintained system. In Section 4, it is shown how the performance of the maintenance policy can be assessed using this

model. Finally, numerical experiments illustrate the performance of the proposed condition-based policy.

2. DETERIORATION MODEL AND MAINTENANCE DECISION FRAMEWORK

2.1. Deterioration model

For many real-world systems, the deterioration process due to wear and tear is intrinsically continuous, e.g. systems subject to erosion (hydraulic structures, dikes), [26–28], corrosion (steel reinforcements of concrete structures, pipelines) [29], consumption (tyres, brakes), cumulative wear (cutting tools) [30]. For these systems, the notion of ‘discrete states’ often used in maintenance and reliability models [1], might be irrelevant whereas the level of deterioration (which can be measured in practice, eventually through a strongly correlated process) has generally a clear and physical signification for the maintenance decision-maker. Accordingly, one of our main objective in this work is to develop a model to assess maintenance decisions based directly on an observed deterioration level, without involving more abstract quantities like one-step transition probabilities in a finite-state process (with artificial states constructed by discretization from the continuous deterioration process) in the modelling procedure, see e.g. Reference [8].

The deterioration behaviour of the unmaintained system is represented by a continuous-state univariate stochastic process $\{X_t, t \geq 0\}$ with initial level of deterioration $X_0 = 0$. The deterioration measure is strictly increasing which means that the system worsens with time due to ageing and accumulated wear or damage. The system failure is signaled by the crossing of a given deterioration threshold L : beyond this level of deterioration, the system can no longer meet the user’s requirements and is considered as failed. This failure can be either an actual ‘hard’ failure of an active system or a pending failure of a passive system or structure. The failure is not assumed to be self-announcing, i.e. it can be detected only by an inspection. After failure, the system remains unavailable until the next scheduled maintenance operation. This usually happens for example in remote systems or structures which are not continuously monitored and require a specific operation to reveal the current deterioration level, in standby safety systems which are rarely activated, or in production systems whose state can be evaluated only through batch quality controls on the products.

The system can be inspected or maintained only at periodic discrete maintenance times $t_k = k\delta t$ and only the discrete-time stochastic process $X_k = X_{t_k}$ can be observed. The maintenance period δt is generally imposed by the system characteristics or environment. Hereinafter, this maintenance period δt is taken equal to 1 (in arbitrary time unit). The maintenance times t_k should be considered only as possible, but not compulsory, time windows for maintenance.

The elementary deterioration increments occurring between two successive maintenance times t_k and t_{k+1} are assumed to be positive, exchangeable and stationary. The positiveness of the increments corresponds to the increasing monotonicity, which is a behaviour observed in physical deterioration process. The properties of exchangeable and stationary increments are very similar (even if weaker) to the properties of stationary and independent increments. The stationarity and independence entail the ‘memoryless’ property that the future increment of deterioration depend neither on the current level of deterioration nor on its age, but only on the

period of time over which the system deteriorates [31, 32]. In practice, even if in some circumstances the deterioration behaviour varies with the state of the system, the independence or exchangeability property can be verified, at least in approximation, in some applications, mostly for systems subject to erosion (dikes), corrosion (steel reinforcements of concrete structures, pipelines), consumption (tyres, brakes) [26–28]. The elementary deterioration increments $\Delta_{(k,k+1)}X$ are assumed to follow a pdf f . The cumulated deterioration quantity during i maintenance periods is then a random quantity:

$$\Delta_{(k,k+i)}X = \sum_{l=1}^i \Delta_{(k+l-1,k+l)}X \quad (1)$$

The associated deterioration pdf is the i th convolution of $f, f^{(i)}$. Always because of the independence and stationarity properties, the distribution f must be infinitely divisible [20, 21, 33].

All these deterioration properties lead to a model based on Lévy processes including Gamma processes [32–36]. In the present work, we choose the deterioration model of [16], i.e. a specific class of Gamma processes (observed on a periodic discrete-time grid) which encapsulate well real-life system properties in a mathematical model while remaining tractable. The deterioration increments follow a negative exponential pdf f with parameter α , and $f^{(k)}$ is an Erlang- k pdf (see Figure 1).

This stationary deterioration model based on Gamma processes has been proposed in several published works for continuously deteriorating systems, e.g. [15, 16, 26–28, 31, 37–43]. It is particularly well adapted to describe a period of steady damage accumulation (i.e. between the breaking-in period and a final period of rapid wear accumulation) of e.g. a machinery for continuous production or a hydraulic structure subject to erosion [41, 44].

The failure intensity function $\rho_X(k) = \mathbb{P}(X_k \geq L | X_{k-1})$ of the maintained system is increasing which justifies both the implementation of a PM policy and the optimality of a decision rule based on a control-limit structure [45, 1, Chapter 3]. The property of an increasing failure intensity function is similar to the classical increasing failure rate property (IFR) in continuous time [46].

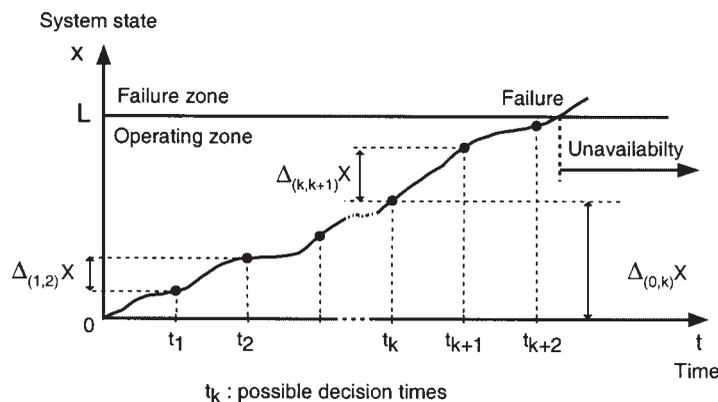


Figure 1. Schematic evolution of the unmaintained system deterioration.

2.2. Maintenance modelling framework

2.2.1. Maintenance operations. Even if general maintenance actions have been considered in some reported research works [18, 24, 25, 47–50, 1, Section 3.7], most of existing CBM models consider only inspections and perfect as good as new replacements, particularly in the case of maintenance models for systems subject to continuous wear [5–9, 13–17]. However, in practice, a maintenance action may not result only in extreme situations (perfect replacement or minimal repair) but in an intermediate one, and the degree of repair has to be considered (see Reference [51] for a review on imperfect maintenance). Imperfect maintenance can be harmful and must be eliminated if it results from errors and wrong maintenance procedures. But, conversely, it can be profitable to adapt the degree of a repair to the deterioration level of the system: a partial imperfect repair is usually faster and cheaper than a perfect replacement, and it may be preferred for a moderately deteriorated system. The problem of determining the optimal depth of repair actions has also been investigated in Reference [1, Section 3.7] for discrete-state maintenance models. In the present work, only improving repairs which return the system in a better state are considered, and we assume that the degree of repair can be controlled (at least partially) by the maintenance decision-maker.

In order to quantify the effect of a maintenance operation, we define its recovery value Δ_y : the operation recovery is the difference of the deterioration level measured between the beginning and the end of this operation. Conditional on the nature of the maintenance action and on the observed level of deterioration, depending on the nature of the maintenance action, the recovery can be either a random or constant quantity.

The durations of the maintenance operations are assumed to be non-negligible. The system is stopped at the onset of a maintenance operation and restarted at the end. The duration (in maintenance periods) of system inactivity due to a maintenance action $\tau(y, y_0)$ is modelled as a discrete random variable function of the operation recovery value $\Delta_y = y_0$ and of the system deterioration level y ; it is assumed to be a stochastically increasing function of the operation recovery [47, 52]. An inspection is less time-expensive, in probability, than a repair and, in the same way, the repair duration is on average shorter than a replacement duration. During the system inactivity due to maintenance operation, a cost is incurred at the rate r_i .

Inspection: The inspection is merely an information-taking maintenance operation which allows to know the current deterioration level y . It is assumed to be perfect and to have no effect on the system deterioration level. An inspection incurs an unit cost $c_i(y)$ which can be function of the current system deterioration level y . Since an inspection does not affect the system state, it can be considered as a maintenance action with a deterministic null recovery $\Delta_y = 0$.

Preventive repair: A preventive repair is a partial and/or imperfect maintenance action performed on the system prior to a failure. It rejuvenates the system by lowering its deterioration level below a pre-specified value. The operation recovery Δ_y is a random function of y with a pdf f_{Δ_y} which characterizes in some sense the maintenance quality and efficiency. Furthermore, we consider that the recovery Δ_y is a stochastically increasing function of the current system state. The deterioration level at the end of the operation when the system is restarted is assumed to be perfectly known. The preventive repair cost $c_r(y, y_0)$ is a function of the deterioration level y measured at the beginning of the operation and of the repair recovery value y_0 , and we have $c_i(y) < c_r(y, y_0) \forall y_0 \leq y$.

Replacement: A replacement is a complete renewal of the system (either an as good as new repair or a true physical replacement). Its recovery is exactly equal to the deterioration level y .

Table I. Maintenance operations characteristics.

Operations	Recovery	Duration	Cost
Corr. replacement	$\Delta_y = y$	r.v. $\tau(y, y)$	$c_c(y)$
Prev. replacement	$\Delta_y = y$	r.v. $\tau(y, y)$	$c_p(y)$
Preventive repair	r.v. $\Delta_y \sim f_{\Delta_y}(y_0)$	r.v. $\tau(y, \Delta_y)$	$c_r(y, \Delta_y)$
Inspection	$\Delta_y = 0$	r.v. $\tau(y, 0)$	$c_i(y)$

A replacement can be performed either preventively on a still running system or correctively ($X \geq L$) on a failed system. The unit preventive (resp. corrective) replacement cost $c_p(y)$ (resp. $c_c(y)$) is a function of the system deterioration level y , and we have $c_i(y) < c_p(y) < c_c(y)$ since $c_c(y)$ also includes all the costs due to the failure. An additional cost is incurred by the time elapsed in the failed state at a cost rate r_u . From the effect modelling point of view, a replacement can be considered as a specific repair with a complete recovery $\Delta_y = y$ but its cost is not necessarily equivalent to the cost of a repair with the same recovery (see Table I).

2.2.2. Structure of the maintenance policy. In the general case (i.e. without any pre-specified structure), the optimization of the maintenance policy with non-periodic inspections, replacements and partial repairs relies theoretically on the dynamic programming equation [32, 53, 54]. In practice, a complex dynamic program for maintenance decision can be difficult to advocate and such an approach can be numerically intractable [55]. To avoid this difficulty, we propose to adapt a simple multi-threshold structure developed in References [16, 46] to the implementation of a condition-based inspection/repair/replacement policy for a repairable system. Such a structure leads to a stationary Markov policy [1,18], depending only on the current observation and well adapted to the Markovian deterioration behaviour of the considered system. We give no theoretical proof of the optimality of this structure and of the associated sequential policy, however it has been shown to exhibit the best performance among classical strategies for discrete-state systems [19]. The maintenance decision is made using $N+2$ thresholds: inspections/maintenance scheduling thresholds (ξ_l , $l = 1, \dots, N-1$), partial repair (ξ_N), preventive replacement (ξ_{N+1}) and restarting (ζ) thresholds.

A maintenance phase always starts by an inspection of the system. Assume that the inspection reveals a deterioration level y . The maintenance decision is built on-line using the following rule, see Figure 2:

- ‘Discarding decision’: if the deterioration level y falls in an ‘inspection-only’ zone, i.e. $y \in [\xi_l, \xi_{l+1})$ with $l = 0, \dots, N-1$: the system is left as it is and the next inspection is scheduled $N-l$ maintenance periods later. As the system deteriorates, the inspection intervals become shorter and shorter.
- ‘Repair decision’: if the deterioration level y belongs to the repair zone, i.e. $y \in [\xi_N, \xi_{N+1})$, a partial repair is performed on the system. The system deterioration level after repair $y - \Delta_y$ is assumed to be known with certainty and to belong to the restarting zone $[0, \zeta)$. The deterioration level $y - \Delta_y$ also belongs to an inspection zone $[\xi_l, \xi_{l+1})$ with $l \in \{0, \dots, N-1\}$ and an inspection is thus scheduled $N-l$ maintenance periods later.
- ‘Replacement decision’: If the deterioration level y is greater than the preventive replacement threshold ξ_{N+1} , an as good as new system replacement is performed (either preventive if

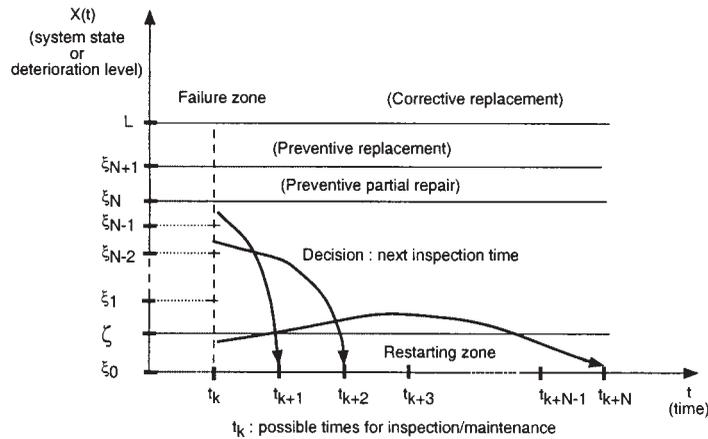


Figure 2. Parametric structure of the maintenance policy.

$y \in [\xi_{N+1}, L)$ or corrective if $y \geq L$). The next inspection is scheduled N maintenance periods later.

Within this structure, the thresholds $\{\xi_1, \xi_2, \dots, \xi_N, \xi_{N+1}, \zeta\}$ define completely the maintenance policy. Roughly speaking, the thresholds $\{\xi_k\}$ control the preventive/corrective maintenance ratio and ζ controls jointly the repair cost and efficiency. These thresholds have to be tuned by the maintenance decision maker in order to optimize the performance of the policy. Note finally that the choice of the structure of the maintenance policy (and of the number of thresholds) does not affect the properties of the deterioration model. On the contrary, in the case of discrete-state (or discretized) models, see e.g. Reference [1, Section 3.7], a maintenance decision is attached to each system state and it is necessary to increase the number of deterioration states in order to extend the set of possible maintenance actions.

In the case of a 4-threshold ($N = 2$) inspection/repair/replacement policy, Figure 3 sketches a representative sample path realization of the maintained system state, featuring an inspection at T_{n+1} , a partial preventive repair ($\Delta_y = y_{0_{n+2}}$) at T_{n+2} and a preventive replacement at T_{n+4} .

The proposed multi-threshold maintenance structure extends classical policies (e.g. periodic preventive replacement, condition-based inspection/replacement) which can be emulated by an appropriate specific choice of the threshold values [17].

2.3. Maintenance performance criteria

We consider two different performance criteria for the maintenance policy on an infinite time span: the long-run expected maintenance cost rate and the asymptotic availability of the system.

Cost criterion: At time t , the cumulative maintenance cost $C(t)$ consists of the three different costs: the cost $\Gamma_m(t)$ due to the maintenance operations themselves, the cost $\Gamma_u(t)$ of system unavailability after a failure and the cost $\Gamma_i(t)$ due to the system inactivity during maintenance.

The long-run expected maintenance cost per unit of time $C_\infty(\Xi)$ is

$$C_\infty(\Xi) = \lim_{t \rightarrow \infty} \frac{\mathbb{E}(\Gamma_m(t) + \Gamma_i(t) + \Gamma_u(t))}{t} \tag{2}$$

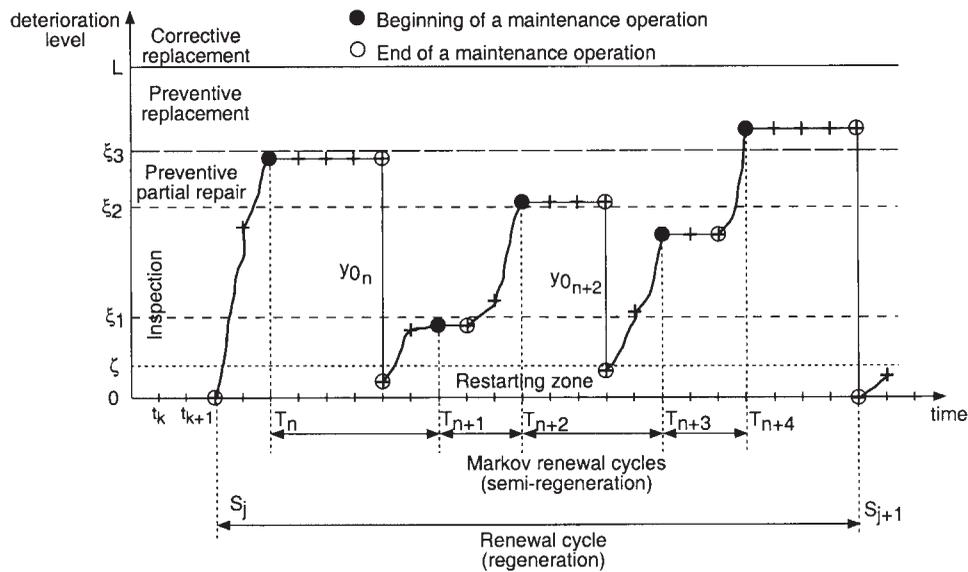


Figure 3. Sample path of the maintained system state for a 4-threshold ($\xi_1, \xi_2, \xi_3, \zeta, N = 2$) policy.

Availability criterion: Let $D_o(t)$ the cumulative time spent in the operating phase (i.e. not in maintenance) and $D_u(t)$ the cumulative time spent in the failed state. The asymptotic (or stationary) availability $A_\infty(\Xi)$ of the maintained system is given by [2]:

$$A_\infty(\Xi) = \lim_{t \rightarrow \infty} \frac{\mathbb{E}(D_o(t) - D_u(t))}{t} \quad (3)$$

In the next section, a stochastic model of the maintained system state at steady state is constructed in order to evaluate these long-term performance criteria (2) and (3).

3. PROBABILISTIC MODELLING OF THE MAINTAINED SYSTEM STATE

3.1. Stochastic properties of the maintained system state

The study of the asymptotic behaviour of the maintained system can be notably simplified because of the (semi-)regenerative properties of the stochastic process describing the evolution of the deterioration level of the maintained system.

Regenerative properties: After a replacement (preventive or corrective) the system is as good as new and its future behaviour does not depend on its past. The process $\{\tilde{X}_k, k \geq 0\}$ describing the evolution maintained system state is a regenerative process with regeneration (or renewal) points equal to the replacement times S_j , see Figure 3. Because of this regenerative property, and following a widely used approach in maintenance modelling based on the renewal theorem, the long-run study (i.e. on an infinite time span) of the deterioration process can be limited to the study of the system state evolution on a single renewal cycle defined by two successive

replacement times. However, because of the possibility of partial system repairs between replacements, even the description of the system evolution on a single renewal cycle can become very complex, and evaluating the maintenance cost can be intractable, see Figure (3). In our case, it is possible to simplify further the analysis using semi-regenerative properties of $\{\tilde{X}_k, k \geq 0\}$.

Semi-regenerative properties: After any maintenance operation (inspection, repair or replacement), the system evolution depends only on the deterioration level y observed at its onset. Conditional on y , the maintenance operation to be performed and the system behaviour until the next maintenance action can be completely characterized independently of the past events. The stochastic process formed by the deterioration levels of the maintained system $\{\tilde{X}_k, k \geq 0\}$ is a semi-regenerative process where the semi-regeneration (or Markov renewal) points are the starting times of every maintenance operation T_n . The successive deterioration levels $\{Y_n = \tilde{X}_{T_n}, n \geq 0\}$ observed at the beginning of the operations form the embedded real state space MC of the semi-regenerative process and the successive times of operation onsets delimit independent cycles, given the MC $\{Y_k, k \geq 0\}$ [15, 49]. For long-run expected performance criteria, using an extended version of the renewal theorem to semi-regenerative process valid under mild conditions, the analysis on an infinite time span can be limited to a single semi-regenerative (or Markov renewal) cycle delimited by two successive maintenance onsets [15, 49, 56]. The analysis of the system behaviour is thus greatly simplified, see Figure 3. Other Markov renewal points can be chosen (e.g. the dates of the end of maintenance operations) but the present choice allows to enumerate and describe easily all the possible maintenance scenarios on a Markov renewal cycle. The price to pay for this simplified analysis is the derivation of the stationary probability law of the MC $\{Y_k, k \geq 0\}$ since all the expectations in the Markov renewal theorem are taken with respect to this law.

3.2. Maintenance scenarios on a Markov renewal cycle

The construction of the stationary probability law of $\{Y_k, k \geq 0\}$, i.e. of the maintained system state at the beginning of a maintenance operation, rests on an exhaustive enumeration and description of its different possible behaviours from the beginning to the end of a Markov renewal cycle (maintenance/deterioration scenarios). Within the considered maintenance structure, any k th Markov renewal cycle is completely characterized by the observed state at the beginning of the maintenance operation $Y_k = y$, and by the following quantities (function of y):

- the operation recovery value y_0 (realization of the random variable Δ_y),
- the operation length $\tau(y, y_0)$ during which the system is assumed to remain at the same level of deterioration,
- the deterioration level of the restarting system $z = y - y_0$,
- the beginning of the next maintenance operation which ends the current Markov renewal cycle and starts the next one (at this time, the system state is $Y_{k+1} = x$).

One out of three exclusive scenarios necessarily explains the transition from a deterioration level $Y_k = y$ at the beginning of a maintenance operation to the deterioration level $Y_{k+1} = x$ at the beginning of the next one, i.e. on a Markov renewal cycle.

- Scenario 1 (inspection): $y \in [\xi_l, \xi_{l+1})$, $l = 0, \dots, N - 1$, an inspection is performed which renders the system unavailable during $\tau(y, 0)$ maintenance periods. At the end of this

inspection, the system remains in the same state y and the next inspection is scheduled $N - l$ maintenance periods later. The pdf value of the cumulative deterioration $x - y$ until the next inspection is $f^{(N-l)}(x - y)$.

- Scenario 2 (repair): $y \in [\xi_N, \xi_{N+1})$, a preventive partial repair is performed. Since after repair the system state z must belong to the restarting zone $[0, \zeta)$, the random repair recovery Δ_y follows a pdf f_{Δ_y} with a finite support $[y - \zeta, y]$. The observed value of the recovery is y_0 . The system is unavailable during $\tau(y, y_0)$ maintenance periods for repair. At the end of the repair, the system is in a known deterioration state $z = y - y_0 \in [\xi_l, \xi_{l+1})$, $l = 0, \dots, N - 1$ and the next inspection is scheduled $N - l$ maintenance periods later. The pdf value of the cumulative deterioration $x - (y - y_0)$ until the next inspection is $f^{(N-l)}(x - y + y_0)$.
- Scenario 3 (replacement): $y \geq \xi_{N+1}$, a replacement (preventive or corrective) is performed. The replacement makes the system unavailable for $\tau(y, y)$ maintenance periods. At the end of the replacement, the system is as good as new and the next inspection is scheduled N maintenance periods later. The pdf value of the cumulative deterioration x until the next inspection is $f^{(N)}(x)$.

3.3. Characteristic probability laws

This section is devoted to the construction of two probabilistic quantities characterizing the maintained system behaviour:

- the stationary law $\pi(x)$ of the MC $(Y_k)_{k \geq 0}$, i.e. of the system state at the beginning of a maintenance operation and
- the discrete probability law $P^i(k, y)$ for the length (in maintenance periods) of a maintenance operation initiated at the deterioration level y .

3.3.1. *Stationary law $\pi(x)$ of the maintained system state.* From the three possible (and exclusive) scenarios listed in Section 3.2, and using the total probability law, the transition probability law $F(x|y)$ from level y to level x of the MC $\{Y_k = \tilde{X}_{T_k}, k \geq 0\}$ can be written:

$$\begin{aligned}
 F(x|y) = & \underbrace{\sum_{l=0}^{N-1} \mathbb{1}_{\{y \in [\xi_l, \xi_{l+1})\}} f^{(N-l)}(x - y)}_{\text{Scenario 1}} \\
 & + \underbrace{\mathbb{1}_{\{y \in [\xi_N, \xi_{N+1})\}} \sum_{l=0}^{N-1} \int_{y - \min(\xi_{l+1}, \zeta)}^{y - \xi_l} f_{\Delta_y}(y_0) f^{(N-l)}(x - y + y_0) dy_0}_{\text{Scenario 2}} \\
 & + \underbrace{\mathbb{1}_{\{y \geq \xi_{N+1}\}} f^{(N)}(x)}_{\text{Scenario 3}}
 \end{aligned} \tag{4}$$

The stationary pdf $\pi(x)$ of the MC $\{Y_k = X_{T_k}, k \geq 0\}$ is then obtained as a solution of the invariance equation [23]:

$$\pi(x) = \int_0^\infty \pi(y) F(x|y) dy \tag{5}$$

which gives finally:

$$\begin{aligned} \pi(x) = & \sum_{l=0}^{N-1} \int_{\xi_l}^{\xi_{l+1}} \pi(y) f^{(N-1)}(x-y) dy \\ & + \int_{\xi_N}^{\xi_{N+1}} \pi(y) \left(\sum_{l=0}^{N-1} \int_{y-\min(\xi_{l+1}, \zeta)}^{y-\xi_l} f_{\Delta_y}(y_0) f^{(N-l)}(x-y+y_0) dy_0 \right) dy \\ & + \left(\int_{\xi_{N+1}}^{\infty} \pi(y) dy \right) f^{(N)}(x) \end{aligned} \tag{6}$$

The integral equation (6) can be numerically solved using an iterative algorithm adapted from classical integration schemes for Volterra equations [57], allowing a numerical evaluation of the stationary pdf $\pi(x)$ [46].

3.3.2. *Discrete probability law of the maintenance operation length $P^i(k, y)$.* At the beginning of a maintenance operation, the system is always inspected and the choice of the nature of the operation (doing nothing, repairing or replacing the system) depends only on the measured deterioration level y . The length of a maintenance operation $\tau(y, \Delta_y)$ is completely determined by y . Since the three possible maintenance options are mutually exclusive, the probability $P^i(k, y)$ that a maintenance phase lasts k maintenance periods, conditional on y , is

$$\begin{aligned} P^i(k, y) = & \underbrace{\mathbb{1}_{\{y \in [0, \xi_N]\}} \mathbb{P}_y(\tau(y, 0) \geq k)}_{\text{Scenario 1}} \\ & + \underbrace{\mathbb{1}_{\{y \in [\xi_N, \xi_{N+1}]\}} \int_{y-\zeta}^y \mathbb{P}_y(\tau(y, y_0) \geq k) f_{\Delta_y}(y_0) dy_0}_{\text{Scenario 2}} \\ & + \underbrace{\mathbb{1}_{\{y \geq \xi_{N+1}\}} \mathbb{P}_y(\tau(y, y) \geq k)}_{\text{Scenario 3}} \end{aligned} \tag{7}$$

Equation (7) can be solved numerically using classical integration schemes, giving $P^i(k, y)$ for any value of k .

Using $P^i(k, y)$ and $\pi(x)$, it is now possible to evaluate the performance criteria (2) and (3) of the maintenance policy.

4. PERFORMANCE ASSESSMENT OF THE MAINTENANCE POLICY

4.1. Evaluation of the performance criteria

Cost criteria: Using the Markov renewal property of the maintained system deterioration process, the criterion (2) can be written as ratio of the expected cumulated cost on a Markov renewal cycle over the expected length of the cycle:

$$C_{\infty}(\Xi) = \frac{\mathbb{E}_{\pi}(C(T))}{\mathbb{E}_{\pi}(T)} = \frac{\mathbb{E}_{\pi}(\Gamma_m(T)) + \mathbb{E}_{\pi}(\Gamma_l(T)) + \mathbb{E}_{\pi}(\Gamma_u(T))}{\mathbb{E}_{\pi}(T)} \tag{8}$$

The explicit expressions of the three components of the expected operating cost $\mathbb{E}_\pi(C(T))$ are given below.

- Cost of the maintenance operations:

$$\mathbb{E}_\pi(\Gamma_m(T)) = \mathbb{E}_\pi(C_{\text{INSP}}(T)) + \mathbb{E}_\pi(C_{\text{REPAIR}}(T)) + \mathbb{E}_\pi(C_{\text{REPL}}(T)). \quad (9)$$

The respective contributions of inspections, repairs and replacements to this cost can be evaluated using the stationary probability $\pi(x)$:

$$\mathbb{E}_\pi(C_{\text{INSP}}(T)) = \int_0^{\xi_N} c_i(y)\pi(y) \, dy \quad (10)$$

$$\mathbb{E}_\pi(C_{\text{REPAIR}}(T)) = \int_{\xi_N}^{\xi_{N+1}} \left(\int_{y-\zeta}^y C_r(y, y_0) f_{\Delta_y}(y_0) \, dy_0 \right) \pi(y) \, dy \quad (11)$$

$$\mathbb{E}_\pi(C_{\text{REPL}}(T)) = \int_{\xi_{N+1}}^L c_p(y)\pi(y) \, dy + \int_L^\infty c_c(y)\pi(y) \, dy \quad (12)$$

- Cost of system inactivity during maintenance, due to the maintenance time $D_i(t)$:

$$\begin{aligned} \mathbb{E}_\pi(\Gamma_i(T)) &= r_i \mathbb{E}_\pi(D_i(T)) = r_i \sum_{k=1}^{\infty} \mathbb{P}(D_i(T) \geq k) \\ &= r_i \sum_{k=1}^{\infty} \mathbb{P}(\tau(y, \Delta_y) \geq k) = r_i \sum_{k=1}^{\infty} \int_0^{\infty} \pi(y) P^i(k, y) \, dy \end{aligned} \quad (13)$$

- Cost of system unavailability after failure, due to the time $D_u(t)$ spent in the failed state:

$$\mathbb{E}_\pi(\Gamma_u(T)) = r_u \mathbb{E}_\pi(D_u(T)) \quad (14)$$

Because of the discrete-time inspections, the exact failure time and $D_u(T)$ are unknown. The unavailability time is approached by an upper-bound $D_u^+(T)$ running from the t_k just before the failure until the next maintenance operation whose expectation is given by

$$\mathbb{E}_\pi(D_u^+(T)) = \sum_{k=1}^n k \mathbb{P}(D_u^+(T) = k) \quad (15)$$

The difficulty to find an explicit expression for $\mathbb{E}_\pi(D_u^+(T))$ results now from the necessity of taking into account all the system maintenance/deterioration histories ending with its breakdown. Mathematical developments lead to the following expression, see Appendix A and Reference [46]:

$$\begin{aligned} \mathbb{E}_\pi(\Gamma_u(T)) &= r_u \sum_{k=1}^N k \left(\sum_{l=0}^{N-k} \int_{\xi_l}^{\xi_{l+1}} \pi(y) G(k|y, N-l) \, dy + G(k|0, N) \int_L^\infty \pi(y) \, dy \right. \\ &\quad \left. + \int_{\xi_N}^L \pi(y) \sum_{l=0}^{N-k} \int_{y-\xi_{l+1}}^{y-\xi_l} f_{\Delta_y}(y_0) G(k|y-y_0, N-l) \, dy_0 \, dy \right) \end{aligned} \quad (16)$$

where $G(k|z, l)$ is the probability that the unavailability time is equal to $k\delta t$ on a Markov renewal cycle, conditional on the deterioration value after maintenance $z \in [\xi_{N-l}, \xi_{N-l+1}]$, $l \geq k$.

Finally, the expected length of a Markov renewal cycle $\mathbb{E}_\pi(T)$ is computed as the sum of the expected lengths of the maintenance phase $\mathbb{E}_\pi(D_i(T))$ (see Equation (13)) and of the operating phase (running or failed state) $\mathbb{E}_\pi(D_o(T))$ defined by the expected date of the next intervention on the system:

$$\begin{aligned} \mathbb{E}_\pi(D_o(T)) = & \sum_{k=1}^N k \left(\int_{\xi_{N-k}}^{\xi_{N-k+1}} \pi(y) dy \right. \\ & + \int_{\xi_N}^{\xi_{N+1}} \left(\int_{y-\zeta}^y \mathbb{1}_{\{y_0 \in [y-\xi_{N-k+1}, y-\xi_{N-k}]\}} f_{\Delta_y}(y_0) dy_0 \right) \pi(y) dy \\ & \left. + \mathbb{1}_{\{k=N\}} \int_{\xi_{N+1}}^\infty \pi(y) dy \right) \end{aligned} \tag{17}$$

All the expressions (10)–(13), (16) and (17) can be numerically evaluated using classical algorithms, giving the expected costs and length of a Markov renewal cycle, allowing the long run expected cost rate $C_\infty(\Xi)$ to be calculated.

Availability criterion: using the Markov renewal property of the maintained system deterioration process, the availability criterion (3) can be written as the ratio of the expected cumulated time spent in a good state on a Markov renewal cycle over the expected length of the cycle:

$$A_\infty(\Xi) = \frac{\mathbb{E}_\pi(D_o(T)) - \mathbb{E}_\pi(D_u^+(T))}{\mathbb{E}_\pi(D_o(T)) + \mathbb{E}_\pi(D_i(T))} \tag{18}$$

The criterion (18) can be calculated using the numerical solutions of Equations (13), (16) and (17).

4.2. Numerical experiments

4.2.1. Further modelling assumptions. For numerical experiments, the following modelling assumptions have been made.

Repair recovery: The repair recovery value Δ_y is chosen to be a uniform random variable on $[y - \zeta, y)$ where y is the observed deterioration level at the beginning of the repair. Since the system deterioration level after the repair should belong to the restarting zone $[0, \zeta]$, the pdf of the random recovery is

$$f_{\Delta_y}(y_0) = \frac{1}{\zeta} \mathbb{1}_{\{y_0 \in [y-\zeta, y]\}} \tag{19}$$

Maintenance operation duration: The maintenance operation durations $\tau(y, y_0)$ are modelled as random Poisson variables with a non-constant parameter depending on the level of deterioration y and on the operation recovery y_0 . In order to obtain an increasing duration average with respect to y and y_0 , the Poisson parameter $\lambda(y, y_0)$ is

$$\lambda(y, y_0) = \frac{y + y_0}{\xi_N} \tag{20}$$

Then, the conditional probability of the operation duration is given by

$$\mathbb{P}(\tau(y, \Delta_y) = l | y, \Delta_y = y_0) = \frac{\left(\frac{y + y_0}{\xi_N}\right)^{l-1}}{(l-1)!} \exp\left(-\frac{y + y_0}{\xi_N}\right) \quad (21)$$

The expectation of $\tau(y, \Delta_y)$, conditional on y and $\Delta_y = y_0$ is

$$\mathbb{E}_\pi(\tau(y, \Delta_y) | y, \Delta_y = y_0) = 1 + \lambda(y, y_0) = 1 + \frac{y + y_0}{\xi_N} \quad (22)$$

The expected system inactivity duration due to maintenance in a Markov renewal cycle at steady-state is given by

$$\begin{aligned} \mathbb{E}_\pi(D_i(T)) &= \mathbb{E}(\mathbb{E}_\pi(\tau(y, \Delta_y) | y, \Delta_y = y_0)) \\ &= 1 + \frac{1}{\xi_N} \left\{ \int_0^{\xi_N} y \pi(y) dy + \int_{\xi_N}^{\xi_{N+1}} \left(2y - \frac{\zeta}{2}\right) \pi(y) dy \right. \\ &\quad \left. + \int_{\xi_{N+1}}^{\infty} 2y \pi(y) dy \right\} \end{aligned} \quad (23)$$

Maintenance costs: The maintenance costs include the operations cost, the system inactivity cost and the system unavailability cost, see Equation (8).

- The operations unit costs are constant, except the repair unit cost which is a function of the observed deterioration level y and the associated recovery value y_0 . For the sake of simplicity, the repair cost is assumed to be a function of the single variable $y_0 : c_r(y, y_0) = c_r(y_0) = \gamma(1 + y_0/L)$ (γ is a given constant). The expected repair cost in Markov renewal cycle is

$$\mathbb{E}_\pi(C_{\text{REPAIR}}(T)) = \int_{\xi_N}^{\xi_{N+1}} \left(\int_{y-\zeta}^y c_r(y_0) f_{\Delta_y}(y_0) dy_0 \right) \pi(y) dy \quad (24)$$

$$= \gamma \int_{\xi_N}^{\xi_{N+1}} \left(1 + \frac{2y - \zeta}{2L} \right) \pi(y) dy \quad (25)$$

- The system inactivity cost rate r_i due to maintenance is assumed to be constant and independent on both the operation nature and the deterioration level.
- The system unavailability cost rate r_u is assumed to be constant and independent on the deterioration level.

4.2.2. Expected maintenance cost. A quasi-Newton algorithm has been implemented to optimize the performance criteria (8) and (18). A convex cost function has been observed in all the performed numerical experiments, which can be explained by the regularization effect of the parametric multi-threshold maintenance structure.

The cost surface in Figure 4 represents the expected maintenance cost (optimized w.r.t. ξ_1 and ξ_2) as a function of ξ_{N+1} (preventive replacement threshold) and ζ (restarting threshold) for a 4-threshold policy ($N = 2$). In each case, the decision thresholds ξ_1 and ξ_2 have been set to their optimal values which minimize the expected maintenance cost rate C_∞ . The surface shape

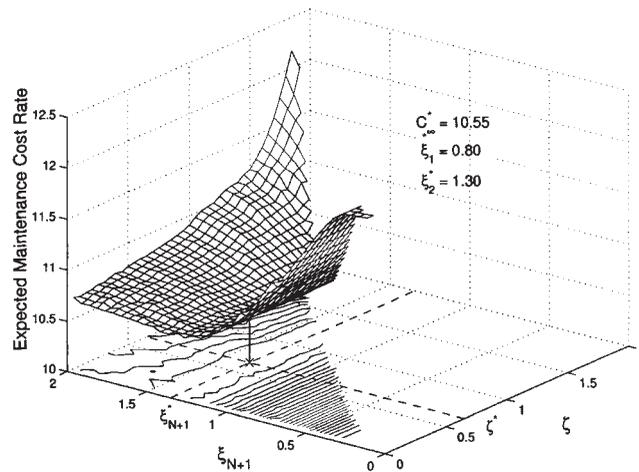


Figure 4. Expected cost surface for a 4-threshold policy—maintenance costs $c_1 = 3$, $\gamma = 40$, $c_p = 35$, $c_c = 100$, $r_u = 5$, $r_i = 10$ —Deterioration characteristics $\alpha = 3$, $L = 2$.

shows that for the considered system configuration (i.e. maintenance costs $c_1 = 3$, $\gamma = 40$, $c_p = 35$, $c_c = 100$, $r_u = 5$, $r_i = 10$ and deterioration characteristics $\alpha = 3$, $L = 2$) the optimal maintenance policy is obtained with $\zeta_1^* = 0.80$, $\zeta_2^* = 1.30$, $\zeta_{N+1}^* = 1.35$ and $\zeta^* = 0.65$. This thresholds configuration leads to a sequential repair/replacement policy with non-periodic inspections exhibiting a narrow repair decision zone. The valley shaped cost function allows several different (but almost equivalent) choices of the maintenance parameters. Finally, the maintenance parameters can be optimally tuned by taking into account the availability criterion as well.

4.2.3. Asymptotic availability. Figure 5 shows asymptotic availability surface curves as a function of ζ_{N+1}^* and ζ for a 6-threshold policy ($N = 4$). Three arbitrary sets of values have been chosen for the other decision thresholds $\zeta_l, l = 1, \dots, 4$. For each of the considered thresholds sets $\zeta_l, l = 1, \dots, 4$, the highest availability is obtained for different values of ζ_{N+1}^* and ζ which correspond in each case to a different PM policy [58]:

- In Figure 5(a), the optimal policy tends towards the condition-based inspection/replacement policy presented in References [16, 17]. The upper availability being reached for $\zeta_{N+1}^* \simeq \zeta_N$, the probability of making a repair tends toward 0 as well as the influence of ζ .
- In Figure 5(b), even if the influence of the threshold ζ remains weak because of a small repair region ($\zeta_{N+1}^* \simeq \zeta_N$), the optimality of the inspection/replacement policy is strengthened by the fact of obliging a return in the new state after repair with restarting threshold ζ equal to zero.
- In Figure 5(c), the availability is maximal for a policy reducing the number of preventive replacements ($\zeta_{N+1}^* \simeq L$) while leaving free the recovery of a repair ($\zeta \simeq \zeta_N$): it is a sequential inspection/repair policy.

The numerical results presented here are only for illustrative purposes. A complete sensitivity analysis of the maintenance policy behaviour with respect to the maintenance parameters and

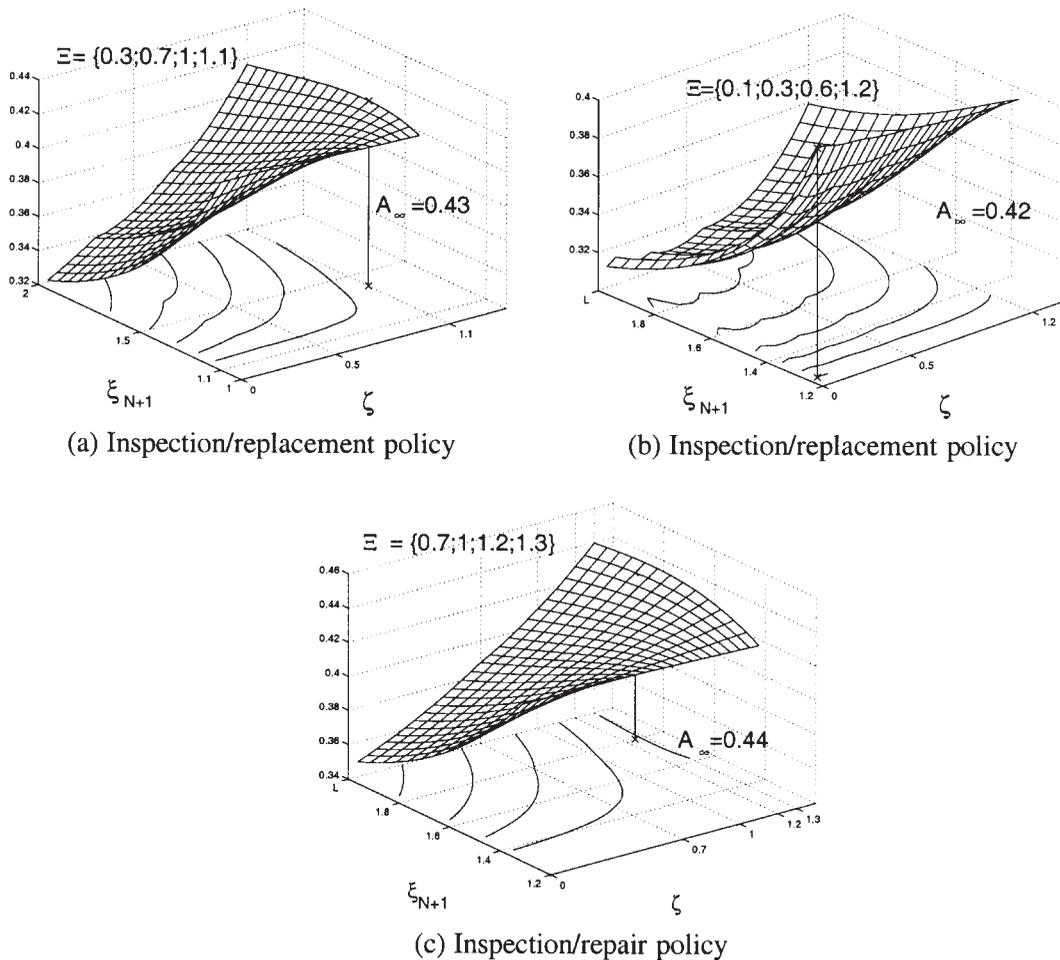


Figure 5. Asymptotic availability surfaces as a function of the restarting threshold ζ and the preventive replacement threshold ξ_{N+1} for a 6-threshold policy—Deterioration characteristics: $\alpha=3$, $L=2$. (a) Inspection/replacement policy, (b) inspection/replacement policy and (c) inspection/repair policy.

deterioration characteristics would require the development of efficient optimization procedures adapted to the high number of parameters and to the multi-criteria formulation of a realistic maintenance optimization problem (which is beyond the scope of this paper).

5. CONCLUDING REMARKS AND FUTURE RESEARCH WORK

We have investigated the problem of inspecting and maintaining a system subject to a stochastic gradual deterioration. We have proposed a parametric CBM structure to schedule non-periodic inspections on the system and to choose the maintenance action to be performed given the

observed level of deterioration. We have shown that the stochastic process describing the deterioration state of the system maintained using this parametric policy exhibits both regenerative and semi-regenerative properties. Taking advantage of these semi-regenerative properties, it has been possible to derive the stationary law of the maintained system state and to construct a mathematical model for the performance assessment of the maintenance policy. One of the main contributions of the work presented in this paper is to show that semi-regenerative stochastic techniques allow to solve numerically rather general maintenance modelling problems, for which very often Monte-Carlo simulation techniques were the only solution. Such a performance assessment model can be used as decision tool to optimize the maintenance policy parameters. In order to extend its applicability to real systems, this maintenance model has to be extended to multi-component systems and to systems whose degradation is only observable through an associated covariate process. We also have found that the optimization of the maintenance policy with classical algorithms represents a heavy computational load. Optimization schemes adapted to the high number of maintenance parameters and to the multi-criteria nature of maintenance problems (cost, availability, safety, etc.) should thus accompany the proposed extensions of the model in order to make it more exploitable.

APPENDIX A

A.1. Expression of the expected unavailability time $E_{\pi}(D_u^+(T))$

Let us consider the system behaviour on a Markov renewal cycle, the time origin t_0 is the beginning of the cycle and maintenance time is discarded. Let $G(k|z, l)$ be the probability that the unavailability time is equal to $k\delta t$ on a Markov renewal cycle, conditional on the deterioration value after maintenance $z \in [\xi_{N-l}, \xi_{N-l+1}[$, $l \geq k$. The level of deterioration just before maintenance is y and the observed recovery value is $y_0(z = y - y_0)$.

Conditional on $z \in [\xi_{N-l}, \xi_{N-l+1}[$, $l \geq k$, the event $[D_u^+(T) = k]$ is equivalent to the composite event: the systems runs during the first $l - k$ maintenance periods of the cycle, fails between t_{l-k} and t_{l-k+1} , and finally remains in the failed state during the last $k - 1$ periods of the cycle.

If the deterioration increments follow an exponential law, we have:

$$G(k|z, l) = \begin{cases} e^{-\alpha(L-z)} & \text{if } l = k \\ 0 & \text{if } l < k \\ \frac{(\alpha(L-z))^{l-k}}{(l-k)!} e^{-\alpha(L-z)} & \text{if } l > k \end{cases} \quad \forall z \in [\xi_{N-l}, \xi_{N-l+1})$$

The probability of an unavailability time equal to k maintenance periods $\mathbb{P}(D_u^+(T) = k)$ is obtained using the total probability law:

$$\begin{aligned} \mathbb{P}(D_u^+(T) = k) &= \mathbb{E}(\mathbb{E}(\mathbb{1}_{\{D_u^+(T)=k\}} | z < \xi_N)) \\ &= \mathbb{E}(\mathbb{P}(D_u^+(T) = k | z < \xi_N)) = \mathbb{E}\left(\sum_{l=1}^N G(k|z, l)\right) \end{aligned} \tag{A1}$$

The observed recovery value y_0 is either equal to 0 (inspection), to y (replacement) or to any intermediate random value following a pdf f_{Δ_y} (repair). Equation (A1) becomes:

$$\begin{aligned} \mathbb{P}(D_u^+(T) = k) &= \mathbb{E} \left[\sum_{l=1}^N G(k|y - y_0, l) \left(\underbrace{\mathbb{1}_{\{y \in [\xi_{N-l}, \xi_{N-l+1}]\}}}_{\text{inspection}} + \underbrace{\mathbb{1}_{\{y \geq L\}}}_{\text{replacement}} \right. \right. \\ &\quad \left. \left. + \underbrace{\mathbb{1}_{\{y \in [\xi_N, L] \cap z = y - y_0 \in [\xi_{N-l}, \xi_{N-l+1}]\}}}_{\text{repair}} \right) \right] \\ &= \sum_{l=0}^{N-k} \int_{\xi_l}^{\xi_{l+1}} \pi(y) G(k|y, N-l) dy + G(k|0, N) \int_L^\infty \pi(y) dy \\ &\quad + \int_{\xi_N}^L \pi(y) \sum_{l=0}^{N-k} \int_{y-\xi_{l+1}}^{y-\xi_l} f_{\Delta_f}(y_0) G(k|y - y_0, N-l) dy_0 dy \end{aligned} \quad (\text{A2})$$

Finally, the expected unavailability time on a Markov renewal cycle is computed as follows:

$$\mathbb{E}_\pi(D_u^+(T)) = \sum_{k=1}^N k \mathbb{P}(D_u^+(T) = k) \quad (\text{A3})$$

Nomenclature

α	parameter of the exponential pdf of the deterioration increments
$A_\infty(\Xi)$	asymptotic availability of the maintained system
$C_\infty(\Xi)$	long-run expected maintenance cost per unit of time
$C(t)$	cumulative maintenance cost at time t
$c_i(y)$	unit cost of an inspection (all costs in arbitrary units)
$c_c(y)$	unit cost of a corrective replacement
$c_p(y)$	unit cost of a preventive replacement
$c_r(y, y_0)$	unit cost of a repair with recovery Δ_y
$D_i(t)$	total duration of system inactivity for maintenance (at t)
$D_o(t)$	total duration of system operation, i.e. not in maintenance (at t)
$D_u(t)$	total duration of system unavailability (at t)
δt	elementary maintenance period
$\Delta_{(k,k+1)}X$	random deterioration increment on a maintenance period δt
Δ_y	random recovery of a preventive partial repair
\mathbb{E}_π	expectation with respect to the law π
$F(x y)$	transition probability law of the MC Y_k
$f(x)$	pdf of $\Delta_{(k,k+1)}X$
$f^{(k)}(x)$	pdf of the cumulative deterioration on k maintenance periods (k -ith convolution)

f_{Δ_y}	pdf of the recovery value
$G(k z, l)$	probability that the unavailability time is equal to $k\delta t$ on a Markov renewal cycle, conditional on the deterioration value after maintenance $z \in [\xi_{N-l}, \xi_{N-l+1}]$, $l \geq k$
γ	repair cost parameter
$\Gamma_i(t)$	cumulated cost due to system inactivity during maintenance (at t)
$\Gamma_m(t)$	cumulated cost of maintenance operations (at t)
$\Gamma_u(t)$	cumulated cost due to system unavailability after failure (at t)
$\mathbb{1}_{Ev}$	indicator function defined by $\mathbb{1}_{Ev} = 1$ if the event Ev is true and $\mathbb{1}_{Ev} = 0$ otherwise
L	failure threshold
N	number of maintenance decision thresholds
$\pi(x)$	stationary law of the MC Y_k
\mathbb{P}_y	probability conditional on y
$P^i(k, y)$	discrete probability law of the length of a maintenance operation initiated at level y
r_i	inactivity cost rate for the system in maintenance
r_u	unavailability cost rate for the failed system
S_k	regeneration (or renewal) times
T_k	semi-regeneration (or Markov renewal) times
$\tau(y, y_0)$	length of a maintenance operation
t_k	k th maintenance decision time ($t_k = k\delta t$)
$\xi_l, l \leq N - 1$	inspection thresholds
ξ_N	preventive repair threshold
ξ_{N+1}	preventive replacement threshold
Ξ	set of maintenance decision parameters
X_k	deterioration process of the unmaintained system (at t_k)
\tilde{X}_k	deterioration process of the maintained system (at t_k)
Y_k	MC describing the system state at the beginning of a maintenance operation
y_0	observed recovery value at the end of an operation
z	observed deterioration level at the end of an operation
ζ	restarting (after repair) threshold

Acronyms

CBM	condition-based maintenance
MC	Markov chain
pdf	probability density function
PM	preventive maintenance

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