

ESTIMATION OF EXTREME STORM SURGES USING A SPATIAL LINKAGE ASSUMPTION.

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ABSTRACT

In the present paper a constant shape parameter over space is used for the Dutch coast on the North Sea to extract results of return levels for a process of daily storm surges, taking account of the degree of spatial dependence among sites. Marginal inference under spatial linkage is conducted for all stations with a ξ -linkage approach, using an algorithm proposed by Butler, (2005). Return levels and their standard errors are extracted under the assumption of spatial linkage and compared to the ones when no parameter linkage is considered. Variance and efficiency gains in the model parameters are estimated and discussed. To calculate these measures, a simple parametric bootstrap method for the case of the ξ -linkage is used.

Keywords: Generalized Pareto Distribution (GPD), ξ -linkage assumption, pseudo-likelihood, parametric bootstrapping, efficiency gains, return level estimation

1. INTRODUCTION

The spatial structure and spatial analysis of extreme events is a really interesting subject, which evolved rapidly within the past few years and receives considerable attention in the present time. The study of the spatial dimension of a process can improve the knowledge of the underlying stochastic process, because it helps to understand the dynamic structure of a typical storm event. For a wide range of scientific applications, the observations are scattered in space, either on a regular grid or at irregularly spaced locations. At some locations there are only few years of observations and so the exploitation of the degree of spatial dependence to other sites can be beneficial for engineering purposes. The need to extrapolate the distributions to locations where observations are not available is a further advantage of a spatial analysis. But it should be noted that the key motivation in favor of such an analysis is the desire to use spatial knowledge to reduce uncertainty in the estimation of marginal (site-specific) extremal characteristics.

Classical approaches to the statistical analysis of spatial extremes are based upon modeling the behavior of sitewise block maxima. Spatial extremes are often analyzed using standard techniques from Multivariate Extreme Value Theory. Tawn (1988) developed parametric models and inferential techniques for the componentwise block maximum model. More recent approaches use threshold excess models and point processes. Zachary et al (1998) consider the estimation of joint extreme quantiles of metocean climate, in particular those of wave height, wave period and wind speed, using observations exceeding appropriately defined thresholds. Coles and Tawn (1991) consider the point process of observations, which are extreme in at least one component. Aspects of the statistical estimation of the resulting models are illustrated with an application to oceanographic data. Coles et al. (1999) use different measures for extremal dependence and produce inference of return levels for extreme events of various data sets utilizing mainly bivariate point processes. Ledford and Tawn (1996, 1997, 1998) discuss the statistical properties of asymptotically independent distributions. Heffernan and Tawn (2004) develop a semiparametric approach,

which overcomes the restrictions imposed by dimensionality, for analysing air pollution data. Geostatistical (kriging) methods were also utilized to incorporate spatial information in the extreme value analysis. These methods model the extreme value parameters as smooth functions of spatial location. (e.g. Holland et al., 2004). Coles and Tawn, (1991) and Coles and Tawn, (1994) make use of a linkage assumption (often referred to as ξ -linkage) to analyze spatial extremes.

From the previous approaches used for the statistical analysis of spatial processes (multivariate processes), it can be noted that two different approaches are used to model spatial extremes: a) one based on a continuous version of the Extremal Types Theorem and b) one related to standard methodology for spatial statistics. Both approaches are computationally intensive and implementations have so far been largely exploratory (Coles, 2001). The first one is based on max-stable processes and the second one on latent spatial process models. Max stable processes are defined with reference to a standard Fréchet marginal distribution. By definition a max-stable process Y in \mathbb{R}^d with unit Fréchet margins is a random field, which has all its higher-order marginal distributions belonging to the class of Multivariate Extreme Value distributions with unit Fréchet margins (Schlather and Tawn, 2003). In the case of latent spatial processes it is assumed that one or more marginal parameters are constant over space (“spatial linkage”) or that the marginal parameter surface is smooth with a known level of smoothness (“spatial smoothing”) (Butler, 2005).

In this work an assumption of a constant shape parameter over space will be used (“spatial linkage”) to extract results of return levels for a process of daily surges, taking account of the degree of spatial dependence among nine sites of the Dutch coast in the North Sea. Based on the results of a previous analysis of bivariate data from the Dutch stations considered (Galiatsatou and Prinos, 2006), different pairs of stations have proven to be consistent with asymptotic dependence (dependence is evident even for stations that are the most remote in space). Marginal inference under spatial linkage is conducted for all stations with a ξ -linkage approach, using an algorithm proposed by Butler, (2005). Return levels are extracted under the assumption of spatial linkage and compared to the ones when no parameter linkage is considered. Variance and efficiency gains in the model parameters are estimated and discussed. To estimate the previously mentioned measures (variance, efficiency gains), a simple parametric bootstrap method for the case of the ξ -linkage is used.

2. THE DATA AND THE GPD MODEL

The data used in the analysis consist of a sequence of 23 years (over the period 1979-2001) of simultaneous three-hourly surge elevations from 9 locations of the Dutch coast. Locations of the existing surge data are presented in Figure 1.

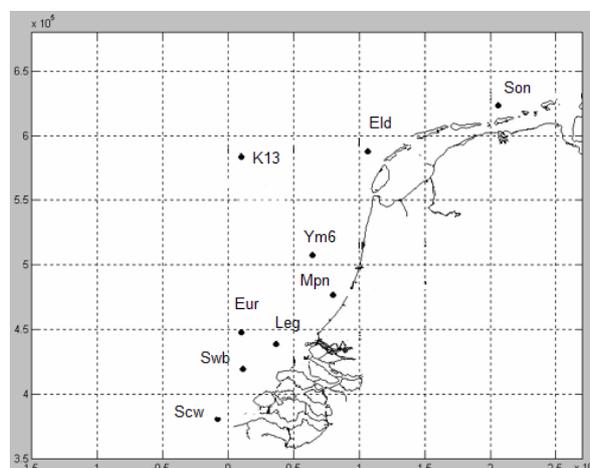


Figure 1. Field stations at the Dutch coast

Details of these data sets are given in www.golfklimaat.nl, of the Dutch National institute RIKZ.

Daily surge maxima are utilized to deal with short range dependence in a rather simple way and to look at the characteristics of an entire storm surge event. It has been proven using plots of $\chi(u)$ and $\bar{\chi}(u)$ (Galiatsatou and Prinos, 2006) that the level of spatial dependence at extreme surge levels is largely insensitive to the degree of temporal aggregation, for aggregation periods of one, two days and a week or more.

If X_1, X_2, \dots, X_n is a series of independent random observations of a random variable X with common distribution function $F(x)$ and Y_1, Y_2, \dots, Y_k ($Y_i = X_i - u$) are the excesses over a high enough threshold u , in some asymptotic sense, the conditional distribution of excesses follows the Generalised Pareto Distribution:

$$G(y) = 1 - \left(1 + \frac{\xi y}{\sigma}\right)^{-1/\xi} \quad (1)$$

where σ, ξ denote the scale and shape parameters, respectively. One of the most interesting features of the maximal generalized Pareto family of distributions is that it is stable with respect to truncations from the left. This property implies that if a given data set and a threshold u_0 is consistent for this law, it will be for any other threshold value $u_1 > u_0$ (Castillo et al., 2005).

An appropriate threshold u is selected, which defines the level upon which an extreme event is defined. Two different methodologies are used for the selection: (a) the mean residual life plot of the excesses of different threshold values and (b) the plots of σ and ξ for a variety of possible threshold values. Too low a threshold is likely to violate the asymptotic basis of the model, leading to bias; too high a threshold will generate few excesses with which the model will be estimated, leading to high variance (Coles, 2001). Thresholds chosen for all nine Dutch locations are close to the 95% quantile of the storm surge at each station.

3. THE ASSUMPTION OF SPATIAL LINKAGE

It is mentioned above that apart from max-stable processes, an alternative approach for analyzing spatial extremes is by the use of latent spatial processes. Compared with max-stable processes, model specification is easier using latent processes. A limitation is that strong dependence cannot be induced in this way (Coles, 2001).

Let $\mathbf{X} = (X_1, \dots, X_d)$ be a (possibly dependent) sequence of random variables and assume that the exceedances $X_i \sim \text{GEV}(\boldsymbol{\theta}_i)$ for each $i \in \{1, \dots, d\}$, where $\boldsymbol{\theta}_i = (\mu_i, \sigma_i, \xi_i)$. We can say that \mathbf{X} has $\boldsymbol{\theta}$ -linkage if $\boldsymbol{\theta}_1 = \boldsymbol{\theta}_2 = \dots = \boldsymbol{\theta}_d$. In many contexts, $\boldsymbol{\theta}$ -linkage is a very strong assumption and it is more sensible to assume than only some of the extreme value parameters are linked. Estimates of the shape parameter ξ can be made more efficient by assuming that the parameter is common across the components of \mathbf{X} , so that $\xi_1 = \xi_2 = \dots = \xi_d$ (Butler, 2005). According to Shi et al. (1992) an estimator $\bar{\boldsymbol{\theta}}$ for the parameter vector $\boldsymbol{\theta}$ of the distribution of a sequence of random variables \mathbf{X} (like the one defined above) is formed by maximizing the likelihood:

$$l_p(\boldsymbol{\theta}; \mathbf{x}) = \sum_{i=1}^d l_i(\boldsymbol{\theta}_i, x_i) \quad (2)$$

Where l_i denote the likelihoods associated with X_i . In the language of Shi et al. (1992) the marginal estimator $\bar{\boldsymbol{\theta}}$ is a pseudo-likelihood estimator for $\boldsymbol{\theta}$. The pseudo-likelihood function is used to split the $(2d+1)$ - dimensional optimization problem into λd separate 2- dimensional optimization problems, where λ is a large number which does not depend on d .

An algorithm developed by Butler, (2005) and implemented to the largest yearly observations of CSX-DNMI storm surge elevations is used here to introduce a spatial linkage (ξ -linkage) to threshold excesses $(X_i - u_i)$ for the nine Dutch stations considered. Equation (2)

is assumed to hold for excesses of appropriately defined thresholds as for r -largest annual maxima and $\theta=(\sigma_i, \xi_i)$. Scale and shape parameters are considered constant over time.

We suggest taking $\xi^1, \xi^2, \dots, \xi^\lambda$ to be a grid of λ regularly spaced points in the range $[\min(\xi^1, \xi^2, \dots, \xi^d), \max(\xi^1, \xi^2, \dots, \xi^d)]=[-0.102, -0.00403]$. For each $i=1, 2, \dots, d$ and $k=1, 2, \dots, \lambda$ the likelihood $l(\sigma_i, \xi_i; x_i)$ is maximized over σ_i to obtain trial estimates $\hat{\sigma}_i^k$. Finally, we obtain Maximum Likelihood (ML) estimates for $(\sigma_1, \dots, \sigma_d, \xi)$ by maximizing: $\sum_{i=1}^d l((\hat{\sigma}_i^k, \xi^k); y_i)$ over k ($y_i=x_i-u_i$).

The time needed to implement the algorithm is proportional to d and so the approach should offer substantial computational savings over a direct optimization approach (unless d is very small).

Figures 2 and 3, show the impact of ξ -linkage on parameter estimates for the nine sites of the Dutch coastline. ‘‘Sitewise’’ estimates are also derived under an assumption of no linkage and are compared to those of the ξ -linkage assumption. Estimates of parameters with and without the ξ -linkage assumption are shown in Table 1.

In Figure 2 discrepancies between parameter σ estimates obtained under ξ - linkage and no linkage assumptions are small. For the sites Son, K13, Leg, Eur and Swb median estimates of the scale parameter are exactly the same for both approaches, while for sites Eld, Ym6, Mpn and Scw differences are larger, but still negligible.

In Figure 3 there are relatively large discrepancies between the two sets of ξ estimates, especially for sites Eld, Ym6, Mpn and Scw, while stations Leg, Son and K13 give estimates very close to the ones obtained under a ξ -linkage assumption. Southern sites give estimates of the shape parameter greater than the ξ -linkage assumption, but still all estimates are negative. Sites at the northern and especially the central part of the Dutch coast give strongly negative estimates. At these sites discrepancies are larger, although in fact they remain small in absolute terms.

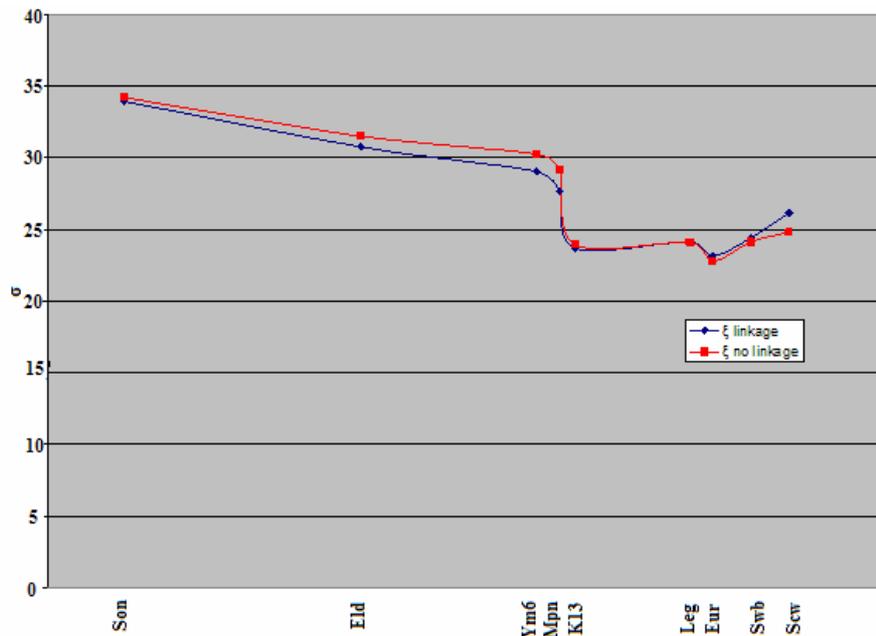


Figure 2. Estimates of the scale parameter (σ) under a no-linkage and a ξ -linkage assumption

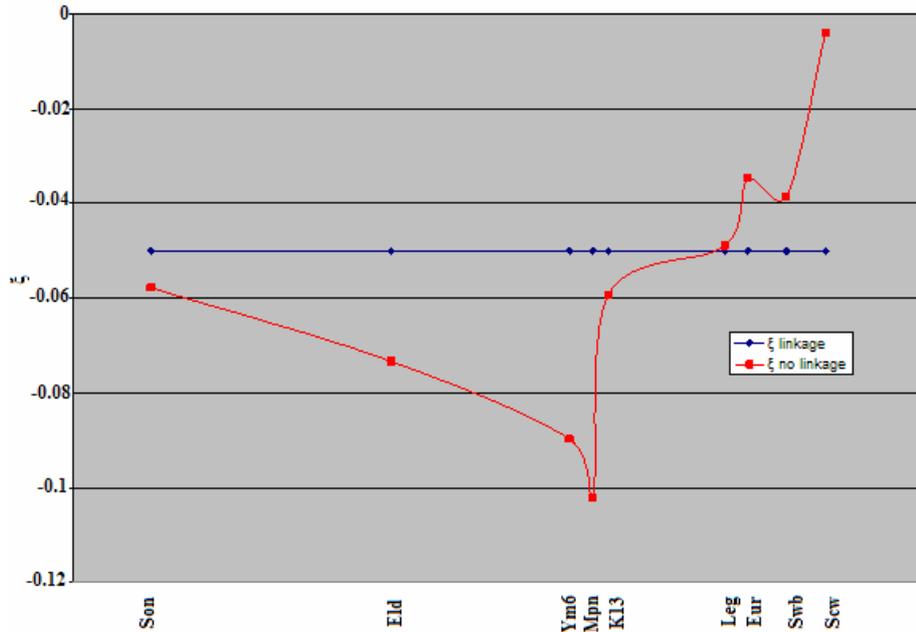


Figure 3. Estimates of the shape parameter (ξ) under a no-linkage and a ξ -linkage assumption

Table1. Estimates of scale and shape parameters under no-linkage and spatial linkage

Station	No-Linkage			ξ -Linkage		
	σ	se(σ)	ξ	σ	se(σ)	ξ
Son	34.232	2.392	-0.058	33.950	1.590	-0.050
Eld	31.511	2.293	-0.073	30.760	1.456	-0.050
Ym6	30.256	2.155	-0.090	29.040	1.418	-0.050
Mpn	29.175	1.865	-0.102	27.660	1.285	-0.050
K13	23.933	1.756	-0.059	23.700	1.194	-0.050
Leg	24.100	1.606	-0.049	24.130	1.126	-0.050
Eur	22.781	1.511	-0.035	23.160	1.091	-0.050
Swb	24.128	1.535	-0.039	24.430	1.134	-0.050
Scw	24.818	1.577	-0.004	26.150	1.209	-0.050

In Table 1, standard errors of the scale parameter are calculated using ML estimation procedure. Standard errors of the shape parameters are omitted, due to the fact that under the ξ -linkage assumption, the shape parameter ξ is kept fixed.

3. ESTIMATION OF CONFIDENCE INTERVALS BY BOOTSTRAPPING

Bootstrap methods offer a simple approach for statistical uncertainty when only a random sample is available. There are in principle two available options for obtaining bootstrap estimates. One is the non-parametric approach, where a purely empirical distribution function (EDF) is established for the data by allocating a probability $1/n$ to each of the observed data points. The other is the parametric bootstrap, which is obtained by assuming that the data (in this case, extreme data are of main concern) have a specified cumulative distribution function (CDF) $F_X(x; u) = \Pr(X \leq x)$, where u denotes a vector of unknown parameters, which determine the CDF (Naess and Clausen, 2001).

Caers and Maes, (1998) developed a semi-parametric bootstrap algorithm, which was used later by Pandey et al. (2003). In this method, a POT sample chosen above a certain threshold u_1 , was further divided into two-samples by choosing another threshold u_2 . For a given set of u_1 and u_2 ($u_2 > u_1$), bootstrap estimates of first and second order tail indices were calculated. Thresholds u_1 and u_2 were varied to minimize mean square errors in an asymptotic sense. Caers and Maes, (1998) generated samples above a threshold u from a smooth parametric distribution, whereas samples below u from the empirical distribution of the original sample. The conditional bootstrap distribution function is given by the formula (Pandey et al., 2003):

$$F_B(x|u) = \begin{cases} (1 - F_E(u))F_P(x) + F_E(x) & \text{for } x > u \\ F_E(x) & \text{for } x \leq u \end{cases} \quad (3)$$

where $F_P(x)$ is the Pareto distribution fitted to POT data by the de Haan or L-Moment estimator. This semi-parametric bootstrap algorithm has been proven to be accurate and appropriate in the analysis of extreme events.

If l is the number of bootstrap samples x_j^* used and \hat{R}_j^* is a maximum likelihood estimator of R_j^* ($j=1, \dots, l$) for each sample, a simple estimator for confidence intervals on \hat{R} is derived by calculating the sample standard deviation s_R^* (Naess and Clausen, 2001):

$$s_R^* = \sqrt{\frac{1}{l-1} \sum_{j=1}^l (R_j^* - \bar{R}^*)^2} \quad (4)$$

where $\bar{R}^* = (1/l) \sum_{j=1}^l R_j^*$. An approximate confidence interval at level $1-q$ is then obtained as

$(\hat{R} - w_{q/2} s_R^*, \hat{R} + w_{q/2} s_R^*)$, where $w_{q/2}$ denotes the standard normal distribution. When estimating such a confidence interval, usually 20-30 samples are sufficient.

In this study a more simple parametric bootstrap approach will be used for the excesses of the chosen threshold u , because we are mainly interested in estimating the efficiency of parameters σ and ξ of the GPD model, used in the previous sections. $L=30$ samples of size $n=1000$ exceedances of the thresholds chosen at each site are generated from each site's distribution under the ' ξ -linkage assumption'. Due to the fact that the log-likelihood of the GPD model is a function of the parameters σ , ξ and u , maximization of the likelihood function for each sample can produce median estimates of $\hat{\sigma}$ and $\hat{\xi}$, which are used in equation (4) to obtain estimates of s_σ and s_ξ .

4. ESTIMATION OF PARAMETER VARIANCE AND EFFICIENCY

The variance of the parameters σ and ξ for the " ξ -linkage" approach is estimated using a parametric bootstrap approach and the square of equation (4). For the non-linkage approach, the variance of the parameters is estimated taking squares of the standard errors from the maximum likelihood estimation (MLE) procedure.

The efficiency of a particular parameter at a particular site is the ratio of the variance obtained under " ξ -linkage" at that particular site to the variance obtained for that site under no linkage (Butler, 2005). Figures 4 and 5 show estimates of the σ and ξ efficiency, respectively for the nine Dutch sites.

Figures 4 and 5 show that ξ -linkage leads to a reduction in the variance of the extreme value parameters σ and ξ and that the degree of reduction varies from parameter to parameter and from site to site. The reduction of the variance of the σ parameter under the ξ -linkage assumption is larger for sites Leg, Eur and Swb, and smaller for sites Mpn and K13. The reduction of the variance of the ξ parameter under the ξ -linkage assumption is larger for site

Leg and lower for sites Ym6 and Son. The variances of ξ - linked estimators for σ and ξ parameters are 25-65% and 20-80% of the variances of the corresponding unlinked estimators. For both parameters there are gains in efficiency. The greatest gains in efficiency are obtained on the northern part of the Dutch coast (especially for the σ parameter) and the least substantial gains are observed on the southern part.

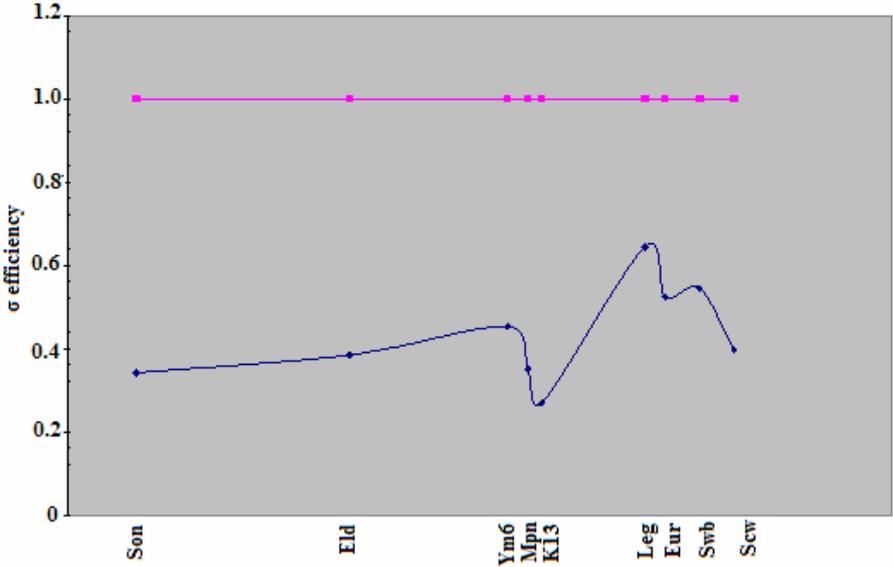


Figure 4. σ -efficiency under ' ξ -linkage' for the nine stations of the Dutch coast

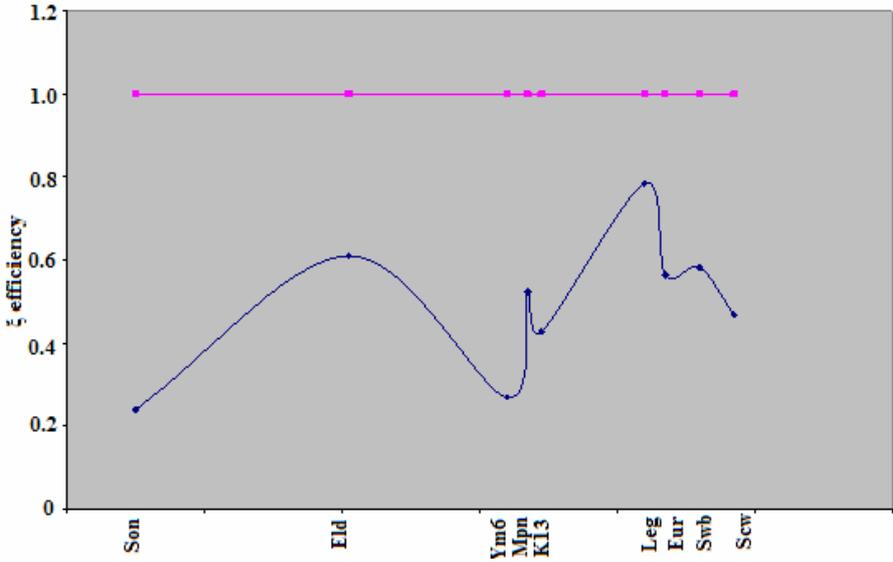


Figure 5. ξ -efficiency under ' ξ -linkage' for the nine stations of the Dutch coast

5. RETURN LEVEL ESTIMATION

If $F(x)$ is the Cumulative Distribution Function (CDF) of the yearly maxima of a random variable X , the return period τ_x , of the event $\{X>x\}$ is the $1/[1-F(x)]$ years. Then the return period τ_x can be estimated for exceedances as:

$$\tau_x = [1 - F(x)]^{-1} \tag{5}$$

The N- year return level is the level expected to be exceeded once every N years. If there are n_y observations per year, this corresponds to the m-observation return level, where $m= N * n_y$.

Note that if an engineering work fails if and only if the event A occurs, then the mean life of the engineering work is the return period of A. The importance of return period in civil engineering is due to the fact that many design criteria use return periods, that is, an engineering work is designed to withstand, on average, return periods of 50, 100, or 500 years (Castillo et al., 2005). The 100-year return levels for all nine stations using a non-linkage and a ξ -linkage assumption are shown in Table 2.

Table 2. Median estimates of 100-year return level using a no-linkage and a ξ linkage assumption

Stations	100-year Return Level (cm)	
	ξ -linkage	no linkage
Son	293.1	289.1
Eld	267.7	258.7
Ym6	259.8	245.3
Mpn	246.2	226.0
K13	205.7	204.2
Leg	217.8	219.6
Eur	206.4	213.3
Swb	219.8	225.6
Scw	226.5	248.3

Estimates of the 100-year return level are close to those obtained under a no linkage assumption for stations Son, K13 and Leg. For the other stations there exist medium to large divergences. The median 100-year return level under a ξ -linkage assumption at different stations is between 96%-109% of the median return level when no linkage is present. Sites for which the shape parameter is estimated higher under a ξ -linkage assumption, give higher estimates of 100- year return levels under spatial linkage compared to no-linkage. This can be noticed for sites at the northern part of the Dutch coast. The contrary is true for sites at the southern part, for which the spatial linkage assumption gives lower estimates of the shape parameter. The shape parameter ξ is dominant in determining the qualitative behavior of the generalized Pareto distribution (GPD). If $\xi < 0$ the distribution of excesses has an upper bound (Coles, 2001) of $u - \sigma/\xi$. If u and σ are kept constant, smaller estimates of ξ will give lower upper bounds and larger, but still negative estimates will give higher upper bounds for the distribution of excesses.

To obtain reliable estimates of the standard errors of the return levels under a ξ -linkage assumption a semi-parametric bootstrap approach, like the one described in Section 4 is needed.

6. CONCLUSIONS

An assumption of a constant shape parameter over nine sites of the Dutch coast on the North Sea was used here to extract results of return levels for a process of daily storm surges. Marginal inference under spatial linkage was conducted for all stations with a ξ -linkage approach, using an algorithm proposed by Butler, (2005). Median values for 100-year return

levels were extracted under the assumption of spatial linkage and compared to the ones when no parameter linkage is considered. Variance and efficiency gains in the model parameters were estimated and discussed. To estimate the previously mentioned measures, a simple parametric bootstrap method for the case of the ξ -linkage was used.

The following conclusions can be derived:

- a. Discrepancies between parameter σ estimates obtained under ξ - linkage and no linkage assumptions are small for all stations, while there are relatively large discrepancies between the two sets of ξ estimates, especially for some sites at the northern part of the coast and one at the south
- b. All estimates of the shape parameter, ξ , (under spatial and no- linkage assumptions) are negative, leading to upper bounded distributions of extreme values.
- c. The variances of ξ - linked estimators for σ and ξ are 25-65% and 20-80% of the variances of the corresponding unlinked estimators.
- d. For both σ and ξ , there are gains in efficiency. The greatest gains in efficiency are obtained on the northern part of the Dutch coast (especially for the σ parameter) and the least substantial gains are observed on the southern part.
- e. The median 100-year return level under a ξ -linkage assumption at different stations is between 96%-109% of the median return level when no linkage is present.
- f. Sites at the northern part of the Dutch coast give higher 100-year return levels, up to 8.94%, under a spatial linkage assumption, while those at the south give lower, up to 8.78%.

7. ACKNOWLEDGMENTS

The authors want to thank the Dutch National institute RIKZ for providing data of the Dutch coast. The work described in this publication was supported by the European Community's Sixth Framework Programme through the grant to the budget of the Integrated Project FLOODsite, Contract GOCE-CT-2004-505420. This paper reflects the authors' views and not those of the European Community. Neither the European Community nor any member of the FLOODsite Consortium is liable for any use of the information in this paper.

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