

SOCIETAL RISK AND THE CONCEPT OF RISK AVERSION

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ABSTRACT

It seems generally accepted that the FN-curve is a fairly accurate description of the societal risk. However in the communication with the public and representative decisionmakers a schematisation of the FN-curve to one or two numbers may bring certain advantages. Various measures like Potential loss of Life, the area under the FN-curve, the Risk Integral etc. are proposed in the literature. Although the formulae look distinctly different at first sight a more thorough inspection reveals, that all schematisations contain as building blocks the two familiar statistical moments of the FN-curve, the expected value of the number of deaths $E(N)$ and the standard deviation $\sigma(N)$. In the paper the linear combination $E(N) + k \cdot \sigma(N)$ is proposed as a simple risk averse measure of the societal risk.

KEYWORDS

Risk analysis, acceptable risk, societal risk, individual risk, risk aversion, decision making.

1 INTRODUCTION

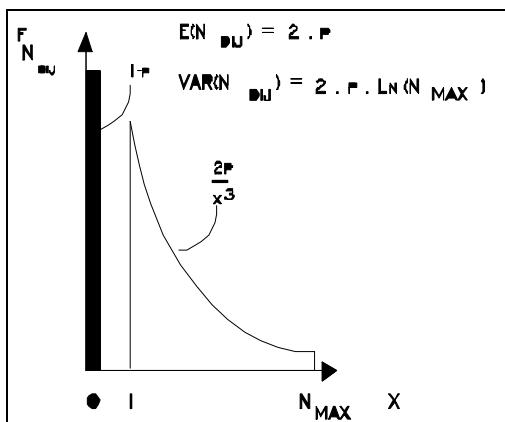
There is general agreement in the literature and in regulatory circles that risk should at least be judged from two points of view (VROM (1988,1992), HSE (1989)). The first point of view is that of the individual, who decides to undertake an activity weighing the risks against the direct and indirect personal benefits. This first point of view leads to the personally acceptable level of risk or the acceptable individual risk, defined in as "the frequency at which an individual may be expected to sustain a given level of harm from the realisation of specified hazards". The specified level of harm is narrowed down to the loss of life in many practical cases.

The second point of view is that of the society, considering the question whether an activity is acceptable in terms of the risk involved for the total population. Commonly the notion of risk in a societal context is reduced to the total number of casualties (VROM (1988,1992), HSE (1989)) using a definition as by IoCE (1985): "the relation between frequency and the number of people suffering from a specified level of harm in a given population from the realisation of specified hazards". If the specified level of harm is narrowed down to the loss of life, the societal risk may be modelled by the frequency of exceedance curve of the number of deaths, also called the FN-curve due to a specific hazard.

The FN-curve can be seen as an exceedance curve with a related probability density function (p.d.f.) of the number of deaths. The p.d.f. of the number of deaths N_{dij} given an accident for activity i at place j can have many forms. A few types are presented here to stimulate further thinking. The first conditional p.d.f. is the Dirac, that limits the outcomes to exactly N fatalities. Other possibilities that allow a larger variation in the outcome, are the exponential and the log-normal p.d.f. The probability of exceedance curves of the number of fatalities, that can be derived from these two forms reflect to some extent the FN-curves found in practical quantitative risk assessment (QRA) studies. A fourth is the inverse quadratic Pareto distribution that coincides precisely with the norm put forward by the Ministry of VROM (1988). The Pareto p.d.f. has no finite standard deviation unless the right tail is truncated (Fig.1).

Exactly the same models could be applied for the material damage that results from a disaster, if the horizontal axis is measured in monetary units. It should be noted that the proposed conditional p.d.f.'s have to be multiplied with the probability p of an accident and that the outcome zero fatalities with probability $(1-p)$ should be added to find the complete p.d.f. of the number of deaths (Fig.1). The classical measures of expected value and standard deviation will appear to be very useful numbers to classify the risk.

probability density function



Inverse quadratic Pareto

probability of exceedance

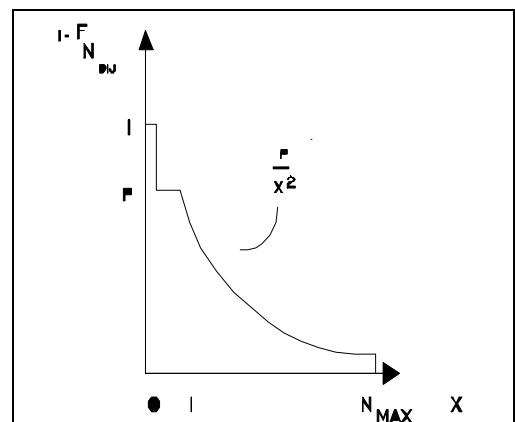


Figure 1: A theoretical p.d.f. and probability of exceedance curve for the number of deaths.

A controversy is found on the way to judge and limit the societal risk. Many apparently different judgemental numbers and normative decision rules can be discerned as will be shown below. Also the question of these numbers and rules include risk aversion is the subject of debate. Some analysis makes however clear that the rules and numbers proposed in the literature fall in two categories: risk neutral or risk averse. The direction of the development seems to be towards risk averse measures, although this trend is somewhat obscured by mathematical complexity.

2 DIFFERENT MEASURES AND LIMITATIONS OF SOCIETAL RISK

It seems generally accepted that the FN-curve is a fairly accurate description of the societal risk. However in the communication with the public and representative decision makers a schematisation of the FN-curve to one or two numbers may bring certain advantages. As to the limitation of the societal risk to acceptable levels many different rules are proposed by scientists and regulatory bodies. Disagreement is especially found on the question if societal risk should be judged with a risk averse or a risk neutral attitude.

One of the oldest simple measures of societal risk is the Potential Loss of Life (PLL), which is defined as the expected value of the number of deaths per year:

$$E(N) = \int x \cdot f_{Ndij}(x) dx \quad (1)$$

where : f_{Ndij} = the p.d.f. of the number of deaths resulting from activity i in place j in one year

Ale (1996) has proposed the area under the FN-curve as a simple measure of societal risk. Although this is not immediately clear, it can be mathematically proven that the area under the FN-curve equals the expected value of the number of deaths (appendix 1).

$$E(N) = \int (1 - F_{Ndij}(x)) dx \quad (2)$$

An absolute limit to the expected value of the number of deaths is not mentioned in the literature. The use of the expected value seems very valuable in the comparison of various alternatives.

VROM(1988) limits the societal risk at plant level by a line that is inversely proportional to the square of the number of deaths. This absolute requirement that formed the basis for the regulation and the siting of hazardous installations or new developments in the Netherlands during the last decade reads:

$$\frac{1 - F_{Ndij}(x)}{x^2} < \frac{10^{-3}}{x^2} \quad (3)$$

for $x \geq 10$ deaths,

where : F_{Ndij} = the c.d.f. of the number of deaths resulting from activity i in place j in one year (the subscript dij will be omitted further on).

The HSE (1989) remarks that the judgement of the societal risk at plant level by the VROM-rule is overly risk averse. HSE proposes to change the value of the exponent in the expression from 2 into 1 in order to form a more even judgement. In recent papers Cassidy (1996) of HSE defined the risk integral RI as an appropriate measure of societal risk that should be further explored:

$$RI = \int x \cdot (1 - F_N(x)) dx \quad (4)$$

A limiting value is however not yet attached to this new concept.

Vrijling (1995) notes that the societal risk should be judged on a national level by limiting the total number of casualties in a given year in the following way:

$$E(N_{di}) + k \cdot s(N_{di}) < b_i \cdot 100 \quad (5)$$

where : k = risk aversion index.

The formula (5) accounts for risk aversion, which will certainly influence acceptance by a community or a society. Relatively frequent small accidents are more easily accepted than one single rare accident with large

consequences, although the expected number of casualties is equal for both cases. The standard deviation of the number of casualties will reflect this difference. The risk aversion is represented mathematically by increasing the expectation of the total number of deaths, $E(N_{di})$, by the desired multiple k of the standard deviation before the situation is tested against the norm.

Rule (5) can be transformed into a similar expression valid at plant level by taking into account the number of independent installations N_A . It can also be transformed mathematically into a VROM-type of rule applicable at plant level as shown in the same paper:

$$I - F_{Ndi}(x) \leq \frac{C_i}{x^2} \quad \text{for all } x \geq 10$$

where $C_i = \left[\frac{b_i 100}{k \sqrt{N_A}} \right]^2$

(6)

For values of $\beta_i = 0.03$, $k = 3$ and $N_A = 1000$ the rule equates exactly to the VROM-rule, which appears to be a specific case in a more general framework.

Bohnenblust (1996) judges the number of casualties after correction with a factor $\varphi(x)$ in an economic framework. Weighing the societal risk SR in the light of the cost of measures to improve safety an optimal decision is reached. Changing the summation into an integral the expression proposed by Bohnenblust reads:

$$SR = \int x \cdot f(x) \cdot f_N(x) dx$$
(7)

Although not explicitly stated by Wehr and Bohnenblust (1995), it can be deduced from a graph in the paper that $\varphi(x) \sim \sqrt{x/10}$. So the SR measure could be expressed as:

$$SR = \int \sqrt{10} \cdot x^{1.5} \cdot f_N(x) dx$$
(8)

3 THE RISK ATTITUDE OF THE VARIOUS RULES

First it should be stated that in this paper a decision based on the expected value only is called risk neutral. Risk neutrality can be modelled with a linear utility function. In case of a risk averse attitude a smaller standard deviation is preferred above a larger in case of equal expected values. In the literature this is frequently modelled by quadratic utility functions. To show the principle the expected utility is evaluated below for a linear and a quadratic utility function:

$$\begin{aligned} \int ax \cdot f_N(x) dx &= a \cdot E(N) \\ \int a x^2 \cdot f_N(x) dx &= a \cdot (E(N)^2 + s(N)^2) \end{aligned}$$
(9)

In case of risk aversion the standard deviation starts to play a role. However the strict application of quadratic utility curves has as a disadvantage that the units become [death² /year], which is difficult to communicate to the public and most probably also to decision makers.

Using the concept of risk attitude the PLL and the area under the FN-curve, that are both equal to E(N), can be classified as risk neutral measures of the societal risk.

The rule proposed by Vrijling(1996), containing $E(N) + k \cdot \sigma(N)$, is clearly risk averse. Consequently the VROM-rule which is proven to be a special case of this rule can be similarly classified as risk averse.

If the exponent of the VROM-rule is changed into 1 as is proposed by HSE only the expected value of the number of casualties is limited, which according to the definition given above should be called risk neutral.

The measure proposed by Bohnenblust (1995,1996) has an intermediate position with an exponent of 1.5.

It is very interesting to note that is can be mathematically proven (appendix 2) that the risk integral proposed by Cassidy (1996) of HSE equals:

$$RI = \int x \cdot (1 - F_N(x)) \cdot dx = \frac{1}{2} (E^2(N) + s^2(N)) \quad (10)$$

Apparently the need for a simple risk averse measure to schematise the FN-curve is also felt in the United Kingdom. A disadvantage of the risk integral RI might be that the units are [death² /year] and some difficulty will be met in formulating an easy to understand limiting value.

4 CASE STUDY

The half of Holland that lies below the sea level is divided in $N_{Ai}= 40$ more or less independent polders surrounded by dike-rings. If it is assumed that at some future date each polder will house $N_{pij}= 1,000,000$ inhabitants, an estimate of the number of casualties in case of flooding can be made. In 1953 approximately 1% of the inhabitants drowned, giving a value of $p_{d|fi}=0.01$. Little is known of the influence of modern technological development on this number, but the failure of energy and communication networks during the minor floods in Limburg point to a limited beneficial influence.

The expected value and the standard deviation of the number of deaths in 40 independent polders per year are equal to:

$$\begin{aligned} E(N_{di}) &= 40p_f 10^{-2} 10^6 \\ s^2(N_{di}) &= 40p_f (1-p_f)(10^{-2} 10^{-6})^2 \end{aligned} \quad (11)$$

If these expressions are substituted in the norm Eqn. 5 the solution for $\beta_i=1$ becomes $p_{fij}=3.10^{-7}$ per year. In case the aversion of the inhabitants against flooding is more extreme and $\beta_i=0.1$ the acceptable probability of failure of the dike ring is $p_{fij}=3.10^{-9}$ per year.

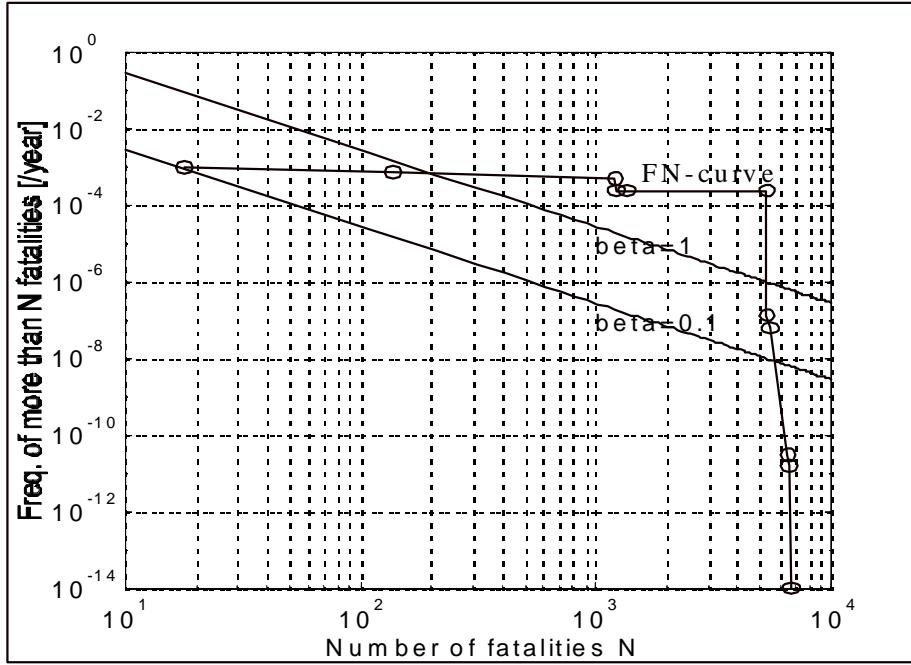


Figure 2: FN-curve for flooding of the Brielse polder

For the Brielse dike ring near Rotterdam a FN-curve (Fig.2) has been drawn estimating the probability of failure of the existing dikes at 10^{-4} per year. The FN-curve shows that there are five equally likely scenario's with death counts varying from 15 to appr. 5000 people. As these scenario's are assumed to be independent, the combinations, that claim even more casualties, are less likely by an order of magnitude.

Developing the local criterion for a dike ring using the values mentioned above, the constant becomes $C_i = 27.8 - 0.278$ for $\beta = 1-0.1$. Thus the present situation based on the philosophy developed by the Deltacommittee (1960) seems insufficiently safe in the light of modern developments. Following the normative framework developed here the acceptable probability of failure of the dike equals $6.3 \cdot 10^{-7}$ to $6.3 \cdot 10^{-9}$ depending on the value of β .

5 CONCLUSIONS

Although the FN-curve, the exceedance curve of the number of deaths, is generally accepted as a clear representation of the societal risk regarding the loss of human life, a search for one simple number to express the societal risk can be observed. Several schematisations of the FN-curve are proposed. Some of these schematisations are completed with a limiting value to provide a decision rule. Although the formulae look distinctly different at first sight a more thorough inspection reveals a common approach that promises a relatively rapid convergence of opinions.

It appears that all schematisations contain as building blocks the two familiar statistical moments of the FN-curve, the expected value of the number of deaths $E(N)$ and the standard deviation $\sigma(N)$. The Potential Loss of Life (PLL) measure and the area under the FN-curve, as a simple measure of the societal risk, are both equal to $E(N)$. However no absolute use of this measure is mentioned and consequently no limiting value is reported in the literature.

Using the concept of risk aversion it was proven that the most recently proposed ways to judge societal risk are all risk averse. The well known VROM-rule, that limits the FN-curve by $10^{-3}/N^2$ appears to be a special case of a more general rule proposed by Vrijling (1995), that limits the societal risk at the Dutch national level by: $E(N) + k \cdot \sigma(N) < \beta \cdot 100$.

The criticism of the VROM-rule, that the exponent of 2 is overly risk averse and that a value of 1 should be preferred, leads to a limitation of $E(N)$ only. Limiting the FN-curve by C/N places an upperbound to $E(N)$ and must be classified as a risk neutral approach.

The risk integral RI, recently proposed by HSE as an alternative measure needing further investigation, is shown to be equal to $0.5\{E(N)^2 + \sigma(N)^2\}$. Thus the risk integral should be classified as a risk averse measure of societal risk. The units [death²/year] in which RI is expressed may hamper the understanding of this measure by the public.

A linear expression like $E(N) + k.\sigma(N)$ with units [death/year] is preferred. In addition the relatively simple relation between this measure and a VROM-type of rule provides the possibility of unification.

Attention should be paid to the fact that some rules (e.g. Vrijling(1995)) limit the societal risk at a national level, while most others (VROM,HSE) address the risk at plant level. Because "many small unrestrained developments could add up to a noticeable worsening of the overall situation" (HSE(1989)), the societal risk should be limited in a concerted way on both national and local level. In the approach of Vrijling (1995) the societal risk is limited at national level and consequently taking the number of hazardous installations into account a VROM-type of rule is derived for plant level.

5 LITERATURE

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APPENDIX 1 (PROOF OF EQN 2)

$$\int_0^{\infty} (1 - F_N(x))dx = \int_0^{\infty} \int_x^{\infty} f_N(u)dudx = \int_0^{\infty} \int_0^u f_N(u)dxdu = \int_0^{\infty} u f_N(u)du = E(N)$$

APPENDIX 2 (PROOF OF EQN 10)

$$s^2(N) = \text{var } N = EN^2 - E^2 N,$$

$$So \quad E^2 N + s^2 N = EN^2,$$

$$\begin{aligned} So \quad \frac{1}{2}(E^2 N + s^2 N) &= \frac{1}{2}EN^2 = \int_0^{\infty} \frac{1}{2}x^2 f_N(x)dx = \int_0^{\infty} f_N(x) \int_0^x u du dx = \int_0^{\infty} \int_0^x u f_N(x) du dx = \int_0^{\infty} \int_u^{\infty} u f_N(x) dx du = \\ &= \int_0^{\infty} u \int_u^{\infty} f_N(x) dx du = \int_0^{\infty} u(1 - F_N(u)) du \end{aligned}$$