An overview of quantitative risk measures and their application for calculation of flood risk

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Summary:

An overview of methods to quantify and limit risks was still missing in literature. Therefore a study of risk literature was performed by the authors. This article gives an overview of quantitative risk measures. A risk measure is defined as a mathematical function of the probability of an event and the consequences of that event. The article focuses on risk measures for loss of life (individual and societal risk) and economic risk. . For every risk measure the most important characteristics are given: the mathematical formulation, the field of application and the standard set in this field. Some of the measures have been applied to a case study, in which the flood risks for an area in the Netherlands have been calculated. The calculated flood risks are compared to the risks in other fields and the existing standards in these sectors.

1 Introduction

Human existence involves exposure to many hazards. Natural disasters such as floods and earthquakes cost thousands of lives every year all over the world. Since the industrial revolution also technical hazards, such as aeroplane crashes, train derailments, tunnel fires and industrial accidents disrupt society on a regular basis.

Long ago people tried to guard themselves from natural hazards with relatively simple methods, for example by building their houses on high grounds to protect them against floods. As society changed protection systems were built, such as dams and dikes. Later new technological inventions, for instance nuclear power plants and aviation, and their accompanying hazards were introduced. Other developments such as population growth and growing levels of production, consumption and transportation have lead to an increase of hazards and the consequences of the accidents. Nowadays large amounts of money are spent to protect society against these disasters. However, in decision- and policymaking these expenditures on safety have to compete with other public interests, for instance public health and the development of new infrastructure.

It is also important to realise that decision making about risks is very complex and that not only technical aspects but also political, psychological and societal processes all play an important role. In this complex decision making process a clear identification of the risks and the effects of risk reducing measures is very useful. From a technical point of view the extent of the risks and the effects of risk reducing measures can be quantified in a quantitative risk assessment (QRA). It is therefore that the QRA can provide a basis for the rational decision making about risks. In literature on quantitative risk assessment generally four phases are distinguished:

- **qualitative analysis:** In this step the system and the scope are defined, and the hazards and failure modes and scenarios are identified and described.
- **quantitative analysis:** The probabilities and consequences of the defined events are determined. The risk is quantified in a risk number or graph as a function of probabilities and consequences
- **risk evaluation:** With the results of the former analyses the risk is evaluated. In this phase the decision is made whether the risk is tolerable or not.
- **risk control and risk reduction measures:** Dependent on the outcome of the risk evaluation measures should be taken to reduce the risk. It should also be determined how the risks can be controlled (for example by inspection, maintenance or warning systems).

Risk measures play an important role in communicating the whole risk assessment process. A risk measure is defined as a mathematical function of the probability of an event and the consequences of that event. This risk measure forms the basis for evaluation of risks by the decision-makers. With limits or standards an acceptable risk level is set. Finally the risk measure can be used as an instrument to show the effect of risk reducing actions.

In the study of flood risk in the Netherlands it was found that an overview of methods to quantify and limit risks was still missing in literature. Therefore a study of risk literature was performed by the first author [1] with the aim to give an overview of quantitative risk measures. Measures that deal with the risk qualitatively were not considered. This article focuses mainly on risk measures that consider loss of life and economic damage as a consequence. A more extensive article with a description of other risk measures (for instance environmental) and methods to quantify the value of human life is being prepared for publication in a journal. The risk measures are categorised in sections based on the consequences they consider:

- Fatalities
  - Individual risk (section 2)
  - Societal risk (section 3)
- Economic damage (section 4)

For every category the most important characteristics of the risk measures are described from a technical perspective. In every section first an overview is given of the risk measures and their mathematical expressions. Consequently the fields of application and the standards used are described. To show the possible applications a case study has been performed in section 5, in which the flood risks have been calculated for an existing area in the Netherlands with some of the described risk measures. The article is concluded in section 6 with a compact overview of the risk measures and their most important characteristics in a table. Finally an evaluation of the case study is given.

2 Individual Risk

2.1 Individual Risk Measures

The first measure is the individual risk (IR) as used by the Dutch Ministry of Housing, Spatial Planning and Environment (VROM). It is defined as the probability that an average unprotected person permanently present at that point location, would get killed due to an accident at the hazardous activity [2].

$$IR = P_f \cdot P_{d|f}$$

In which:

P_f probability of failure

$P_{d|f}$ probability of dying of the individual in the case of failure, assuming the permanent unprotected presence of the individual

The IR is thus a property of the place and as such useful in spatial planning. A slightly different definition, in which the actual presence of the individual is considered, is used by Dutch Technical Advisory Committee on Water Defences (TAW) [3] and by Bohenblust [4] to describe the actual personal risk.

An overview of measurements to express the individual risk is given by Bedford and Cooke [5]. Apart from the individual risk as mentioned above four other expressions are described. The *loss of life expectancy* shows the decrease of life expectancy due to various causes. The *delta yearly probability of death* computes the intensity at which a given activity is performed (at suitable units) in order to increase the yearly probability of death by 10^{-6}. The *activity specific hourly mortality rate* reflects the probability per time unit of dying while engaged in a specified activity. An example is the Fatal Accident Failure Rate (FAFR) which gives the number of fatalities per 1000 hours of exposure to a certain risk. A variant is the *death per unit activity*, which replaces the time unit by a unit measuring the amount of activity. The risks of travel by car, train or aeroplane are often expressed in the form of the number of deaths per kilometre travelled.

2.2 Fields of application and standards

The measure of individual risk is used in the Netherlands to determine the risks of hazardous installations, transport routes and airports. Locations with equal individual risk levels are shown on a map with so-called risk contours, that facilitate land use planning purposes. In figure 1 the risk contours for a hazardous installation and a transport route are shown.

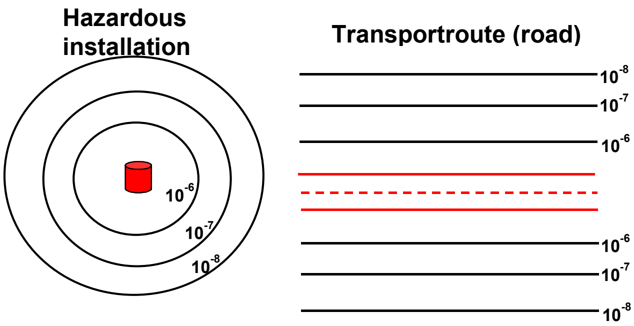


Figure 1: Characteristic individual risk contours for a hazardous installation (point source) and a transport route (line source)

To limit the risks the Dutch Ministry of Housing, Spatial planning and Environment (VROM) has set the following standard for populated areas [2].

$$IR < 10^{-6}\left(yr^{-1}\right)$$

Risks lower than 10^{-6} per year should always be reduced to a level as low as is reasonably achievable (ALARA). In Canada and UK similar standards for the individual risk for hazardous installations have been set. These standards are set for more or less involuntary imposed risks related to the siting of hazardous activities. The method of TAW [3] gives the opportunity to limit a broader set of risks ranging from voluntary activities, such as mountaineering, to more involuntary risks, such as those of hazardous installations. The following standard is proposed by TAW:

$$IR < \beta \cdot 10^{-4}\left(yr^{-1}\right)$$

In this expression the value of the policy factor β varies according to the degree of voluntariness of the activity and with the benefit perceived. It ranges from 10 in the case of complete freedom of choice (for mountaineering), to 0.01 in case of an imposed risk without any perceived direct benefit. This method has been used in case studies concerning various risks.

The background of the standard proposed by Bohnenblust [4] is comparable to the TAW standard. Bohnenblust limits the acceptable IR taking into account the voluntariness and the degree of self-control of the activity. Four risk categories have been determined ranging from voluntary to involuntary. The proposed limits of Bohnenblust and TAW are shown in figure 2. Bohnenblust studies the safety of the railway system in Germany [4].

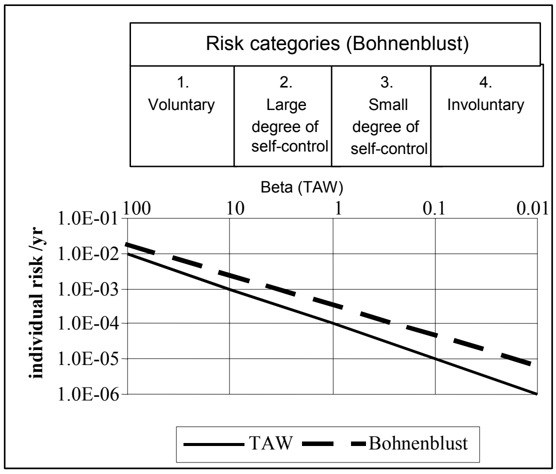


Figure 2: Individual risk standard according to Bohnenblust and TAW

3 Societal Risk

Societal risk is defined by Ichem [6] as "the relationship between frequency and the number of people suffering from a specified level of harm in a given population from the realisation of specified hazards". Where individual risk gives the probability of dying on a certain location, the societal risk gives a number for a whole area no matter where precisely within that area the harm will occur. The difference is shown in figure 3.

3.1 Societal Risk Measures

The aggregated weighted risk (AWR) as described by Piers [7] is calculated by multiplying the number of houses inside a certain area with their IR level:

$$AWR = \iint_A IR(x,y) \cdot h(x,y) \cdot dxdy$$

In which:

IR(x,y)	individual risk on location (x,y)
h(x,y)	number of houses on location (x,y)
A	area for which the AWR is determined

By integrating the individual risk levels and the population density the expected value of the number of fatalities can be determined [8]:

$$E(N) = \iint_A IR(x,y) \cdot m(x,y) \cdot dxdy$$

In which:

E(N) expected value of the number of fatalities per

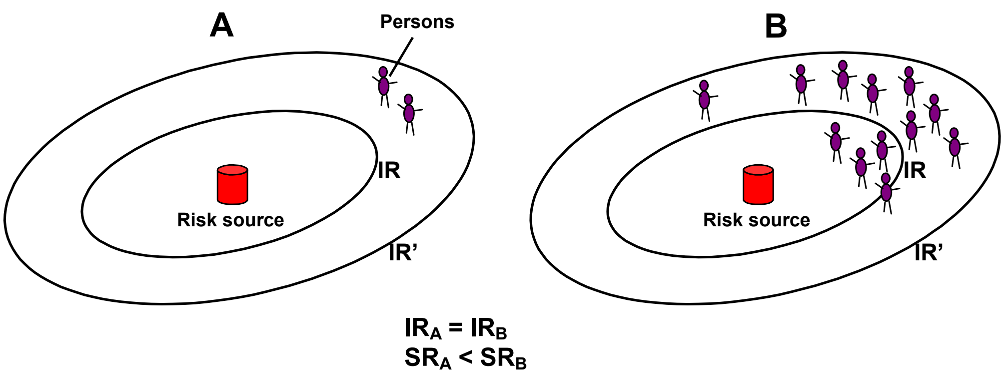


Figure 3: The difference between individual and societal risk. Both situations have equal individual risk levels (shown by IR' and IR). Because of the larger population density of situation B, B has a larger societal risk, based on Stallen et al. [9]

year
m(x,y) population density on location (x,y)

The scaled risk integral (SRI), proposed by Carter and Riley [10], takes into account the individual risk level and other characteristics of the location:

$$SRI = \frac{P \cdot IR \cdot T}{A} \quad \text{where} \quad P = \frac{n + n^2}{2}$$

In which:

IR	individual risk per million year (cpm)
T	proportion of time the area is occupied by n persons
A	the surface of the area in hectares
P	population factor
n	number of persons in the area

Note that the SRI is not dimensionless: (person + person^2)/(hectare·10^6·year).

Societal risk is often graphically shown in the form of a FN-curve. This curve displays the probability of exceedance as a function of the

$$1 - F_N(x) = P(N > x) = \int_x^\infty f_N(x)$$

number of fatalities, on a double logarithmic scale.

In which:

$f_n(x)$	the probability density function (pdf) of the number of fatalities per year
$F_N(x)$	probability distribution function of the number of fatalities per year

A simple measure for societal risk is the expected value of the number of fatalities per year, E(N), in literature often referred to as the Potential Loss of Life (PLL):

$$E(N) = \int_0^\infty x \cdot f_N(x) \cdot dx$$

It has been shown by Vrijling and van Gelder [11] that this measure equals the expected number of fatalities per year. The British Health and Safety Executive (HSE) proposes the risk integral as a measure for societal risk [12]:

$$RI = \int_0^\infty x \cdot (1 - F_N(x)) \cdot dx$$

In [11] it is mathematically proven that the RI can be expressed in two characteristics of the pdf of the number of fatalities, the expected value E(N) and the standard deviation σ(N):

$$RI = \frac{1}{2}(E^2(N) + \sigma^2(N))$$

Smets [13] proposes an expression in which the aversion towards accidents with many deaths is represented by a coefficient α, which can range between 1 and 2.

$$\int_1^{1000} x^\alpha \cdot f_N(x)dx$$

If the integration boundaries are not taken into account the expression of Smets equals the expected value for α=1. If α=2 the expression equals second moment of the pdf:

$$\int x^2 \cdot f_N(x) \cdot dx = E(N^2)$$

Bohnenblust [4] presents the perceived collective risk R_p as a measure

$$R_p = \int_0^\infty x \cdot \varphi(x) \cdot f_N(x) \cdot dx$$

for societal risk:

In which:

ϕ(x)	risk aversion, a function of the number of fatalities

In this measure the expected value of the number of fatalities is weighed with a risk aversion function ϕ(x). From the risk aversion values proposed by Bohnenblust it can be deduced that $\varphi(x) \approx \sqrt{0{,}1 \cdot x}$, [11]. The expression can now be written in the following form:

$$R_p = \int_0^\infty \sqrt{0{,}1} \cdot x^{1{,}5} \cdot f_N(x) \cdot dx$$

A similar measure, the expected disutility of a system U_{sys}, is proposed by Kroon and Hoej [14]:

$$U_{sys} = \int_0^\infty x^\alpha \cdot P(x) \cdot f_N(x)dx$$

Again the weighting factor α has been applied together with a risk aversion factor P(x), which shows the expected disutility as a function of the number of fatalities. Note that the risk integral and the measures proposed by Smets, Bohnenblust and Kroon and Hoej can all be described with the following general formula.

$$\int x^\alpha \cdot C(x) \cdot f_N(x) \cdot dx$$

Different choices have been made by the various authors for the values of α (ranging from 1 to 2) and the factor C, which is a constant or a function of x.

The measure of Total Risk as proposed by Vrijling et al. [15] is constructed out of the expected value of the number of fatalities and the standard deviation which is multiplied by a risk aversion factor k:

$$TR = E(N) + k \cdot \sigma(N)$$

The total risk takes into account a risk aversion index k and the standard deviation and is therefore called risk averse. The standard deviation is relatively high for accidents with low probabilities and high consequences.

A quick overview of the societal risk measures learns that two types of expressions can be distinguished. The FN curve and the expected value are directly derived from the pdf and are therefore called risk neutral. Risk aversion can be modelled by weighing the expected value with a factor α, taking into account a risk aversion factor (P(x) or φ(x)) or by involving the standard deviation in the equation (α=2).

3.2 Fields of application and standards

In the decision-making process about the risks of the Netherlands' national airport Schiphol first the area under the FN curve (= expected value) was proposed as a risk measure. After that the AWR was put forward as measure for the risks. In the past an agreement has been made that AWR levels were no longer allowed to increase, the so-called standstill principle [7].

For expected value determined by the individual risk contours no current use of this method has been found, neither has a standard been proposed.

The SRI has been proposed to use in the decision making about new developments nearby hazardous installations in the UK. For the purposes of decision-making comparison values are used, for examples see Carter and Riley [10].

The FN curve is used in various countries to express and limit the risks of mainly hazardous installations. As part of the Dutch external safety policy the so-called group-risks are determined for various activities on a national level. These are shown in the FN curve in figure 4.

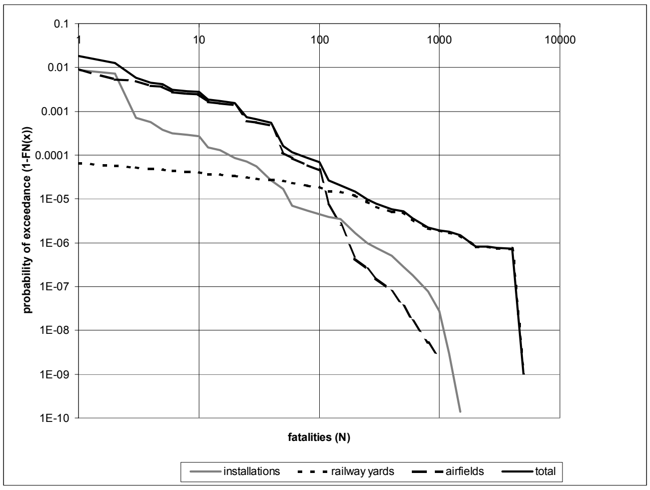


Figure 4: FN curve for the risks of various activities in the Netherlands in 1999 (source: RIVM, NL)

In several countries the risks of various hazardous activities are limited by a FN criterion line. These standards can be described with the following general formula:

$$1 - F_N(x) < \frac{C}{x^n}$$

In which

n	steepness of the limit line
C	constant that determines the position of the limit line

A standard with a steepness of n=1 is called risk neutral. If the steepness n=2, the standard is called risk averse. In this case larger accidents are weighted more heavily and accepted with a relatively lower probability. Some international FN standards are given in figure 5:

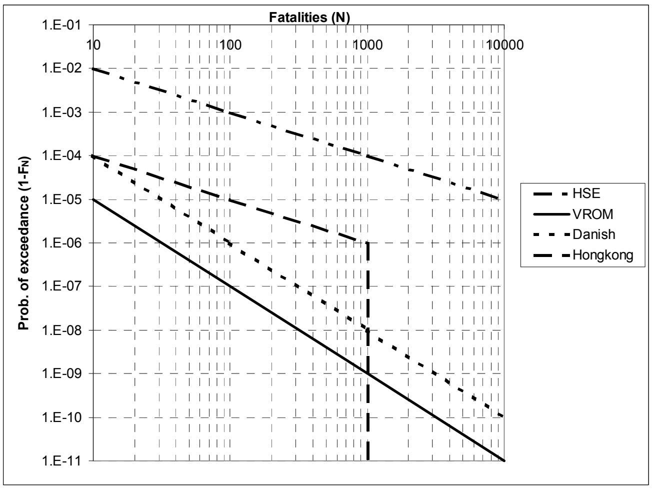


Figure 5: Some international standards in FN format

As a part of the standard usually an ALARA (or ALARP) region has been determined below the limit line, in which the risk should be reduced to a level as low as reasonably achievable (or possible).

The expected number of fatalities is used in the regulation of the risks of dams. Standards have been proposed by British Columbia Hydro [16] and the United States Bureau of Reclamation [17].

BC Hydro: $\mathrm{E(N)} < 10^{-3}\left(\mathrm{fat/yr}\right)$

USBR: $\mathrm{E(N)} < 10^{-2}\left(\mathrm{fat/yr}\right)$

The risk integral (RI = 1/2 ($E(N)^2 + \sigma(N)^2$) has been developed by the Health and Safety Executive for land use planning in the United Kingdom. However, no limiting value is yet attached to it.

In literature no current use of the method of Smets can be found. The limit proposed depends on the steepness of the utility function. For a quadratic utility function (α=2) the following limit has been proposed [9]

$$\int_1^{1000} x^2 \cdot f_N(x) < 10^{-2}$$

No standard has been proposed for the measure of Bohnenblust. It has been used as part of a method, which takes into account individual, societal risk and economic aspects in studies concerning the investments in railway safety in Germany [4].

The measure of Kroon and Hoej has been used for evaluation of the risk attitude for tunnels in an OECD/ PIARC study concerning transport of dangerous goods through tunnels [14].

A standard for the acceptable risk is presented by Vrijling et al. [15]. It limits the risks on a national level considering the policy factor β, which is has already been presented in section 2.2.

$$TR < \beta \cdot 100$$

This national criterion for acceptable risk can be translated into a standard for a single installation or location. This criterion has the typical form of a FN limit, with a quadratic steepness (α=2):

$$1 - F_N(x) < \frac{C}{x^2}$$

Assume that the expected value of the number of fatalities is much smaller than its standard deviation (which in general is true for accidents with low probabilities and high consequences) and a Bernoulli distribution. The factor C can now be written as a function of the number of installations on a national level (N_A), the risk aversion factor (k), and the policy factor (β):

$$C = \left[\frac{\beta \cdot 100}{k \cdot \sqrt{N_A}}\right]^2$$

From this equation it can be concluded that the Dutch limit for societal risk ($C=10^{-3}$, α=2), as presented in figure 6, is a special case of the total risk limit described above. The method of total risk has been used in several case studies, for example in [15].

FN limit lines can suffer from a peculiar inconsistency, which is illustrated by Bedford and Cooke [5]. While the risks of installations are each acceptable by the FN limit line, the risks of the "probabilistic mixture" of these installations can exceed the limit, although the number of installations has not changed. A similar problem can also be recognised. If a risk criterion is defined on a installation level, the height of the national criterion is determined by the number of locations. An increase in the number of installations, each of them acceptable by the local limit, can therefore lead to an unacceptable high-risk level on a national scale. To prevent these problems it is proposed to set a limit on a national level and distribute the acceptable risk over the locations.

4 Economic Risk

Besides the danger of loss of life due to certain activities the economic risks play an important role in the decision making. In this section four approaches to quantify economic risks and their applications are described.

4.1 Economic Risk Measures

The FD curve displays the probability of exceedance as a function of the economic damage. The FD curve and the expected value of the economic damage can be derived from the pdf of the economic damage ($f_D(x)$).

$$1 - F_D(x) = P(D > x) = \int_x^{\infty} f_D(x) \cdot dx$$

$$E(D) = \int_0^\infty x \cdot f_D(x) \cdot dx$$

In which:

$F_D(x)$	the probability distribution function of the economic damage
E(D)	expected value of the economic damage

The problem of the acceptable level of risk can also be formulated as an economic decision problem. In the method of economic optimisation the total costs in a system (C_{tot}) are determined by summing up the expenditure (I) for a safer system and the expected value of the economic damage. In the optimal economic situation the total costs in the system are minimised:

$$\min(C_{tot}) = \min(I + E(D))$$

With this criterion the optimal probability of failure in a system can be determined, when investments (I) and the expected economic damage (E(D)) are a function of the probability of failure. By Slijkhuis et al. [18] it has been shown how uncertainty and risk aversion can modelled in the method of economic optimisation. Investments and economic damage are modelled as stochastic parameters. The standard deviation of the total costs and a risk aversion factor (k) are taken into account in the determination of the optimal level of protection. The attitude towards uncertainty and the risk aversion can be varied by the value of the risk aversion factor k. The economic optimum is now found by:

$$\min(\mu(C_{tot}) + k \cdot \sigma(C_{tot}))$$

4.2 Fields of application and standards

An example of the use of a FD curve is a study on the economic risks in the Russian region Novgorodsky [19]. Jansen [20] has tried to obtain a financial economic risk limit in the form of a FD curve. However, from research of the economic risks in various fields (nuclear energy, aviation, floods) no consistent risk limit could be found.

The expected value of the economic damage is used as a part of cost benefit analysis of flood prevention measures in the UK [21] and the Netherlands [22]. In both approaches the benefits of a measure are determined by calculating the expected value of the economic damage before and after the measure has been taken. The difference between these two values results in the benefits, which can be weighed against the costs of the measures. A limit for the expected economic damage per year for dams has been proposed by BC Hydro [23], in which the financial risks for one dam should not exceed:

$$E(D) < \$10.000 \cdot \left(yr^{-1}\right)$$

The method of economic optimisation has originally been applied by van Danzig and Kriens [24] to determine the optimal level of flood protection (i.e. dike height) for Central Holland (this polder forms the economic centre of the Netherlands). The total investments in dike heightening (I_{tot}) are determined by the initial costs (I_0) and the variable costs (I'). The dike is heightened X metres, the difference between the new dike height (h) and the current dike height (h_0).

$$I_{tot} = I_0 + I' \cdot X \qquad \text{and} \qquad X = h - h_0$$

The risk expected value of the economic damage can be calculated from the probability of flooding (P_b), the damage caused by the flood (D), and the discount rate (r'). The flood level h is modelled as exponentially distributed with parameters A and B.

$$E(D) = \frac{P_b \cdot D}{r'} \qquad \text{and} \qquad P_b = e^{-\frac{h-A}{B}}$$

Now the total costs are formulated as the sum of investments and the expected value of the economic damage. The economic optimum is found by minimising the total costs. By taking the derivative of the total costs and the dike height, the optimal flooding probability ($P_{b,opt}$) and the optimal dike height (h_{opt}) are found.

$$P_{b,opt} = \frac{I' \cdot B \cdot r'}{D} \qquad \text{and} \qquad h_{opt} = A - B \cdot \ln(P_{b,opt})$$

The method of economic optimisation has also been applied for the design of various hydraulic structures, for example for breakwaters in Vrijling and van Gelder[25].

The economic optimisation, which takes into account uncertainty, has been in used in a case study to determine optimal dike height for Central Holland Slijkhuis et al. [18]. The investments and the economic damage caused by inundation are modelled as stochastic parameters. The influence of these uncertainties causes the economic optimal failure probability and dike height to rise quite substantially (compared with the results of van Danzig and Kriens).

5 Case study: Flood risk calculated with different risk measures

To show the possible application of some of the described risk measures a case study has been performed. In this study [1] the flood risk is calculated for an existing polder in the Netherlands, with the following risk measures:

- Individual Risk, compared to VROM and TAW standards
- Societal Risk: FN curve, expected value of the number of fatalities, risk integral, total risk
- Economic Risk: FG curve, expected value of the economic damage, economic optimisation

The studied area, "Betuwe, Tieler- en Culemborger Waarden" (BTCW), is situated in the eastern part of the Netherlands and measures about 80 by 25 kilometres (figure 6). The polder is inhabited by approximately 360.000 persons and has an estimated economic value of about 40 billion Euro. The polder is threatened by river floods, in the north by the Lek river and in the south by the Waal river. Dikes that surround the whole polder (or dike-ring as it is called in the Netherlands) achieve protection against high water (see figure 6). The number of fatalities and the damage caused by a flood are mainly determined by the location of the initial breach. Therefore the total dike ring is divided in fourteen distinct sections, each will lead to a distinctly different flood pattern [26]. For every section the probability of flooding has been determined with a model that takes into account the different failure modes of the dike (for example overtopping or instability of the dike) [27]. This results in an overall probability of flooding of the area of once in a thousand years. For every dike section breach simulations of the flood pattern are made. These results are used as input for the damage model [28]), which calculates the number of fatalities and the economic for each flood. It has to be noted that the fatality modelling involves a lot of uncertainties, evacuation has for instance not been modelled in it. It is found that these uncertainties have a major impact on the magnitude of the calculated individual and societal risks.

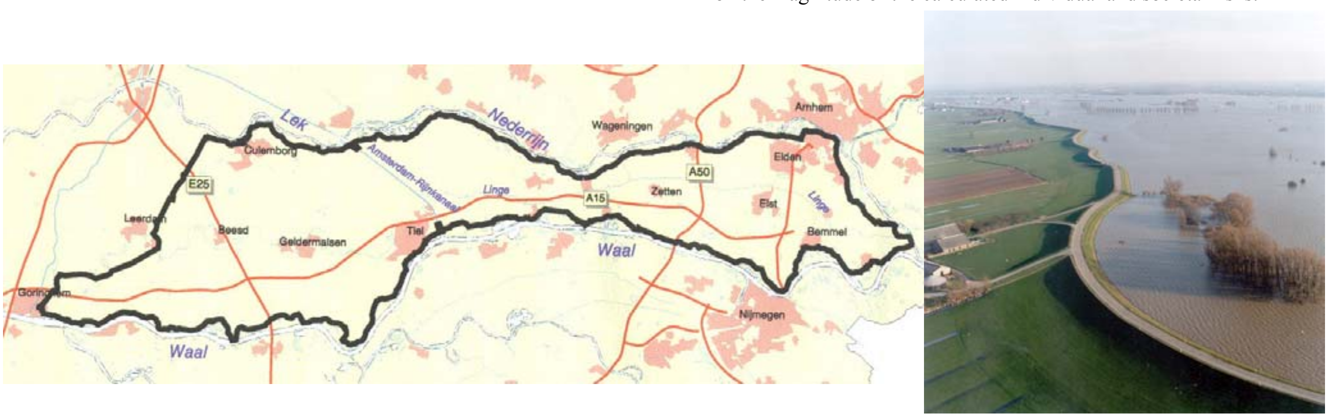


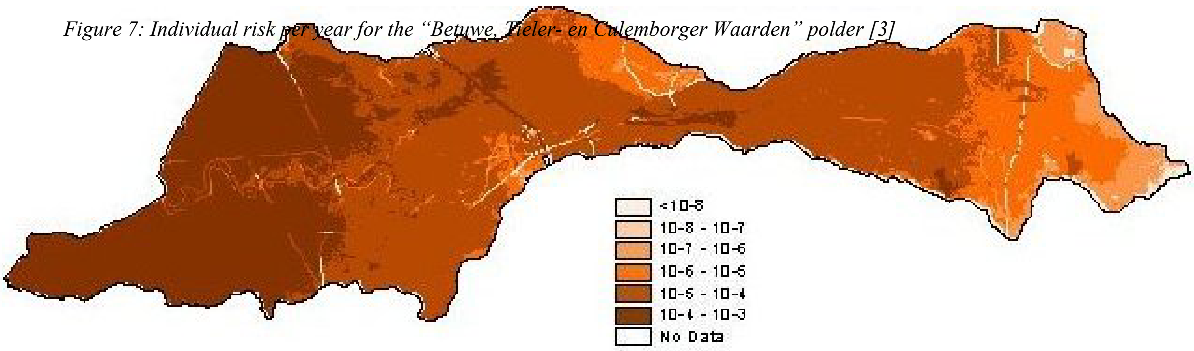
Figure 6: The Betuwe, Tieler- en Culemborger Waarden area and a picture of the flood protection with dikes

The individual risk is calculated with the data from the different flood simulations. For all the flood scenario's the probability of drowning at every location in the polder given a certain flood ($P_{d|i}$) is determined. By multiplication with the probability of the flood (P_i) and summing up the values for all n defined scenario's, the individual risk for every location (IR(x,y)) can be calculated. The calculated individual risk levels for the area are shown in figure 7.

$$IR(x,y)=\sum_{i=1}^{n} P_i \cdot P_{d|i}(x,y)$$

The IR of the polder can be compared to standards of VROM (accepting an individual risk of 10^{-6} / year) and TAW. By TAW a β value of 0,1 for flood prone areas has been proposed [3] which results

Figure 7: Individual risk per year for the "Betuwe, Tieler- en Culemborger Waarden" polder [3]



in an acceptable IR of 10^{-5} per year. In figure 8 it has been shown in which areas the calculated IR for the polder would not be acceptable by the VROM and the TAW standards (the dark areas exceed the limit)

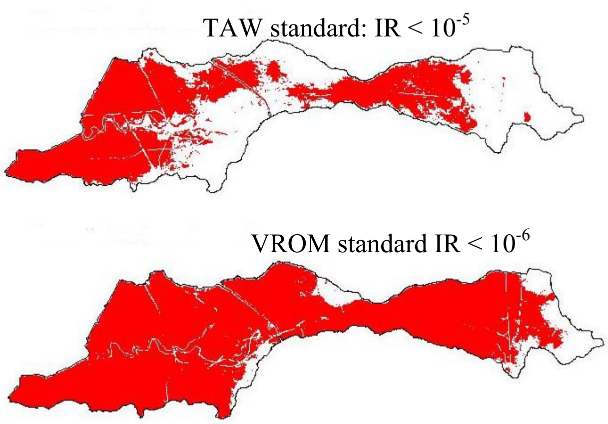


Figure 8: Individual risk of the Betuwe polder compared to VROM standard and the TAW standard, dark areas exceed the limit [1]

For every flood scenario the probability of occurrence and the number of fatalities are determined. With this data a pdf of the number of fatalities is formed, from which a FN curve can be derived. The FN curve for the flooding of the polder can be compared to some other risks in the Netherlands. Figure 9 shows that the risks of a flood of the polder are larger than the sum of the risks for airports, hazardous installations and railway yards in the Netherlands.

From the pdf the expected value of the number of deaths per year and the standard deviation are derived:

$$E(N)=2{,}4(fat/y) \qquad \sigma(N)=104{,}4(fat/yr)$$

The risk integral (RI) can be calculated with the expected value and the standard deviation of the number of fatalities:

$$RI=\frac{1}{2}(E^2(N)+\sigma^2(N))=\frac{1}{2}(2{,}4^2+104{,}4^2)=5452(fat/yr)^2$$

A lso the total risk is determined. For a value of the risk aversion index k of 3 this leads to:

The TAW criterion limits the total risk to a value of β·100, resulting in an acceptable total risk of 10 fatalities per year.

For every flood scenario the economic damage is determined. With information from the different scenarios a pdf is formed. From this pdf an FD curve [1] and the expected value of the economic damage can be derived.

Also the method of economic optimisation has been applied for the

$$TR=E(N)+k\cdot\sigma(N)=2{,}4+3\cdot 104{,}4=315{,}6(fat/yr)$$

BTCW polder. In figure 10 the investments in dike improvement, the expected value of the economic damage and their sum the total costs are shown as a function of the reliability index. This index can be

$$E(D)=\int_0^\infty x\cdot f_D(x)\cdot dx=13{,}2(\text{million Euro})$$

converted to a probability of flooding. The economic optimum is found when the total costs are minimal. The situation occurs for a reliability index of 4, which equals a probability of flooding of $3{,}16\cdot 10^{-5}$ per year (or once in 32.000 year).

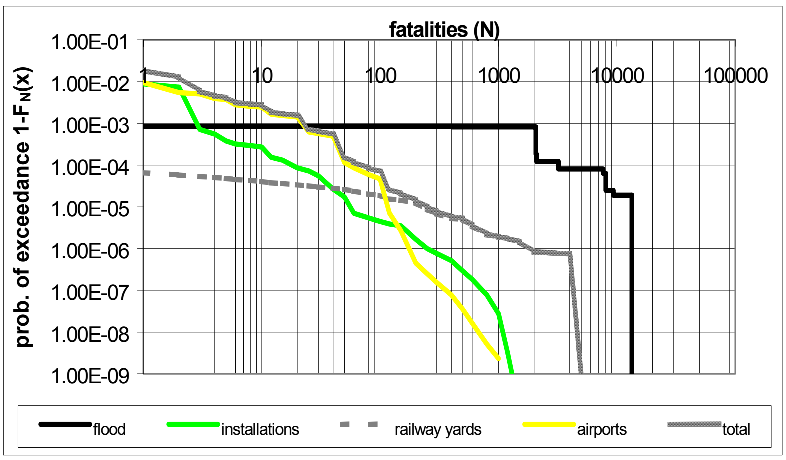


Figure 9: FN curve for the polder "Betuwe, Tieler- en Culemborger Waarden" compared to the FN curves of some other actvities [1]

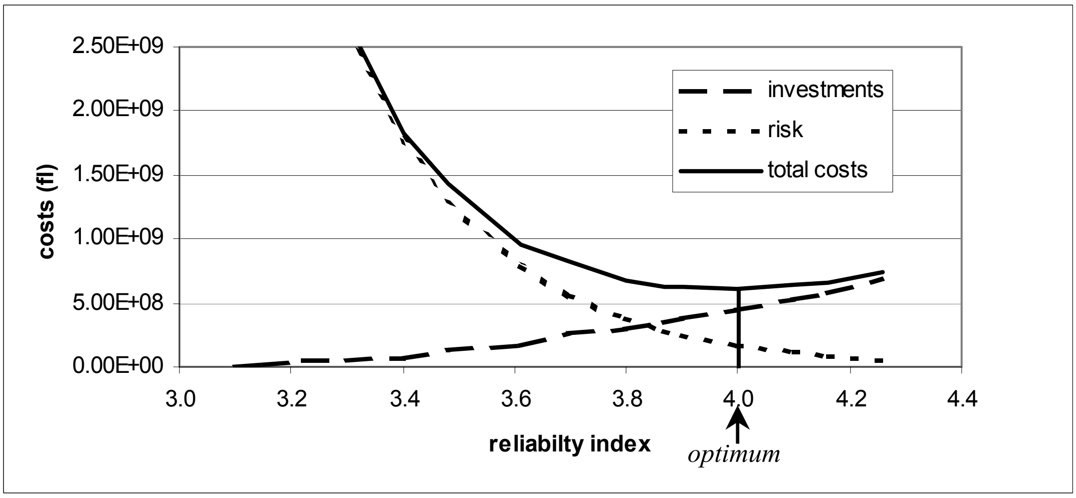


Figure 10: Economic optimisation of the "Betuwe, Tieler-, Culemborger Waarden"

6 Conclusion

In this article an overview is given of various risk measures used in quantified risk analysis. Such an overview was still missing in literature. In table 1 in the appendix the most important characteristics of the described risk measures are summarized: the mathematical expression, the field of application, the standards and a reference to literature. Some other important aspects following from the overview and case study are outlined below.
A risk measure is defined as a mathematical function of the probability of an event and the consequences of that event. In general risk measures can thus be expressed with a mathematical expression. Most risk measures limit itself by considering one type of consequences. This means that different characteristics and dimensions of the risk are modelled into one number (or graph). Although some argue that this oversimplifies the complex nature of risks, such a risk number can prove of significant use in the decision making process.

It can be shown that, although the formulae look different, a majority of the risk measures can be expressed with similar characteristics [11]. A more thorough inspection reveals that most societal risk measures can be derived from the probability density function and often contain the expected value and standard deviation as building blocks. Also economic risk measures, such as the FD curve and the expected value of the economic damage (E(D)), are based on the pdf.

In a case study it has been shown that it is possible to calculate the flood risk for an existing polder in the Netherlands, the "Betuwe Tieler-Culemborger Waarden", with individual, societal and economic risk measures. The calculation of the risks is performed with different flood scenarios for which the probabilities and consequences (fatalities and economic damage) are determined. The modelling of evacuation has major influence on the magnitude on the magnitude of the calculated risks. It has been found that the calculated risks of flooding of the Betuwe area are high compared to the risks of other activities in the Netherlands (hazardous installations, railway yards, airports). The calculated risks also exceed the current Dutch standards for acceptable individual risk for hazardous installations. However, in judging the tolerability of the flood risk it has to be noted that also the nature of activity has to be taken into account. Limits set for protection of the population against hazardous installations cannot directly be applied to limit the flood risk. Differences in the nature of the activity (technical vs. natural) and in attitude of the population towards the type of risk have to be considered in the acceptance of risks.

Recommendations
The answer to the question "how safe is safe enough?" should come from a broad judgement of all relevant aspects. Therefore co-operation with other fields of science is necessary in the study of risks and risk measures. Psychological studies can for instance be of interest in determining societal risk acceptance (of flood risks) and the accompanying quantitative criteria.
For a better insight in flood hazards in the Netherlands the flood risks of other areas (polders) in the Netherlands have to be determined. Also more research has to be done in the field of fatality and evacuation modelling.

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Appendix: Table 1: Overview of risk measures

Risk Measure	Mathematical expression	Field of application	Limit	Literature
Individual Risk (IR)				
Individual Risk (IR)	$IR = P_f \cdot P_{d\|f}$	hazardous activities: Netherlands, UK, Canada	$<10^{-6}$	[2], [5]
IR – TAW	$IR = P_f \cdot P_{d\|f}$	Studies, for example floods	$<\beta \cdot 10^{-4}$	[3]
IR – Bohnenblust	$IR = P_f \cdot P_{d\|f}$	studies railway safety in Germany)	see figure 3	[4]
Societal Risk				
Aggregated weighted risk (AWR)	$GGR = \iint_A IR(x,y) \cdot h(x,y) \cdot dxdy$	Netherlands: Schiphol	stand still (no increase AWR)	[7]
Expected value of the number of fatalities from IR contours	$E(N) = \iint_A IR(x,y) \cdot m(x,y) \cdot dxdy$	-	-	[8]]
Scaled risk integral	$SRI = \dfrac{P \cdot IR \cdot T}{A}$	HSE (UK): land use planning	-	[10]
FN curve	$1 - F_N(x) = \int_x^\infty f_N(x) \cdot dx$	International: hazardous activities (installations)	$1\text{-}F_N(x)<C/x^\alpha$	[9]
Expected value number of the number of fatalities E(N)	$E(N) = \int_0^\infty x \cdot f_N(x) \cdot dx$	US, Canada: dams	USBR:$< 10^{-2}$ BC Hydro: $< 10^{-3}$	[11], [16], [17]]
Risk integral	$RI = \int_0^\infty x \cdot (1 - F_N(x)) \cdot dx = \frac{1}{2}(E^2(N) + \sigma^2(N))$	HSE (UK): land use planning	-	[12], [11]]
Smets	$\int_1^{1000} x^\alpha \cdot f_N(x) \cdot dx$	-	$< 10^{-2}$, for α=2	[9], [13]
Bohnenblust	$R_p = \int_0^\infty x \cdot \varphi(x) \cdot f_N(x) dx$	studies railway safety in Germany	-	[4]
Kroon and Hoej	$\int_0^\infty x^\alpha \cdot P(x) \cdot f_N(x) \cdot dx$	OECD / PIARC study on tunnel safety	-	[14]
Total risk	$TR = E(N) + k \cdot \sigma(N)$	NL: studies External safety	$<\beta \cdot 100$	[15]
Economic Risk				
FD curve	$1 - F_D(x) = \int_x^\infty f_D(x) \cdot dx$	display various economic risks	Proposed by Jansen [20]	[19]]
Expected value of the economic damage	$E(D) = \int_0^\infty x \cdot f_D(x) \cdot dx$	UK and NL: cost benefit analysis floods, US: dams	USBR: E(D)<\$10.000	[21], [22], [23]
Economic optimisation	$\min(C_{tot}) = \min(I + E(D))$	NL: flood protection	Economic Optimum	[24], [1]
Economic optimisation and uncertainty	$\min(\mu(C_{tot}) + k \cdot \sigma(C_{tot}))$	-	Economic Optimum	[18],[25]