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Synthesis of a multivariate extreme sea climate

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ABSTRACT: This paper offers a method to describe the joint probability distribution of a sea climate. Various methods to obtain such models are mentioned, followed by the treatment of a so-called mixed method. This type of statistical models based on physical laws are used for describing wave heights, wind setup, wave periods and wind speed. The focus in this paper is on the selection and calibration of parametric physical models for the use in the description of the joint probability distribution of hydraulic loads. For the validation and calibration of these models, observations have been used. An application is presented about the Rotterdam harbour extension.

1 INTRODUCTION

Sufficient protection of coastal areas against flooding in many cases is created by coastal structures, like breakwaters, dikes, dams and dunes. It is important for design to determine the hydraulic loads on these structures. The origin of these loads are the different states of the hydraulic climate. To make probabilistic design and risk-based optimisation possible a performance measure has to be defined. In calculating the actual performance the multivariate statistics of hydraulic boundary conditions play an important role. In many cases designers have to deal with a prescribed small probability of failure for the design. This leads to the need of extreme event data, from which subsequently a probability density function can be obtained. The lack of extreme event data implies extrapolation of the available data and makes it difficult to decide which probability distribution is suitable and will describe the processes in a proper way.

In this paper extrapolation guided by mathematical models based on physical laws is used to solve this problem. In this way implementation of physical knowledge in statistical extrapolation methods is achieved. An overview of some other known methods are summed up in section 3.

The chosen method is applied on a case study of the Rotterdam harbour extension. Suitability for the design process is an important requirement for the overall model. Designers generally require models that can give them quick but reliable insight in the sensitivity of a design to different variables. Comparing different design alternatives in an early stage of projects has to be possible. Consequently the models need to be as simple as possible, but in correspondence with reality.

The probability theory of extremes, the peaks-over-threshold method of estimation and comparison with the direct use of annual maxima and many related subjects are well described in many articles, of which Cox et al. (2002) shows a nice overview.

2 PERFORMANCE MEASURE

In any design process it is necessary to define a measure for the performance of the structure. Subsequently, a target level for the performance of the structure can be set. Design alternatives performing below the target can then be recognised as not suitable for the situation at hand. Essential in reliability-based design is the adoption of the reliability of the structure, or its complement the failure probability, as a measure of the performance of the structure.

The failure probability is, among others, a function of the geometry of the structure. If a maximum value of the failure probability is given, the set of acceptable design alternatives is conceptually given by:

$$D = \left\{ \mathbf{z} \mid P_f\left(\mathbf{z}\right) \le P_{f;max} \right\} \tag{1}$$

Where $\mathbf{z}$ is a vector of design variables describing the geometry of the structure, P_f is the calculated

failure probability and $P_{f;max}$ is the maximum acceptable failure probability.

Equation (1) indicates that in reliability-based design it is necessary to quantify the probability of failure for a given design alternative. Next to the geometry of the structure (denoted by the vector **z**), the failure probability is a function of a set of random environmental quantities, denoted **x**. The probability of failure is mathematically given by:

$$P_f(\mathbf{z},\mathbf{x}) = \int_{g(\mathbf{z},\mathbf{x})<0} f_{\underline{\mathbf{x}}}(\mathbf{x})\,\mathrm{d}\mathbf{x} \qquad (2)$$

Where $g(\bullet)$ is an indicator function that takes a negative value if the structure fails and $f_{\underline{\mathbf{x}}}(\mathbf{x})$ denotes the joint probability density function (JPDF) of random quantities describing the loads on the structure and the strength of the structure.

Equation (2) is effectively the volume integral of the joint probability distribution f over the failure domain. The shape of the integrated volume is determined by the shape of the failure domain and the shape of the joint probability distribution f. Therefore, the calculated probability of failure of the structure depends both on the shape of the failure domain and on the shape of the joint probability distribution.

In the coastal zone, the shape of the joint probability distribution f is dominated by the shape of the joint probability distribution of the hydraulic loads. The hydraulic boundary conditions that are relevant for determining the hydraulic loads are the water level (h) and the following long-term wave characteristics: significant wave height (H_s), peak period (T_p) and main wave direction (T_{h0}). Hydraulic loads in the coastal zone show a considerable dependence because under extreme conditions the hydraulic loads are all wind-driven. Wind is the common source. A description of the joint probability distribution needs to take account of this dependence.

3 METHOD OVERVIEW

Over the years a number of methods have been proposed. It appears that two families of methods can be distinguished (see figure 1):

- Fully statistical methods;
- Mixed methods; methods based on a mix of statistical models and parametric-physical models.

Fully statistical methods distinguish themselves by the fact that the distributions of the hydraulic quantities and the dependence between them is fully based on the available data. Mixed methods distinguish themselves by the fact that parametric-physical models are used to establish the dependence between the variables.

A second distinction that can be made is in the set of variables that is described by the method. It appears that a wide variety of methods for the description of the joint probability distribution of wave height and wave period are available. The number of methods available for the description of the joint probability distribution of water level and wave conditions appears to be much more limited to the mixed method of Vrijling and Bruinsma (1980) and its successors Webbers (2000) and Voortman (2003) and to the statistical methods of de Haan and de Ronde (1997) and Hawkes et al. (2002).

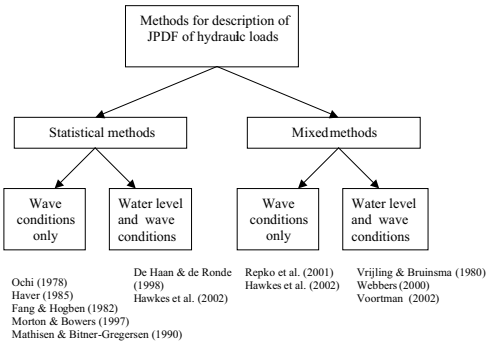

Figure 1. Overview of methods for the description of the long-term distribution hydraulic loads

Figure 1 is limited to the description of the long-term statistics of hydraulic loads. A considerable amount of methods may be found for the description of short-term distributions of sea-state parameters. Useful though they are, those methods are not relevant to the present study.

4 APPLIED PHILOSOPHY

The applied methods in this paper are based on the philosophy of the mixed method of Vrijling and Bruinsma (1980).

The main complication in the description of the joint probability distribution of hydraulic loads is the fact that the loads are dependent.

For use in combination with probabilistic methods a description of the loads in terms of stochastically independent variables is preferable from a practical point of view. The mixed method of Vrijling and Bruinsma (1980) uses parametric-physical models to write the dependent hydraulic loads as an explicit function of a set of stochastically independent input variables. The basis of the method is a conceptual model of the physical dependence of the hydraulic loads on the meteorological and tidal conditions (see also figure 2).

Well-known parametric models can be applied to establish the dependence between different variables. A dependence model in combination with the

distribution of the input variables provides the distribution of a dependent variable, conditional on the input variables. In that respect the approach is similar to the methods of Haver (1985) and Mathisen and Bitner-Gregersen (1990).

A major advantage of basing the dependence structure on physical concepts over a pure statistical method is the fact that well-known and well-supported physical models may be expected to hold also under unobserved extreme conditions. Adopting a physics-based dependence model therefore provides information on the dependence structure outside the interval of the observed conditions. The description of the joint distribution of wave height and wave period by Hawkes et al (2002) is in line with the philosophy of Vrijling and Bruinsma.

5 FRAMEWORK PHYSICAL RELATIONS HYDRAULIC CLIMATE

To obtain a transparent approach to the problem the following steps are taken. Firstly a proper description of all relevant physical processes is developed, which forms the theoretical framework for describing the hydraulic climate.

Secondly, physical models are identified that describe the hydraulic conditions as a function of a input variable. In general these models describe the dependence between pairs of variables, like water level and wave height or wave height and wave period. A selection of models needs to be made such that all relevant variables are described.

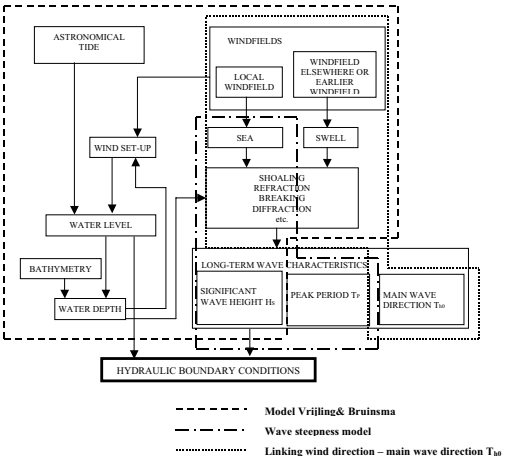

Figure 2. Diagram of the theoretical framework of physical relations (processes) used for the derivation of the description of the hydraulic climate (after Vrijling and Bruinsma (1980))

Thirdly, the models are implemented, also consisting the calibration and validation based on field data.

Fourthly, the different models are linked together to find the description of the overall model.

In figure 2 the processes are shown, which determine the hydraulic climate.

The water level can be seen as built up from two stochastically independent variables, the wind set-up and the astronomical tide. The wind-set up and the long-term wave characteristics (H_s, T_p, T_{h0}) are caused by the wind field. Another influence on the wave characteristics and the wind set-up is the water depth. This water depth can be seen as built up of the bathymetry and the water level.

6 CHOICE OF PARAMETRIC PHYSICAL MODELS

The dependence models chosen to describe parts of the hydraulic climate, are listed below.

6.1 *Water level h – significant wave height H_s*

To describe the strong dependence in the North Sea between the water level and the significant wave height a parametric model is applied, which is proposed by Vrijling and Bruinsma (1980) for the design of the Eastern Scheldt Storm Surge Barrier.

The main assumptions in this model are:

- The astronomical tide can be seen as stochastically independent of wind set-up (shown by Schalkwijk (1947) and Weenink (1958)), which leads to:

$$\underline{h} = \underline{a} + \underline{s} \tag{4}$$

  where a represents the astronomical tide during a maximum high water level in [m +NAP] and s represents the wind set-up in [m]. The underlining of a variable means that the variable is stochastic.
- The significant wave height and the wind set-up are caused by the same common cause, the wind fields, so that the wind speed may be used to establish a relation between water levels and wave heights

The wind set-up model described by Weenink (1958) leads to the following expression for each wind direction:

$$u = \sqrt{s \cdot g / \alpha} \tag{5}$$

where u represents the wind speed, g represents the acceleration of gravity in [m/s^2] and α is a dimensionless constant depending on the wind direction.

By combining the wind set-up model described by Weenink and the astronomical tide, the water level as a function of wind speed can be derived for each wind direction.

By using a wave growth formula of Bretschneider (1952, 1958) for the wind driven part of the waves and using the wave height of swell for calibration, the following wave growth formula can be obtained for the overall wave height:

$$H_s = \sqrt{H_{s,swell}^{\ 2} + \left[0.283 \cdot \frac{u^2}{g} \cdot \tanh\left[0.53 \cdot \left(\frac{g \cdot d}{u^2}\right)^{0.75}\right] \cdot \psi\right]^2}$$

$$\text{in which } \psi = \tanh\left[\frac{0.0125 \cdot \left(\frac{g \cdot F}{u^2}\right)^{0.42}}{\tanh\left[0.53 \cdot \left(\frac{g \cdot d}{u^2}\right)^{0.75}\right]}\right] \qquad (6)$$

where
F = fetch [m]
u = wind speed [m/s]
d = water depth [m]
$H_{s,swell}$ = significant wave height of swell

By applying characteristic wind speed as the combining variable, and for d the average water depth of the North Sea basin and for F the possible fetch in the main wind direction, the relation between the significant wave height and the water level is achieved. Still possible breaking of waves must be checked.

6.2 *Significant wave height H_s -peak period T_p (wave steepness model)*

To find the dependence between H_s en T_p the wave steepness model proposed by Vrijling (1996) and applied by Repko (2001) is implemented. The main assumptions in this model are:

- The deep water wave steepness s_{0p} is stochastically independent of the significant wave height H_s;
- The significant wave height of swell is negligible compared to the significant wave height of wind driven waves during extreme storm events.

Repko uses the deep water wave steepness defined by:

$$s_{0p} = H_S \Big/ \left(g \cdot T_p^{\ 2} / 2\pi\right) \qquad (7)$$

Another more theoretical model based on a universal relation between total wave energy (actually the variance of the surface elevation), the peak frequency of the energy spectrum and the wind speed, for wind driven waves, is proposed by Mitsuyasu et al (1980). A comparison is made between this model and the first assumption of the wave steepness model mentioned above.

Based on Mitsuyasu et al (1980), Battjes (1984) derived the following equation:

$$H_s^2 T_p^{-3} = c \cdot g \cdot u \qquad (8)$$

where c equals $0.99 \cdot 10^{-4}$.

Substituting equation (7) in equation (8) the deep water wave steepness can be written as:

$$s_{0p} = \left(2\pi \cdot c^{2/3} \Big/ g^{1/3}\right) \cdot \left(u^{2/3} \Big/ H_s^{\ 1/3}\right) \qquad (9)$$

Equation (9) is used for the actual comparison.

6.3 *Main wave direction T_{h0} - main wind direction*

The combination of extreme water levels and extreme wave heights are assumed to be caused by wind fields with a similar characteristic wind direction as the wind field with the largest possible fetch. The main wave direction T_{h0} is assumed to equal this main wind direction.

Figure 3 also shows the chosen models covering different parts of the theoretical framework.

The JPDF of the hydraulic climate can be derived by combining the marginal distribution of a chosen input variable, the dependence models based on physical relations and model uncertainties.

In this case, the water level is used as the input variable (see also Vrijling and Bruinsma, 1980). The value of the input variables is not known with certainty. Therefore they need to be described by probability models. Long records of water level observations, also in extreme situations (for instance the extreme storm surge 1953) are available, has led to this choice. Thus, the JPDF can be derived:

$$f_{\underline{h},\underline{H}_S,\underline{T}_p,\underline{\theta}_{h0}}\left(h, H_S, T_p, \theta_{h0}\right) = f_{\underline{T}_p|h,H_s,\theta_{h0}}\left(T_p \,|h, H_s, \theta_{h0}\right) \cdot f_{\underline{H}_s|h,\theta_{h0}}\left(H_s \,|h, \theta_{h0}\right) \cdot f_{\underline{\theta}_{h0}|h}\left(\theta_{h0} \,|h\right) \cdot f_{\underline{h}}\left(h\right) \qquad (10)$$

If the interest is solely in the marginal distribution of the wind setup for example, it can be found from the calibrated model by:

$$F_{\underline{s}}(\eta) = \iint_{s(v,\delta)<\eta} f_{\underline{u}}(v) f_{\underline{\Delta}}(\delta) dv \, d\delta \qquad (11)$$

Where $f_{\underline{u}}(\bullet)$ denotes the marginal probability density function of the wind speed and $f_{\underline{\Delta}}(\bullet)$ the marginal distribution of the model uncertainty.

Similar expressions apply for the wave height and wave period. Because the calibrated model is intended to reflect the processes in nature, the marginal distributions derived from the model should correspond to the marginal distribution of the observations.

7 CALCULATION OF THE PROBABILITY OF JOINT OCCURRENCES OF HYDRAULIC CONDITIONS

The mixed method is calibrated such that it accurately reproduces the empirical distributions of wind setup, wave height and wave period at the measurement location in the south of the North Sea. Because the hydraulic conditions are written as a function of the wind speed, the joint probability of exceedance of pairs of variables are easily calculated by applying a well-known reliability calculation method. The approach will be explained for the joint exceedance probability of a combination of wind setup and wave height, but the method may be applied to any other pair of variables in the same way.

Reliability methods are generally aimed at estimating the probability of occurrence of the following event:

$$g(\mathbf{x}) < 0 \tag{12}$$

Where g is an indicator function and $\mathbf{x}$ is a vector of random input variables. The function g is generally denoted a limit state function.

Exceedance of a predefined threshold level by the wind setup is written as:

$$s(u,\Delta) > s_{thr} \tag{13}$$

Where s_{thr} is the threshold level, u is the wind speed and Δ is the model uncertainty. Note that the wind setup model in the right-hand side of equation (5) is developed and calibrated Rearranging expression (13) leads to:

$$s_{thr} - s(u,\Delta) < 0 \tag{14}$$

Which is an inequality in the form needed for a reliability method. The probability of exceedance of the wind setup threshold can now be quantified by implementing the wind setup model and using the distributions of wind speed and model uncertainty as input. The probability of exceedance of a given wave height is found in the same way. The probability of joint exceedance of a wind setup and a wave height threshold is conceptually given by:

$$P\left(\eta - s(u,\Delta_s) < 0 \cap \zeta - H_s\left(u,\Delta_{H_s}\right) < 0\right) \tag{15}$$

Equation (15) describes a parallel system of two failure modes. To calculate the probability of occurrence of the parallel system of equation (15) accounting for partial dependence requires the application of a numerical method. Level III methods are slow but robust methods that can deal with any limit state function and any type of distribution. Much quicker are level II methods that use a transformation to the space of standard-normal variables to quickly find an estimate of the probability of occurrence of a failure mode. Level II methods are exact if the limit state function is linear and the input variables are described by normal or log-normal distributions. In other cases, the failure probability estimate obtained by a level II method is an approximation to the exact solution.

8 IMPLEMENTATION AND VALIDATION OF MODELS

8.1 *Case description*

In order to face future needs of space in the Rotterdam harbour, a land reclamation is under study. The project area of this planned harbour extension, called "Maasvlakte 2", is located along the southern North Sea coast (figure 3).

The ultimate objective of the case study is to describe the hydraulic climate at Maasvlakte 2 by a mixed method as mentioned before. The focus in this paper is on the selection and calibration of parametric physical models. For the validation and calibration of these models, observations will be used.

Figure 3. Location of project area Maasvlakte 2

In the case of Maasvlakte 2 no hydraulic measurements are available at the location of the project area. Nineteen years of simultaneous observations of hydraulic conditions are available on relatively deep water about 70 km offshore the Dutch coast, the Euro-0 platform. Wind observations are available from the location of Hook of Holland (near the project location).

The research in this case study is limited to the so-called Ultimate Limit State (ULS), which describes the states in which a structure collapses. For ULS the extreme events (like storms) need to be described. In case of Maasvlakte 2 only the combination of extremely high waves and extremely high water levels are studied.

In most cases only relatively deep water observations are available. The reason for performing measurements offshore is that nearshore data are generally not homogeneous, which makes statistical calculations useless. In a future stage of research a

translation step has to be carried out to the shore (Maasvlakte 2) to obtain the description of the JPDF at this location. A possible alternative for the translation step could be for example to use the JPDF at relatively deep water as input for a physical-numerical model like SWAN (*Simulation of Waves Nearshore,* Booij et al. (1999)). This translation step is left out of consideration in this research but still has to be accomplished in further research.

8.2 *Data selection*

From the available data 10 independent storms are extracted. These storms can be characterized by extremely high water levels and extremely high waves. The selection takes place by using the widely used Peak-Over-Threshold (POT) method and comparing the results with the method of depression route selection as proposed in the Report of the Delta Committee (1960). The POT method is used for starting variable *h*.

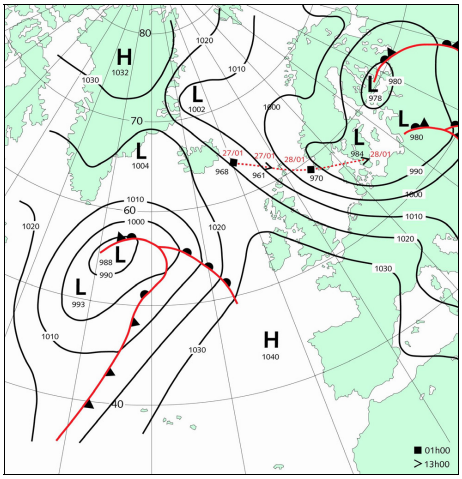

Figure 4. Map with the route depressions and the air pressure distribution above the North Sea (RIKZ, 1994))

For every independent observation of *h*, simultaneously measured values are selected of the following variables: significant wave height H_s, peak period T_p, main wave direction T_{h0}, wind speed *u,* main wind direction w_0.

After analysing the selected storms it appeared that those storms were all from the same North Westerly direction, which all were caused by fixed routes of the depression cores. The cores of the depression, causing the 10 North Westerly storms, all follow a very similar route from the region somewhere between Iceland and Scotland towards the area Denmark and North Germany. This is in harmony with the route Bouws (1978) described as characteristic for the extreme wave climate.

The route of one of the selected storms is shown in figure 4 and the development of the wind and wave direction is shown in figure 5. In the Netherlands, extreme storm events are reported by RIKZ (see RIKZ, various years).

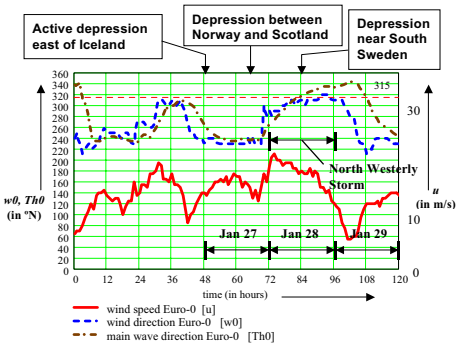

Figure 5. Development in time of the wind speed, wind direction and main wave direction during a storm at Euro-0 (28th January 1994), including comment from RIKZ (1994)

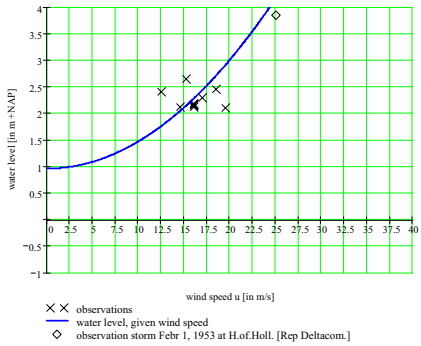

Figure 6. Water level as a function of wind speed (platform Euro-0)

Figure 6 shows the result of using the wind set-up model described by Weenink (1958) combined with the average astronomical tide as proposed by Vrijling and Bruinsma (1980). Considerable uncertainty exists on the used wind set-up-wind speed relation, may be caused by using the wrong characteristic wind speed.

By using the same wind speed, the North Westerly wind direction in the wave growth formula of Bretschneider (eq. 6), figure 7 can be obtained, in which the amount of swell is varied. The fetch *F* is 800 km and the water depth *d* of the North Sea equals 35m. Apparently breaking of waves can be neglected at Euro-0 (Webbers 2000).

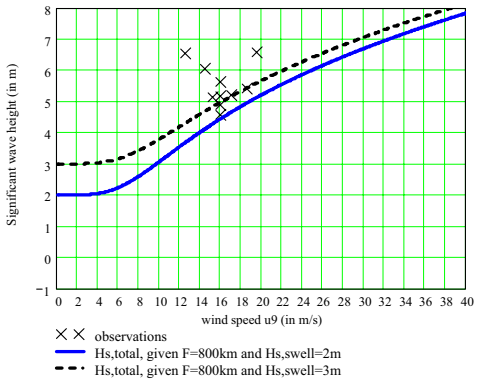

Figure 7. Significant wave height as a function of wind speed (platform Euro-0)

According to the model of Vrijling and Bruinsma (1980) the characteristic wind speed is applied as the common variable. Subsequently the relation between the significant wave height and the water level is found (figure 8).

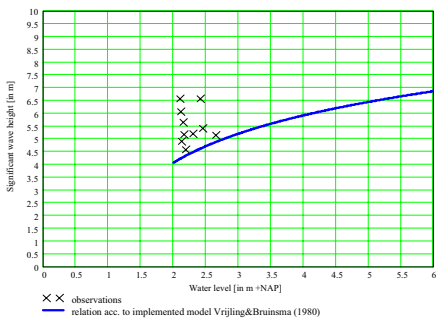

Figure 8. Significant wave height as a function of water level (platform Euro-0)

Comparing the observations to the parametric model, it appears that the model gives a underestimation of the wave heights. It is assumed that this underestimation is caused by neglecting the spatial variability of the wind speed. In fact a false wind speed is taken into account, namely the speed at the shore (as mentioned in *Case description*). It is very likely that the characteristic wind speed is higher, since the wind speed above the sea is actually causing the hydraulic climate and not at the boundary.

Before implementing the model of Repko et al (2001), the assumption of independence is verified (figure 9). The assumption is not falsified by the calibration data. The data is too limited to infer the shape of the distribution with certainty, but a normal distribution is not rejected.

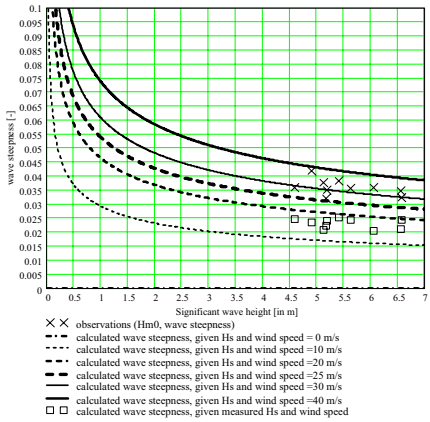

Figure 9. Deep water wave steepness as a function of significant wave height (Euro-0)

Also the model by Mitsuyasu et al (1980) as proposed by Battjes (1984), is considered for comparison. The results of using relation (9) are represented by the curves in figure 9. It appears that in the extreme area (high waves) the curves become more and more horizontal.

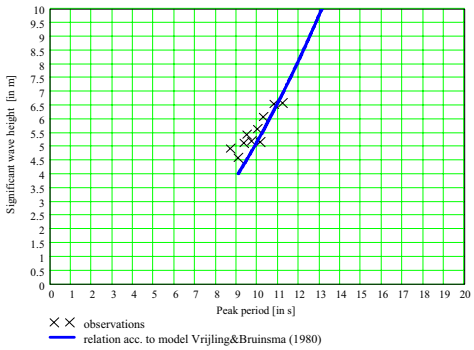

Figure 10. Peak period as a function of significant wave height (platform Euro-0)

It appears that the Mitsuyasu model supports the assumption of stochastically independent wave height and steepness during extreme storms. In this case a systematic underestimation of the wave steepness seems to appear, probably due to neglecting the spatial variability of the wind speed above the North Sea as mentioned before. Using the model of Repko et al, the relation between the significant wave height and the peak period can be determined (Fig. 10).

9 CONCLUSIONS AND RECOMMENDATIONS

The description of the JPDF of the hydraulic boundary conditions by linking the proposed statistical models based on physics seems to be promising after applying it to the case study of Maasvlakte 2. Also the different shortcomings in the overall model has been identified. Important conclusions:

- Apparently extreme high waves in combination with the high water levels are caused by North Westerly storms (fixed routes depression cores).
- During the efforts to calibrate and validate the bivariate model of water level and significant wave height with field data, it turned out that there is a systematic underestimation of the wave height, probably caused by neglecting the spatial variability of the wind speed. The utilisation of wind data at the whole North Sea during the storms is a very strong recommendation.
- Other models support the relatively simple model of wave steepness to describe the dependence between significant wave height and peak period.

Still the derivation of the multivariate probability distribution function has to take place. Also the check on consistency of the method with the empirical distribution function of the data is an important step of the validation and calibration. Furthermore, the translation step to the shore (Maasvlakte 2) still has to be established. Also comparison with physical-numerical models like SWAN would be still interesting. Finally, when also more years with observations (also near the shore) are available, it is highly recommended to use them in further research.

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