3.1 Design applications: state of the art of probabilistic design tools[1]

3.1.1 INTRODUCTION

In this section the probabilistic design tools will be presented. This encompasses the methods to combine the inherent uncertainty of the natural boundary conditions, the uncertainty due to lack of information of the natural environment, the quality of the structure and the engineering models into the measure of a failure probability that expresses the reliability of a structural system. Also a decision has to be taken whether the structural reliability is sufficient in view of the economic and societal functions of the structure. As an aid to this decision a safety philosophy has been formulated.

As the application of the probabilistic design method requires considerable effort and resources, a simpler approach using partial safety factors is presented within the probabilistic framework.

This section aims at designers with an interest in the probabilistic design method and the background of a partial safety factor code. It may also serve as an introduction for researchers who want to familiarise themselves with the theoretical backgrounds.

[1] Prof. J.K. Vrijling and Dr. P.H.A.J.M. van Gelder, TU Delft, Delft, The Netherlands, Dr. H.G. Voortman, ARCADIS, Amersfoort, The Netherlands

3.1.2 GENERAL INTRODUCTION OF PROBABILISTIC METHODS

3.1.2.1 *Introduction*

The most important and most clear difference of probabilistic design compared to conventional (deterministic) design is that in probabilistic design one takes explicitly account of the uncertainties involved in the behaviour of the structure under consideration. Over the years considerable progress has been made in the development of probabilistic methods. This section attempts to introduce shortly the probabilistic working method, independent of the application. For further reading, several textbooks have been written. As a start for further reading the following references can be used:

- Thoft-Christensen & Baker, 1982;
- Madsen, et al. 1986;
- Ditlevsen & Madsen, 1996.

Mentioning these three references is in no way meant to imply any judgement of the value of other references on the same subject.

3.1.2.2 *Limit state equations and uncertainties*

3.1.2.2.1 *The concept of limit states*

The first step in a reliability analysis of any structure is defining its functions. When the functions of the structure are defined, the ways in which malfunctioning of the structure can occur are defined. These ways of malfunctioning are called failure modes. The failure modes are described in such a way that they are fit for mathematical treatment. A function that describes functioning or failure of a structure or one of its components is called a reliability function or limit state equation. A general limit state equation is denoted *g* and can be written as:

$$M = g(\mathbf{X}) = R(\mathbf{X}) - S(\mathbf{X}) \tag{3.1}$$

In which $\mathbf{X}$ is a vector of random variables describing the geometry of the structure, the loads that are applied, the strength of materials etc; M is a random variable, usually referred to as the safety margin; $R(\mathbf{X})$ is the strength (Resistance) of the structure as a function of $\mathbf{X}$; $S(\mathbf{X})$ is the load (Solicitation) on the structure as a function of $\mathbf{X}$.

The limit state equations are defined in such a way that negative values of realisations of M indicate failure and positive values indicate safe states.

For a general structure several limit states can be defined. Take for example the statically determined concrete beam with a point load in the middle in Figure 3.1.

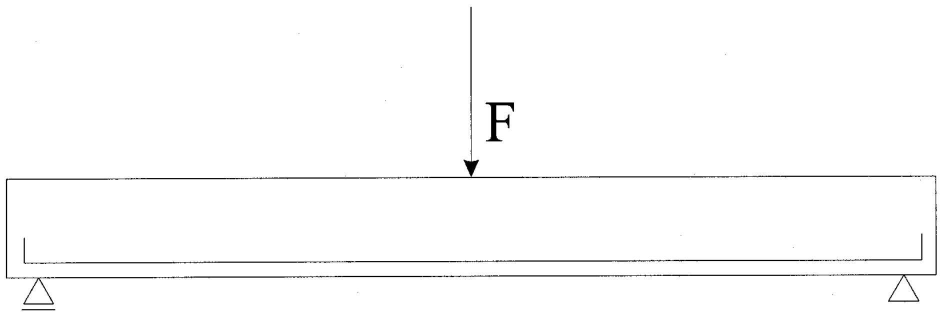

Figure 3-1. Statically determined concrete beam.

In the situation where the position of the load is fixed, the following limit states can be defined for this simple structure:

- Exceedance of the ultimate bending moment in the middle of the beam;
- Exceedance of the shearing strength near one of the supports;
- Exceedance of the admissible deformation in the middle of the beam;
- Cracking of the concrete on the lower side of the beam, which may lead to corrosion of the reinforcement bars in an aggressive environment;
- Chloride ingress through the uncracked concrete, followed by corrosion of the reinforcement.

Observation of these five failure modes shows that the consequences are not of equal magnitude in all cases. The first two failure modes lead to immediate collapse of the beam, while the other three threaten its functioning (inadmissible deformation) or may introduce failure after a period of time in which repair can still be made (cracking and chloride ingress).

Therefore, in general four kinds of limit states are defined (see e.g. Eurocode 1, Basis of Design, 1994):

- Ultimate Limit States (ULS), describing immediate collapse of the structure;
- Serviceability Limit States (SLS), describing loss of function of the structure without collapse (a beam with a too large deformation might not be able to support a load which is sensitive to this deformation, while the beam is still able to withstand the load);
- Accidental Limit States (ALS), describing failure under accident conditions (collisions, explosions).

The acceptable probability of failure for a certain limit state depends on its character. Usually the highest safety requirements are set for Ultimate Limit States. Accepted failure probabilities for Serviceability Limit States might be considerably higher, especially if the effects of failure are easily reversed. Accidental Limit States can be treated like Ultimate Limit States, or the probability of occurrence of the accident can be taken into account. The acceptable probability of failure also depends on the time in which it is possible that a certain failure mode occurs. For failure during the construction phase, this time is considerably shorter than for the other types of failure. Therefore, for the construction phase characteristic values with a smaller return period are defined.

3.1.2.2.2 *Uncertainties related to the limit state formulation*

Basically, the limit state equation is a deterministic model indicating functioning or failure of the structure. Uncertainties are generally related to the input of the limit state equation. The following types of uncertainty are discerned (see also Vrijling & van Gelder, 1998):

1. Inherent uncertainty;
2. Model uncertainty;
3. Statistical uncertainty.

Ad 1: The uncertainty that is part of the described physical process is called inherent uncertainty. This uncertainty exists even if unlimited data is available. For instance: even if the wave height at a certain location is measured during an infinite period of time, the wave heights will still be uncertain in the future. A probability distribution can be used to describe the inherent uncertainty.

Ad 2: Model uncertainty can be distinguished into two subtypes. The first type of model uncertainty is related to the limit state equation itself. The model describing the physical process is a schematisation of the true process. Due to the schematisation, parts of the process are left out under the assumption that they are not important to the final result. This leaving out of parts of the process introduces a scatter (uncertainty) when comparing the model to measurements. Using a more sophisticated model, which describes more accurately the physical process, can reduce this type of uncertainty. The second kind of model uncertainty is related to the distribution function of the input variables. Also parametric distribution functions are schematisations of some real (unknown) distribution. Model uncertainty related to the input variables means that the chosen model might not be the true or best model. This type of uncertainty is reduced if more measurements are available, since then the correct distribution type becomes more clear.

Ad 3: When fitting a parametric distribution to limited data, the parameters of the distribution are also of random nature. The uncertainty in the parameters is generally referred to as statistical uncertainty. This type of uncertainty reduces when the number of data points increases.

3.1.2.3 *Reliability analysis on level II and III*

3.1.2.3.1 *Introduction*

The reliability of a structure or component is defined as the probability that the structure or component is able to fulfil its function. Reversibly the probability of failure is defined as the probability that the structure does not function. The properties of the structure (load, strength, geometry) are modelled by a vector of random variables, called the basic variables. The space of the basic variables is divided in a safe set and a failure set by the limit state equation(s) (Figure 3-2).

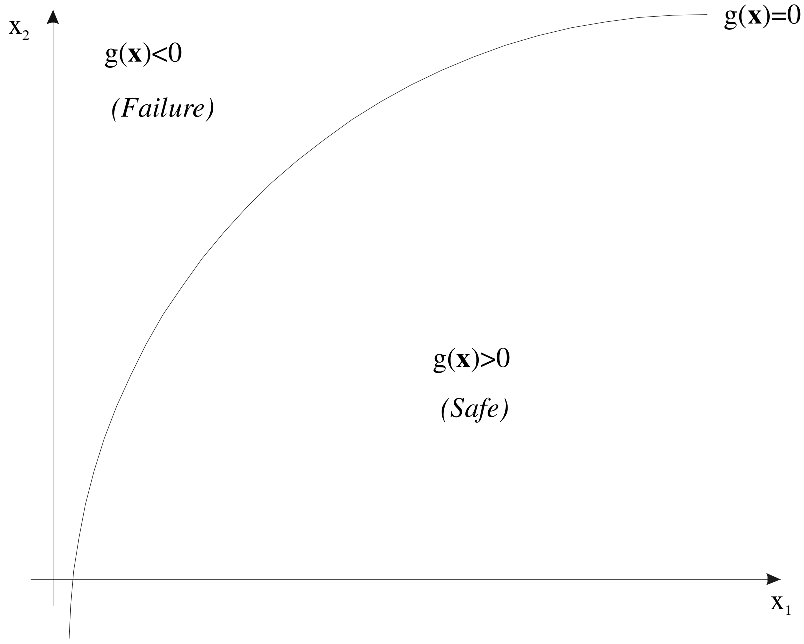

Figure 3-2. Limit state equation, safe set and failure set in the space of basic variables.

The probability of failure equals the probability that a combination of values of the basic variables lies in the failure domain. In formula:

$$P_f = P(\mathbf{X} \in F) \tag{3.1-1}$$

In which $\mathbf{X}$ is the vector of basic variables; and F is the failure domain.

Evaluation of this probability comes down to the determination of the volume of the joint probability density function of the basic variables in the failure domain. In formula:

$$P_f = \int_F f_{\mathbf{X}}(\mathbf{x})d\mathbf{x} = \int_{g(\mathbf{x})\le 0} f_{\mathbf{X}}(\mathbf{x})d\mathbf{x} \tag{3.2}$$

In which $f_{\mathbf{x}}(\mathbf{x})$ is the joint probability density function of the basic variables; and $g(\mathbf{x})$ is the limit state equation.

In general it is not possible to solve the integral analytically. Several numerical methods have been developed in the past. Section 3.1.2.3.2 introduces the direct integration methods. Section 3.1.2.3.3 introduces approximating methods. More details are given in (Thoft-Christensen & Baker, 1982; Madsen et al. 1986; Ditlevsen & Madsen, 1996).

3.1.2.3.2 *Direct integration methods (Level III)*

Riemann integration

Standard numerical integration methods can be applied to Equation (3.2) In that case the probability of failure is estimated by:

$$P_f \approx \sum_{i_1=0}^{m_1}\sum_{i_2=0}^{m_2}...\sum_{i_n=0}^{m_n} 1(g(\mathbf{x})) f_{\mathbf{X}}(x_{01} + i_1\Delta x_1, x_{02} + i_2\Delta x_2, ..., x_{0n} + i_n\Delta x_n)\Delta x_1 \Delta x_2 ... \Delta x_n \tag{3.3}$$

In which m_i is the number of steps for variable number i; n is the number of basic variables; $1(g(\mathbf{x}))$ is the indicator function defined as

$$1(g(\mathbf{x})) = 1, \quad \text{if } g(\mathbf{x}) \le 0; \tag{3.4}$$

$$1(g(\mathbf{x})) = 0, \quad \text{if } g(\mathbf{x}) > 0 \tag{3.5}$$

The calculation time depends on the number of basic variables (n) and the number of calculation steps to be taken (m). The total number of iterations can be written as:

$$N = \prod_{j=1}^{n} m_j \tag{3.6}$$

This indicates that the calculation time increases rapidly with an increasing number of basic variables. Furthermore the calculation time as well as the accuracy of the method depend strongly on the number of calculation steps per variable. Importance sampling methods have been proposed to increase the calculation speed as well as the accuracy of the calculation method. These methods are not elaborated upon here. Reference is made to (Ouypornprasert, 1987; CUR, 1997).

Monte Carlo simulation

A different method which uses the joint distribution of the basic variables is Monte Carlo simulation. In this method a large sample of values of the basic variables is generated and the number of failures is counted. The number of failures equals:

$$N_f = \sum_{j=1}^{N} 1\left(g\left(\mathbf{x}_j\right)\right) \tag{3.7}$$

In which N is the total number of simulations. In Equation (3.7) the same indicator function is used as in Equation (3.3).

The probability of failure can be estimated by:

$$P_f \approx \frac{N_f}{N} \tag{3.8}$$

The coefficient of variation of the failure probability can be estimated by:

$$V_{P_f} \approx \frac{1}{\sqrt{P_f N}} \tag{3.9}$$

In which P_f denotes the estimated failure probability.

The accuracy of the method depends on the number of simulations (CUR, 1997). The relative error made in the simulation can be written as:

$$\varepsilon = \frac{\frac{N_f}{N} - P_f}{P_f} \tag{3.10}$$

The expected value of the error is zero. The standard deviation is given as:

$$\sigma_\varepsilon = \sqrt{\frac{1-P_f}{NP_f}} \tag{3.11}$$

For a large number of simulations, the error is Normal distributed. Therefore the probability that the relative error is smaller than a certain value E can be written as:

$$P(\varepsilon < E) = \Phi\left(\frac{E}{\sigma_{\varepsilon}}\right) \tag{3.12}$$

$$N > \frac{k^2}{E^2}\left(\frac{1}{P_f} - 1\right) \tag{3.13}$$

The probability of the relative error E being smaller than $k\sigma_{\varepsilon}$ now equals $\Phi(k)$. For desired values of k and E the required number of simulations is given by:

Requiring a relative error of $E = 0.1$ lying within the 95 % confidence interval ($k = 1.96$) results in:

$$N > 400\left(\frac{1}{P_f} - 1\right) \tag{3.14}$$

Equations (3.13) and (3.14) show that the required number of simulations and thus the calculation time depend on the probability of failure to be calculated. Most structures in coastal engineering possess a relatively high probability of failure (i.e. a relatively low reliability) compared to structural elements/systems, resulting in reasonable calculation times for Monte Carlo simulation. The calculation time is independent of the number of basic variables and therefore Monte Carlo simulation should be favoured over the Riemann method in case of a large number of basic variables (typically more than five). Furthermore, the Monte Carlo method is very robust, meaning that it is able to handle discontinuous failure spaces and reliability calculations in which more than one design point are involved (see below).

The problem of long calculation times can be partly overcome by applying importance sampling. This is not elaborated upon here. Reference is made to (Bucher, 1987; Ditlevsen & Madsen, 1996; CUR, 1997).

3.1.2.3.3 *Approximating methods (Level II)*

First Order Reliability Method (FORM)

In the FORM-procedure the value of the volume integral (Equation(3.2)) is estimated by an approximating procedure. The following procedure is followed to estimate the probability of failure:

- A transformation $\mathbf{X} = T(\mathbf{U})$ is carried out, mapping all the random variables in the space of standard normal-distributed variables (U-space);

- The reliability function is also transformed to the *U*-space and is replaced by its first-order Taylor approximation in a certain point;

Hasofer & Lind (1974) have shown that the calculated probability of failure is invariant for the formulation of the limit state equation, if the limit state is linearised in the point with the highest value of the joint probability density of the basic variables (design point, see Fig. 3-3).

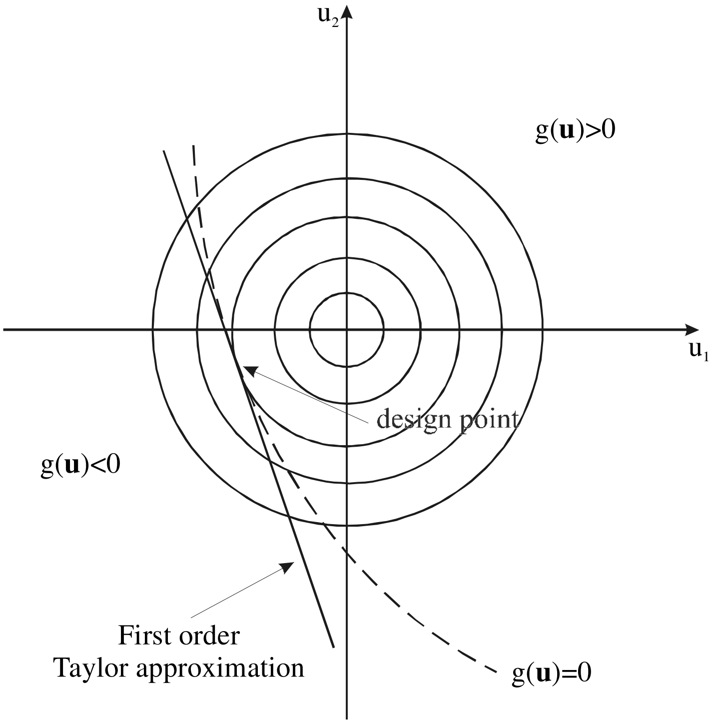

Figure 3-3. Design point, real failure boundary and linearised failure boundary in the space of the standard-normal variables (*U*-space)

In the *U*-space this point coincides with the point at the failure boundary with minimum distance to the origin. This distance is called the Hasofer-Lind reliability index (β_{HL}). For the calculation of the reliability index several methods have been proposed. Reference is made to (Thoft-Christensen & Baker, 1982; Madsen et al. 1986; Ditlevsen & Madsen, 1996). As a result of the transformation to *U*-space, the safety margin $M = g(\mathbf{x})$ is linearised in *U*-space. Therefore, the probability of failure can be approximated by:

$$P_f = P(M < 0) \approx P(\beta_{HL} - \alpha^{\mathrm{T}}\mathbf{U}) = \Phi(-\beta_{HL}) \tag{3.15}$$

This solution is exact only if the reliability function is linear in the basic variables and if the basic variables have Normal distributions. As noted above, the elements in the α-vector are a measure of the importance of the random variables. $\alpha_i^2 \cdot 100\,\%$ gives the relative importance in % of the random variable x_i.

Generally, the limit state equation is dependent on a number of deterministic parameters, which can be collected in the vector **p**. The elements in **p** can be statistical parameters like expected values and standard deviations and it can be e.g. geometrical quantities with negligible uncertainty. The limit state equation in U-space is written as:

$$g(\mathbf{U}, \mathbf{p}) = 0 \tag{3.16}$$

For the reliability index β_{HL} the sensitivity with respect to the parameter p_j can easily be obtained in the form, see e.g. Madsen et al (1986):

$$\frac{\mathrm{d}\beta_{HL}}{\mathrm{d}p_j} \tag{3.17}$$

FORM calculations generally provide estimates of the failure probability in relatively short calculation times. This is a big advantage over level III methods in general. However, if the reliability function is highly non-linear, FORM-estimates of P_f may possess a considerable error. In Figure 3-3 a curved limit state equation is shown together with its first order approximation. In this case the estimate of the probability of failure obtained with FORM underestimates the real failure probability. In case of discontinuous failure spaces, FORM procedures may fail to give a correct failure probability at all.

Second Order Reliability Method (SORM)

The disadvantages of FORM estimates are partly overcome by the Second Order Reliability Method (SORM). Instead of calculating the probability of failure directly from the reliability index, a formula is applied in which the curvature of the limit state equation at the design point is used to get a better estimate of the probability of failure. This working method implies that a reliability index and a design point are known. Therefore, in SORM at first a FORM calculation is performed. For more details reference is made to (Breitung, 1984). Regarding the description of discontinuous failure spaces, the same disadvantages as for FORM apply.

3.1.2.4 *Fault tree analysis*

3.1.2.4.1 *General system analysis by fault tree*

The methods for reliability calculations given in the previous section are all based on the analysis of one limit state only. However, even for a very simple structure generally several failure modes are relevant. A set of limit states can be presented as a series system, a parallel system or a combination thereof. The probability of failure of the system is determined by the properties of the system as well as by the individual failure probabilities of the components (limit states).

If the order of occurrence of the failure modes is not of consequence for the failure of the structure, the system of failure modes can be described by a fault tree. An example is given in Figure 3-4. The system representation is also given.

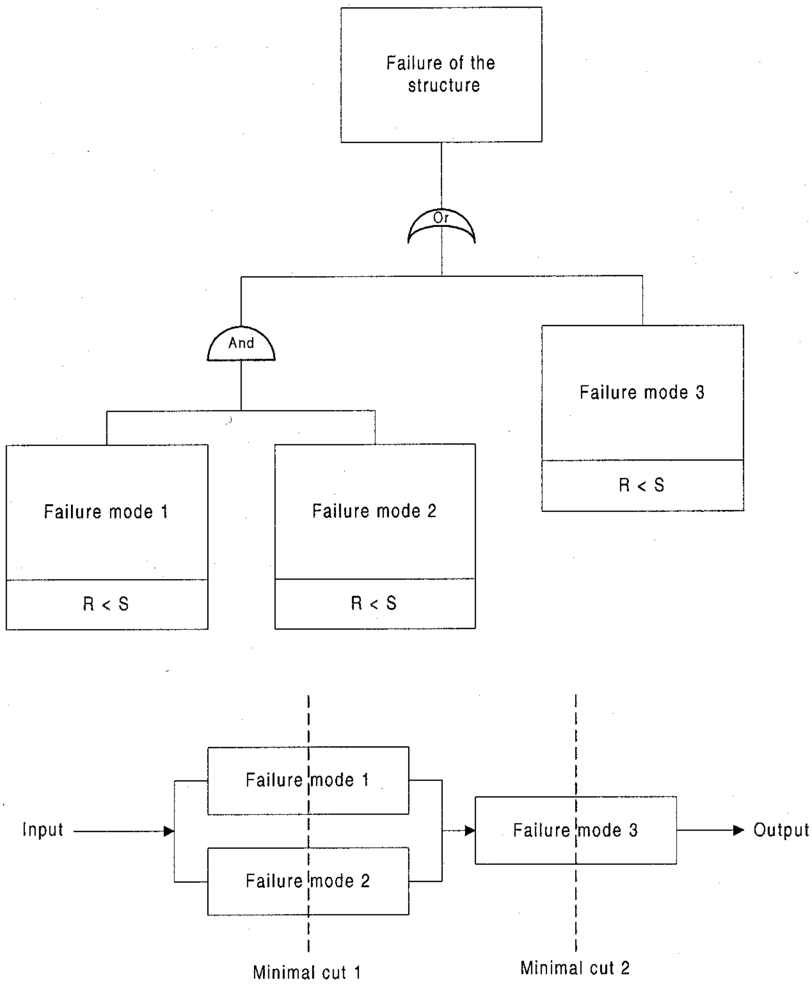

Figure 3-4. Fault tree and system representation of a system of three failure modes.

The two presentations of the system are completely equivalent. An and-gate coincides with a parallel system of failure modes and an or-gate coincides with a series system of failure modes.

An overview of the notation used in the fault trees is given in Table 3-1 and Table 3-2.

Table 3-1. Overview of gate symbols in fault trees (Andrews & Moss, 1993).

Symbol	Description
	"And" gate
	"Or" gate
	Voting gate
	Inhibit gate
	Exclusive "or" gate
	Priority "and" gate

Table 3-2. Overview of event symbols in fault trees (Andrews & Moss, 1993).

Symbol	Description
	Base event
	Event not further developed in the tree
	Compound event I
	Compound event II
	Conditional event (used in combination with inhibit gate)
	Normal event (house event)
	Reference symbol

3.1.2.5 *Calculation of system probability of failure*

3.1.2.5.1 *Introduction*

In the previous section the fault tree presentation of a system of failure modes is introduced. The fault tree indicates whether failure modes are part of a parallel system or part of a series system of failure modes. An analytical expression for the probability of system failure is possible only if the failure modes are uncorrelated or fully correlated. The formulae are given in Table 3-3.

Table 3-3. Overview of system probability of failure for combinations of correlation and system type.

System type	Upper bound	Independent components	Lower bound
Series	$P_f = \sum_{i=1}^{n} P_{f_i}$	$P_f = \prod_{i=1}^{n} P_{f_i}$	$P_f = \max\left(P_{f_i}\right)$
Parallel	$P_f = \min\left(P_{f_i}\right)$	$P_f = 1 - \prod_{i=1}^{n}\left(1 - P_{f_i}\right)$	$P_f = 0$

The formulae given in Table 3-3 are also bounds on the real system probability of failure. For a series system the upper bound is given by the uncorrelated case and the lower bound by the case with full correlation. For a parallel system the upper bound is given by the case with full correlation and the lower bound by the uncorrelated case.

For arbitrary correlation between limit state equations, an analytical solution is no longer possible. In that case one has to use numerical methods to obtain the system probability of failure. Section 3.1.2.5.2 introduces direct integration methods. Section 3.1.2.5.3 introduces a few approximating methods for system failure.

3.1.2.5.2 *Direct integration methods for systems*

<u>Riemann integration</u>

The Riemann process introduced in section 3.1.2.3.2 can also be applied to evaluate the probability of system failure. Similar to the case with on limit state equation the probability of failure is estimated by:

$$P_f \approx \sum_{i_1=0}^{m_1}\sum_{i_2=0}^{m_2}\ldots\sum_{i_n=0}^{m_n} 1(\mathbf{g}(\mathbf{x})) f_{\mathbf{X}}\left(x_{01} + i_1\Delta x_1, x_{02} + i_2\Delta x_2, \ldots, x_{0n} + i_n\Delta x_n\right)\Delta x_1 \Delta x_2 \ldots \Delta x_n \tag{3.18}$$

In which $\mathbf{g}(\mathbf{x})$ is a vector of limit state equations; m_i is the number of steps for variable number i; and n is the number of basic variables.

Since now the system probability of failure has to be calculated, the indicator function is defined in a different way. For a series system the indicator function is given as:

$$1(\mathbf{g}(\mathbf{x})) = \begin{cases} 1 \text{ if } \min\limits_{i=1,n}\left(g_i(\mathbf{x})\right) \le 0 \\ 0 \text{ if } \min\limits_{i=1,n}\left(g_i(\mathbf{x})\right) > 0 \end{cases} \tag{3.19}$$

For a parallel system the indicator function is:

$$1(\mathbf{g}(\mathbf{x}))=\begin{cases}1 \text{ if } \max\limits_{i=1,n}(g_i(\mathbf{x}))\le 0\\ 0 \text{ if } \max\limits_{i=1,n}(g_i(\mathbf{x}))>0\end{cases} \tag{3.20}$$

In general the evaluation of more limit state equations in a calculation step will not take much extra time. Therefore, the performance of the Riemann integration for more limit states is comparable to the performance for one limit state (section 3.1.2.3.2).

Monte Carlo simulation

Like Riemann integration, Monte Carlo simulation is also applicable for systems. The number of failures is determined by applying Equation (3.7). Also in this case the indicator function is replaced by one of the system indicator functions given in the previous section. The evaluation of extra limit states is in general not very time consuming. Therefore, the performance of the Monte Carlo method is not heavily influenced by the application to more limit states. Also for systems the Monte Carlo method proves to be a very robust, but not very fast method.

3.1.2.5.3 *Approximating methods for systems*

Fundamental bounds on the system probability of failure

When the failure probabilities per limit state are known, it is always possible to provide a lower and upper bound for the failure probability (see Table 3-3). In several cases these bounds prove to give a reasonable range in which the real probability of system failure is to be found. In some cases however, the fundamental bounds provide a too wide range. In that case one has to use more advanced methods.

Ditlevsen bounds for the system probability of failure

An alternative calculation method for the bounds of the probability of failure of a series system is developed by (Ditlevsen, 1979). The bounds according to Ditlevsen are given by:

$$\begin{aligned}&P(g_1(\vec{x})<0)+\sum_{i=2}^{n}\max\left[\left(P(g_i(\vec{x})<0)-\sum_{j=1}^{i-1}P\big(g_i(\vec{x})<0\cap g_j(\vec{x})<0\big)\right),0\right]\le P_f\\ &P_f\le\sum_{i=1}^{n}P(g_i(\vec{x})<0)-\max_{j<i}\big(P\big(g_i(\vec{x})<0\cap g_j(\vec{x})<0\big)\big)\end{aligned} \tag{3.21}$$

These bounds are more narrow than the fundamental bounds, but for a large number of limit states they may still be too wide.

First-order method for systems
An approximating procedure which is able to provide a more accurate estimate of the system probability of failure of a system is proposed by (Hohenbichler & Rackwitz, 1983). In this method the correlated limit state equations are transformed to a set of uncorrelated limit states. The system probability of failure can then be calculated using fundamental rules for the probability of system failure and first or second order estimates of the probability of failure per limit state. Generally, an approximation is obtained for $\Phi_n(\beta,\rho)$ in which β denotes the vector of reliability indices for every limit state and ρ denotes the correlation between the failure modes. The failure probability of a parallel system is then given by:

$$P_{f;parallel} \approx \Phi_n(-\beta,\rho) \tag{3.22}$$

And for a series system by:

$$P_{f;series} \approx 1-\Phi(\beta,\rho) \tag{3.23}$$

This method in general has shorter calculation times than the level III methods. However, due to the approximations estimates of the failure probability might show considerable errors (Schuëller & Stix, 1987). As with all approximating methods one should be aware of this disadvantage.

3.1.2.6 *Choice of safety level*

To construct a breakwater that is always performing its function and is perfectly safe from collapse is at least an uneconomical pursuit and most likely an impossible task. Although expertly designed and well constructed, there will always be a small possibility that the structure fails under severe circumstances (Ultimate Limit State). The acceptable probability of failure is a question of socio-economic reasoning.

In a design procedure one has to determine the preferred level of safety (i.e. the acceptable failure probability). For most civil engineering structures the acceptable failure probability will be based on considerations of the probability of loss of life due to failure of the structure.

In general two points of view for the acceptable safety level can be defined (Vrijling et al., 1995):

The individual accepted risk. The probability accepted by and individual person to die in case of failure of the structure; In Western countries this probability is of the order 10^{-4} per year or smaller

The societal accepted risk. Two approaches are presented, depending on the relative importance of the total number of lives lost in case of failure on the one hand and the total economic damage on the other. If the number of potential casualties is large the likelihood of failure should be limited accordingly. The accepted probability of occurrence of a certain number of casualties in case of failure of a structure is then restricted proportional to the inverse of the square of this number (Vrijling et al., 1995). If the economic damage is large, an economic optimisation equating the marginal investment in the structure with the marginal reduction in risk should be carried out to find the optimal dimensions of the structure.

The two boundary conditions based on the loss of human lives form the upper limits for the acceptable probability of failure of any structure. In case of a breakwater without amenities the probability of loss of life in case of failure is very small. In that case the acceptable probability of failure can be determined by economical optimisation, weighing the expected value of the capitalised damage in the life of the structure (risk) against the investment in the breakwater. The next section provides more background on this concept.

If, for a specific breakwater, failure would include a number of casualties, the economic optimisation should be performed under the constraint of the maximum allowable probability of failure as defined by the two criteria related to loss of life.

The explicit assessment of the acceptable probability of failure as sketched above is only warranted in case of large projects with sufficient means. For smaller projects a second approach is generally advised. This second approach to the acceptable safety level is based on the evaluation of the safety of existing structures supplemented by considerations of the extent of the losses involved in case of failure. Consequently the assumption is made that the new structure should meet the safety requirements that seem to be reasonable in practice. This approach is found in many codes where a classification of the losses in case of failure leads to an acceptable probability of failure. Most structural codes provide safety classes for structures (NKB 1978, Eurocode 1). The structure to be designed might fit in one of these safety classes providing an acceptable probability of failure. It should be noted however that for most structural systems loss of life is involved contrary to breakwaters. Therefore the following classification and table with acceptable probabilities of failure was developed especially for vertical breakwaters:

- *Very low safety class*, where failure implies no risk to human injury and very small environmental and economic consequences
- *Low safety class*, where failure implies no risk to human injury and some environmental and economic consequences
- *Normal safety class*, where failure implies risk to human injury and significant environmental pollution or high economic or political consequences

- *High safety class*, where failure implies risk to human injury and extensive environmental pollution or very high economic or political consequences

Table 3-4. Overview of safety classes.

Limit state type	Safety class			
	Low	**Normal**	**High**	**Very high**
SLS	0.4	0.2	0.1	0.05
ULS	0.2	0.1	0.05	0.01

3.1.2.7 *Reliability based design procedures*

3.1.2.7.1 *General formulation of reliability based optimal design*

Generally, in a design process one pursues the cheapest design that fulfils the demands defined for the structure. The demands can be expressed in two fundamentally different ways:

- The total expected lifetime costs of the structure consisting of the investment and the expected value of the damage costs are minimised as a function of the design variables;
- If a partial safety factor system is available, one can optimise the design by minimising the construction costs as a function of the design variables under the constraint that the design equations related to the limit state equations for all the failure modes are positive.

The minimisation of the lifetime costs can be formalised as follows (Enevoldsen & Sørensen, 1993):

$$\begin{aligned} &\min_{\mathbf{z}} C_T(\mathbf{z}) = C_I(\mathbf{z}) + C_{F;ULS} P_{F;ULS}(\mathbf{z}) + C_{F;SLS} P_{F;SLS}(\mathbf{z}) \\ &\text{s.t} \quad z_i^L \le z_i \le z_i^U \qquad i = 1,\dots,m \\ &\qquad P_{F;ULS}(\mathbf{z}) \le P_{F;ULS}^U \\ &\qquad P_{F;SLS}(\mathbf{z}) \le P_{F;SLS}^U \end{aligned} \tag{3.24}$$

In which:

$\mathbf{z} = (z_1, z_2, \dots, z_m)$:	The vector of design variables;
$C_T(\mathbf{z})$:	The total lifetime costs of the structure;
$C_I(\mathbf{z})$:	The investment in the structure as a function of the design variables $\mathbf{z}$;
$C_{F;ULS}$:	The damage in monetary terms in case of ULS failure;
$C_{F;SLS}$:	The damage in monetary terms in case of SLS failure;

$P_{F;ULS}(\mathbf{z})$: The probability of ULS failure as a function of the design variables;

$P_{F;SLS}(\mathbf{z})$: The probability of SLS failure as a function of the design variables;

z_i^L, z_i^U : The lower and upper bound of design variable i;

$P_{F;ULS}^U, P_{F;SLS}^U$: The upper bound of the failure probability for ULS failure and SLS failure respectively.

Generally the design variables will be subjected to constraints. For instance, all geometrical quantities should be greater than zero. Furthermore, the failure probabilities can be subject to constraints, especially for structures where human lives are involved. In that case the maximum failure probabilities are enforced by regulations. In cases that loss of human lives is not involved in case of failure of the structure, formally the constraint on the failure probabilities can be set to 1 and the acceptable failure probability as well as the optimal design are completely decided by the lifetime costs only. If relevant, maintenance costs and inspection costs can be added to the total expected lifetime costs.

Obtaining accurate assessments of the damage in case of failure is not always practically possible. In that case, the optimal design can be found by minimising a cost function which only comprises the investment and imposing a constraint on the failure probability which expresses a qualitative idea of the economic optimal failure probability.

If the design is performed using a code based partial safety factor system, the following optimisation problem is applicable:

$$\begin{aligned} &\min_{\mathbf{z}} C_T(\mathbf{z}) = C_I(\mathbf{z}) \\ &\text{s.t} \quad z_i^L \le z_i \le z_i^U \\ &\qquad G_i\left(\mathbf{z}, \mathbf{x}^{\mathbf{C}}, \gamma\right) > 0 \end{aligned} \tag{3.25}$$

In which:

$C_I(\mathbf{z})$: The investment in the structure as a function of the design variables $\mathbf{z}$;

$G_I(\mathbf{Z},\mathbf{X}^{\mathbf{C}},\boldsymbol{\gamma})$: The limit state function for failure mode i as a function of the design variables $\mathbf{z}$, the characteristic values of the random variables as defined in the partial safety factor system $\mathbf{x}^{\mathbf{c}}$ and the vector of partial safety factors $\boldsymbol{\gamma}$.

Generally, partial safety factors are available for several target probabilities of failure or safety classes (see below). Since the choice of the safety factors involves implicitly the choice of a target probability of failure and expected costs of failure, the same optimal design should be obtained from (23) and (24).

3.1.2.7.2 *Cost optimisation*

If loss of life in case of failure of the structure is not an issue for the structure under consideration, no constraint is set on the failure probability and the acceptable probability of failure equals the economic optimal probability of failure. A procedure for probabilistic optimisation of vertical breakwaters has been developed (Sørensen et al, 1994; Voortman et al, 1998).

The optimisation for a vertical breakwater can be written as:

$$\min_{\mathbf{z}} C(\mathbf{z}) = C_{I;0} + C_I(\mathbf{z}) + \sum_{n=1}^{N}\left(\frac{365 C_{F;SLS} P_{F;SLS}(\mathbf{z}) + C_{F;ULS} P_{F;ULS}(\mathbf{z})}{(1+r'-g)^n} + \frac{C_{maint}}{(1+r')^n}\right) \quad (3.26)$$

$$\text{s.t} \quad 0 \le z_i$$

In which:

$\mathbf{z}$:	The vector of design variables;
$C_{I;0}$:	Initial costs, not depending on the design variables;
$C_I(\mathbf{z})$:	Construction costs as a function of the design variables;
$C_{F;SLS}$:	Costs per day in case of serviceability failure;
$P_{F;SLS}(\mathbf{z})$:	The probability of serviceability failure per day;
$C_{F;ULS}$:	Costs per event in case of ultimate limit state failure;
$P_{F;ULS}(\mathbf{z})$:	The probability of ultimate limit state failure per year;
C_{maint}:	Maintenance costs for the breakwater per year;
r':	The net interest rate per year;
g:	The yearly rate of economical growth, expressing growth and development of the harbour;
N:	The lifetime of the structure in years.

Inspection of Equation (3.26) shows that the total lifetime costs consist of investment costs and the expected value of the damage costs. In principle, for every year of the structure's lifetime, the expected damage has to be taken into account. Not all the costs are made at the same time. Therefore, the influence of interest, inflation and economical growth has to be taken into account in order to make a fair comparison of the different costs (van Dantzig, 1956).

The expected value of the damage costs is a function of the failure probability. The failure probability is a function of the design variables. Therefore, minimisation of Equation (3.26) results in the optimal geometry and at the same time the optimal failure probability. Ready at hand minimisation algorithms can be applied to find the optimal set of design variables.

When implementing the cost function in any programming language, the failure probability as a function of the design variables has to be included. For this part of the cost function one of the probabilistic procedures introduced in section 3.1.2.3 or section 3.1.2.5 can be used. Due to the specific character of the optimisation process, the choice of the probabilistic procedure is not an arbitrary one. One should be aware of the following points:

- The minimisation process comprises a large number of evaluations of the cost function, each evaluation involving a probability calculation. Therefore, time-consuming methods should be avoided;
- The values of the cost function for any given point should be stable. Especially the Monte Carlo procedure provides probability estimates that contain an error, which is inherent to the procedure. This (small) error generally presents no problem, but in this case it causes variations of the cost function, which disturb the optimisation process (see Fig. 3.5).

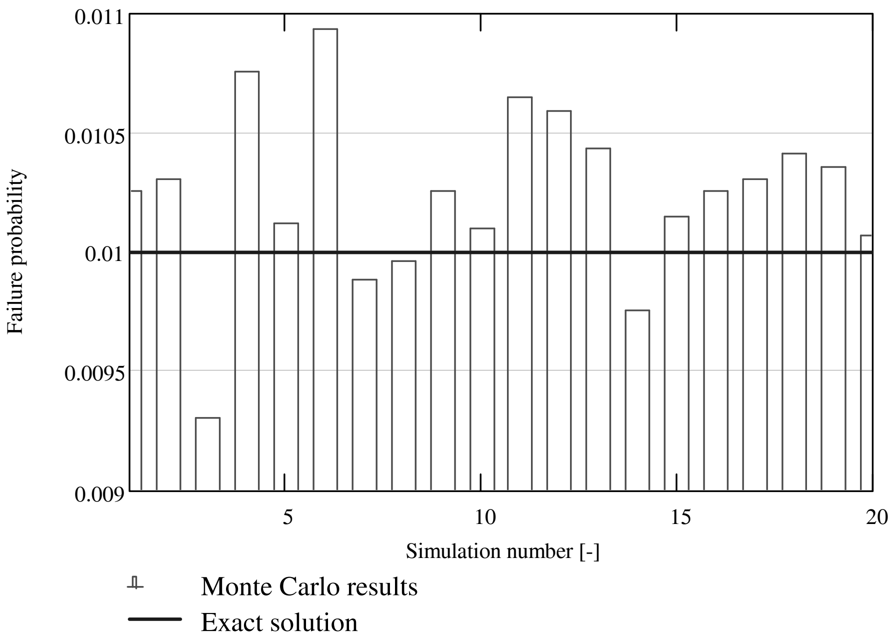

Figure 3-5. Result of 20 calculations of failure probability by Monte Carlo.

The points of attention mentioned above lead to the conclusion that level II methods are suitable for application in an optimisation process. Level III methods will generally lead to too much computational effort or will disturb the optimisation process.

The procedure described above has been applied to a fictitious design case of a vertical breakwater in a water depth of 25 m. Three design variables are considered: the height and width of the caisson and the height of the rubble berm.

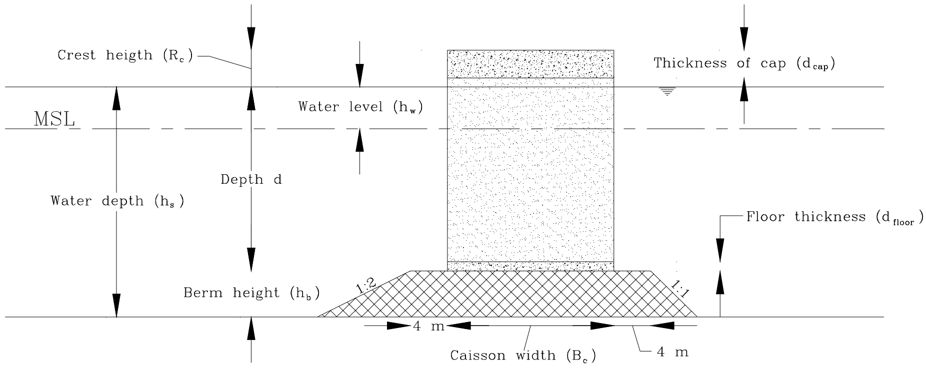

Figure 3-6. Overview of conceptual breakwater design for economic optimisation.

As a first step a deterministic optimisation for chosen wave heights has been performed. This step is meant to show the connection between the deterministic optimisation for a given safety level and the full probabilistic approach. Because of this, the choice of the input values (comparable to the characteristic values in Equation (3.25) is not corresponding to the choice made for the partial safety factor system (see below). Furthermore, all safety factors have been set to 1 and the berm height is fixed at a value of 6 m. For this situation it is possible to find a minimum caisson width as a function of the crest height for every single failure mode. Once the crest height and the caisson width are known, the construction costs of the caisson breakwater can be calculated (see Fig. 3-7).

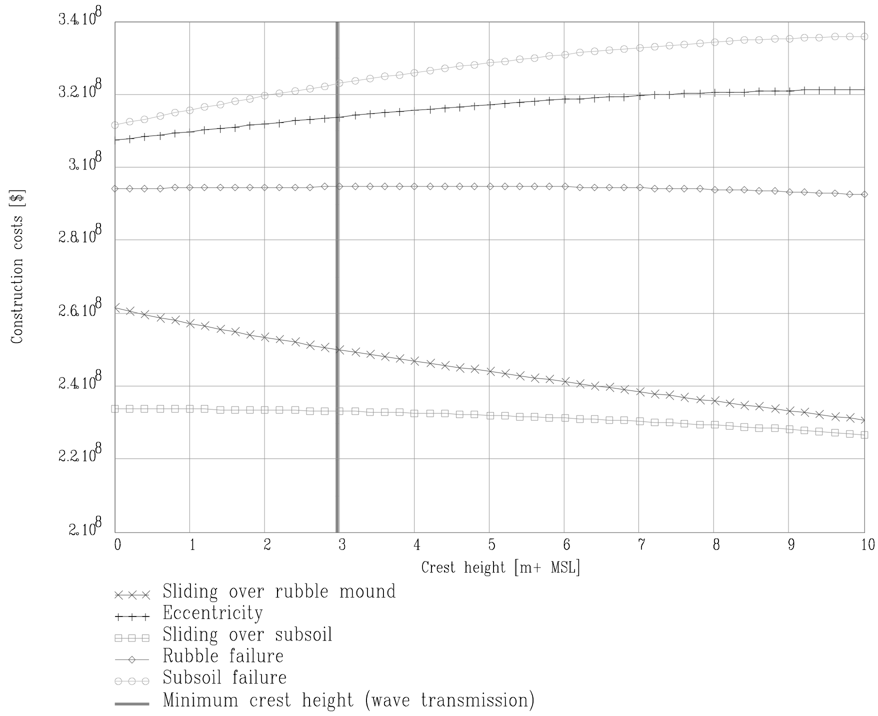

Figure 3-7. Construction costs of the breakwater as a function of the crest height (Berm height 6 m).

Generally, bearing capacity failure of the subsoil shows the largest minimum caisson width. Furthermore, the results show that in general a lower crest height leads to a more narrow caisson and thus to lower construction costs. However, the minimum crest height required is determined by wave transmission. In the deterministic approach the minimum crest height related to wave transmission imposes a constraint on the crest height. Thus, the optimal geometry is decided by wave transmission and by bearing capacity failure of the subsoil.

While at first sight it seems reasonable to have an equal failure probability of failure for all the failure modes in the system, probabilistic optimisation shows that, like in the deterministic approach, bearing capacity failure of the subsoil largely determines the probability of ultimate limit state failure (see Figure 3-8).

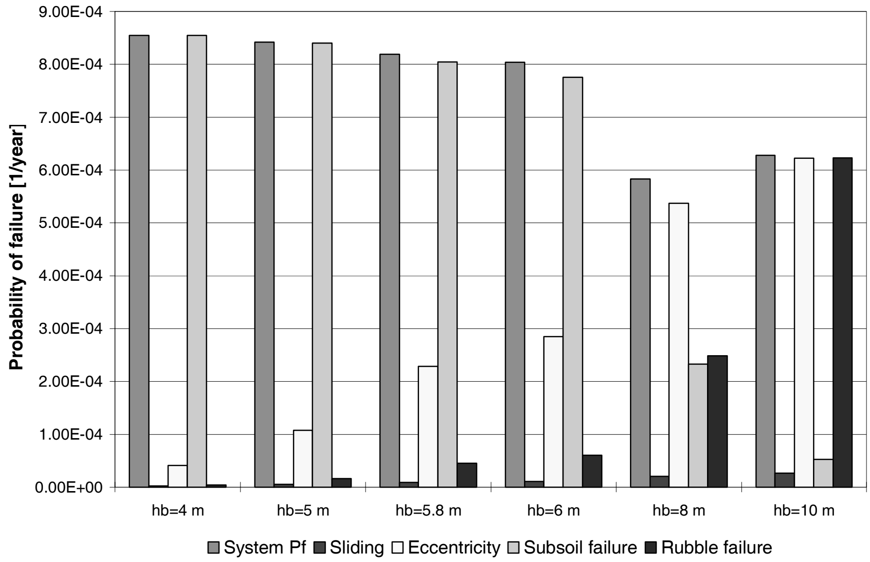

Figure 3-8. Overview of ULS failures probabilities for several berm heights.

Inspection of the lifetime costs as a function of crest height and caisson width indicates that also in the probabilistic approach the crest height is limited by wave transmission (see Fig. 3-9).

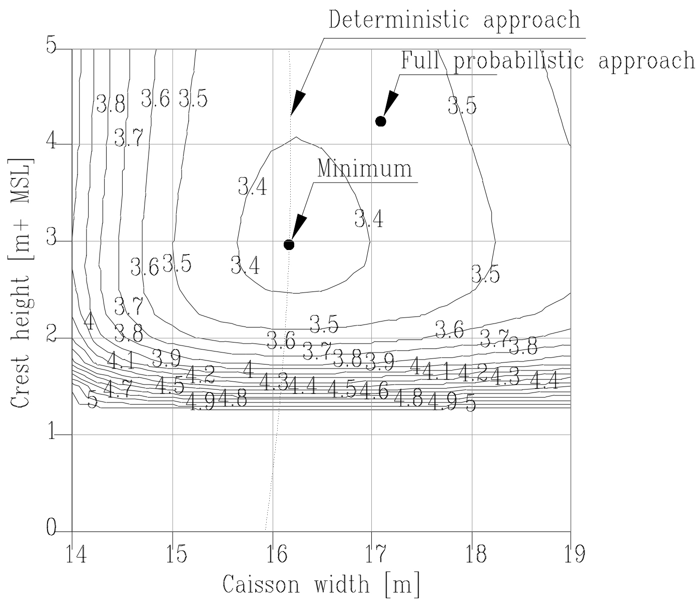

Figure 3-9. Contour plot of total lifetime costs in 10^8 US $ (Random wave height only) and optimal geometries for different levels of modelling. Taken from Voortman et al (1998).

The optimal probability of system failure is quite low in comparison to existing structures ($8 \cdot 10^{-4}$). This could be caused by the choice of the cost figures or by a limited spreading of the random variables.

3.1.2.7.3 *Partial Safety Factor System*

Partial safety factors used in design of vertical wall breakwaters can be calibrated on a probabilistic basis. The calibration is performed for a given class of structures, materials and/or loads in such a way that for all structure types considered the reliability level obtained using the calibrated partial safety factors for design is as close as possible to a specified target reliability level. Procedures to perform this type of calibration of partial safety factors are described in for example Thoft-Christensen & Baker, 1982, Madsen et al. (1986) and Ditlevsen & Madsen (1996).

A code calibration procedure usually includes the following basic steps:

1) definition of scope of the code; here : vertical wall breakwaters
2) definition of the code objective; here : to minimise the difference between the target reliability level and the reliability level obtained when designing different typical structures using the calibrated partial safety factors
3) selection of code format, see below
4) selection of target reliability levels, see section 3.1.2.6
5) calculation of calibrated partial safety factors, see below

6) verification of the system of partial safety factors, see below

The partial safety factors can be calibrated as follows. For each failure mode one or more limit state functions are established:

$$g_i(\mathbf{x},\mathbf{z}) = 0 \tag{3.27}$$

where $\mathbf{x} = (x_1,...,x_n)$ are realisations of n stochastic variables $\mathbf{X} = (X_1,...,X_n)$ and $\mathbf{z} = (z_1,...,z_N)$ are the deterministic design variables.

Corresponding to (5-26) a design equation is established from which $\mathbf{z} = (z_1,...,z_N)$ are determined:

$$G_i(\mathbf{x}^c,\mathbf{z},\gamma) = 0 \tag{3.28}$$

where $\mathbf{x}^c = (x_1^c,...,x_n^c)$ are characteristic values of $\mathbf{X} = (X_1,...,X_n)$ and $\gamma = (\gamma_1,...,\gamma_M)$ are partial safety factors. Usually design values for loads are obtained by multiplying the characteristic with the partial safety factors and design values for resistances are obtained by dividing the characteristic values with the partial safety factors. On the basis of the limit state functions in (1) element reliability indices β_i or a system reliability index β_s can be determined for the structure considered, see e.g. Madsen et al. (1986).

The partial safety factors γ are calibrated such that the reliability indices corresponding to L example structures are as close as possible to a target reliability index β^t. This is formulated by the following optimisation problem

$$\min \quad W(\gamma) = \sum_{i=1}^{L} w_i \left(\beta_i(\gamma) - \beta^t\right)^2 \tag{3.29}$$

where w_i , $i = 1,2,...,L$ are weighting factors indicating the relative frequency of appearance of the different design situations. Instead of using the reliability indices in Equation (3.29) to measure the deviation from the target, the probabilities of failure can be used. $\beta_i(\gamma)$ is the system reliability index for example structure i with a design $\mathbf{z}$ obtained from the design equations using the partial safety factors γ and the characteristic values. Usually the partial safety factors are constrained to be larger than or equal to 1.

The code format and thus the partial safety factors to be used in design of vertical breakwaters can in principle depend on:

a) the uncertainty related to the parameters in the relevant limit states
b) the safety class
c) the type of limit state
d) the expected lifetime of the structure
e) if laboratory model tests have been performed
f) the amount of quality control during construction

Ad a) uncertainty related to parameters: the uncertainties related to the parameters in the limit state functions are taken into account in a deterministic design through partial safety factors. The partial safety factors are calibrated in such a way that a large partial safety factor is used in the case of large uncertainties and a small partial safety factor is used when the uncertainties are relatively small.

Ad b) safety class: the safety classes in section 3.1.2.6 are used.

Ad c) type of limit state: two types of limit states are considered, namely: ULS (Ultimate Limit State; e.g. foundation failure, failure of significant part of caisson concrete structure) and SLS (Serviceability Limit State, e.g. overtopping, settlement of foundation soil). Acceptable probabilities of failure could be as indicated in Table 5-4.

Ad d) expected lifetime: the expected lifetime T_L for vertical breakwaters can be quite different. Therefore three different expected lifetimes are considered: T_L =20 years, T_L =50 years and T_L =100 years.

Ad e) model tests: sometimes laboratory tests are performed in order to estimate the wave loads more accurately. In that case the uncertainty related to the wave loads is reduced and it is therefore reasonable to decrease the partial safety factors. Similarly, also detailed field and laboratory tests are performed to determine the soil parameters. In that case the uncertainty related to the soil strength parameters is usually reduced and the partial safety factors can be reduced.

Ad f) quality control: finally, the uncertainty can also be dependent on the amount of control at the construction site.

The characteristic values are suggested to be the mean value for self weight and permanent actions, for wave heights the expected largest significant wave height in the design lifetime T_r, 5 % fractiles for structural strength parameters and the mean values for geotechnical strength parameters.

An example of the partial safety factor system is shown in Table 3-5.

Table 3-5. Partial safety factors.

p.s.f.	Parameter	Tentative values - γ
Loads		
γ_{G_1}	Self weight	1.0
γ_{G_2}	Permanent actions, e.g. ballast	1.1
γ_H	Wave load	See below
Strength		
γ_ϕ	Effective friction angle	See below
γ_{C_u}	Undrained shear strength	See below
γ_c	Concrete strength	1.6
γ_r	Reinforcement	1.3
γ_{scour}	Scour failure	See below
γ_{armour}	Armour layer failure	See below

The partial safety factor for the wave load is determined from :

$$\gamma_H = \gamma_{H_0}\gamma_T\gamma_{H_2} \tag{3.30}$$

Where:

γ_{H_0} takes into account the uncertainty related to the wave load

γ_T takes into account the influence of the expected lifetime. For example γ_T=1 for T=50 years, $\gamma_T > 1$ for T=100 years and $\gamma_T < 1$ for T=20 years.

γ_{H_2} takes into account the effect of model tests used to estimate the wave load. γ_{H_2}=1 if no model tests are performed

For partial safety factor for the geotechnical parameters are determined from :

$$\gamma_m = \gamma_0\gamma_1\gamma_2\gamma_3 \tag{3.31}$$

Where:

γ_0 takes into account the uncertainty related to the strength parameter

γ_1 takes into account the effect of safety class. γ_1=1 for normal safety class

γ_2 takes into account the effect of model tests used to estimate the strength parameters. γ_2=1 if no model tests are performed

γ_3 takes into account the amount of control. γ_3=1 if normal control is used.

The factors in equations (3.30) and (3.31) are calibrated using representative values for the soil strength parameters and with wave climates from Bilbao, Sines, Tripoli and Fallonica. The full stochastic model used in the calibration is shown in Table 3-5.

The calibrated partial safety factors corresponding to normal safety class in ULS and high safety class in SLS:

Wave load:	γ_{H0}=1.1
Effective friction angle:	γ_0=1.2
Undrained shear strength:	γ_0=1.3
Scour failure:	γ_0=2.2
Armour layer failure:	γ_0=0.6

The following factors defined in are derived (no factors are derived for γ_2 taking into account the effect of model tests used to estimate the strength parameters and γ_3 taking into account the amount of control). The factors for γ_1 can be used to obtain partial safety factors for all safety classes in ULS and SLS.

Table 3-6. Safety factor γ_T, which takes into account the influence of the expected lifetime *T*.

T	**20 years**	**50 years**	**100 years**
γ_T	0.98	1.0	1.05

Table 3-7. Safety factor γ_{H_2} that takes into account the effect of model tests used to estimate the wave load.

Model	**1**	**2**
γ_{H_2}	1.0	0.85

Table 3-8. Safety factor γ_1 , which takes into account the effect of the safety class.

	Safety class				
ULS		**Low**	**Normal**	**High**	**Very high**
SLS	**Low**	**Normal**	**High**	**Very high**	
P_f	0.4	0.2	0.1	0.05	0.01
γ_1	0.75	0.9	1.0	1.1	1.25

The safety classes correspond to the ones given in Table 3-5.

REFERENCES

Andrews, J.D.; Moss, T.R. 1993. Reliability and risk assessment. Harlow, U.K.: Longman Scientific and Technical, 368 pp.

Battjes, J.A.; Janssen, J.P.F.M. 1978. Energy loss and set-up due to breaking of random waves. *Proceedings International Conference Coastal Engineering (ICCE)*, Hamburg, Germany, no. 16, Volume 1, pp. 569-587.

Breitung, K. 1984. Asymptotic approximations for multinormal integrals. *Journal of Engineering Mechanics*, ASCE, Engineering Mechanics Division, vol. 110, pp. 357-366.

Bruining, J.W. 1994. Wave forces on vertical breakwaters. Reliability of design formula. *Delft Hydraulics Report*, Emmeloord, The Netherlands, 65 pp.

Bucher, G.C. 1987. Adaptive sampling, an iterative fast Monte Carlo procedure. *Internal Report, University of Innsbruck*, Innsbruck, Austria.

Christiani, E. 1997. Application of reliability in breakwater design. *Ph.D. thesis, Hydraulics & Coastal Engineering Laboratory, Department of Civil Engineering, Aalborg University*, Aalborg, Denmark, Series Paper 14, 179 pp.; 1 Appendix.

CUR 1997. Risk analysis in civil engineering. *Centre for Civil Engineering Research, Codes and Specification*, CUR Research Committee E10 "Risk Analysis", no. 190, Vol. 1: Probabilistic Design in Theory, Gouda, The Netherlands. In Dutch.

Ditlevsen, O. 1979. Narrow reliability bounds for structural systems. *Journal of Structural Mechanics*, vol. 7, pp. 453-472.

Ditlevsen, O.; Madsen, H.O. 1996. Structural reliability methods. Chichester et al.: John Wiley & Sons, 372 pp.

Enevoldsen, I.; Sørensen, J.D. 1993. Reliability-based optimisation of series systems and parallel systems. *Proceedings IFIP WG 7.5 Conference on Reliability and Optimisation of Structural Systems*, Thoft-Christensen, P.; Ishikawa, H., Takamatsu, Kagawa, Japan, no. 5, pp. 31-46.

Eurocode 1 1994. Basis of design and actions on structures. Part 1: Basis of Design.

Goda, Y. 1969. Re-analysis of laboratory data on wave transmission over breakwaters. *Report. Port and Harbour Research Institute (PHRI)*, Tokyo, Japan, vol. 8, no. 3.

Hasofer, A.M.; Lind, N.C. 1974. An exact and invariant first order reliability format. *Journal of the Engineering Mechanics Division*, ASCE, vol. 100, pp. 111-121.

Heijn, K.M. 1997. Wave transmission at vertical breakwaters. *Master Thesis, Delft University of Technology*, Delft, The Netherlands.

Hohenbichler; Rackwitz, R. 1983. First-order concepts in system reliability. *Structural Safety*, vol. 1, pp. 177-188.

Lancellotta, R. 1995. Geotechnical engineering. Rotterdam, The Netherlands: Balkema, rev. and updated edition, 436 pp.

Madsen, H.O.; Krenk, S.; Lind, N.C. 1986. Methods of structural safety. Englewood Cliffs, N.J., USA: Prentice Hall Inc., 403 pp.

Martinelli, L. 1998. Risk analysis of Genoa Voltri breakwater: the case of Genoa Voltri. *Ph.D. Thesis, Università di Bologna*, 15 December 1998, Bologna, Italy.

NKB 1978. Recommendations for loading and safety regulations for structural design. *NKB report*, no. 36.

Oumeraci, H. 1994. Review and analysis of vertical breakwater failures - lessons learned. *Coastal Engineering, Special Issue on 'Vertical Breakwaters'*, Eds.: Oumeraci, H. et al., Amsterdam, The Netherlands: Elsevier Science Publishers B.V., vol. 22, nos. 1/2, pp. 3-29.

Ouypornprasert, W. 1987. Adaptive numerical integration in reliability analysis. *Internal Report, University of Innsbruck*, Innsbruck, Austria.

Schueller, G.I.; Stix, R. 1987. A critical appraisal of methods to determine failure probabilities. *Structural Safety*, Amsterdam, The Netherlands: Elsevier Science Publishers B.V., vol. 4, pp. 293-309.

Sørensen, J.D.; Burcharth, H.F.; Christiani, E. 1994. Reliability analysis and optimal design of monolithic vertical wall breakwaters. *Proceedings IFIP WG 7.5 Conference on Reliability and Optimisation of Structural Systems*, Rackwitz, R.; Augusti, G.; Borri, A. (eds.), Assisi, Italy, no. 6, Part 2: Technical Contributions, Paper 27, pp. 257-264.

Takayama, T.; Ikeda, N. 1992. Estimation of sliding failure probality of present breakwaters for probabilistic design. *Report. Port and Harbour Research Institute (PHRI)*, Tokyo, Japan, vol. 31, no. 5, pp. 3-32.

Tanimoto, K.; Takahashi, S. 1994. Design and construction of caisson breakwaters - the Japanese experience. *Coastal Engineering, Special Issue on 'Vertical Breakwaters',* Eds.: Oumeraci, H. et al., Amsterdam, The Netherlands: Elsevier Science Publishers B.V., vol. 22, nos. 1/2, pp. 57-77.

Thoft-Christensen, P.; Baker M.J. 1982. Structural reliability theory and its applications. Berlin/Heidelberg/New York: Springer, 267 pp.

Van Dantzig, D. 1956. Economic decision problem for flood prevention. *Econometrica*, vol. 24, pp. 276-287.

Van der Meer, J.W.; d'Angremond, K.; Juhl, J. 1994. Probabilistic calculations of wave forces on vertical structures. *Proceedings International Conference Coastal Engineering (ICCE)*, ASCE, Kobe, Japan, vol. 23.

Van Gelder, P.H.A.J.M. 1998. Analysis of wave impact data on vertical breakwaters. *Proceedings Task 4 Workshop, MAST III, PROVERBS-Project: Probabilistic Design Tools for Vertical Breakwaters*, Delft, The Netherlands, Version 27/28 August 1998, Annex IV, 4 pp.; 8 figs.

Voortman, H.G.; Kuijper, H.K.T.; Vrijling, J.K. 1998. Economic optimal design of vertical breakwaters. *Proceedings International Conference Coastal Engineering (ICCE)*, ASCE, Copenhagen, Denmark, no. 26, 14 pp.

Vrijling, J.K.; Bruinsma, J. 1980. Hydraulic boundary conditions. In: *Hydraulic aspects of coastal structures: Developments in hydraulic engineering related to the design of the Oosterschelde storm surge barrier in the Netherlands*, Delft, The Neth, pp. 109-133.

Vrijling, J.K.; Van Hengel, W.; Houben, R.J. 1995. A framework for risk evaluation. *Journal of Hazardous Materials*, Amsterdam, The Netherlands: Elsevier Science B.V., vol. 43, pp. 245-261.

Vrijling, J.K. 1996. Evaluation of uncertainties and statistical descriptions. *Proceedings Task 4 Workshop Hannover, MAST III, PROVERBS-Project: Probabilistic Design Tools for Vertical Breakwaters*, Annex 3, Hannover, Germany, 15 pp.; 1 Annex.