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An evaluation of the self-determined probability-weighted moment method for estimating extreme wind speeds

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Abstract

For the estimation of probability distribution parameters, the method of self-determined probability-weighted moments (SD-PWM) has previously been introduced as a refinement on the original method of probability-weighted moments (PWM). Tables have been created summarizing the solution of the relevant equations for certain probability distributions, but application of these is awkward. In addition, certain associated algorithms are difficult to interpret and contain formulations that do not appear to properly enforce the definitions of self-determined probability-weighted moments. Therefore, new algorithms have been developed to both clarify and simplify the determination of SD-PWM parameter estimates. As an application of the SD-PWM algorithms, the estimation of extreme wind speeds is considered using the Gumbel and generalized extreme value (GEV) distributions. The estimation results are compared to similar results obtained via PWM, the method of moments and the maximum likelihood method. The analyses suggest SD-PWM may be a reasonable tool for analyzing the ability of a particular distribution to describe a sample. Relative to the method of moments and PWM estimates, the SD-PWM estimates compare well based on fits of the cumulative distributions. While the SD-PWM estimates exhibit increased variability relative to the method of moment (MOM) estimates, SD-PWM wind speed estimates are generally conservative relative to the MOM estimates.

Keywords: Self-determined probability-weighted moments; Extreme wind speeds; Parameter estimation; Numerical algorithms; Extreme value analysis

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1. Introduction

A common task in wind engineering is the estimation of extreme wind speeds based on probabilistic models of wind events and records of past observances. To use probabilistic models to describe the behavior of a set of observations, or a sample, estimates are determined for the distribution parameters such that the probability distribution best describes the general behavior of the sample. Various methods exist for determining a probability distribution's parameter estimates from a given sample of extreme wind speeds. Common parameter estimation methods include, but are not limited to, the method of moments, the maximum likelihood method, and the method of probability-weighted moments. (See [1] for descriptions of these methods.) Other estimation methods employed in the literature include the de Haan method [2], used by Simiu and Heckert [3] to describe wind speed behavior at 44 stations in the United States; the Gumbel-Lieblein BLUE method [4] and modified Gumbel method [5], used by Harris [6] to study the accuracy of 50-year wind speed estimates based on typical UK data; and the method of L-moments [7], used by Pandey et al. [8] to study the same US wind speed data. Issues with parameter estimate variability and sensitivity to data record size have been noted, particularly when peaks over threshold approaches are used. Given the vital need to design structures for extreme events and the relative lack of wind speed data for such events, finding a parameter estimation technique that produces reliable extreme wind speed estimates remains a concern.

Introduced by Haktanir [9], the method of self-determined probability-weighted moments (SD-PWM) is an estimation method intended to improve upon the original method of probability-weighted moments (PWM) as defined by Greenwood et al. [10]. The PWM method is thought to be less affected by sampling variability and be more efficient at producing robust parameter estimates from small samples [11]; by extension, SD-PWMs should enjoy the same property. Haktanir suggests that two strengths of the method of SD-PWMs are (1) the ability to directly consider outliers that may be present in a given sample, and (2) the ability to detect whether a particular distribution is appropriate for describing the behavior of a sample. Haktanir applied this method to five distributions—Gumbel, generalized extreme-value (GEV), log-logistic, three-parameter lognormal, and Pearson type three—and developed calculation algorithms for each distribution to determine the SD-PWM parameter estimates for a sample. These algorithms, however, can be complex to apply, particularly for the three-parameter lognormal and Pearson type three distributions. Additionally, the algorithms can require multiple inputs from numerical tables developed by Song and Hou [12] and Ding et al. [13], which relate distribution functions to parameter estimates. In the case of the three-parameter lognormal algorithm, this process is complicated by the fact that Song and Hou's numerical table is not readily available. Thus, it is desirable to produce computational algorithms that implement the method of self-determined probability-weighted moments so that their possible relevance to extreme wind speed estimation can be assessed.

To increase its practical application, SD-PWM algorithms have been developed for the five distributions considered by Haktanir and have been implemented using Mathwork's computational software Matlab® [14]. For the Gumbel and GEV distributions, algorithms similar to those given by Haktanir were used to estimate distribution parameters and extreme wind speeds for a sample of the sets considered by Simiu and Heckert. To examine the relative performance of the SD-PWM estimations, parameters and extreme wind speeds were also estimated with the PWM method as well as the method of moments and the maximum likelihood method. The suggested strengths of the SD-PWM method were assessed as they apply to extreme winds, and some conclusions were drawn. In the sections below, we discuss the application of the self-determined probability-weighted moment method to wind speed estimation. Section 2 provides necessary background into the theory of SD-PWM, while Section 3 discusses algorithm development and its associated issues. Section 4 presents the results of applying these algorithms to the problem of estimating extreme wind speeds, and Section 5 gives a discussion of the results and some conclusions as to the usefulness of this method.

2. Theory of self-determined probability-weighted moments

As mentioned above, the method of self-determined probability-weighted moments was developed as a modification of the original method of probability-weighted moments, which was originally designed to facilitate parameter estimation for those distributions that are analytically expressible only in inverse form (e.g., the Wakeby distribution, Tukey's lambda distribution, etc.). In general, the probability-weighted moments (PWMs) of a distribution are defined as

$$M_{p,r,s} = E[X^p F^r (1-F)^s] = \int_0^1 [x(F)]^p F^r (1-F)^s \mathrm{d}F, \tag{1}$$

where $F = F(x; \phi_1, \phi_2, \ldots \phi_k) = P(X \leqslant x)$ is the cumulative distribution function, $\phi_1, \phi_2, \ldots \phi_k$ are the parameters of the distribution, $x(F)$ is the inverse cumulative distribution function, and $p, r,$ and s are real numbers. The most commonly used PWMs are $M_{1,0,s}$ and $M_{1,r,0}$; for convenience, these are re-expressed as follows:

$$\begin{aligned}
\alpha_s \equiv M_{1,0,s} &= \int_0^1 x(F)(1-F)^s \, \mathrm{d}F \\
&= \int x[1 - F(x; \phi_1, \phi_2, \ldots, \phi_k)]^s f(x; \phi_1, \phi_2, \ldots, \phi_k) \, \mathrm{d}x, \\
\beta_r \equiv M_{1,r,0} &= \int_0^1 x(F) F^r \, \mathrm{d}F \\
&= \int x F(x; \phi_1, \phi_2, \ldots, \phi_k)^r f(x; \phi_1, \phi_2, \ldots, \phi_k) \, \mathrm{d}x.
\end{aligned} \tag{2}$$

In each expression, $f(x; \phi_1, \phi_2, \ldots \phi_k)$ denotes the probability density function, while the lower and upper bounds for x in the second integrals are such that $F(x) = 0$

and $F(x) = 1$, respectively. For a k-parameter probability distribution, α_s and β_r are, typically, non-linear functions of the distribution parameters $\phi_1, \phi_2, \ldots \phi_k$ due to the generally non-linear dependence of F and f on these parameters. It can be shown [11] that the PWM sets $\{\alpha_s : s = 0, 1, 2, \ldots\}$ and $\{\beta_r : r = 0, 1, 2, \ldots\}$ are linear combinations of each other; thus, the definition from (2) that is analytically preferable may be used without loss of generality.

To relate the probability-weighted moments of a distribution to an ordered sample $x_1 < x_2 < \cdots < x_{n-1} < x_n$, an estimator of α_s or β_r is needed. In the usual PWM method, unbiased estimators of α_s and β_r are obtained by using pre-determined non-exceedance probabilities. Following [9], for the n-element sample one defines

$$P^j_{\mathrm{nex},i} = \binom{i-1}{j} \Big/ \binom{n-1}{j}, \quad j = 0, 1, 2, \ldots, n-1, \tag{3}$$

where $P_{\mathrm{nex},i}$ is a reasonable estimate for the non-exceedance probability of the ith event. The PWM estimators can then be expressed as

$$\alpha'_s = \frac{1}{n}\sum_{i=1}^{n}(1 - P_{\mathrm{nex},i})^s x_i, \quad s = 0, 1, 2, \ldots \tag{4a}$$

and

$$\beta'_r = \frac{1}{n}\sum_{i=1}^{n} P^r_{\mathrm{nex},i} x_i, \quad r = 0, 1, 2, \ldots \tag{4b}$$

Note that Eq. (4a) is true only if the polynomial in $P_{\mathrm{nex},i}$ is expanded prior to the substitution of Eq. (3). Other authors [1, 10] have suggested that the plotting position formula

$$F_i = (i - 0.35)/n \tag{5}$$

may be used in place of $P_{\mathrm{nex},i}$ in Eq. (4), which gives biased estimators but has been shown to yield better estimates for both distribution parameters and quantiles in certain cases [7]. In either case, the PWM parameter estimates $\phi'_1, \phi'_2, \ldots, \phi'_k$, are those values such that

$$\alpha'_s = \alpha_s(\phi'_1, \phi'_2, \ldots, \phi'_k), s = 0, 1, \ldots k - 1 \tag{6a}$$

or

$$\beta'_r = \beta_r(\phi'_1, \phi'_2, \ldots, \phi'_k), r = 0, 1, \ldots k - 1. \tag{6b}$$

Eqs. (6a) or (6b) represent k non-linear equations to solve for the distribution parameters; in general, iterative methods are required to obtain the solutions.

An inspection of the above derivation shows that the non-exceedance probabilities are essentially fixed a priori; the only dependence is upon the size of the data sample. More importantly, any non-uniformity of spacing in the data cannot be reflected in the non-exceedance probabilities, which leads to questions about the ability of this method to handle outliers appropriately. The method of self-determined probability-weighted moments deviates from the method of probability-weighted moments in that it is assumed the ordered sample follows a particular distribution. Accordingly, the non-exceedance probabilities are assigned using the corresponding cumulative distribution function with no prior assumptions about the spacing between sample points. The SD-PWM estimators α''_s and β''_r for α_s and β_r, respectively, are defined as

$$\alpha''_s = \frac{1}{n}\sum_{i=1}^{n}[1 - F(x_i, \phi_1, \phi_2, \ldots, \phi_k)]^s x_i, \ \ \beta''_r = \frac{1}{n}\sum_{i=1}^{n} F^r(x_i, \phi_1, \phi_2, \ldots, \phi_k)x_i. \quad (7)$$

The SD-PWM parameter estimates $\phi''_1, \phi''_2, \ldots, \phi''_k$ for the given distribution and a given sample are defined as those values such that

$$\alpha''_s(\phi''_1, \phi''_2, \ldots, \phi''_k) = \alpha_s(\phi''_1, \phi''_2, \ldots, \phi''_k), s = 0, 1, \ldots, k-1, \quad (8a)$$

or

$$\beta''_r(\phi''_1, \phi''_2, \ldots, \phi''_k) = \beta_r(\phi''_1, \phi''_2, \ldots, \phi''_k), r = 0, 1, \ldots, k-1. \quad (8b)$$

Again, Eq. (8) represents a set of k non-linear equations that must be solved for k unknowns using iterative methods. Typically, these equations are more complicated than those of Eq. (6), since non-linear functions of the parameters are present on both sides of the equality.

Although the set of equations to be solved increases in complexity when moving from PWMs to SD-PWMs, Haktanir suggests that the assignment of non-exceedance probabilities directly from the cumulative distribution function yields two advantages over traditional PWMs [9]. First, by computing non-exceedance probabilities directly from the cumulative distribution function, probabilities are directly affected by the value of sample points. The parameter estimates thus should reflect more accurately the presence of irregularly spaced data, particularly in the tails. Haktanir also suggests that the direct assigning of non-exceedance probabilities from a particular distribution causes the corresponding results to reflect the ability of the selected distribution to describe the sample. If a distribution is appropriate for describing a sample, errors in fitting the sample to the distribution should be less than errors generated by other methods, since the distribution parameter estimates directly reflect the influence of the distribution. Alternatively, if a distribution is inappropriate for describing a sample, errors generated by the method of SD-PWM should be greater than errors generated by other methods. Both of these ideas are explored further in Sections 4 and 5.

3. Algorithm development and usage

For the two distributions considered here—Gumbel and generalized extreme value—the SD-PWM equations and algorithms generally follow the equations and algorithms given in [9]. Deviations from Haktanir's algorithms, as well as the significant changes needed to implement the SD-PWM method for other distributions such as three parameter lognormal and Pearson type three, are discussed in [15]. In the rest of this section, we discuss some general issues associated with development and implementation of the SD-PWM method, and we provide an illustration of its use.

3.1. General issues

As previously mentioned, the method of SD-PWM requires a system of non-linear equations to be simultaneously solved. For each distribution, an associated Matlab® script employing the iterative root-finding function *fsolve* has been developed to determine the SD-PWM parameter estimates for a given sample set. By default, *fsolve* employs its "large scale" algorithm to solve for the zeros of the relevant equations. This algorithm is a subspace trust region method based upon a version of Newton's method. (See [16] as well as the Matlab® help menu for *fsolve.*) To improve performance, constraint settings on the algorithm providing reasonable results for each script were determined and implemented. In general, this entailed using full bandwidth on the initial preconditioner for the preconditioned conjugate gradient iterations as well as requiring a very tight tolerance on the termination value. See [15] for further information on these scripts.

The iterative techniques used in the algorithms require that initial guesses on the parameter estimates be made. Since the method of SD-PWM follows from the method of PWM, the PWM parameter estimates are reasonable candidates for these initial guesses. Thus, when appropriate, the PWM parameter estimates are used for the required initial guesses. However, for some distributions (e.g., GEV), the PWM method itself requires an iterative solution process. In such instances, approximations for the PWM parameter estimates are used to establish reasonable initial guesses for the SD-PWM algorithm. For example, the PWM estimate of the GEV shape parameter a comes from the iterative solution of the following equation [9]:

$$\frac{3\beta_2' - \bar{x}}{2\beta_1' - \bar{x}} = \frac{1 - 3^{-a}}{1 - 2^{-a}}, \tag{9}$$

where β_1' and β_2' are computed as indicated in Eq. (4b). Alternatively, Hosking et al. [17] give the following approximation for Eq. (9):

$$a = 7.8590r + 2.9554r^2, \quad r = \frac{2\beta_1' - \bar{x}}{3\beta_2' - \bar{x}} - \frac{\ln 2}{\ln 3}. \tag{10}$$

For $-\frac{1}{2} < a < \frac{1}{2}$, Eq. (10) gives absolute errors of less than 0.0009 compared to Eq. (9) [17]. Because Eq. (10) allows for a direct estimate of a, and since the errors are reasonably small, this equation is used to determine the initial estimate a_0 for the SD-PWM algorithm.

It can be shown that $\alpha_0 = \beta_0 = \mu$, the expected value of the distribution. The corresponding SD-PWM estimators α_0'' and β_0'' can be shown to equal $\bar{x}$, the sample mean. Since μ typically depends upon the distribution parameters in a simpler manner than the higher order SD-PWMs, it is usually possible to solve the equation $\mu = \bar{x}$ directly for one of the parameters in terms of the sample mean and the other distribution parameters. This means that, for a k-parameter distribution, the k equations to be solved will reduce to a set of $k - 1$ equations that are solved iteratively, and one equation whose solution follows trivially. This trend is observed for all distributions presented here, and it was used to simplify the parameter estimation algorithms.

3.2. Illustrative example

As an example of how the SD-PWM method works, we illustrate the performance of the method for obtaining parameter estimates for the Gumbel distribution. For the Gumbel distribution, the cumulative distribution function is

$$F(x) = \exp\{-\exp[-b(x-c)]\}, \tag{11}$$

where b is related to the scale parameter, c is the location parameter, and $-\infty < x < \infty$. Since there are two parameters, the first two PWMs can be used for estimation. Greenwood et al. [10] considered the set α_s and found

$$\alpha_0 = c + \frac{\gamma}{b}, \tag{12a}$$

$$\alpha_1 = \frac{c}{2} + \frac{\gamma - \ln 2}{2b}, \tag{12b}$$

where Euler's constant $\gamma \approx 0.5772157$. Using Eqs. (7) and (8a) and recalling that $\alpha_0'' = \bar{x}$, we obtain the following equations for the Gumbel parameters:

$$b = \frac{\ln 2}{\bar{x} - 2\alpha_1''}, \tag{13a}$$

$$c = \bar{x} - \frac{\gamma}{b}, \tag{13b}$$

$$\alpha_1'' = \frac{1}{n}\sum_{i=1}^{n}[1 - F(x_i, b, c)]x_i = \frac{1}{n}\sum_{i=1}^{n}[1 - \exp\{-\exp[-b(x_i - c)]\}]x_i. \tag{13c}$$

We see that c is a simple function of the sample mean and b; thus, we can eliminate c from Eq. (13c) and substitute the result into Eq. (13a) to obtain

$$0 = ln\,2 - b\left[\bar{x} - \frac{2}{n}\left(\sum_{i=1}^{n}[1 - \exp\{-\exp[-b(x_i - \bar{x}) - \gamma]\}]x_i\right)\right]. \tag{14}$$

The roots of this equation are the SD-PWM estimates for the parameter b; the parameter c is obtained via Eq. (13b) once b is known.

Denoting the right-hand side of Eq. (14) by $f(b)$, we notice that this function is highly non-linear and sensitive to the sample values; thus, it is not surprising that the roots of $f(b) = 0$ can exhibit complex behaviors. Refer to [15] and [18] for further discussion of issues related to finding roots of the SD-PWM equations. In general, two distinct behaviors were observed. As illustrated in Fig. 1, different choices of data sets led to either a pair of roots or no roots being generated by solving $f(b) = 0$ (see Section 4 for an explanation of the meaning of the threshold value for each data set). A total of 132 data sets were analyzed using the SD-PWM method for the Gumbel distribution, and 75 of these (57%) had roots generated in pairs. The remaining 57 data sets produced no roots for Eq. (14); no instances of "double roots" were observed. In every case in which multiple roots were possible, the SD-PWM algorithm converged to the larger root (denoted by b_2 in Fig. 1). There is, however, no a priori reason to assume that the lower root b_1 is not a legitimate parameter value for the Gumbel distribution. Thus, it is necessary to check the two possible solutions to determine which one performs best in fitting the data. Fig. 2 shows typical results of a comparison of the cumulative distribution functions obtained using the b_1 root, the b_2 root, and the sample set data (F values for the data were estimated using Eq. (3) with $j = 1$). In all cases, the higher root b_2 produced a

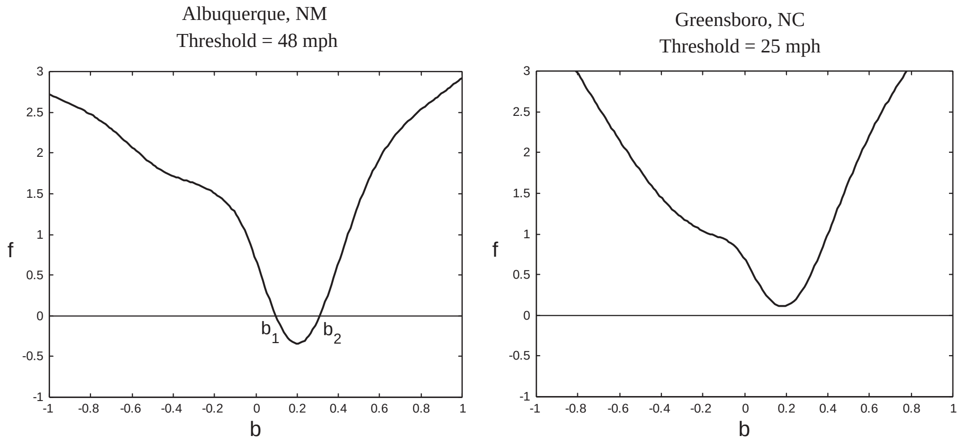

Fig. 1. Typical behaviors of Eq. (14) for different data sets.

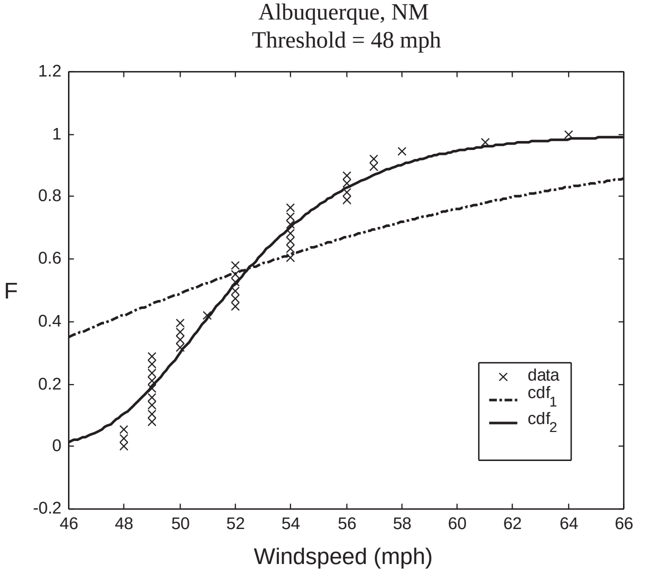

Fig. 2. Gumbel cumulative distribution functions for different roots of Eq. (14).

better fit to the sample data, leading to the conclusion that the SD-PWM algorithm was converging to the proper parameter estimate.

While the above discussion is specific to the Gumbel distribution, it does reflect many of the features seen in the development and use of the SD-PWM algorithms for other distributions. The most significant distinction presented by the other distributions considered in [15] is that they are all three-parameter distributions, whereas Gumbel is two-parameter. Thus, Eqs. (7) and (8) will produce a pair of equations to be solved for roots, rather than the single equation observed in Eq. (14). The additional complexity induced by solving non-linear equations simultaneously can lead to even more complicated root behavior, thus making it very important to check the quality of the fit for the cdf arising from the parameter estimates.

4. Sensitivity study

As part of the process of determining the applicability of the SD-PWM method to the estimation of extreme wind speeds, we investigated the claims of Haktanir, discussed in Section 2, regarding the sensitivity of this method. More specifically, we performed numerical experiments using synthetic data sets to ascertain (1) whether the SD-PWM method could be used to diagnose the appropriateness of an assumed probability distribution for describing the behavior of a given data set, and (2) whether the SD-PWM method performed more robustly in the presence of outliers in comparison to the PWM method. The results of these trials provide support for some of Haktanir's claims, but not all aspects of his claims were deemed to hold true. Below, we discuss the details of these investigations.

In order to study the ability of the SD-PWM method to indicate an inappropriate choice for assumed distribution of a set of data, the following performance test was undertaken. Multiple data sets were generated numerically using the GEV distribution

$$F(x) = \exp\left\{-\left[1 - \tfrac{a(x-c)}{b}\right]^{1/a}\right\} \tag{15}$$

with a fixed value $c = 40$ for the location parameter and various choices for the shape parameter a and scale parameter b (Recall that $a = 0$ corresponds to the Gumbel distribution). Parameter estimation was then performed on each set, incorrectly assuming that the underlying distribution was a Gumbel distribution. Four different estimation techniques were used: SD-PWM, PWM, maximum likelihood (ML), and method of moments (MOM). A corresponding Gumbel data set was generated from the estimated parameters of each technique, and this data set was compared to the original GEV sample utilized in the estimation process. Errors between the actual GEV distribution and the assumed Gumbel distribution were measured by performing the Kolmogorov-Smirnov (K-S) test [19] on the two data sets and computing the value of the discrepancy statistic via Matlab's *kstest2* function. This statistic computes the maximum absolute value of the difference between the two empirical cdfs over the relevant range of inputs; a larger value of this statistic implies a worse fit between the distributions of the two input data sets. Note that this version of the K-S test can be used to perform hypothesis testing on the two sample sets, with the null hypothesis being that the two sets come from the same distribution. By recording the results of the hypothesis testing along with the statistic value, we obtain a more complete picture of the "closeness" of the real GEV and assumed Gumbel distributions.

The results of this test are shown in Table 1. The shape and scale parameters for each GEV case are listed first, and the average values of the K-S test statistic are listed in columns indicating which estimation technique was used to obtain the Gumbel parameters used to generate the comparison data set. From the table, we see clearly that statistic values associated with the SD-PWM estimates of the Gumbel parameters are larger than the corresponding values obtained via use of the method of moments or PWM techniques. This distinction exists even as the shape parameter a approaches zero (that is, as the underlying GEV distribution approaches a true Gumbel distribution), although the differences do diminish in size. In addition, maximum likelihood statistic values were generally smaller than SD-PWM values, although some exceptions to this were observed (Many of these exceptions corresponded to particularly bad fits between the actual GEV and ML-estimated, assumed Gumbel distributions). Finally, the statistic values in bold indicate situations in which the null hypothesis of the K-S test (i.e., that the two distributions are the same) could not be rejected at a 95% significance level in a majority of test runs. We note that the estimates of the Gumbel parameters obtained via the SD-PWM method were more likely to lead to rejection of the null hypothesis than any other estimation method, indicating that the K-S test could still distinguish a significant difference between the actual and assumed distributions. These

Table 1
Average K-S test statistic values from a comparison of actual GEV and assumed Gumbel distributions

GEV $[a, b]$	Estimation technique				GEV $[a, b]$	Estimation technique			
	SD-PWM	ML	MOM	PWM		SD-PWM	ML	MOM	PWM
[0.2,4]	0.2508	0.6612	0.0504	0.0494	[−0.2,4]	0.1939	0.0488	0.0804	0.0549
[0.2,2]	0.3634	0.2048	0.0528	0.0500	[−0.2,2]	0.1392	0.0488	0.0826	0.0565
[0.2,1]	0.1887	0.0990	0.0555	0.0544	[−0.2,1]	0.1151	0.0941	0.0844	0.0560
[0.1,4]	0.1883	0.6041	0.0292	0.0287	[−0.1,4]	0.1526	0.0247	0.0316	0.0318
[0.1,2]	0.0903	0.1857	0.0290	0.0267	[−0.1,2]	0.0680	0.0247	0.0332	0.0287
[0.1,1]	0.0721	0.0510	0.0286	0.0264	[−0.1,1]	0.0568	0.0503	0.0334	0.0302
[0.05,4]	0.0794	0.0127	0.0166	0.0150	[−0.05,4]	0.0607	0.0143	0.0166	0.0139
[0.05,2]	0.0434	0.3578	0.0165	0.0131	[−0.05,2]	0.0327	0.0143	0.0186	0.0189
[0.05,1]	0.0361	0.0276	0.0160	0.0148	[−0.05,1]	0.0277	0.0256	0.0151	0.0150
[0.025,4]	0.0410	**0.0090**	**0.0101**	**0.0083**	[−0.025,4]	0.0276	**0.0090**	**0.0092**	**0.0075**
[0.025,2]	0.0245	**0.0090**	**0.0111**	**0.0087**	[−0.025,2]	0.0163	**0.0090**	**0.0088**	**0.0090**
[0.025,1]	0.0205	0.0158	**0.0093**	**0.0086**	[−0.025,1]	0.0140	0.0133	**0.0082**	**0.0092**
[0.01,4]	0.0220	**0.0071**	**0.0090**	**0.0076**	[−0.01,4]	**0.0087**	**0.0058**	**0.0072**	**0.0082**
[0.01,2]	0.0130	**0.0071**	**0.0060**	**0.0045**	[−0.01,2]	**0.0063**	**0.0058**	**0.0060**	**0.0060**
[0.01,1]	0.0111	**0.0085**	**0.0058**	**0.0054**	[−0.01,1]	**0.0059**	**0.0069**	**0.0056**	**0.0058**

observations support the conclusion that the SD-PWM method can diagnose an inappropriate choice of underlying distribution, as the errors between the real GEV and assumed Gumbel distributions are typically larger and more significant than those based on the other estimation techniques.

Further confirmation of this observation can be seen in Table 2, which shows typical results from a performance test similar to the one described above except that the GEV distribution was taken as both the real and assumed distribution, with GEV parameters estimated by the various techniques. We now observe that the K-S test statistic values associated with all of the various estimation techniques are generally of the same order, and in a majority of cases the estimated parameters led to distributions that could not be distinguished from the real distribution at 95% significance (the relatively higher errors associated with the maximum likelihood estimations when the shape parameter is negative seem to stem mainly from difficulties in obtaining converged parameter estimates; the method failed to converge for the two starred cases). Thus, when the "proper" distribution is assumed, all estimation methods were (generally) equally adept at finding the proper parameters, which highlights the importance of the larger errors for the SD-PWM method when an "improper" distribution choice is made.

To analyze the abilities of the SD-PWM and PWM methods to perform estimation in the presence of outliers, the following investigation was pursued. Once again, comparisons of synthetic data sets were performed; in each comparison one set was generated from a known Gumbel or GEV distribution (with predetermined parameters) while the other was generated using parameters estimated using one of the two techniques. The parameter estimation was performed on a series of "dummy" data sets in which the predetermined distribution parameters were used in combination with non-exceedance probability values restricted in lie in various fixed ranges. By varying the upper boundary of the range of non-exceedance probabilities, dummy data sets with more extreme values (and hence more outliers) were produced. Multiple trials were executed for each combination of parameters and probability range, and average discrepancy statistic values from the comparisons were computed.

Table 3 shows the results of this investigation. In this table, two K-S test statistic values are listed for each distribution/range combination: the top value represents

Table 2
Average K-S test statistic values from a comparison of actual and assumed GEV distributions

GEV $[a,b]$	Estimation technique				GEV $[a,b]$	Estimation technique			
	SD-PWM	ML	MOM	PWM		SD-PWM	ML	MOM	PWM
[0.2,2]	**0.0053**	**0.0053**	**0.0053**	**0.0052**	[−0.2,2]	**0.0051**	*	0.0148	**0.0052**
[0.1,2]	**0.0053**	**0.0053**	**0.0053**	**0.0051**	[−0.1,2]	**0.0051**	*	**0.0060**	**0.0051**
[0.05,2]	**0.0052**	**0.3578**	**0.0165**	**0.0131**	[−0.05,2]	**0.0052**	0.0169	**0.0053**	**0.0051**
[0.025,2]	**0.0052**	**0.0090**	**0.0111**	**0.0087**	[−0.025,2]	**0.0052**	0.0125	**0.0052**	**0.0051**
[0.01,2]	**0.0052**	**0.0085**	**0.0058**	**0.0054**	[−0.01,2]	**0.0052**	**0.0079**	**0.0052**	**0.0051**

Table 3
Average K-S test statistic values for PWM and SD-PWM estimations using various ranges

Distribution	Non-exceedance probability range					
	[0.002,0.96]	[0.002,0.98]	[0.002,0.99]	[0.002,0.996]	[0.002,0.998]	[0.002,0.999]
Gumbel $b = 0.25$	0.0526	0.0557	0.0526	0.0567	0.0521	0.0566
	0.2074	0.1310	0.1024	0.0681	0.0600	0.0541
Gumbel $b = 0.5$	0.0520	0.0529	0.0531	0.0546	0.0507	0.0555
	0.1232	0.0976	0.0837	0.0594	0.0562	0.0554
Gumbel $b = 1.0$	0.0549	0.0529	0.0509	0.0493	0.0487	0.0566
	0.0975	0.0851	0.0706	0.0571	0.0591	0.0539
GEV $a = 0.05$, $b = 4$	0.0477	0.0537	0.0527	0.0601	0.0504	0.0564
	0.0457	0.0472	0.0463	0.0556	0.0564	0.0609
GEV $a = 0.05$, $b = 2$	0.0495	0.0452	0.0486	0.0545	0.0584	0.0563
	0.0461	0.0453	0.0584	0.0572	0.0520	0.0571
GEV $a = -0.05$, $b = 4$	0.0568	0.0419	0.0450	0.0451	0.0405	0.0422
	0.0503	0.0486	0.0456	0.0365	0.0408	0.0475
GEV $a = -0.05$, $b = 2$	0.0561	0.0413	0.0504	0.0449	0.0375	0.0503
	0.0539	0.0454	0.0455	0.0419	0.0483	0.0412

the average K-S test statistic obtained using PWM estimates of the distribution parameters, while the bottom number corresponds to the value obtained via SD-PWM estimation (For all distributions studied, the location parameter was again set to $c = 40$). We observe that, for the GEV distributions, discrepancy statistic values are approximately equal under the two estimation methods, with neither showing appreciable trends as the range size increases. For the Gumbel distributions considered here, we note that the discrepancy statistic values associated with PWM parameter estimation are typically smaller than or (roughly) equal to those associated with SD-PWM parameter estimation. The PWM values remain more or less constant as the range size increases, while the SD-PWM values show a noticeable decrease with increasing range size. These results do not support Haktanir's assertion regarding SD-PWM versus PWM performance in the presence of outliers, as increasing the number of extreme values in the samples did not lead to noticeably better fits for distributions based on SD-PWM parameters. It should be noted, however, that these results do not definitively rule out the possibility of most robust estimation by the SD-PWM method, as other factors such as sample size may play an important role.

The results do, however, indicate that the choice of probability-weighted moment estimator may influence the quality of the estimation. The SD-PWM Gumbel algorithms were created using the moment estimator α_s'', as indicated in Section 3.2, while the GEV algorithms make use of the moment estimator β_r''. Recall that each estimator weights a different end of the probability range more heavily—α_s'' weights the lower end preferentially, while β_r'' weights the upper end. While it is not immediately obvious that the SD-PWM estimated Gumbel distribution fits should improve as the upper end of the probability range is increased, it does indicate that the linear dependence of the two types of estimators does not translate into equivalence of the estimations. Further study of this issue, and the associated issue of proper formats for SD-PWM Eqs. (8a) and (8b), will be pursued.

5. Application to extreme wind speed estimation

5.1. Background

Now, we consider the problem of estimating extreme wind speeds from the daily fastest mile wind speed recordings used in [3]. Twelve stations were considered in this study; results from three of those stations (Albuquerque, NM, Concord, NH, and Pocatello, ID) are presented here as representative of the overall findings. As suggested in [3], these data sets were subjected to an "intervalling" process that produces filtered data sets that are preferentially populated by higher daily wind speeds separated by a minimum period, thus reducing the chance of correlated data. In this study, a four-day interval is used, ensuring that all data points are separated by at least two days. Simiu and Heckert [3] observed insignificant variations in results for data sets produced by using four-day and eight-day intervals, so the shorter period is used here. Details of the intervalling process are given in [15]. The effects of intervalling on data from the representative stations are seen in the histograms shown in Fig. 3. The vast majority of points removed by the filtering process are lower wind speed recordings—clearly non-extreme events—while the majority of the higher wind speed recordings are retained.

In addition to filtering the wind speed records, a peaks-over-threshold approach [20] is used here. This approach, which uses only events above a prescribed threshold to estimate distribution parameters, ensures that the majority of points used for estimation are extreme values. The maximum threshold is taken here as the highest wind speed that yields a sample containing at least 30 points. To analyze the effects of thresholds on parameter and quantile estimates, a set of eleven thresholds was employed for each filtered data set, starting with the aforementioned maximum threshold and including the next ten lower wind speeds. Fig. 4 shows the influence that the thresholds have upon sample size for filtered data from the representative stations. It was judged that virtually all results associated with data sets containing more than 250 points were unreliable, showing significantly higher errors (i.e., Kolmogorov-Smirnov discrepancy statistic values) when the estimated distributions were compared to the actual data. Other researchers (e.g., [21]) have stressed the importance of avoiding mixed populations of wind sources when performing extreme wind speed estimation; this phenomenon is most likely the source of the problems observed in this study, since insufficient information exists to distinguish the daily maximum wind speeds by source. Thus, no results are shown below for data sets containing more than 250 points, which corresponds to using only eight thresholds per station.

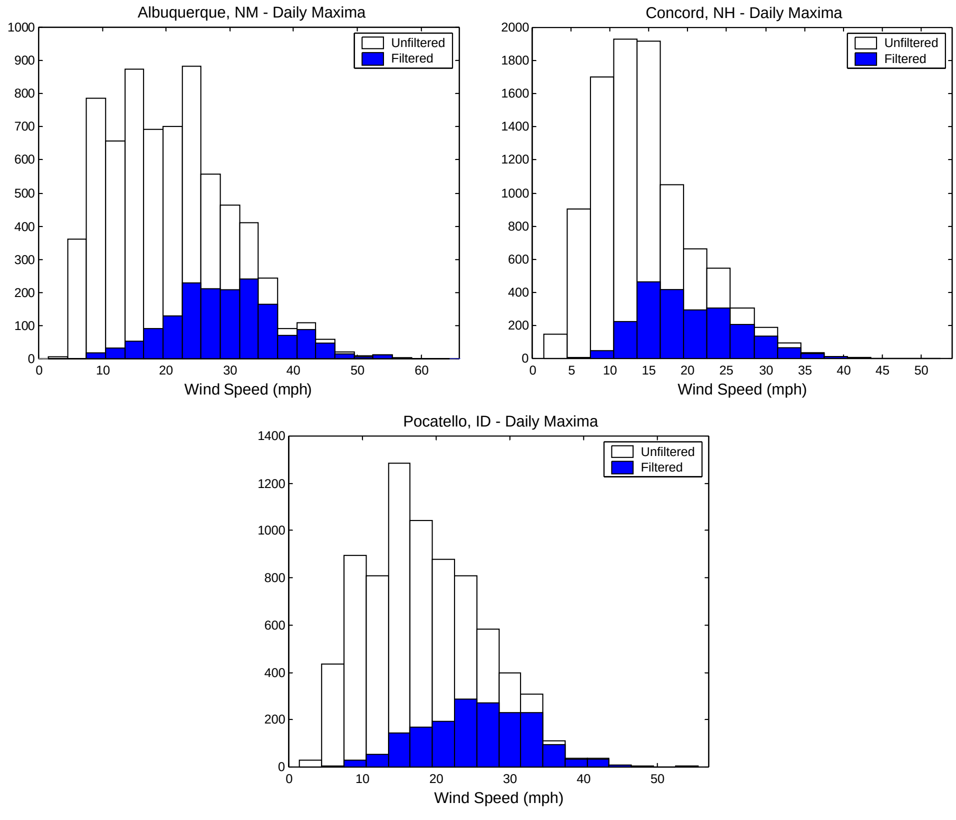

Fig. 3. Histograms of representative data sets before and after filtering via the intervalling process.

5.2. Results

We first tested whether self-determined probability-weighted moments could be used to diagnose the appropriateness of a given probability distribution for describing the behavior of samples of extreme winds. In Fig. 5, values of the K-S test statistic are plotted from a comparison of real wind speed data from Pocatello, ID with synthetic wind speeds generated using estimated Gumbel and GEV distribution parameters. Various thresholds were considered for the wind speeds, and the method of SD-PWM was used to estimate the distribution parameters in each case. We note that the errors associated with the Gumbel distribution are, for the most part, equal to or greater than the errors associated with the GEV distribution. This trend, which is typical of all the stations considered, indicates that the Gumbel distribution was less accurate at fitting these wind data. Moreover, the method of SD-PWM was found to be more likely to produce large errors in comparison to the other estimation techniques. This is seen in Fig. 6, in which the same error measure is plotted for data from Concord, NH assuming that the Gumbel distribution applies and using all four

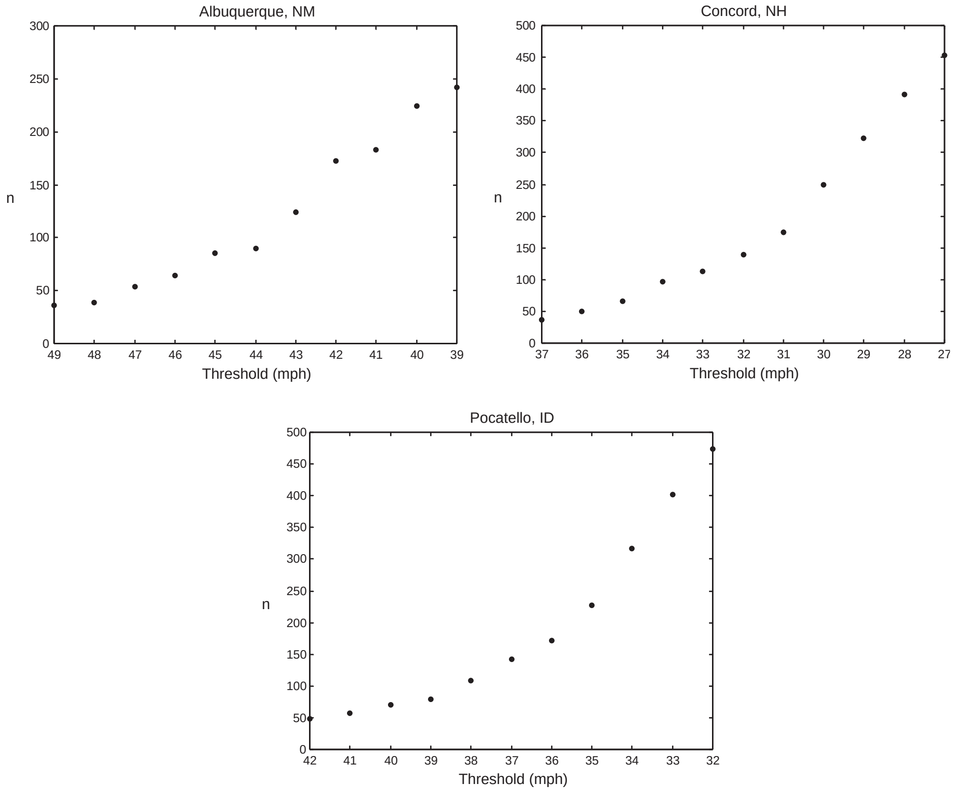

Fig. 4. Sample size versus threshold for representative stations.

estimation techniques. In a majority of cases here (and for other stations studied), the SD-PWM method gave K-S test statistic values as large as or larger than those of the other methods when applied to the Gumbel distribution. This trend disappeared, however, when other distributions were assumed; this is seen in Fig. 7 for the Albuquerque, NM data under the assumption that the GEV distribution applies (Note that the maximum likelihood method failed to converge for these estimation cases, so no results from this method are shown). Comparing with Fig. 6, we note that the SD-PWM statistic values are noticeably lower for the GEV distribution, whereas the values for the other estimation methods have essentially the same range for the two distributions, particularly for the higher thresholds. These results, which are similar to those observed in the sensitivity test discussed in Section 4, point to the SD-PWM method being able to distinguish the Gumbel distribution as being less suitable for use in describing extreme wind speed behavior for these data sets. While this is not definitive evidence against the use of Gumbel in extreme wind speed estimation, it is consistent with the observation by Harris [6] that the Gumbel distribution performs poorly unless improvements are applied.

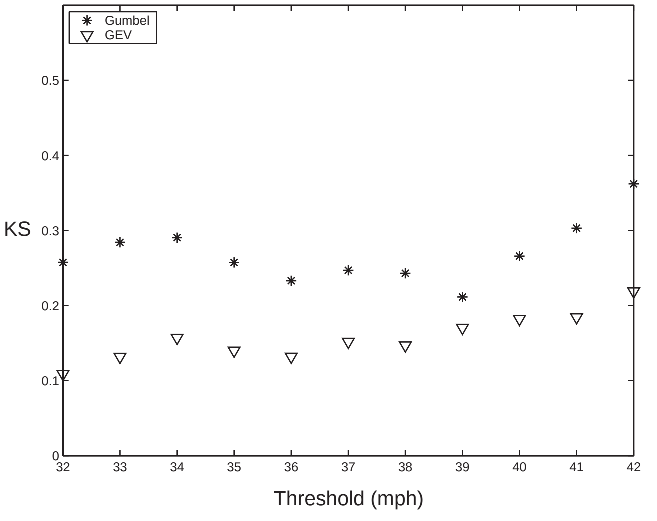

Fig. 5. SD-PWM K-S test statistic values for various distributions at Pocatello, ID.

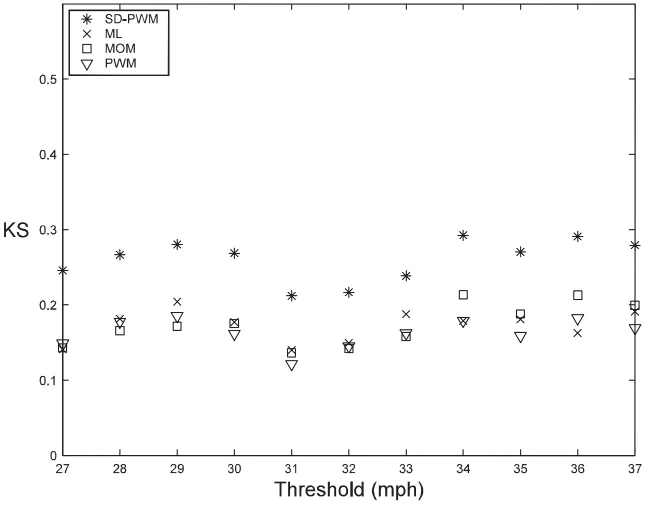

Fig. 6. Gumbel K-S statistic values for various estimation methods at Concord, NH.

Concerning Haktanir's contention on the more robust estimations given by the SD-PWM method in the presence of outliers, little supporting evidence was seen. As documented in Fig. 7, K-S test statistic values at high thresholds (associated with more extreme wind speeds) did not show a noticeable improvement when the SD-PWM estimated parameters were used versus those parameters of the PWM or MOM techniques. This conclusion is also in keeping with the observations of the sensitivity test.

Next, we turn our attention to the behavior of the estimated parameters. To illustrate typical behavior, we plot in Fig. 8 the results for the estimated GEV shape parameter a and estimated GEV scale parameter b using data from the Concord, NH station. Particularly notable here is the significant variability of the shape parameter estimate over the thresholds for estimates obtained via the SD-PWM method. Such variability is felt to be inappropriate, since it indicates excessive sensitivity upon the actual data used to obtain the estimate. Similar variability of the shape parameter was reported in [3] and was identified as a source of significant fluctuations in the estimated extreme wind speeds, especially for very long return periods. In contrast,

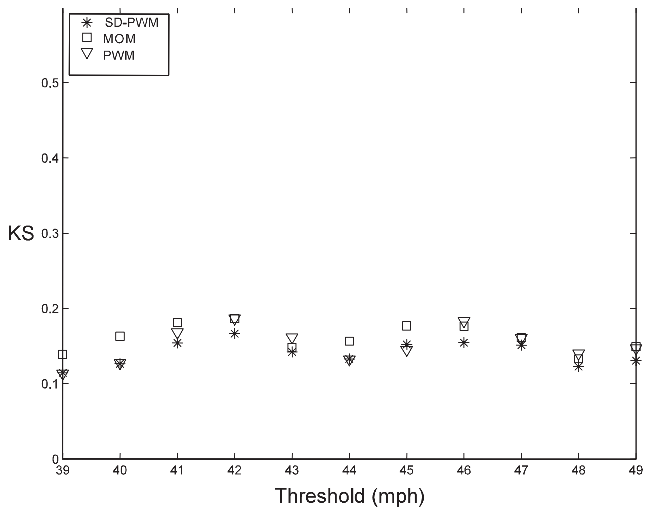

Fig. 7. GEV K-S test statistic values for various estimation methods at Albuquerque, NM.

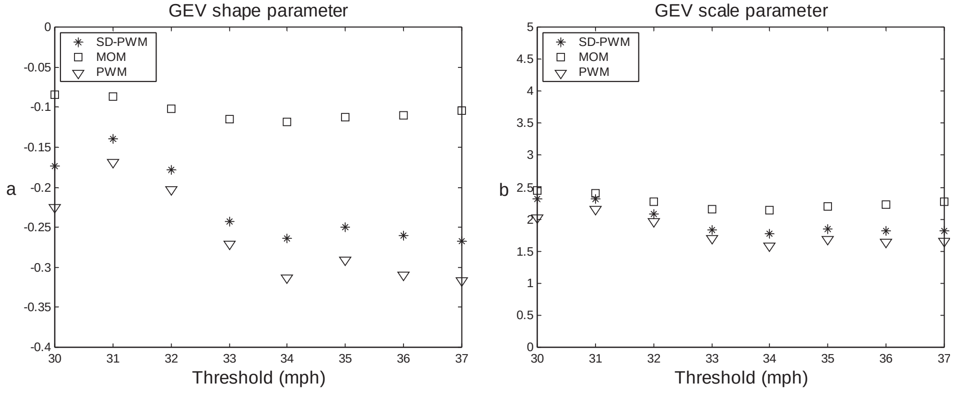

Fig. 8. Estimated GEV parameters for various estimation methods at Concord, NH.

the shape parameter estimates found using the method of moments are remarkably consistent, displaying little dependence upon threshold. Such behavior does not necessarily indicate better performance as an estimation method (recall the error trends indicated in Fig. 7); however, it is more in line with the expectations of estimation performance. Similar (though less dramatic) variability trends can be noted in the scale parameter estimates using SD-PWM and MOM. These results call into question the ability of the SD-PWM method to robustly estimate distribution parameters for small samples, although the influence of non-extreme wind speed data at the lower thresholds may also be contributing to the observed variability. Future research is planned that will seek to clarify this issue.

Two comments about the parameter estimates should be made at this point. First, the GEV estimates of the shape parameter a are negative for almost all stations and all thresholds considered in this study. The method of moments did produce some slightly positive shape parameter values (max value ≈ 0.02) for the Albuquerque data at very high thresholds, but no other data set gave similar results. Such results are consistent with the assumption that the data follows a Fisher-Tippett Type II extreme value distribution, also known as a Fréchet distribution. The proper choice of extreme value distribution is a controversial topic in wind engineering, with various authors arguing in favor of a Type I or Gumbel distribution (e.g., [6]) and also a Type III or reverse Weibull distribution (e.g., [21]). At this point, we do not believe that the results of this investigation provide definitive evidence concerning this question; the issues of mixed wind speed populations in the data and slow convergence rates need to be addressed before any claims can be made. Second, Fig. 9 indicates that the method of moments provides noticeably different parameter estimates than both PWM and SD-PWM. This sensitivity to estimation procedure was unexpected, particularly in light of the error estimates for these methods. More sensitive measures of goodness of fit may be necessary to discern the appropriateness of a given set of parameter estimates.

To examine the influence of the parameter estimations upon the resulting extreme wind speeds, two sets of wind speed estimates were produced—one associated with a 50 year mean recurrence period, the other associated with a 500 year mean recurrence period. Representative results obtained by assuming a GEV distribution for all stations are shown in Fig. 9. There are several features worth noting in these results. First, the estimates obtained using the SD-PWM method are consistently higher than those obtained using the method of moments for both the 50-year and 500-year speeds. This implies that the SD-PWM estimates are producing more conservative wind speed estimates from a design standpoint, since the higher speeds will create larger wind loads that must be resisted by a structure. Next, both the 50-year and 500-year wind speed estimates exhibit significant variability across thresholds for almost all cases, rendering the assigning of a specific value problematic. Some stability across high thresholds is observed in the 50-year SD-PWM wind speed estimates and the 500-year MOM wind speed estimates for Albuquerque, but this was not observed for most stations. Finally, the difference in actual estimated wind speed between the SD-PWM method and the method of moments is relatively small for the 50-year estimates but can be substantial for the

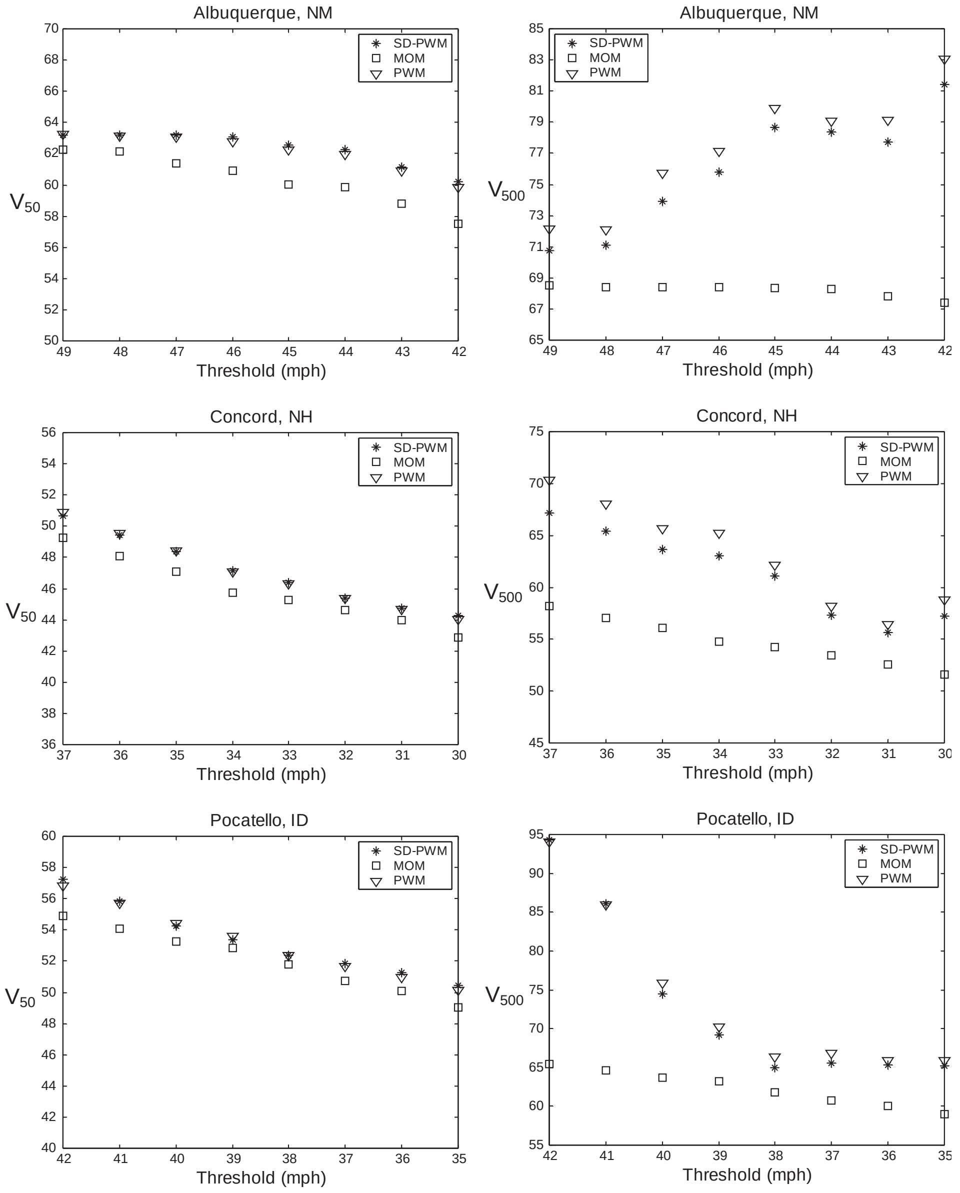

Fig. 9. GEV estimated wind speeds for various estimation methods at representative stations.

500-year estimates. This may indicate that the 50-year estimated wind speeds are ultimately insensitive to the estimation technique used. The 500-year estimates, however, do not seem to have this feature, placing doubt on the validity of their use.

6. Conclusions

Algorithms developed to implement the determination of SD-PWM parameter estimates for various distributions have been applied to the problem of extreme wind speed estimation. The estimation of extreme wind speeds has been considered using the Gumbel and GEV distributions. The analyses suggest that SD-PWM is capable of analyzing the ability of a particular distribution to describe a sample, as originally hypothesized by Haktanir. To this end, the Gumbel distribution seems inappropriate for the estimation of extreme winds unless other techniques are applied. No definitive evidence was found, however, to support the conclusion that the SD-PWM method performs more robustly in the presence of outliers. Relative to the MOM and PWM estimates, the SD-PWM estimates compare well based on overall discrepancies in the cumulative distributions. While the SD-PWM parameter estimates exhibit increased variability versus the MOM estimates, SD-PWM wind speed estimates are generally more conservative. Behavior of the SD-PWM algorithm for estimating 50-year extreme wind speeds is comparable to PWM and MOM, although longer period wind speed estimates appear to be less reliable.

While it appears that the method of self-determined probability-weighted moments has features that make it very useful for extreme wind speed estimation, more work must be done before it can be considered as a fully viable technique. The robustness of its parameter estimates needs to be firmly established, particularly for small data sets, using data that is uninfluenced by mixed wind populations. This will help address the question of wind speed estimate variability across thresholds. Also, SD-PWM algorithms need to be created for other distributions recommended for extreme wind speed estimation, such as the generalized Pareto distribution [21]. Previous attempts by the authors to create an appropriate generalized Pareto distribution have not been successful, due to problems in obtaining convergent numerical integration. However, new approaches to the integration are being considered that hopefully will resolve this problem. Finally, comparisons with other estimation techniques are necessary, especially those associated with the Gumbel distribution [6]. This could provide a useful insight into the appropriate distribution to use when performing extreme wind analyses.

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