

Optimisation of Structural Systems by Appropriately Assigning Probabilities of Failure. Application to Rubble Mound Breakwaters

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Abstract: An appropriate assignment of probabilities of failure to subsystems and components in a structural system can bring a minimum of costs and risk. In this paper, a method for economic optimisation of rubble mound breakwaters using pre-assigned probabilities of failure is presented. Application to a design case shows that the proposed method is useful in estimating the optimal design variables in a conceptual design.

Keywords: rubble mound breakwater, probabilities of failure, lifetime cost, economic optimisation.

1. INTRODUCTION

The design of engineering structures is a trial-and-error process that requires much practical experience. Designers often start with guess values, then iterate until an optimised structure is obtained under some criteria. Criteria to optimise the design can be the minimisation of loss of life, economic consequences, operational conditions or environmental aspects. To design structures of which the probability of loss of life due to their failure is very small, the economic optimisation is suitable and sufficient, although ethical issues concerning the value of human life come up. Different techniques have been used to economically optimise structural systems in general and breakwaters in particular. Some authors apply a straightforward probabilistic optimisation for breakwater design ([11,12]), while decomposition techniques and failure rate constraints techniques can be found in [1,6]. Reliability-based and risk-based optimisation are presented in [5,9].

This paper aims at deriving a rough estimate of optimal design variables of a rubble mound breakwater cross-section by appropriately assigning probabilities of failure, given the constraints of design variables and operational conditions.

In this paper, the costs of risk and investment are used in the objective function. The costs of a subsystem are described as functions of an assigned probability of failure. The constraints of design variables and operational conditions creates probabilities of failure bounds within which a trial-and-error iteration is made until an assignment of the probabilities of failure to subsystems and components is determined for which the total costs are minimal.

The paper is structured as follows. Section 2 presents the failure mechanisms and reliability analysis of breakwater systems. The proposed economic optimisation method is demonstrated in Section 3. A case study applying the proposed method is presented in Section 4. Finally, Section 5 ends the paper by some conclusions.

2. RELIABILITY ANALYSIS

2.1. General Introduction to Structural Reliability Analysis

A complex structure can be modelled by a system including subsystems and components. When analyzing a structural system in the view of probabilistic and reliability, the system failure is of main concern. A failure is defined as a condition in which the structure loses its specified functionality.

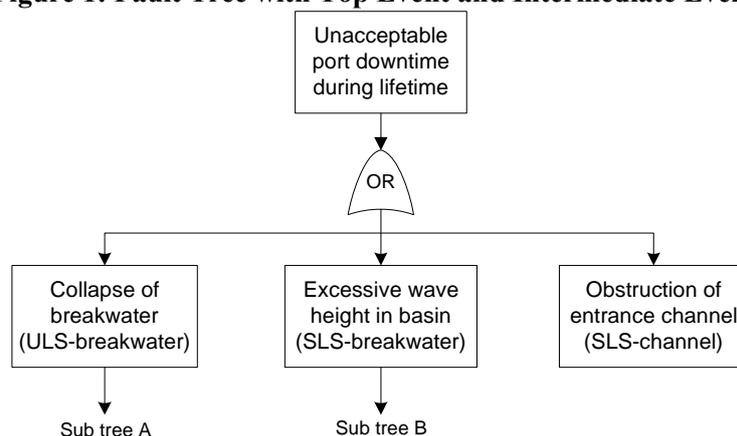
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Therefore, in order to define failures, the structure functions must be defined beforehand. Each failure mode is described by a formula. The interaction (or correlation) between failure modes and the contributions of each one to the system failure can be defined in a fault tree. A fault tree describes the relationships between the failure of the system and the events leading up to this failure. The fault tree gives a logical succession of all events that lead to one unwanted “top event”. The probability of failure of the whole system, equal to the probability that the top event occurs, can be calculated based on probabilities of failure of components and their correlations. Theories in structural system analysis can easily be found in many classical books and documents on probabilistic and reliability, e.g. in [8,14].

2.2. Failure Mechanisms and Reliability Analysis of Rubble Mound Breakwaters

A breakwater system can have several objectives, e.g. to protect the harbour basin or to be a temporary berth for ships. In this study, the objective of a breakwater is “to protect the harbour basin against unacceptable wave action”. Within this objective, the top event in the fault tree of a breakwater system is the “port downtime”, i.e. the stoppage of the port operations due to malfunction of the breakwater system. Three major mechanisms causing this top event can be distinguished: i) collapse of the breakwater; ii) excessive wave height in the basin; iii) obstruction of the entrance channel. Fault tree of a breakwater describing the major mechanisms is illustrated in Figure 1.

Figure 1: Fault Tree with Top Event and Intermediate Events



The first mechanism is the collapse of the breakwater which belongs to the Ultimate Limit State (ULS). As a result of collapse, the protective function of the breakwater will be lost. In this study, it is assumed that port operations are only active during normal conditions. Whenever a storm comes, the port stops its operation and ships have to leave the berths and flee the storm. Therefore, “port downtime” is neglected during a storm, but the collapse of the breakwater leads to no longer basin protection afterward.

The second mechanism is the excessive wave height inside the harbour basin during normal weather condition. The port operations at the lee side, e.g. ship manoeuvring, berthing, loading and unloading, can only be carried out within an allowable wave height in the harbour. Whenever the wave height exceeds the maximum allowable height, the breakwater system can be considered to fail. This failure mechanism occurs without severe collapse, and can be regarded as Serviceability Limit State (SLS).

The last mechanism is not closely related to the protective function of the breakwater. Obstruction of the entrance channel can be the consequence of a bad breakwater layout. Ships runs aground in the navigational channel will cause traffic obstructions and affect the port operations.

2.2.1. Ultimate Limit State of a Breakwater

Breakwater structures are composed of many parts, e.g. armour layer, concrete cap, toe structure. Collapse of one of these parts will weaken the resistance of other parts and, eventually, the whole

structure. ULS failures occur under an extreme condition, in this case a typhoon.

Failure modes in cross-section of a rubble mound breakwater can be the instability of the primary armour layer, the erosion of the toe, the excessive wave overtopping, the slip circle, the excessive settlement. The first two failure modes are briefly presented here.

To calculate the hydraulic stability of a Tetrapod armour layer, the following Van der Meer's formula is applied (see [2]):

$$\frac{H_s}{\Delta D_n} = f_i \left(3.75 \frac{N_{od}^{0.5}}{N^{0.25}} + 0.85 \right) s_{om}^{-0.2} \quad (1)$$

where H_s is the significant wave height at the location of the breakwater [m], N_{od} is the number of displaced units within a strip with width D_n , N is the number of wave, f_i is the coefficient denoting the difference between slope angle of the tested model and the real design, s_{om} is the wave steepness, D_n is the characteristic diameter of armour elements [m] and Δ is the relative density of material.

D_n and Δ can be calculated as follows:

$$D_n = \sqrt[3]{V} = \sqrt[3]{\frac{M}{\rho_c}} \quad (2)$$

$$\Delta = \frac{\rho_c}{\rho_w} - 1 \quad (3)$$

in which V is the volume of blocks, M is the mass of blocks [ton], ρ_c and ρ_w are the density of concrete and water, respectively [ton/m³].

The following limit state function can then be derived:

$$Z = f_i \left(3.75 \frac{N_{od}^{0.5}}{N^{0.25}} + 0.85 \right) s_{om}^{-0.2} \left(\frac{\gamma_c}{\gamma_w} - 1 \right) D_n - H_s \quad (4)$$

The erosion of the toe is one of the major failure mechanisms. According to Van der Meer et al. [10], the following formula is valid:

$$\frac{H_s}{\Delta D_{n50}} = \left(0.24 \frac{h_t}{D_{n50}} + 1.6 \right) N_{od}^{0.15} \quad (5)$$

The equation then is reformulated to a limit state function as the following:

$$Z = \left(0.24 \frac{h_t}{D_{n50}} + 1.6 \right) N_{od}^{0.15} \left(\frac{\rho_r}{\rho_w} - 1 \right) D_{n50} - H_s \quad (6)$$

where h_t is the water depth at the toe, ρ_r is the density of rock.

2.2.2. Serviceability Limit State of a Breakwater

The SLS of a breakwater relates to the tranquillity of the port basin. During normal conditions, the breakwater system can be considered to fail if the wave height inside the basin exceeds a maximally allowable height.

The wave inside the port basin is a combination of wave refraction - diffraction via the entrance, wave transmission through and overtopping the breakwater, and locally generated wave. The locally wave generated by wind or ship can be considered as minor and negligible.

The wave height in the port basin can be calculated by the following formula:

$$H_{basin} = (K_{dif} + K_t) H_{sea} \quad (7)$$

where H_{sea} is the wave height outside the basin, K_{dif} is the diffraction coefficient and K_t is the transmission coefficient.

According to [3], K_t can be calculated as follows:

$$K_t = 0.46 - 0.3 \frac{R_c}{H_s} \quad (8)$$

where R_c is the crest freeboard and H_s is the incident wave height.

The limit state function for the excessive wave height is the following:

$$Z = H_{allow} - H_{basin} \quad (9)$$

where H_{allow} is the allowable wave height inside the port basin.

3. ECONOMIC OPTIMISATION

3.1. Cost Quantification

An economic optimal design is defined as the design for which the total lifetime costs are minimal with some design constraint. The total generalised cost includes the costs of construction, the expected costs of failure and the costs of maintenance.

The costs of construction can be formulated as follows:

$$I_{cons} = I_0 + I(d) \quad (10)$$

where I_0 is an initial cost which does not depend on the design variables (e.g. mobilisation cost or cost for design), d is the vector of design variables and $I(d)$ is the construction cost that is a function of the design variables.

The expected costs of failure consist of the economic damage and the costs of repair. The expected costs of failure are calculated for the total lifetime of the breakwater, including the costs of repair and the economic losses caused by the suspension of harbour operations. In case of a collapse of the breakwater, the failure costs per one event are multiplied by the probability of ULS failure $P_{f,ULS}$. In case of intranquillity, the cost per year of interrupted operation is multiplied by the probability of SLS failure per year $P_{f,SLS}$.

The following formula is applied to calculate the expected costs of failure:

$$I_{failure} = \sum_{i=1}^N \frac{C_{ULS} P_{f,ULS}(d)}{(1+r)^i} + \frac{365 \cdot C_{SLS} P_{f,SLS}(d)}{(1+r)^i} \quad (11)$$

where C_{ULS} is the damage cost per event and C_{SLS} are the damage cost per day in case of ULS and SLS failure, r is the interest rate, N is the reference time in which ULS and SLS failures can occur. The factor 365 is needed to convert the damage cost per day to a damage cost per year.

The cost of maintenance depends on the maintenance strategy, which in turn depends very much on the port authorities, the port's budget distribution and other aspects. In this paper the maintenance cost is neglected.

The total cost now becomes:

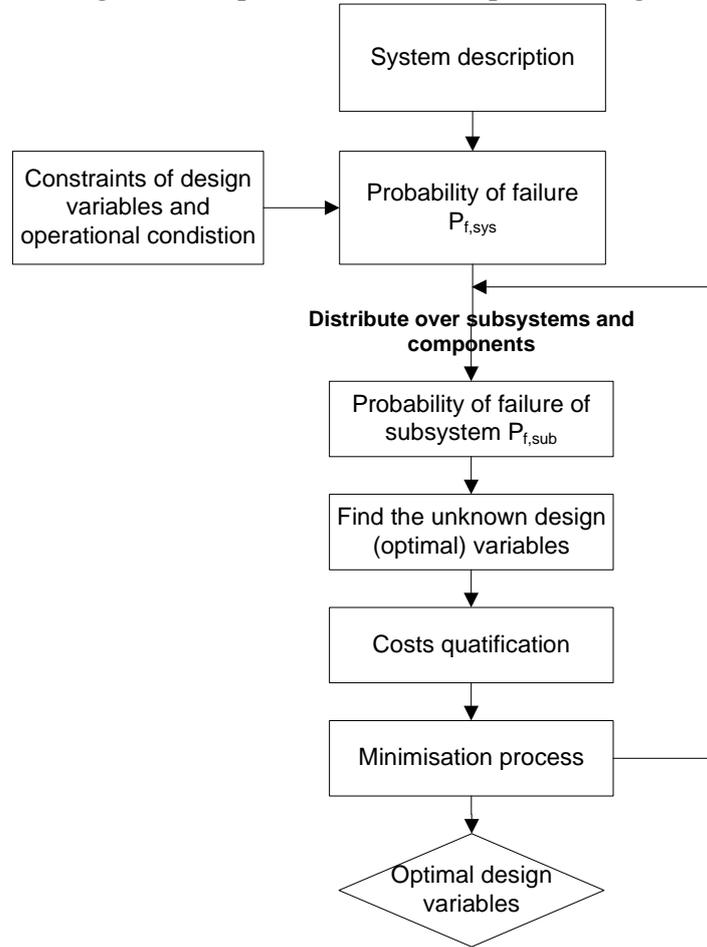
$$I(d) = I_0 + I(d) + \sum_{i=1}^N \frac{C_{ULS} P_{f,ULS}(d)}{(1+r)^i} + \frac{365 \cdot C_{SLS} P_{f,SLS}(d)}{(1+r)^i} \quad (12)$$

3.2. Proposed Method for Optimal Design

In stead of starting the design process with some guess values, the proposed method starts by calculating the probability of failure and distributes it over subsystems and components. The proposed optimisation method is described in Figure 2.

Two problems that should be emphasized in this scheme are the distribution of probability of system failure and the determination of unknown design variables.

Figure 2: Proposed Scheme for Optimal Design



The probability of system failure can be distributed over subsystems and components following the rule of equilibrium that a difference should not be more than 2 or 3 orders of magnitude. Formulae of system reliability calculation are applied based on the form of the system. For example, a series system including some elements are considered. If the elements are mutual exclude and independent, the following formula is valid:

$$P_{f,sys} = \sum_{i=1}^n P_{f,i} \quad (13)$$

If the strengths of the elements are statistically independent, the following formula is valid:

$$P_{f,sys} = 1 - \prod_{i=1}^n (1 - P_{f,i}) \quad (14)$$

where $P_{f,sys}$ is the probability of the system, $P_{f,i}$ is the probability of element i^{th} among n elements.

The determination of unknown design variables is a reverse of the failure probability calculation process, which is presented in [8,14]. In this process, the probability of failure is described under the form of reliability index β . Given the LSF $g = g(X_1, X_2, \dots, X_j, \dots, X_n)$ and the probability of failure P_f , the process is as follows.

Step 1: calculate the reliability index by the following formula:

$$\beta = -\Phi^{-1}(P_f) \quad (15)$$

where Φ is the standard normal distribution function.

Step 2: express the LSF in terms of reduced variables Z_i

$$Z_i = \frac{X_i - \mu_{X_i}}{\sigma_{X_i}} \quad (16)$$

$$g = g(Z_i) \quad (17)$$

Step 3: express the LSF in terms of reliability index β and influence factors α_i

$$Z_i = \beta \alpha_i \quad (18)$$

$$g = g(\beta \alpha_i) \quad (19)$$

Step 4: calculate the mean value of the unknown design variable X_j using Eq.16, Eq.17:

$$\mu_{X_j} = f(\beta, \alpha_i) \quad (20)$$

Step 5: calculate the α_i values, using the following formula:

$$\alpha_i = \frac{-\frac{\partial g}{\partial Z_i}}{\sqrt{\sum_{k=1}^n \left(\frac{\partial g}{\partial Z_k}\right)^2}} \quad (21)$$

Step 6: solve the system of (n+1) equations in Step 4 and 5 to find μ_{X_j} and α_i .

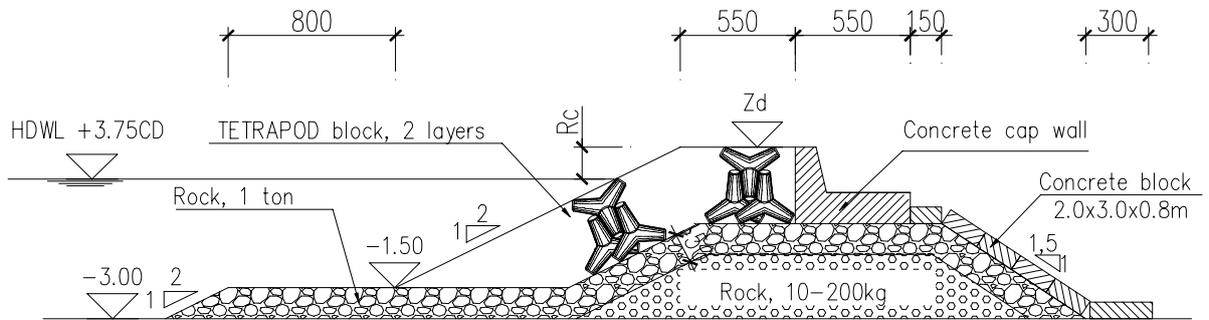
The (n+1) equations system in Step 6 can be solved by hand by iteration method or by mathematical computer programs.

The cost quantification in the scheme follows Eq.10, 11, 12.

4. THE DESIGN CASE

In this paper, a fictitious design case is considered. A rubble mound breakwater is designed in the location of the South of Dason, Haiphong, Vietnam. An overview of the conceptual design cross-section is given in Figure 3. The master plan of the breakwater system is fixed with a total length of 6000 meters. The requirement is to find the optimal weight of Tetrapod blocks and the optimal crest height. The operational constraint in this case is that the allowable port downtime is no more than 5 days per year.

Figure 3: Cross-section of the Design Case



The design parameters and cost indication are originated from a similar study for optimal conceptual design of a rubble mound breakwater in the same location (see [4,7]). An overview of the relevant input data and parameters are presented in Table 1. Damage costs are summarized in Table 2.

Table 1: Overview of Relevant Input Parameters

No.	Variables and Parameters	Type	Distribution Parameters	
			μ	σ
1.	H_s (ULS -during extreme conditions)	Gumbel	3.55 [m]	0.4
2.	S_{om}	normal	0.045	0.004
3.	N	normal	3000	200
4.	H_s (SLS - during normal conditions)	Weibull	1.6 [m]	0.2
5.	h_t	normal	6.25 [m]	0.5
6.	G_c	normal	5.5 [m]	0.5
7.	ρ_c	normal	2.2 [ton]	0.1
8.	ρ_r	normal	2.6 [ton]	0.05
9.	N_{od} (Tetrapod layer)	deterministic	0.5	
10.	N_{od} (toe)	deterministic	2	
11.	ρ_w	deterministic	1.025 [ton]	
12.	H_{allow}	deterministic	0.8 [m]	
13.	K_{dif}	deterministic	0.325	
14.	f_i	deterministic	1.1	

Table 2: Summary of Damage Costs

Failure Type	Damage Description	Damage Amount (US \$)
SLS	Economic damage per day	1.12×10^6
ULS	Structural damage per event	600,000 + 20% of construction costs of the breakwater
	Economic damage per event	320×10^6

In the reliability analysis of the breakwater, three failure modes are considered. They are the instability of the armour layer, the erosion of the toe and the excessive wave height inside the basin. Eq.1-9 are applied.

Results of the proposed optimisation process with regards to the block weight (represented by D_n) and the crest height Z_d are given in Figures 4 in case the port downtime is 5 days per year. Figure 5 presents the total generalized cost in case the port downtime varies from 1 to 5 days per year.

Figure 4 : Costs as a Function of Design Variables

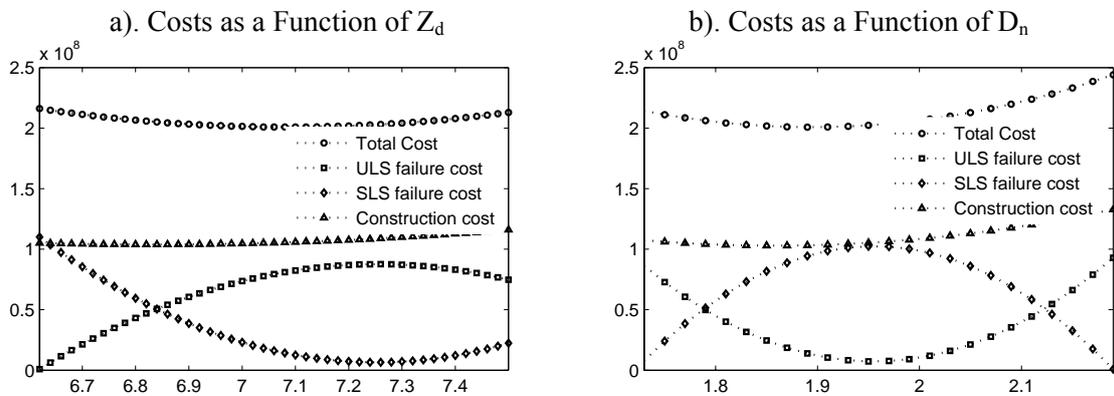


Figure 5: Costs Corresponding to Port Downtime

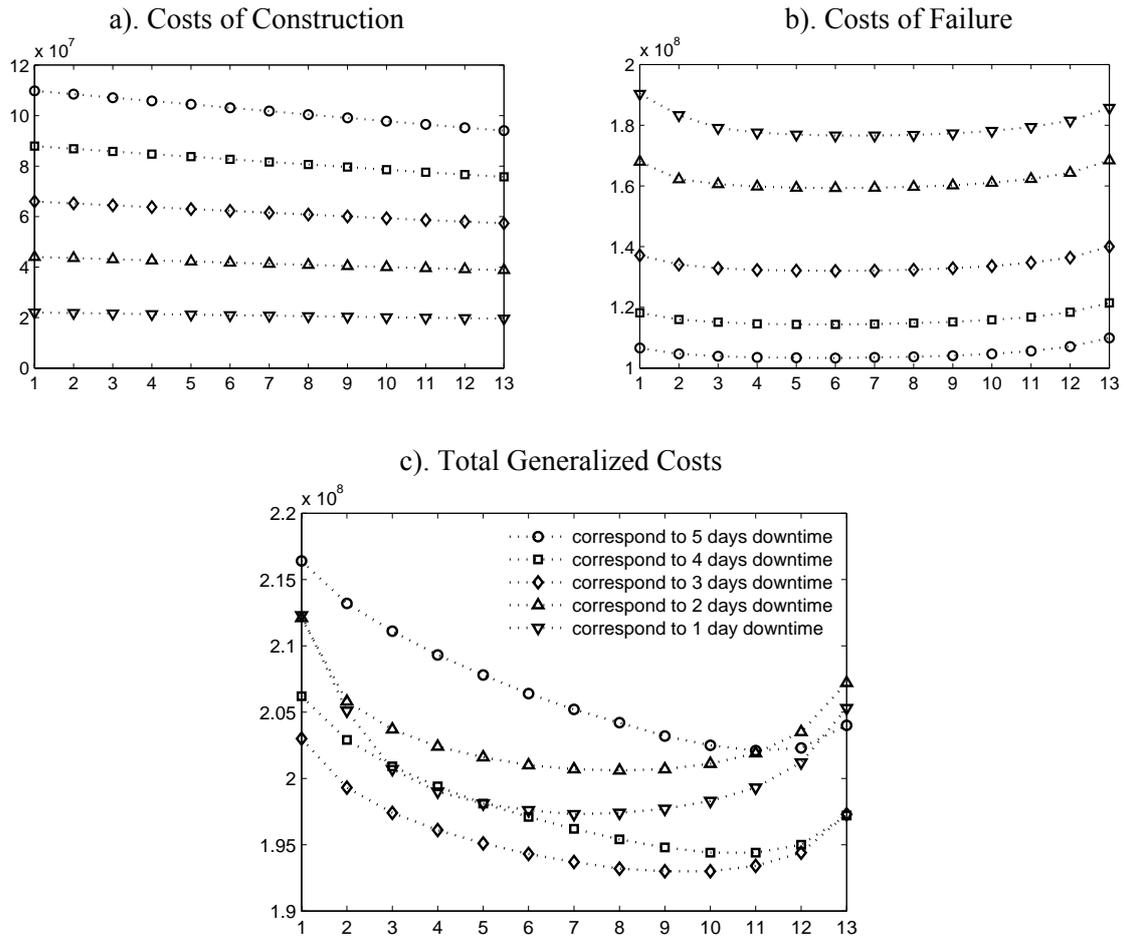


Figure 5 shows clearly the relation between the design variables and the costs. In this figure, each point in the cost curves corresponds to a pair of D_n and Z_d which satisfies the constraint of the port downtime. The horizontal axis describes the varying trend of the design variables, D_n and Z_d . The left side of the axis corresponds to a larger value of D_n and a smaller value of Z_d , and vice versa. When the port downtime decreases, the costs of failure reduces but the cost of construction increases. Therefore an optimal solution is found when a balance between the costs of failure and the costs of construction is obtained. In this case, the optimal solution is found at $Z_d = 7.02$ m and $D_n = 1.82$ m which is equivalent to a Tetrapod of 13.3 tons. The optimal port downtime is 3 days per year.

5. CONCLUSIONS

A method for economic optimization of rubble mound breakwater cross sections, conditioned to constraints of design variables and operational conditions, has been demonstrated in this paper. The method proves to be useful to achieve an estimate of optimal design variables. The provided example illustrates how this method can be applied in practice and how to analyze the results. In the example, the Tetrapod block weight and the crest height are optimized. Other design variables can also be optimized following the same approach.

The proposed method enables designers to find directly design variables through pre-defined probabilities of failure. The more constraint to the designers are provided, the more accurate the design variables are. Such optimized design variables can be used as good starting point in a conceptual design. Appearance of costs of failure in the proposed method allows a better economic efficiency in a long lifetime of structures. In order to avoid an intensive calculation, only relevant

design variables should be optimized. Other design variables can be determined following design codes and guidelines.

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