COPING WITH RISK AND UNCERTAINTY IN URBAN WATER INFRASTRUCTURE REHABILITATION PLANNING

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ABSTRACT

The level of investment required for urban water infrastructure renewal is often very high (measured in billions of dollars) whilst decision-making has large, long lasting consequences and is fraught with risk and uncertainty. This paper deals with new methodologies for optimal design/rehabilitation of urban water systems under the condition of inherent uncertainty. The methodologies find solutions that are more "robust" than the classical deterministic optimisation designs, which assume that all model parameters are known with certainty. Several approaches to dealing with risk and uncertainty are analysed in this paper, including: the use of the standard safety margins (redundant design methodology); the development of stochastic robustness/risk evaluation models; and the consideration of the rehabilitation and reliable operation of system within a single model. Both single-objective and multiobjective optimization methodologies are verified on two case studies and compared to well known deterministic solutions from the literature. The results of both case studies clearly demonstrate that neglecting uncertainty in the design process may lead to serious under-design of water distribution networks

1. INTRODUCTION

In the recent past the UK media have been highlighting concerns about the state of the nation's water and wastewater networks and the amount of money being spent to look after them. The statement, like the following from The Guardian newspaper: '*We are sitting on a time bomb in terms of decaying infrastructure beneath our feet*' (2 September 2003) may seem sensationalist,

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but the fact that the rates of activity on infrastructure renewal (reported in the ten years up to 2002) suggesting that the average sewer has to last around 700 years (Gee, 2004), indicates how difficult is the task facing the water industry. The level of investment required is often very high, with countries such as USA estimating annual costs for investment (for the years 2000 to 2019) to average between \$11.6 and \$20.1 billions for drinking water systems and between \$13 and \$20.9 billions for wastewater systems (CBO, 2002). In the Netherlands one billion Euros is invested every year on sewer rehabilitation alone. Similarly, water industry authorities and analysts in other developed countries believe that maintaining the high-quality drinking water and wastewater services will require a substantial increase in spending over the next two decades. However, in addition to extremely high costs, decision-making for water distribution and sewer system rehabilitation has large, long lasting consequences and is fraught with risk and uncertainty. For example, uncertain information about the condition and hydraulic performance of the existing systems could lead to their future inadequate performance due to them being oversized, undersized or even obsolete.

Risk and uncertainty are, therefore, inescapable factors in potable water supply and sanitation provision. Similarly to many other areas of everyday life, water managers take and manage risks, balancing potential rewards against uncertain losses. However, the problem of risk management and decision-making under risk and uncertainty is difficult and highly complex with part of the difficulty stemming from an incomplete appreciation of the different kinds of risk and part from the inadequate consideration of the different mindsets with which people respond to risks. Therefore, no common definition of risk exists, but many refer to it as a combination of the frequency, or probability, of occurrence and the consequence of a specified hazardous event. Most definitions comprise the elements of probabilities and consequences. For the purposes of this paper risk is defined as the product of the probability of some failure (e.g. sewer collapse or pump failure) and the consequence of that failure (e.g. property damage due to flooding).

Uncertainty is also a term that has caused much controversy among scientists. For example, the famous distinction between risk and uncertainty was offered by Knight (1921) where risk refers to situations where the decision-maker can assign mathematical probabilities to the randomness which he/she is faced with. In contrast, Knight's "uncertainty" refers to situations when this randomness cannot be expressed in terms of specific mathematical probabilities. However, in this paper the definition of Korving et al. (2002) is used where two types of uncertainty exist: inherent and epistemic. Inherent (or aleatoric) uncertainty consists of random fluctuations that are intrinsic to the problem being studied (e.g. temporal/spatial water demand variability); this type of uncertainty is irreducible and is usually characterised using statistical tools, e.g. probability density functions (PDFs). Epistemic uncertainty results from a limited knowledge of fundamental phenomena, e.g. rainfall-runoff processes or in-sewer processes. The two main types of epistemic uncertainty are model uncertainty (due to lack of understanding of the problem physics) and statistical uncertainty (due to lack of sufficient data); in general, this type of uncertainty can be reduced when the relevant knowledge/data becomes available.

This paper deals with new methodologies for optimal design/rehabilitation of urban water systems under the condition of inherent uncertainty. The methodologies develop solutions that are more "robust" than the classical deterministic optimisation designs, which assume that all model parameters are known with certainty. Robustness is here defined narrowly as *the ability of the system to maintain a level of performance even if the actual parameter values are different from the assumed values.* Therefore, designs less sensitive to changes in input parameters should be more robust (Uber et al., 1991). It should be noted here that the term robustness, as defined in this paper, is closely related to the term reliability in water distribution systems. Because reliability mainly refers to the ability of the network to provide consumers with adequate and high quality supply, under normal and abnormal conditions, robustness could be viewed as some kind of "reliability". The reliability of water systems has been further classified by researchers as mechanical reliability and hydraulic reliability. Mechanical reliability usually refers to failures of system components, such as pipe breakage or pump being out of service. Hydraulic reliability, on the other hand, refers to consequences of uncertainties, such as nodal demand, pipe roughness and reservoir and tank levels. Therefore, one can argue that the term robustness is not only closely related to hydraulic reliability of water distribution system design, but could be considered equivalent to it. However, since the methodologies presented in this paper are of general type, i.e., could be applied to any pipe network system, and because hydraulic reliability for, say, urban water systems could have completely different meaning, the term robustness is more appropriate for studies presented in this paper. However, like with reliability, the question that still remains unanswered is "What is the best measure of robustness (reliability) and what level of robustness (reliability) is acceptable?"

2. OPTIMAL DESIGN AND REHABILITATION

Water distribution system design optimisation is one of the most heavily researched areas in the hydraulics profession (see Walski, 1985; Goulter, 1992; Lansey, 2000 for detailed review). Design of new water distribution networks is often viewed as a least-cost optimization problem with pipe diameters, tank characteristics and pump characteristics being the most commonly considered decision variables. However, ageing water distribution systems experience, with time, many problems such as structural failures, drop in carrying capacity and poor water quality, which, in the absence of any adequate rehabilitation programme, become more pronounced over the years. Similarly to design optimization, rehabilitation planning, whether by replacement, duplication, cleaning and relining of pipes, is often viewed as a least-cost optimization problem. Therefore, the terms "optimal design" and "optimal rehabilitation planning" of water distribution networks will be used interchangeably in this paper to describe the process of finding the best value for some goal associated with the model of a water distribution (or wastewater) system. Pipe layout, connectivity and imposed minimum (and maximum) head constraints at pipe junctions (nodes) and, if applicable, velocity constraints at each pipe are considered known.

Recently, genetic algorithms (GA) have become the preferred water system design optimisation technique for many researchers (Dandy et al., 1996; Savic and Walters, 1997). This method uses full network model simulation to evaluate how good a design solution is and, therefore, may require substantial computing time when real networks are considered.

2.1 DETERMINISTIC PROBLEM FORMULATION

The following mathematical statement of the optimal design/rehabilitation problem is presented for a general water supply network. Here, the objective is to design a water distribution network while minimizing the cost and meeting the pressure requirements at nodes. The problem is solved if the solution satisfies two requirements: a) minimum cost; b) all nodal pressures exceed a required minimum.

The mathematical formulation of the problem is as follows:

$$f(D_1, D_2, ..., D_N) = \sum_{i=1}^{N} c(D_i, L_i) \rightarrow \min \quad (1)$$

$$H_j \geq H_j^{\min}, j = 1, ..., M \quad (2)$$

$$D_i \in D(i = 1, ..., N_d) \quad (3)$$

where: $c(D_i, L_i)$ – cost of pipe i with diameter D_i (which is chosen from a discrete set of available diameters) and length L_i, H_j – head at junction j, $H_j^{\min}$ – minimum allowable head at node j, N - number of pipes in the system selected as potential rehabilitation candidates, N_d – number of decision variables, and M - number of junctions.

To calculate the state of the system for any given configuration and fixed demands [Q_d] one has to solve the non-linear system of equations. Although there are several approaches to setting up those equations, one can, for example, solve a system consisting of continuity equations at nodes:

$$\sum Q_j^{in} - \sum Q_j^{out} = Q_{d,j}, \ j = 1, ..., M \quad (4)$$

and energy conservation equations around each elementary loop:

$$\sum h_f - \sum E_p = 0 \quad (5)$$

where: Q_j^{in} - flow into a junction, Q_j^{out} - flow out of a junction, $Q_{d,j}$ –demand at junction j, h_f – pipe head-loss term which can be expressed using the Hazen-Williams or the Darcy-Weisbach formula, E_p – energy input by a pump. What makes the problem difficult to solve is that pressure heads at nodes are non-linear and implicit functions of pipe diameters and volumetric demands. For example, if the Hazen-Williams formula is used the h_f term is expressed as:

$$h_f = \omega \left(\frac{Q}{C} \right)^a \frac{L}{D^b} \tag{6}$$

where: ω – numerical conversion constant which depends on the units used, C – pipe Hazen-Williams roughness coefficient, Q – flow through the pipe, L and D – length and diameter of the pipe respectively. It should be noted that all parameters in equations (1)-(6) are considered known with certainty, i.e. they are deterministic.

2.2 PROBLEM FORMULATION CONSIDERING UNCERTAINTY

The above deterministic approach assumes that all model input variables (e.g. demand, pipe friction characteristics, etc) are precisely known from observations or forward projections, which is not true for real-life systems. While it is possible to estimate present water demands reasonably well (Obradovic and Lonsdale 1998), the situation becomes much worse when future demands need to be predicted. Pipe roughness coefficients are usually estimated or derived from model development and calibration studies, and will change significantly with time – depending on pipe material and corrosion factors. Given the above, there is a clear need to consider uncertainty in input parameters and model output results when seeking the best design of water distribution systems.

Lansey et al. (1989) were the first to develop a methodology for the least-cost design of water distribution systems considering both input and output data of a simulation model as uncertain variables. They formulated the optimization problem with a single objective, accommodating the uncertainties as constraints. The problem is then solved using the Generalised Reduced Gradient (GRG2) technique. Xu and Goulter (1999) used a probabilistic model in a network-design optimisation for the first time. The uncertainties of the model simulation results were quantified through the analytical technique known as the First-Order Reliability Method (FORM). This technique requires repetitive calculations of first-order derivatives and inversion of matrices and, consequently, requires substantial computational effort - even for small networks. Furthermore, prior studies have shown that the GRG2 method may have a tendency to converge to a local minimum. The application of this method also requires that the decision variables (i.e. pipe diameters) be modelled as continuous variables, which does not reflect the reality of pipes being manufactured in a set of discrete-sized diameters. In order to overcome the above limitations, Kapelan et al. (2003) and Babayan et al. (2005a) developed two new robust design methodologies, which make use of GAs. The former technique uses a sampling-based methodology to quantify the relevant uncertainties while the latter uses an analytical methodology for the same purpose

There are several possible ways of approaching the robustness issues related to uncertainty in model input parameters. The first one is to design redundancy into the system by adding 'safety margins' to parameters which are considered uncertain and then to solve the resulting deterministic optimisation problem. This approach works well when the number of uncertain parameters is fairly small (e.g. less then 10), otherwise it can lead to designs which are too expensive. An alternative way of dealing with uncertainty is to incorporate it directly into the problem formulation. This requires using one of the uncertainty quantification methodologies which can, in general, be divided into two groups: (1) methodologies that use stochastic simulation (sampling) and (2) methodologies that use analytical techniques (Haldar and Mahadevan, 2000). Sampling-based techniques, even though more universal, straightforward and accurate, are more time consuming, being typically several orders of magnitude slower than analytical approaches, even when using advanced sampling methods (Haldar and Mahadevan, 2000, Zhao and Ono, 2001). However, for optimal design, it is possible to exploit the stochastic nature of the GA by incorporating sampling techniques within the optimization process (Kapelan et al. 2005a), which will also be demonstrated in this paper.

The formulation of a robust design problem for both of the above uncertainty quantification techniques differs from the formulation of the deterministic problem (1)-(6) in the way the constraint of Eq. (2) is expressed. The minimum pressure constraint now becomes the target robustness constraint:

$$P\left(H_j \geq H_j^{\min}, j = 1,...,M\right) \geq P_{\min} \tag{7}$$

where: P – robustness, i.e. probability that nodal heads are above the minimums required and P_{min} – minimum required (i.e. target) level of robustness. The form of this constraint imposed on the network as a whole is usually termed as a chance-constrained formulation and is traditionally used in construction engineering. However, it is not the only possible formulation. In construction engineering failure of a single element is often irreversible and can lead to the collapse of the whole structure. In water network design a more flexible approach can be more appropriate in some cases. For example, condition (7) can be replaced with the set of constraints imposed on separate nodes allowing the analyst to adjust the desirable level of robustness in accordance with each node's importance. Although in the present paper only condition (7) is used for robustness optimisation, the proposed methodology can handle other chance-constrained formulations.

2.2.1 Redundant Design Approach

Probably the simplest approach to solving the network design problem under uncertainty is to design redundancy into the system by adding safety margins to uncertain parameters. Here, the system is designed assuming that demands at all nodes and all pipe roughness values are larger than expected:

$$\tilde{Q}_{d,i} = (1+\alpha_i)Q_{d,i},\ i = 1,...,M \tag{8}$$

$$\tilde{C}_j = (1+\beta_j)C_j,\ j = 1,...,N \tag{9}$$

and then solving the problem (1, 3-7) with stochastic constraint (7) replaced by the deterministic one (2). In the above equations C_j denotes the pipe roughness coefficient and α_i and β_j are the coefficients which determine the degree of redundancy in the resulting design ($\alpha_i \geq 0$; $\beta_j \leq 0$ for Hazen-Williams pipe roughness coefficients, which decrease with increasing roughness).

The main question which faces the engineer is how to choose the magnitude of the 'redundancy coefficients' – too small and the system will not be robust enough, too large and the system will become over-redundant with unnecessarily high costs. For simplicity, consider the case of uncertain demands only. It is well-known that nodal pressure dependency on demands is non-linear and monotonic. Therefore, if pressure constraints $H_j \geq H_j^{\min}$ are satisfied for some value of demand Q_d at a particular node, then they will also be satisfied for any value of the demand less than Q_d (assuming demands at all other nodes are fixed). As a consequence, it is clear that for only *one* node with uncertain demand, the 'ideal' value of the redundancy coefficient can be chosen for that node and the system designed with the desired level of robustness. For example, assume that the aforementioned demand node has a normally distributed demand Q_d with standard deviation of 10% of the mean value Q_m and that the target robustness level P_{min}=95%. The optimal design problem can then be solved as a deterministic optimisation problem where: (1) the demand at the node is assigned a value of $\tilde{Q}_d = (1+0.165)Q_m$ (assuming normal distribution) and (2) the set of deterministic pressure constraints is used. Note that $(1+\alpha)=1.165$ corresponds to $P(Q<\tilde{Q}) = 95\%$ which, in turn, leads to: $P\left(H_i \geq H_{i,\min}, i = 1..N_n\right) \geq 95\%$ (because nodal pressures depend on demand Q_d monotonically).

For multiple uncertain variables, however, redundancy coefficients cannot be determined in a straightforward manner. Firstly, there is no unique set of α_i for which $P(Q_{d,i} < \alpha_i Q_{i,m}) = P_{\min}$. In fact, all possible vectors $[\boldsymbol{\alpha}]$ which satisfy this condition constitute an N_n dimensional hypersurface. Secondly, fluctuation of demands at different nodes has a different effect on the system robustness depending on the system design (which has yet to be determined). Babayan et al. (2005b) developed a possible way of finding the optimal combination of α_i is to use the following iterative procedure. Let $S(\boldsymbol{\alpha})$ denote a deterministic solution of problem (1)-(6) corresponding to a fixed vector of $\boldsymbol{\alpha}$ and let $P\left(S(\boldsymbol{\alpha})\right)$ denote the robustness of that solution (which can be computed using some sampling technique like Monte Carlo Simulation - MCS). The following algorithm can then be used to design the system with a target level of robustness P_{min}:

1. First, find interval $\left[\boldsymbol{\alpha}_L^0, \boldsymbol{\alpha}_R^0\right]$ within which optimal vector $\boldsymbol{\alpha}$ is located. Assign $\boldsymbol{\alpha}_L^0 = 0$ and some initial value to $\boldsymbol{\alpha}_R^0$. Specify the robustness accuracy ξ and the minimum size of the search interval ε.

2. Find $S(\boldsymbol{\alpha}_L^0)$ and calculate $P\left(S(\boldsymbol{\alpha}_L^0)\right)$. If $P\left(S(\boldsymbol{\alpha}_L^0)\right) \ge P_{\min}$ then the problem is solved.
3. Find $S(\boldsymbol{\alpha}_R^0)$ and $P\left(S(\boldsymbol{\alpha}_R^0)\right)$. If $P\left(S(\boldsymbol{\alpha}_R^0)\right) \le P_{\min}$ then $\boldsymbol{\alpha}_L^0 = \boldsymbol{\alpha}_R^0$, $\boldsymbol{\alpha}_R^0 = \mu\boldsymbol{\alpha}_R^0$ where $\mu > 1$. Repeat this process until $P\left(S(\boldsymbol{\alpha}_L^0)\right) \le P_{\min} \le P\left(S(\boldsymbol{\alpha}_R^0)\right)$.
4. At the i-th step of the optimisation process assign $\boldsymbol{\alpha}_M^i = (\boldsymbol{\alpha}_L^{i-1} + \boldsymbol{\alpha}_R^{i-1})\gamma_i$ (where $0 < \gamma_i < 1$) and determine $S(\boldsymbol{\alpha}_M)$ and $P\left(S(\boldsymbol{\alpha}_M^i)\right)$. If $P\left(S(\boldsymbol{\alpha}_M^i)\right) < P_{\min}$ then $\boldsymbol{\alpha}_L^i = \boldsymbol{\alpha}_M^i$, $\boldsymbol{\alpha}_R^i = \boldsymbol{\alpha}_R^{i-1}$, otherwise $\boldsymbol{\alpha}_R^i = \boldsymbol{\alpha}_M^i$, $\boldsymbol{\alpha}_L^i = \boldsymbol{\alpha}_L^{i-1}$.
5. Repeat step 4 until $P_{\min} < P(\boldsymbol{\alpha}_M^i) < P_{\min} + \xi$ or $\left\|\boldsymbol{\alpha}_R^i - \boldsymbol{\alpha}_L^i\right\| < \varepsilon$.
6. The final solution is $S(\boldsymbol{\alpha}_R^i)$.

2.2.2 Uncertainty Quantification Approaches

The main drawback of the algorithm described above is the need to make assumptions about how uncertainty in input parameters affects response variables before the start of the optimisation process. Another way of addressing this problem is to take into account the influence of uncertainty for every potential solution considered during the optimisation process. This could be done following Babayan et al. (2005a), by, first, replacing the stochastic constraint (7) with the following set of deterministic constraints:

$$\xi_{H_j} \ge H_j^{\min} + \alpha\sigma_{H_j},\; i = 1,\ldots,M \tag{10}$$

where: ξ_{H_j} and σ_{H_j} – mean value and standard deviation of the head at node j (which depends on vectors of demands and pipe roughnesses in the whole network), α – parameter which determines the level of system robustness. Because of the implicit relationship between demands, pipe roughnesses and heads it is impossible to calculate the mean and standard deviation in (10) directly. Straightforward numerical evaluation requires an unreasonable computational effort. Therefore a simplified method of evaluating σ_{H_j} is used, based on the assumptions outlined here below.

First, assume the validity of the superposition principle:

$$\begin{aligned} &H_i\left(\xi_{X1}+t_1,\xi_{X2}+t_2,\ldots,\xi_{XK}+t_K\right)-\left(\xi_{X1},\xi_{X2},\ldots,\xi_{XK}\right)\approx \\ &\sum_{j=1}^{K}\Big(H_i\left(\xi_{X1},\ldots,\xi_{Xj}+t_j,\ldots,\xi_{XK}\right)-H_i\left(\xi_{X1},\xi_{X2},\ldots,\xi_{XK}\right)\Big) \end{aligned} \tag{11}$$

where: K – number of uncertain variables X and $\boldsymbol{\xi}_X$ – vector of their mean values. Note that the probability density function (PDF) of an uncertain variable is non-zero within a limited region (e.g. uniform distribution) or decreases exponentially with distance from the mean value (e.g.

normal distribution). Hence, all that is needed is that a superposition principle is satisfied in some region around $\boldsymbol{\xi}_X$ – about two standard deviations being enough in most cases. Taking into account (11) and the assumption of independent nodal demands and pipe roughnesses, the following approximation for nodal head mean and standard deviation is obtained:

$$\xi_{H_i} \approx H_i(\xi) + \sum_{j=1}^{K} \tau_{ij} \tag{12}$$

$$\sigma_{H_i}^2 \approx \sum_{j=1}^{K} \int_{-\infty}^{\infty} \left(H_i(X_j) - H_i(\xi) - \tau_{ij} \right)^2 \eta_j(X_j) dX_j \tag{13}$$

$$\tau_{ij} = \int_{-\infty}^{\infty} \left(H_i(X_j) - H_i(\xi) \right) \eta_j(X_j) dX_j \tag{14}$$

It is clear from (14) that τ_{ij} in (12) and (13) accounts for non-symmetry of heads around the mean values of uncertain variables. In the case of dependent uncertain variables, the expected value can still be computed using equation (12).

Integrals in equations (13) and (14) are one-dimensional integrals and can be calculated using conventional numerical formulae. To estimate the standard deviation for all N_n nodes in a network with K uncertain parameters using equations (13) and (14) one needs to perform $(n\text{-}1)K+1$ model runs, where n is the (odd) number of points in the formula for numerical integration (for odd n the point in the centre, corresponding to ξ_X, is common for all dimensions and the value of the function at this point need be computed only once). For $n = 3$ the computational complexity of the Integration method is approximately equal to the complexity of the FOSM method. However, the former provides a much better approximation of the response function. Equation (13) allows the estimation of the relative contribution of each model input variable's uncertainty to each nodal head's uncertainty. This information can be used to build up the list of 'significant' input variables and model the rest of the network as certain, leading to significant computational time savings. To summarize, the following algorithm is proposed to solve the least cost design problem under uncertainty using GA:

1. *Identify the sets of critical nodes and significant input variables*. Take some initial configuration of the network (e.g. the existing network configuration). Then, compute the mean and standard deviation for each nodal head using equations (12)-(14). Add the node to the set of "critical nodes" (Ω) if constraint (10) is violated for that node. In addition to this, identify those uncertain input variables whose relative contribution to the standard deviation of head at critical nodes is more then some prescribed level (e.g. 5%) and add them to the set of "significant input variables" (Λ). Note that this step requires $K \times n$ state estimate calculations (i.e. hydraulic solver runs), where K is the number of uncertain input variables and n is the number of points in the quadrature formula used.

2. *Run GA to find the optimal robust design.* Use the GA to obtain the solution of problem (1) with the following penalty term added:

$$Penalty = pr \cdot \sum_{i=1}^{Nn} \max\left(0, H_{i,\min} + \alpha\sigma_{Hi} - \xi_{Hi}\right) \qquad (15)$$

To compute the penalty function value for the current system configuration [D], ξ_{Hi} and σ_{Hi} need to be computed first. This can be done using equations (12)-(14). Note that the total number of state estimate calculations (i.e. hydraulic solver runs) necessary to obtain the value of fitness function is $|\Lambda|(n-1)+1$ where Λ is the number of significant variables and n is the number of quadrature formula points. Finally, note that as the GA run progresses, the sets of critical nodes and significant variables may have to be updated periodically. This can be done using the procedure described in step 1. Note also that the first step of the above algorithm enables identification of a list of significant variables. This information could also be used in the Redundant Design method, for choosing the initial vector $\boldsymbol{\alpha}_R$.

A different approach, using a sampling technique, has been developed by Kapelan et al. (2003, 2004) to quantify uncertainties. Similarly to Babayan et al. (2005a) the deterministic optimisation problem (1)-(6) is changed by introducing the robustness constraint (7) instead of the minimum head constraint (2). The new problem is then solved using a two-level procedure that can be visualised as where a sampling level procedure is located within the optimisation level procedure. The latter is responsible for finding the optimal solution, i.e. optimal decision variable values while the former is responsible for the evaluation of the potential solution candidates by propagating the uncertainty in model inputs (demands, pipe roughnesses) to relevant model outputs (nodal heads). A conventional approach to solving the optimisation problem would be to use some standard optimisation method, e.g. standard GA in the optimisation level procedure and a standard Monte-Carlo sampling technique in the sampling level procedure. For this conventional approach to work, the number of the Monte Carlo samples must be large (usually several thousands or more) which is, obviously, extremely computationally demanding. To overcome this problem, a new methodology is developed Kapelan et al. (2003, 2004). The objective is to reduce the computational effort as much as possible while preserving the solution evaluation accuracy. This is achieved by using an improved sampling technique, the so called Latin Hypercube (LH) sampling technique, to increase the efficiency of the MC simulation (LH is used here with the ability to determine design robustness using fewer samples than the standard MC simulation) and a modified GA (where a solution's robustness evaluation is spread over multiple generations, i.e. over the effective life of the corresponding chromosome).

The *sampling level procedure* involves the following steps: each time the value of the robustness P in (7) is calculated, a total of N_s sets (i.e. samples) of all random uncertain input variables (nodal demands, pipe roughnesses) are generated. The corresponding uncertain output variables (nodal heads) are then obtained by running the deterministic hydraulic solver for each of the samples. Finally, the robustness P in (7) is estimated as the fraction (i.e. percentage) of the total number of samples N_s for which the minimum head requirement condition is met simultaneously at all network nodes. The typical number of samples N_s used in the methodology is very small, e.g. 5-50. Such a small number of samples is not enough to accurately estimate the value of robustness P. In the LH sampling technique, the values of uncertain input variables are generated in a random yet constrained way. First, the range of each uncertain input variable is divided into N_s non-overlapping intervals on the basis of equal probability. After that, a single random value is selected from each interval. This process is repeated for all uncertain variables. Once that is done, the N_s values obtained for the first stochastic variable are paired in a random manner with N_s values obtained for the second stochastic variable and so on. The drawback associated with the use of the LH sampling technique when compared to the MC sampling technique is the increased computational effort required to generate the same number of samples. However, for the problem considered, the time required for the sample generation constitutes a negligible fraction of the total time required for the fitness evaluation, i.e. hydraulic solver runs.

The *optimization level procedure* is using a modified standard GA capable of performing an efficient search under uncertainty. The GA, the robust chance constrained GA (rccGA) developed by Kapelan et al. (2003) is based on (but not identical to) the robust GA developed by Chan-Hilton and Culver (2000) and the chance constrained GA developed by Loughlin and Ranjithan (1999). The main idea behind the rccGA is to breed solutions capable of surviving over multiple generations which enables calculation of each chromosome's objective as the average of past objective values over that chromosome's age. As a consequence, even if a small number of samples (e.g. 5-20) is used for each evaluation, the chromosomes are effectively evaluated using a larger number of samples (e.g. 100-400 if the chromosome survived for 20 generations). The rccGA search procedure is as follows:

1. Create the initial GA population at random and initialise the age of each chromosome to zero. Evaluate the fitness of each chromosome by calculating the fitness vale. When calculating the robustness value P, use a small number of LH samples (e.g. 5-20 samples).
2. Sort the chromosomes in term of their fitness values. Identify the best chromosome as the one with the lowest value of the cost plus the penalty term. Keep a separate record of the best chromosome identified so far.
3. Create the next generation population as follows:
    - 3.1. Directly copy better half of the (sorted) current generation population. Increase the age of these chromosomes by one.
    - 3.2. Fill in the rest of the population by using the standard GA selection, crossover and mutation operators applied to the existing generation population. Set the age of these chromosomes equal to zero.
    - 3.3. Calculate the fitness values for all chromosomes in the new population. When doing so, use equation (?) with the robustness value P equal to the averaged robustness value over each chromosome's age.
4. Sort the chromosomes in terms of their fitness values and identify the fittest chromosome. Update the best chromosome record if the newly identified best chromosome: (a) has a better fitness value than the best chromosome identified so far and (b) is of age equal to or higher

than the minimum required age (i.e. has survived for at least the minimum required number of generations).

5. Repeat steps 3-4 until some GA convergence criterion is met. Once the GA has converged, re-evaluate the best solution found using a large number of Monte Carlo samples (100,000 in the case study shown here).

The rccGA presented here is effectively exploiting the fact that the GA search process is of a stochastic nature with a population of solutions evaluated at each generation. It is a well known fact that GA determines (near) optimal solutions by combining highly fit building blocks of population chromosomes (Goldberg 1989). As the search progresses, the population is likely to have more and more chromosomes containing highly fit building blocks. As a consequence, a relatively large number of indirect evaluations of these building blocks are likely to be found in the population even if a small number of samples are used to evaluate each chromosome's fitness.

2.2.3 Fuzzy Logic Approach

Finally, the last approach to the robust design of water distribution systems to be considered in this paper involves not just the water network design phase, but also the operation phase. This approach differs substantially from the above two approaches (e.g. the constraint (7) is treated differently) and will be described further in this section. Farmani et al. (2005) developed a new formulation that works at two levels. In the upper-level optimization, the problem is set as a deterministic optimization problem for infrastructure design and operational scheduling of water system based on projected (deterministic) demands. At this level design variables are the pipe rehabilitation decisions, tank sizing, tank sitting and pump operation schedules. The tank volume values are treated as independent design variables whose values are assumed to be independent of any uncertainty or noise and they determine the structure of the system. The maximum and minimum heads identified for each tank over 24 hour operation time are interpreted as the maximum and minimum operating levels for the corresponding tanks. All tanks should empty and fill over their operational ranges during the specified average demand day, leaving the specified emergency volumes untouched. The mismatch between the tank heads calculated as above and those generated randomly is used as the tank operating level difference (TLD) constraint. The performance of each candidate design solution is evaluated through simulation of the network flows. An extended period hydraulic network solver is used to determine the heads at all the system nodes for all design conditions, and the accumulated sum of the nodal pressure shortfalls (NPS) is used as the head deficit constraint. For a configuration, generated at the upper level, to be considered at the lower level optimization it is necessary to satisfy the above constraints (TLD and NPS) fully. The cost of the solution, including the capital costs of pipes, pumps and tanks as well as the present value of the energy consumed during a specified period, is considered as one of the main objectives in the overall optimization process.

In the lower-level optimization process, for each fixed configuration that satisfies the hydraulic requirements at the upper level, the demand at the critical node is considered as a random variable and the probabilistic characteristic of the system is analyzed using the probability theory. Monte Carlo simulation technique is one way of generating different scenarios of nodal demands. However, it requires a large number of trials to ensure results of reasonable accuracy. The random nodal demands generated here use uniform distribution with 100 sample points with the mean being equal to the deterministic demand at the node and the standard deviation equal to 10% of the mean value. At the lower level, design variables are pump operation schedules whose values are dependent on the optimal values of the design variables and realization of uncertain parameters and they adjust level of service in response to disruption in the system. A solution to such an optimization problem is considered robust if it remains feasible for any uncertain scenario. However, it is unlikely that a solution will remain feasible and optimal for every scenario. Hydraulic failure of water distribution networks is somewhat more ambiguous in practice and a crisp modelling of this type may not adequately represent the engineering realities of the design and operation of a water distribution network. Therefore, at the lower level, fuzzy rules are used to quantify the degree of feasibility.

Fuzzy rules refer to a set of IF-THEN rules with linguistic values and variables. The values are specified as fuzzy concepts which are defined by membership functions. A fuzzy set is not a set with crisp rules of membership (clearly defined boundary) but it can contain elements with only a partial degree of membership. Fuzzy sets are functions that map a value that might be a member of the set to a number between zero and one indicating its actual degree of membership. A degree of zero means that the value is not in the set and a degree of one means that value is completely representative of the set. The most common membership functions are; triangular, trapezoidal, gauss-like and sigmoid. In the paper by Farmani et al. (2005) a sigmoid membership function (S-curve) has been used that corresponds to the increasing and decreasing nonlinear surfaces. The decline S-curve is used in quantification of the degree of feasibility for TLD, NPS, the number of nodes with deficiency and the initial time that the first deficiency occurs over the design period. To evaluate the overall feasibility membership function value for each configuration, the mean aggregation approach is used which calculates the mean value of the feasibility membership function of all constraints.

2.3 RISK UNCERTAINTY AND ROBUSTNESS

Risk is a commonly used term, which as a concept, is generally well understood. However, it is a term which has been widely defined within many industries, such that there can be some misunderstanding of the terms clear usage within a specified context. Also the perception of risk and the 'viewpoint' from which risk is being regarded is important. Kane (1992), for example, states: '*Risk is a frequently used term. However, it is also one of the most widely defined; what is risk to some is the perception of risk to others*.' In this paper a risk is the likelihood that a hazard (anything with the potential to cause harm) will cause a specified harm to someone or something. Risk can then be quantified by the expression that takes into account that risk is a function of severity (of the possible harm that can result from the considered hazard) and probability of occurrence (i.e., frequency and duration, probability of occurrence of hazardous event, possibility of avoiding or limiting the harm).

If risk is used instead of robustness, the problem formulation is basically the same as in the aforementioned robustness-based approach. Kapelan et al. (2005) showed that the only difference is that instead of the system robustness constraint in equation (7), total system risk (of water not being delivered at acceptable pressure) is lower than some acceptable (i.e. maximum allowed) risk level, $R_{\max}$. Therefore, the constraint in equation (7) is replaced with the following:

$$\sum_{j=1}^{M} P\left(H_j \le H_j^{\min}\right) \times CF_j \le R_{\max} \tag{16}$$

where: CF_j - consequence of pressure failure at j-th node, surrogated here as a fraction of the total mean system demand:

$$CF_j = \frac{\overline{Q}_{d,j}}{\sum_{j=1}^{M} \overline{Q}_{d,j}} \tag{17}$$

where: $\overline{Q}_{d,j}$ - the mean demand at node j. Consequence of failure can be expressed in several other ways, e.g. as a number of customers affected by the pressure failure or the financial loss experienced, etc.

2.4 MULTIOBJECTIVE OPTIMIZATION

Many real-world decision making problems need to achieve several objectives: minimise risks, maximise reliability, maximise robustness, minimise deviations from desired levels, minimise cost, etc. The main goal of single-objective optimisation is to find the "best" solution, which corresponds to the minimum or maximum value of a single objective function that lumps all different objectives into one. This type of optimisation is useful as a tool which should provide decision makers with insights into the nature of the problem, but usually cannot provide a set of alternative solutions that trade different objectives against each other. On the contrary, in multiobjective optimisation with conflicting objectives, there is no single optimal solution. The interaction among different objectives gives rise to a set of compromised solutions, largely known as the trade-off, non-dominated, non-inferior or Pareto-optimal solutions (Savic, 2002). The consideration of many objectives in the design or planning stages provides three major improvements to the procedure that directly supports the decision-making process:

(1) A wider range of alternatives is usually identified when a multiobjective methodology is employed.

(2) Consideration of multiple objectives promotes more appropriate roles for the participants in the planning and decision-making processes, i.e. "*analyst*" or "*modeller*"– who generates alternative solutions, and "*decision maker*" - who uses the solutions generated by the analyst to make informed decisions.

(3) Models of a problem will be more realistic if many objectives are considered.

Single-objective optimisation identifies a single optimal alternative, however, it can be used within the multiobjective framework. This does not involve aggregating different objectives into a single objective function, but, for example, entails setting all except one of them as constraints in the optimisation process. Those objectives expressed as constraints are assigned different levels of attainment of their respective objective functions (e.g. minimum reliability levels) and several runs are performed to obtain solutions corresponding to different satisfaction of constraints. However, most design and planning problems are characterised by a large and often infinite number of alternatives. Thus, multiobjective methodologies are more likely to identify a wider range of these alternatives since they do not need to pre-specify for which level of one objective a single optimal solution is obtained for another. A very simplified view of the decision-making process is that it involves two types of actors: analysts (modellers) and decision makers. This is a crude simplification of the process since many stakeholders and actors may be involved, but simple enough to demonstrate shortcomings of assuming that in general one person can assume both (or many more) roles. Analysts are technically capable people who provide information about a problem to decision makers who decide which course of action to take. Modelling and optimisation techniques are tools which analysts may use to develop useful information for the decision makers. However, single-objective models require that all design objectives must be measurable in terms of a single fitness function. This in turn requires some a priori ordering of different objectives (i.e., a weighting scheme) to allow easy integration of them into a single function of same units. Thus, single-objective approaches place the burden of decision making squarely on the shoulders of the analyst. For example, it is the analyst who must decide the cost equivalent of a specific risk of failure. Even if the decision makers are technically capable and willing to provide some a priori preference information, the decision making role is taken away from them. By providing a trade-off curve between different objectives and alternative solutions corresponding to the points on this curve, multiobjective approaches allow for the responsibility of assigning relative values of the objectives to remain where it belongs: with the decision maker!

3. CASE STUDIES

Although the case studies presented in this paper deal with optimal rehabilitation of water distribution networks, most of the methodologies presented are general enough to be applied to wastewater networks too. The basic assumption of the paper is that a number of inputs to hydraulic models of a system being studied are uncertain, which necessitates modelling of risk as an uncertain category (e.g. uncertain demands lead to uncertain achieved pressure, which in turn, leads to uncertain water delivery to customers). The design methodologies presented here are tested on two commonly used benchmark problems (Farmani et al., 2003), the New York Tunnels problem (Schaake and Lai, 1969) and the Anytown problem (Walski et al., 1987). The New York Tunnels strengthening problem (see Figure 1) is considered first. The original (deterministic) problem formulation was to minimise the cost of network rehabilitation while satisfying the minimum head requirements at all nodes. The network consists of 21 pipes, each of which can be left unchanged or duplicated with a new one having one of 15 available diameters (in the range 91-518 cm). There are, therefore, a total of 16 options for each pipe in the network (including the "do nothing" option) and the total number of possible solutions, i.e. network configurations is $1621=1.9\times10^{24}$ which makes using a full enumeration approach for solving this problem practically impossible. To demonstrate that the methodologies presented here are capable of handling uncertainties in different types of input parameters and for different associated PDFs, two cases are considered. In the first case, nodal demands are assumed to be uncertain variables following Gaussian PDFs with means equal to deterministic demand values and standard deviations equal to 10% of the corresponding mean values. In the second case, uncertainty in pipe roughness coefficients is considered in addition to the uncertainty in nodal demands. The friction coefficients in all existing pipes are assumed to be uniformly distributed stochastic variables on the interval ±10% of deterministic value. The robustness of all the solutions obtained was always checked using MCS with 100,000 samples.

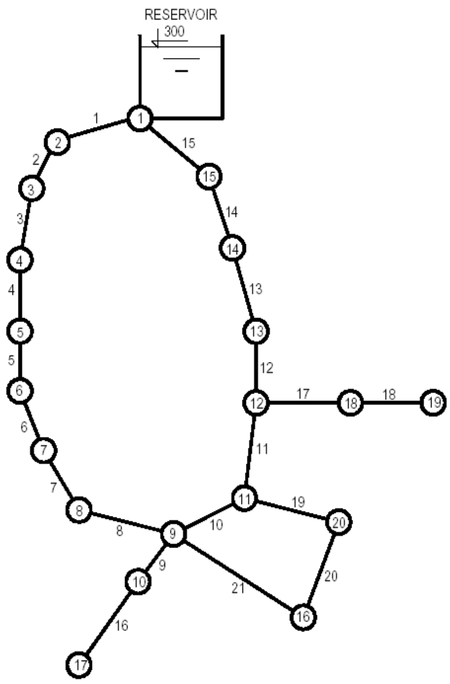

Figure 1 - Layout for the New York Tunnels system

The Anytown water distribution system (see Figure 2) was set up by Walski et al. (1987) as a realistic benchmark problem on which to compare and test network optimization methodologies. The network has features and problems typical of those found in many real systems. The system has developed around an old central part of Anytown. There is a surrounding residential area, and a planned industrial expansion to the north. Additional links (dashed lines) are generally treated as optional, except one. Each node should at least be connected to two pipes for redundancy. The design optimisation problem is to partially renew and expand the water distribution network, by adding new pipes or pipes in parallel (duplication of existing links), by optional cleaning and lining of existing old pipes, by adding new tanks (selecting number, size and location) and by adding new pumps, if necessary, alongside the 3 existing ones. The target is to minimise construction and energy costs, while meeting increasing water demands, pressure requirements, and urban expansion. Node demands, daily water use pattern, topographical layout, existing diameters and roughness coefficients are all given as data. There are two existing elevated tanks, at nodes 65 and 165 respectively. Tanks can be added at all nodes, except the nodes that are directly connected to existing tanks, and their costs given as a function of tank capacity.

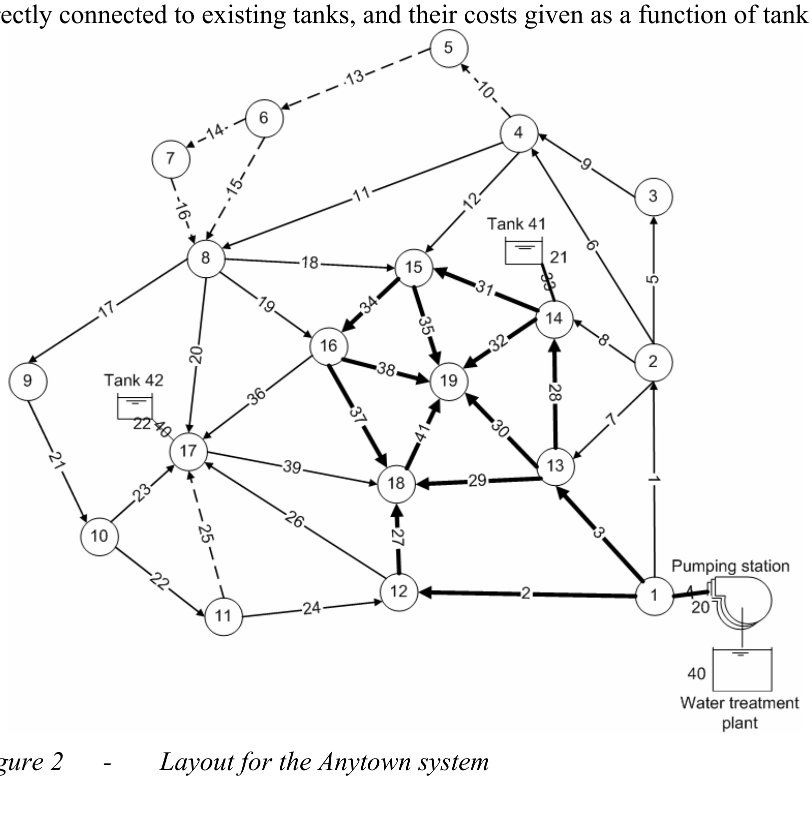

Figure 2 - Layout for the Anytown system

The Anytown problem was originally tackled by participants at the Battle of the Network Models workshop, and has since been examined by Murphy et al. (1994) and Walters et al. (1999). Farmani et al. (2005) also investigated the multi-objective optimization of the Anytown water system under 5 loading conditions. A minimum pressure of at least 40 psi (28.2m) must be provided at all nodes at average day flow as well as instantaneous peak flow which is 1.8 times the average day flow. The system is also subject to fire flow under which it must supply water at a minimum pressure of at least 20 psi (14.1m). The demand at the critical node is considered as a random variable and the probabilistic characteristic of the system is analyzed using the probability theory. The Monte Carlo simulation technique is one way of generating different scenarios of nodal demands. However, it requires a large number of trials to ensure results of reasonable accuracy. The random nodal demands generated here use uniform distribution with 100 sample points with the mean being equal to the deterministic demand at the node and the standard deviation equal to 10% of the mean value.

4. RESULTS

4.1 NEW YORK TUNNELS PROBLEM

In the case of the Integration method the described optimisation problem is solved for the iteratively determined values α of 1.5, 2.0, and 2.8. During the first step of the algorithm the set of significant uncertain variables, consisting of demands at 7 nodes (9, 11, 16, 17, 18, 19 and 20) and pipe roughness coefficients in two pipes (13 and 14) was identified. Therefore, each fitness function evaluation required 15 and 19 runs of the hydraulic network solver for the cases of uncertainty in demands only and in demands and pipe roughnesses respectively.

In the Redundant Design method the following two ways of choosing the initial vectors $\boldsymbol{\alpha}_R$ and $\boldsymbol{\beta}_R$ were considered: 1) worst case scenario: $\boldsymbol{\alpha}_{R,i} = 0.1$ ($i = 1..N_n$), $\boldsymbol{\beta}_{R,j} = -0.05$ for the existing pipes and $\boldsymbol{\beta}_{R,j} = 0$ for the new pipes (note: all uncertain parameters are changed in an unfavourable way simultaneously); 2) $\boldsymbol{\alpha}_{R,i} = 0.1$ ($i = 1..N_n$), $\boldsymbol{\beta}_{R,j} = -0.05$ if the corresponding uncertain parameter is on the list of significant uncertain parameters (identified in the first step of the Integration approach). Parameter γ_i was assigned a fixed value of ½ during the whole iteration process, and the value of ε was set equal to 10^{-4}. After every iteration, the robustness of the resulting design was estimated using Monte Carlo Simulations with 10,000 samples. It took from 3 to 9 iterations for the algorithm to converge with the typical number of iterations around 5 (and therefore it required about one third of the time needed to get a solution with the Integration approach).

Before solving the problem using the Sampling methodology, a limited number of trial rccGA runs were performed where the number of LH samples and the minimum required chromosome age were varied. The following values were identified as suitable for the optimisation problems solved here: 5, 10 and 20 LH samples for 90%, 95% and 99% target robustness, respectively. The best value for the minimum required chromosome age was found to be 20 (for all cases). Therefore, each fitness function evaluation (6) required 5-20 hydraulic solver calls regardless of the uncertainty case considered (demands only or both demands and pipe roughnesses).

4.1.1 Uncertain Demands Only

The number of uncertain (demand) parameters for this problem is 19. The best results obtained are shown in Figure 3. The curve "Full Sampling" represents the solutions obtained by solving the optimisation problem using a standard GA with 1000 LH samples for each fitness evaluation. It can be seen that: (1) both the Integration method and the Sampling method identified solutions which are almost identical to the ones obtained with the Full Sampling approach (which requires 50 times more runs of the hydraulic solver); (2) the Redundant Design method was significantly less successful in identifying the optimal solutions. Also, the results for the worst case scenario are worse than for the case where safety margins were applied to significant nodes only; (3) in the latter case the results are better for the highly robust solutions (99%) – this is because system robustness is affected almost solely by demand fluctuation at a few significant nodes (to which the safety margins were applied); (3) As expected, the cost of solutions rises exponentially with robustness level within the analysed interval (90% to 99%).

As expected, the optimal deterministic solution identified by Murphy et al. (1993) is significantly less expensive then all stochastic (i.e. robust) solutions, \$38.8M. At the same time, the cost of the most expensive robust network with robustness level 99% is still less than the cost of the most expensive of the "optimal" deterministic configurations (Quindry et al., 1981), \$63.6; Robustness of both deterministic solutions is low, 33.4% (Qundry et al., 1981) and 34.6% (Murphy et al., 1993) despite the fact that the solution by Quindry et al is more than 50% more expensive than the one found in Murphy et al. . This is the consequence of deterministic optimisation which left little or no redundancy in the system to cope with demand fluctuations. This suggests that uncertainty in input parameters should be included in the problem formulation during the optimisation in order to obtain a robust network design. Nodes 16, 17 and 19 seem to be the most critical nodes in terms of satisfying the minimum pressure constraint (when considering nodal robustness values). This is true for both deterministic and stochastic designs. Almost the same set of pipes that are duplicated in the optimal deterministic case need to be duplicated in all stochastic cases to achieve the target level of robustness. The main difference, however, is that the duplication pipe diameters for the optimal stochastic designs are larger than the corresponding duplication pipe diameters for the optimal deterministic design.

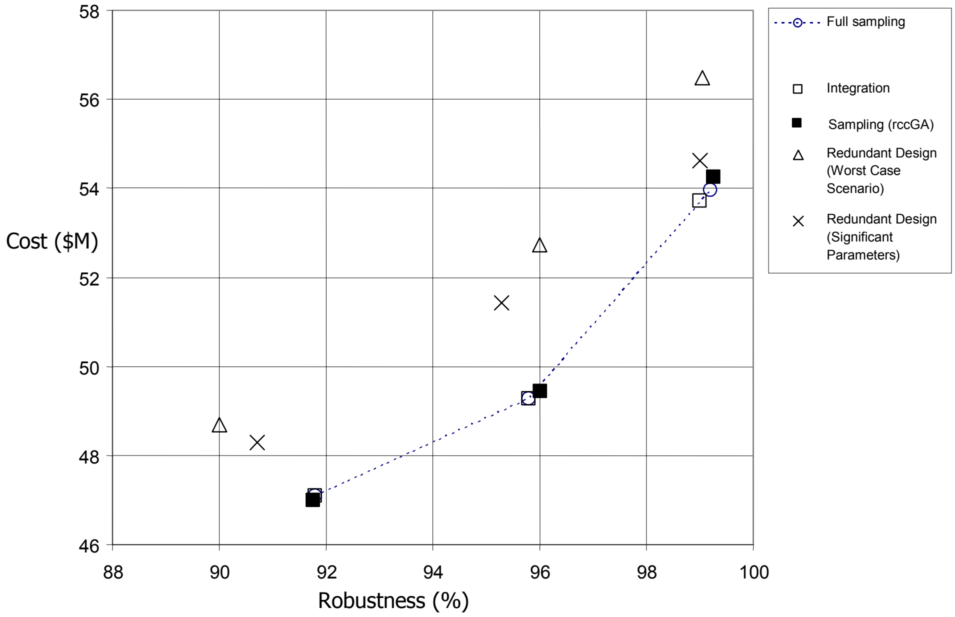

Figure 3 - Robustness vs. Cost Trade-off Curve for Uncertain Demands

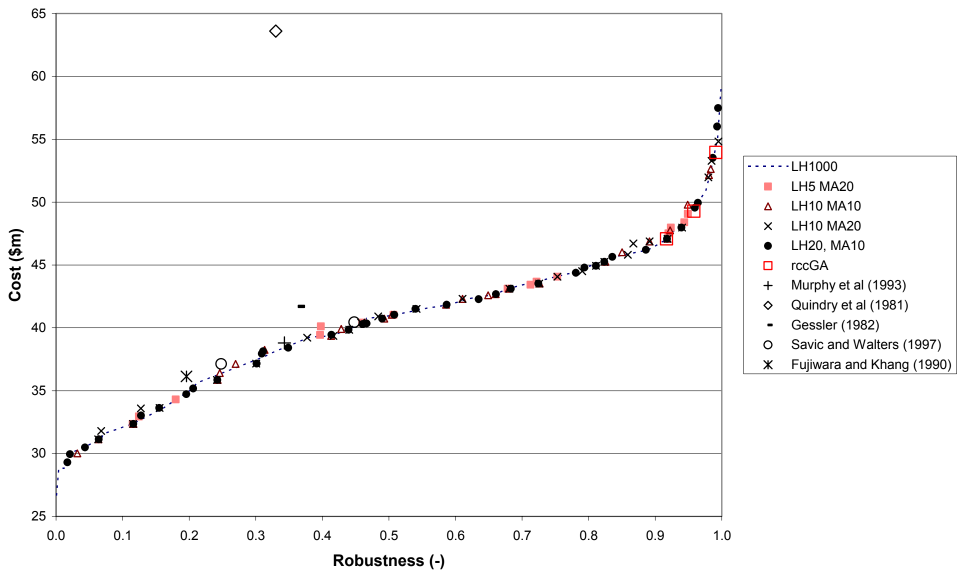

Figure 4 - Robustness-based Pareto Optimal Fronts

To demonstrate the need and advantages of considering pipe network design as a multiobjective optimisation problem under uncertainty the New York tunnels problem was considered and results compared to previously obtained. The approach uses the sampling methodology and two alternative problem formulations, as reported by Kapelan et al. (2005b). The first objective in both formulations is to minimise the total rehabilitation cost. The second objective is to either maximise the overall robustness or to minimise the total risk. While the robustness was defined as in the previous examples, the total risk is defined here as the sum of nodal risks, where nodal risk is defined as the product of the probability of pressure failure at that node and consequence of such failure. The optimal rehabilitation problem is solved using the newly developed "robust NSGAII" (rNSGAII) method which is a modification of the well-known NSGAII optimisation algorithm (Deb et al. 2000). Similarly to rccGA, in rNSGAII a small number of demand samples are used for each fitness evaluation.

The Pareto optimal solutions obtained for the robustness-based problem formulation are shown in Figure 4. The curve "LH1000" represents the solutions obtained using the standard classical, full sampling approach. In this approach the standard NSGAII is used with 1000 LH samples for each fitness evaluation. In addition to this, the three points/solutions labelled rccGA represent the solutions obtained using the rccGA methodology. All Pareto optimal fronts obtained using small number of LH samples (LH5-LH20) are in a very good agreement with the full sampling approach front (LH1000). The coverage and spread quality of the identified Pareto optimal fronts increases with the number of LH samples used. While the Pareto optimal front obtained for 5 LH samples is not very good in terms of both coverage (not uniform) and spread (both front tails not identified at all) of the solutions obtained, the Pareto front for 10 samples and especially the front for 20 LH samples has excellent coverage and uniform spread of solutions. The three optimal single-objective solutions obtained using the rccGA optimisation method (see also Figure 3) are in excellent agreement with corresponding Pareto optimal solutions. Most of the previously identified 'optimal' deterministic solutions are located close to the Pareto optimal front but are grouped in the low robustness zone (20-45%). The cost of rehabilitation solutions rises in an exponential manner for increasingly high robustness levels (approximately 90% and above).

The Pareto optimal solutions obtained for the risks-based problem formulation are shown in Figure 5. The shape of the Pareto optimal front obtained is significantly different from the shape of the Pareto optimal front obtained using the robustness-based model. This is, basically, a consequence of the definition of risk adopted in equations (16)-(17). All Pareto optimal fronts obtained using small number of LH samples (LH5-LH20) have good coverage and spread of points and are in very good agreement with the full sampling approach (LH1000). The cost of rehabilitation solutions rises in an exponential manner for decreasingly low risk values (approximately 0.02 and below).

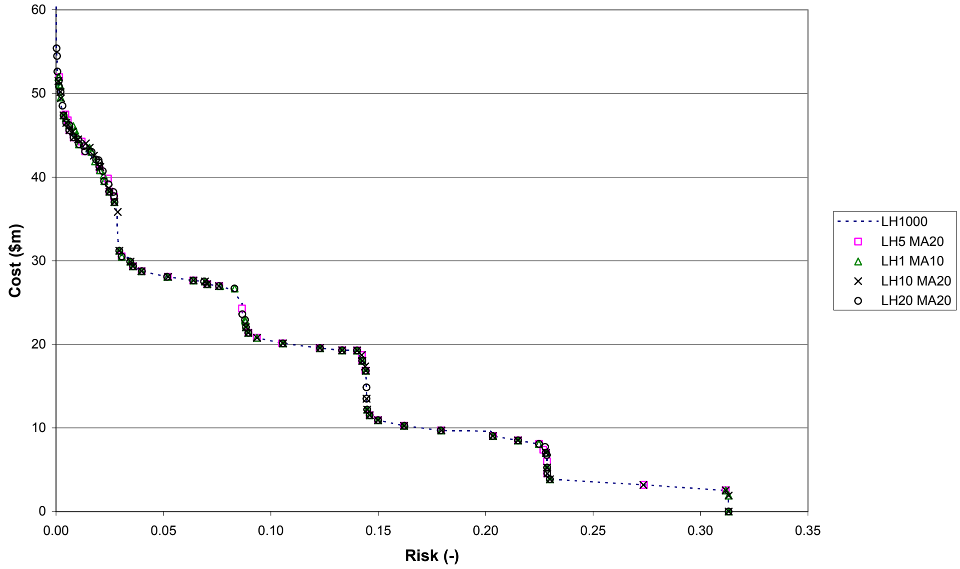

Figure 5 - Risk-based Pareto Optimal Fronts

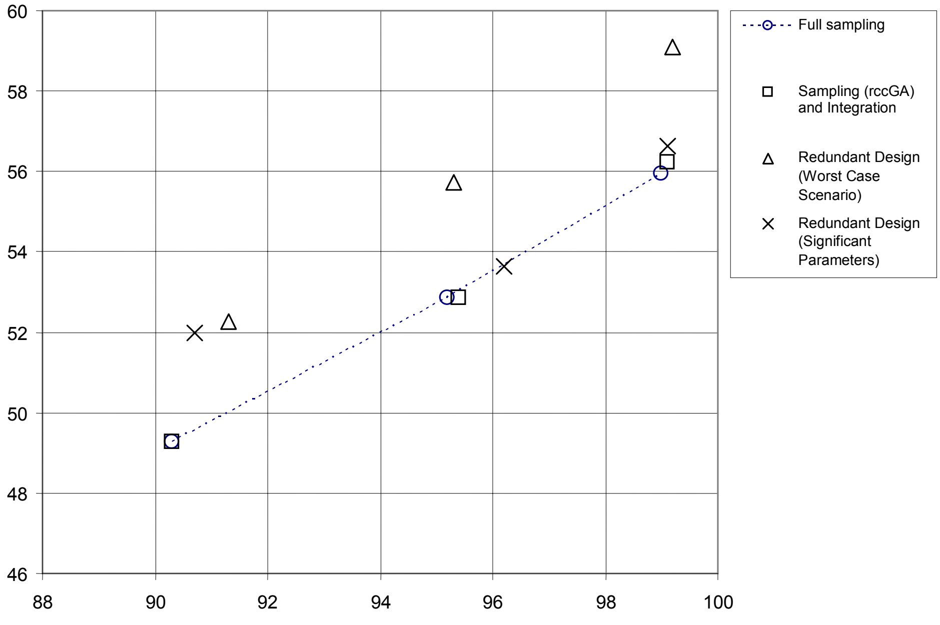

Figure 6 - Robustness vs. cost trade-off curve for uncertain demands and pipe roughnesses

4.1.2 Uncertain Demands and Pipe Roughnesses

In this case the number of uncertain parameters is equal to 40 (19 uncertain demands and 21 uncertain pipe roughness coefficients). The analysis performed in the first step of the Integration method shows that only two (friction coefficients for pipes 13 and 14) out of the 21 new uncertain parameters have a noticeable effect on the system robustness. As a consequence, although the optimal network obtained for this problem has a cost higher than in the case of uncertain demands only, the difference is less then 5%.

Within the analysed interval of robustness (90% to 99%) the relationship between the cost of solution and robustness level is close to linear (see Figure 6). This is because duplication of pipes eliminates the influence of uncertainty in these pipes' roughnesses. In particular, duplication of pipe 14 leads to a stepwise increase in the system robustness. Again, the optimal robust solutions identified using the Sampling (rccGA) and Integration methods are very similar (the Sampling method managed to identify solutions which are slightly closer to the desired level of robustness). The Redundant Design method for the worst case scenario found, again, more expensive solutions. However, for robustness levels higher than 95% with safety margins applied to significant uncertain parameters only, the results are quite close to solutions identified by the Full Sampling, Sampling (rccGA) and Integration methods. As expected, in comparison with the previous case (uncertain demand only) one has to spend more money to provide the same level of robustness in the resulting network. Robustness of the deterministic solutions for uncertain demands and pipe roughnesses is lower than for uncertainty in demand only.

4.2 ANYTOWN NETWORK PROBLEM

The performance of the NSGAII method has again been investigated here for the solution of Anytown optimization problem. In this optimization problem, the pay-off is investigated between the total cost and the system robustness. Figure 7 gives the pay-off characteristic between the total cost and the system robustness for the Anytown network. The search results indicate that the NSGAII has the potential to find Pareto optimal solutions for water distribution networks. The limited number of solutions on the pay-off curve shows that the problem is highly constrained. The distance between the solutions represents discrete nature of the optimization problem. In order to determine the pay-off characteristic between the total cost and system robustness the multi-objective algorithm was run with a population size of 100 sample solutions, and was allowed to run for 5000 generations. Detailed inspection of the solutions along the pay-off curve shows that one new tank with different volume size is included at node 8 for all solutions. Details of the design costs and system robustness for all the solutions on Pareto front are summarized in Table 1.

Solution No→	1	2	3	4	5	6	7
Pipes	6.6737	6.8736	7.3753	7.7716	8.3222	8.5874	8.7950
Tanks	0.8595	0.7915	0.7476	0.7683	0.7455	0.8008	0.742328
Pumps	6.1396	6.0756	6.0461	6.0496	5.9827	5.9618	6.05649
Total	13.6728	13.7407	14.169	14.5895	15.0504	15.3500	15.5938
Robustness (%)	58	61.7	62.6	63.1	71.7	71.8	74.3

Table 1 - Summary of Design Costs ($M) and robustness (reliability)

It is evident that main difference in the solution costs originates from pipe costs. This indicates that in order to provide higher resilience and flexibility in the network, the network capacity has to increase to respond to pipe breaks or demands that exceed design values. The design and operation of network under multiple loading conditions provides degree of flexibility for the solutions. This can explain minimum robustness value of above 50% for the solutions on the Pareto front. Also, the low value of the highest robustness can be due to the fact that introduction of substantial redundancy in the network results in poor performance of the network under five loading conditions. The most important difference among the solutions that affects the degree of robustness is the number of hours pumps are in operation. For solutions with low robustness (1, 2, 3 and 4) the pump operation time over 24 hour design period is higher than for the solutions with high robustness (5, 6 and 7). For solutions 5, 6 and 7 there is an excess pump capacity to respond to disruptions in the system.

5. SUMMARY AND CONCLUSIONS

Several approaches to dealing with risk and uncertainty are analysed in this paper, including: the use of the standard safety margins (redundant design methodology); the development of stochastic robustness/risk evaluation models; and the consideration of the rehabilitation and reliable operation of system within a single model. The uncertain parameters considered in the robustness/risk evaluation model include nodal demands and pipe friction characteristics. The robust/risk resilient design problem is formulated as a stochastic, constrained optimization problem. Both single-objective and multiobjective optimization methodologies are verified on two case studies, for one of which, optimal robust solutions obtained are compared to well known deterministic solutions from the literature. The results of both case studies clearly demonstrate that *neglecting uncertainty in the design process may lead to serious under-design of water distribution networks*.

The methodologies proposed are of a general type, in that any PDF can be used to model uncertain demands and other parameters(such as pipe roughnesses). Unlike in the First Order Reliability Method (Xu and Goulter, 1999), the methodology shown here does not require existence and calculation of the first-order derivatives of nodal heads ($\partial H_i/\partial Q_j$). This is an important advantage, especially in the case of complex networks with various control devices where such derivatives may be very difficult or, in some cases, impossible to obtain. The disadvantage of the Integration methodology shown here is that the target level for design robustness cannot be specified explicitly in the problem formulation phase, but has to be specified indirectly (as the target value of parameter P_{min}). As a consequence, the actual level of robustness can be calculated only once the optimization process has converged and the final solution is obtained. However, the Sampling method does not suffer from this problem.

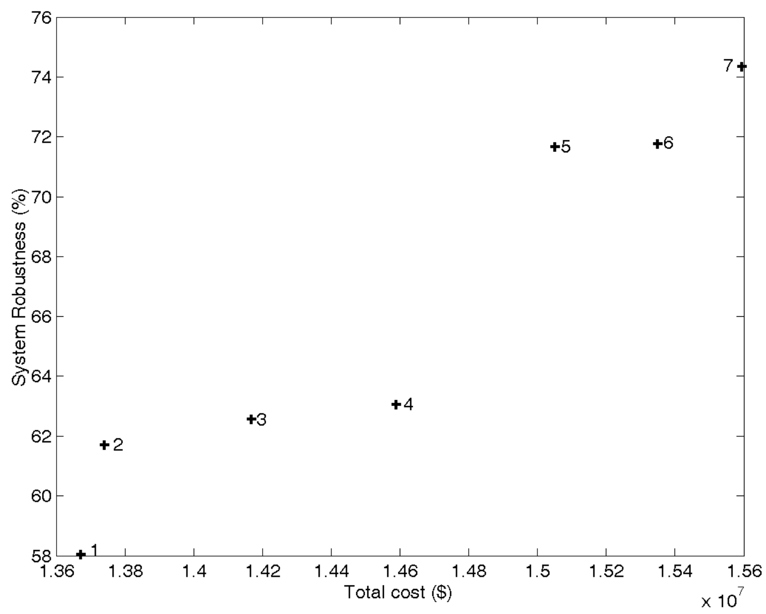

Figure 7 - Robustness vs. cost trade-off curve for the Anytown system

Both robust design methodologies (i.e. Integration method and Sampling method) presented here seem to be capable of identifying (near) optimal least-cost design solutions under uncertain input variables while achieving significant computational savings when compared to the full sampling technique. The computational savings obtained (using either of the methodologies) are in the range of *two orders of magnitude.* While the Integration method can be used with any type of optimization methodology, the Sampling method works only with population based search algorithms because it actually takes advantage of these search techniques and reduced MCS samples. The computational complexity of the Integration method does not increase with the target level of robustness but it does increase with the increased number of uncertain input variables. It also provides the means of estimating the relative contribution of each of the uncertain parameters to the system robustness, allowing identification of the most important ones. Its main disadvantage is that it addresses the target robustness indirectly, and therefore, may require updating of the list of critical/significant nodes during the optimisation process (time consuming). Also, unlike the Sampling method which can easily handle the hydraulic model non-linearity, the Integration method it is based on the assumption of the validity of the superposition principle, which may not be true for all networks and therefore could limit its applicability.

In addition to the above characteristics of the new methodologies applied in a single-objective context, the multiobjective design methods presented here (i.e. robustness based and risk based) are capable of identifying (near) Pareto optimal fronts under uncertain demands while achieving significant computational savings (approximately two orders of magnitude) when compared to the full sampling technique. This is important because in a single run the whole Pareto front is obtained which allows for the responsibility of assigning relative values of the objectives to remain with the decision maker. Even though the high quality solutions (i.e. solutions with high robustness / low risk) obtained using both robustness and risk driven models are similar (but not identical), the risk based approach is preferred to the robustness based approach because it takes into account the consequence of a potential failure in addition to the probability of that failure.

Similarly, the two-level optimization of the Anytown problem considering total cost and network robustness as objective functions and design parameters and operation as decision variables resulted in high quality solution networks. The solutions presented along the Pareto curve are the fully feasible solutions operating under 5 loading conditions over the 24-hour design period without violating any constraints. They also have a degree of flexibility built in to respond to uncertain scenarios affecting the system. Further research is underway comparing the performance of different probability distributions for demand uncertainty and also different fuzzy membership functions for quantification of the degree of feasibility for solutions at the lower level optimization.

6. ACKNOWLEDGEMENTS

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