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PREDICTABILITY OF STREAMFLOW PROCESSES OF THE YELLOW RIVER

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Understanding the predictability of a streamflow process is important for making decision for flood control and water management. However not much attention has been paid to the predictability study of streamflow processes so far. Predictability may be broken down into two categories: (1) model predictability and (2) potential predictability. The widely used model performance measure, coefficient of efficiency (*CE*), can act as a measure of model predictability for stationary series, however, it could be misleading about the model performance when being applied to seasonal processes which have seasonal mean values. Therefore, an adjusted coefficient of efficiency (*ACE*) is proposed for evaluating model predictability for seasonal time series. The model predictability of two daily river flow processes of the upper and middle Yellow River are studied based on linear ARMA models and measured in terms of *ACE*.

INTRODUCTION

Understanding the predictability of a streamflow process is important for risk analysis for flood control and water management, and the predictability of the river flow processes can also be used as an important feature for categorizing different rivers and making hydrological regionalization. However not much attention has been paid to the predictability of streamflow processes so far, whereas the predictability of atmospheric variables has been intensively studied by the meteorology community.

The predictability of a time series depends on the relationship between the future value and the past values. Because of inaccuracy of the model description, disturbance of measurement noise and instability of the system under study, the predictability of any real world process is limited. Common sources of the predictability of a process could be (1) autocorrelation; (2) relations between the process and influencing factors, such as the relationship between streamflow processes and rainfall, temperature and sea surface temperature, etc.; and (3) the performance of the forecasting method. Correspondingly, predictability of a process may be broken down into two categories: (1) model predictability and (2) potential predictability. Model predictability mainly focuses on the accuracy of the forecasting method. It is studied based on numerical and/or statistical models fitted to the process under study and measured according to the resulting forecast error growth. Model predictability can be further divided into two categories: (1) linear model predictability that is assessed according to linear model fitting, and (2) nonlinear model predictability according to nonlinear model fitting. Potential predictability mainly focuses on the autocorrelation and relationship with exogenous variables. It is studied based on the decomposition of temporal variability of a process into a noise caused variability and another part assumed to be at least potentially predictable (e.g., Zheng et al., 2000) [1]. And the potential predictability can be measured as the fraction of the total variability accounted for by the latter part. To quantify the predictability for a k-step-ahead prediction of a time series, we need some measures. Clearly, whatever predictability measures are used, they are sensitive to model fitting procedures in the former case, or sensitive to how the separation of variance is performed in the latter case.

Granger and Newbold (1978) [2] proposed a definition of predictability for covariance stationary series, patterned after the familiar R^2 of linear regression, as the ratio of the variance of the optimal prediction to the variance of the original time series. For estimating the predictability measure, Bhansali (1992) [3] proposed an estimation procedure for estimating the variance of the mean squared error of prediction. In fact, the predictability measure used by the above-mentioned authors is essentially the same as the coefficient of efficiency (*CE*) which is proposed by Nash and Sutcliffe (1970) [4] and widely used as model fitting criterion in the hydrology community.

We only study linear model predictability in this study. We focus on the predictability of two daily river flow processes of the upper and middle Yellow River based on the autocorrelation of the series.

THE ESTIMATION OF PREDICTABILITY MEASURE *CE*

Given a stationary process $\{x_t\}$, suppose the k-step-ahead observed value is x_{t+k}, and its linear least-squares prediction is $\hat{x}_{t+k}$. The corresponding variance of k-step prediction error is given by $\sigma_k^2 = E\{[x_{t+k} - \hat{x}_{t+k}]^2\}$. Then the ratio $W(k) = \sigma_k^2 / E(x_t^2)$ measures the proportion of variance of x_{t+k} that is unexplained by $\hat{x}_{t+k}$, and the quantity

$$CE(k) = 1 - W(k) = 1 - \sigma_k^2 / E(x_t^2) \tag{1}$$

measures the proportion of variance of x_{t+k} that can be explained by $\hat{x}_{t+k}$. By analogy with the classical regression theory, the *CE* provides, in an R^2 sense, a measure of predictability of the future values of the process from a knowledge of the infinite past (Bhansali, 1992) [3].

We can see from equation (1) that, the estimation of the variance of the k-step-ahead prediction error σ_k^2 is crucial in the estimation of the k-step-ahead predictability measure $CE(k)$. Bhansali (1992) [3] gave several estimators of σ_k^2 based on the estimate of autoregressive parameters for a stationary process. However, for practical model evaluation purpose, with k-step-ahead predictions given by whatever models, the *CE* value for k-step-ahead predictions equation (1) can simply be expressed as

$$CE(k) = 1 - \frac{\sum_{t=1}^{n}(x_{t+k} - \hat{x}_{t+k})^2}{\sum_{t=1}^{n}(x_{t+k} - \overline{x})^2} \tag{2}$$

where $\overline{x}$ is the average of $x_{1+k}, \ldots, x_{n+k}$.

This is the well-known formula of *CE* by the hydrology community and is widely used for evaluating the performance of hydrologic models. *CE* reflects the fraction of the total sum of the squares of the observations explained by a model. However, there are several problems existing when interpreting the meaning of *CE* for a model fitted to a seasonally stationary series, such as streamflow processes.

First, we know that a value of zero for the coefficient of efficiency indicates that the observed mean is as good a predictor as the model, while a negative value indicates that the observed mean is a better predictor than the model (Wilcox et al., 1990) [5]. But for hydrological time series which usually have strong seasonality, what we concern is whether the model is better than seasonal mean values of the series rather than overall observed mean. This is a question that cannot be answered by *CE* value obtained from equation (2).

Second, there is an interesting phenomenon that when we assess model performance in terms of *CE*, the *CE* value calculated for the whole year will be higher than the average of *CE* values calculated for separate seasons, which illogically indicates that the model performance for the whole year is better than for most separate seasons. In fact, this problem will also be encountered when applying another commonly used goodness-of-fit criterion - coefficient of determination.

These problems arise from the inadequacy of the definition of *CE* for dealing with seasonal processes. As aforementioned, *CE* is defined under the assumption that the process of interest is stationary (Bhansali, 1992), whereas hydrologic time series are usually strongly seasonal. When seasonality exists, especially when the mean value changes with season, for most of the seasons (such as days or months) in a year, the value of overall standard deviation is larger than the values of seasonal standard deviation, as in the case of daily streamflow at Tangnaihai (Figure 1). Therefore, when only considering overall mean value, the denominator of the right-most item in equation (2), which indicates the standard deviation of the observed series, is larger than if seasonal means are taken into account. Take the case of daily streamflow at Tangnaihai for instance, the overall standard deviation (calculated with overall mean) is about 559.5 m^3/s, while the average of daily standard deviations (calculated with daily mean) is about 275.7 m^3/s. Therefore, using overall mean gives rise to a larger standard deviation, and therefore a larger *CE*, that is, the *CE* value is exaggerated with equation (2) when seasonality exists.

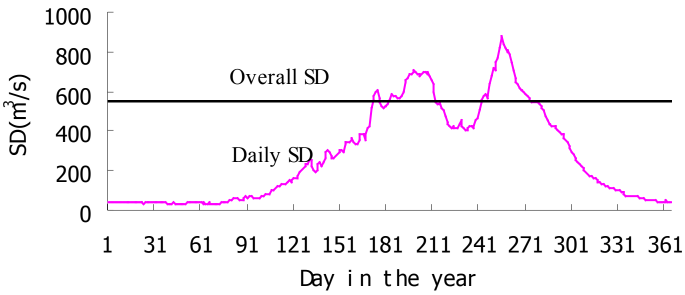

Figure 1.The value of overall standard deviation (SD) compared with the values of seasonal SD for daily streamflow at Tangnaihai

To overcome these problems, seasonal mean values must be considered for calculating *CE*, by replacing $\bar{x}$ in equation (2) with seasonal mean value which varies depending on what season x_{t+k} is in. So, we get the adjusted *CE* (*ACE*) as

$$ACE(k) = 1 - \frac{\sum_{t=1}^{n}(x_{t+k} - \hat{x}_{t+k})^2}{\sum_{t=1}^{n}(x_{t+k} - \bar{x}_m)^2} \tag{3}$$

where $m = (t+k)$ mod S (mod is the operator calculating the remainder), ranging from 0 to S-1; and S is the number of seasons; $\bar{x}_m$ is the mean value of each season. When a deterministic trend existing, the trend should be included in $\bar{x}_m$. With this modification, a value of zero for the coefficient of efficiency indicates that the mean value of each season is as good a predictor as the model. Similar modification can be applied to the calculation of coefficient of determination (R^2) for evaluating seasonal time series, which will not be discussed here.

We use predictable time length to represent the predictability of a process under evaluation. The predictable time length is defined as the future step at which the prediction is no better than the mean value for a stationary process or the seasonal mean value for a seasonal process. Therefore, for stationary time series, we define the predictable time length as the step just before the *CE* value of the prediction reaches 0 for the first time; and for seasonal processes, we define the predictable time length as the step just before the *ACE* value of the prediction reaches 0 for the first time.

Note that, model predictability is the ability of a model that makes better prediction than the no-model overall mean or seasonal means, and possibly plus trend if existing, while seasonality in mean and the trend are considered as a part of potential predictability. This is one of the differences between model predictability and potential predictability.

PREDICTABILITY OF THE STREAMFLOW OF THE YELLOW RIVER

Case study areas

Two streamflow processes of the Yellow River at Tangnaihai and Tongguan are studied.

Tangnaihai is sited at the headwaters of the Yellow River on the Tibet Plateau. The contributing area of the gauging station at Tangnaihai is more than 2500 m above sea-level, with an area of 133,650 km^2, including a permanently snow-covered area of 192 km^2. Snowmelt water contributes about 5% of total runoff. Precipitation is concentrated in summer, usually in the form of continuous rain. The headwater area is sparsely populated, and there is no major hydro-construction, therefore, the daily streamflow process at Tangnaihai is in a basically natural status.

Tongguan is sited at the middle reaches of the Yellow River. The hydrologic characteristics of the middle reaches of the Yellow River are very different from that of the headwaters. The precipitation is highly concentrated in summer, and is usually in the form of storm-rain. The runoff responses fast to rainfall because of the special loess geomorphology and poor vegetation-cover, therefore, streamflow processes are often influenced severely by irregular rainfall. Water withdrawal for industry and agriculture utilization is also very influential on river flow processes. Furthermore, there are many hydro-constructions in the main channel. Therefore, the daily streamflow process is severely intervened by human activities.

Predictability of simulated AR processes

For comparison, we first evaluate the predictability of several simulated AR(1) processes of the form

$$x_t = \Phi x_{t-1} + \varepsilon_t$$

where Φ is the autoregressive coefficient, ε_t is Gaussian noise with mean zero and variance 1. We make 10 simulations for 6 AR(1) processes with $\Phi = 0.2, 0.4, 0.6, 0.8, 0.9$ and 0.95 respectively. The size of each simulation is 3000. For each simulation, we fitted an AR(1) model to the first 2000 points and make *k*-step-ahead forecast for the left 1000 points, then calculate $CE(k)$ with equation (2). Table 1 lists the steps just before which the value *CE* reaches 0 for the first time, which represents the predictable time lengths of these simulated processes.

Table 1 The predictable time length for AR(1) processes

Φ	0.2	0.4	0.6	0.8	0.9	0.95
Minimum steps	1	3	3	5	12	23
Maximum steps	3	6	10	19	39	39
Average steps	1.6	4	5.5	9.2	22.1	32.9

The average steps in Table 1 can be approximately considered as the average predictable time lengths of the AR(1) processes. We plot the values of Φ versus the average predictable time lengths in Figure 2. We find that there is an exponential or high order polynomial relationship between the autoregressive coefficients and the average predictable time length. Moreover, this relationship is not influenced significantly by the variance of the noise component ε_t. For instance, for the process $x_t = 0.8x_{t-1} + \varepsilon_t$, if the variance of the ε_t is 0.25, the average predictable steps of 10 simulations is 10.4.

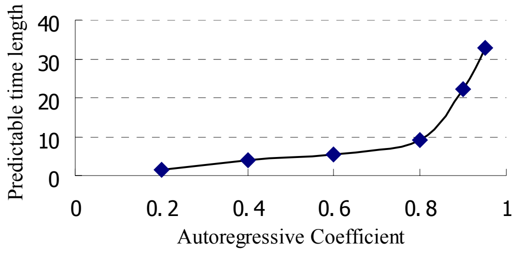

Figure 2 The relationship between the autoregressive coefficients and the average predictable time length of AR(1) processes

Predictability of the streamflow of the Yellow River

Linear ARMA models are fitted to the daily streamflow series at Tangnaihai and Tongguan separately, and forecast accuracy of the two models are evaluated using *ACE* so as to determine the model predictability.

Because there is a significant downward trend in the streamflow process at Tongguan, the trend is fitted by simple linear regression and then removed. Then, both daily streamflow series at Tangnaihai and Tongguan are logarithmized and deseasonalized by subtracting the daily mean values and divided by daily variance Following the model construction procedure proposed by Box and Jenkins (1976) [6], an ARMA(19,1) and an ARMA(9,0) are fitted to deseasonalized flow series at Tangnaihai and Tongguan separately. The plot of *CE*/*ACE* values versus predict time steps is shown in Figure 3. When calculating *ACE* for the streamflow at Tongguan, trend is considered.

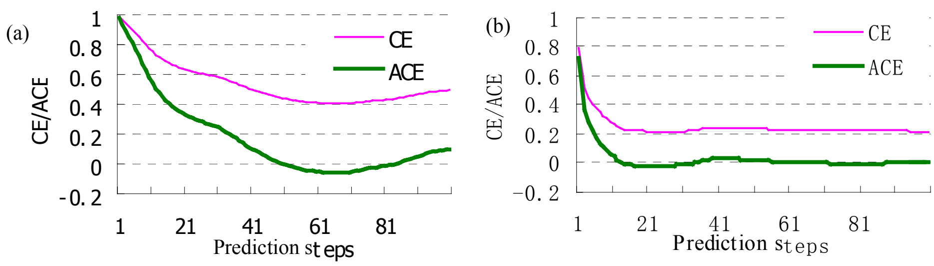

Figure 3 *CE*/*ACE* values versus predict time steps for daily streamflow at (a) Tangnaihai and (b) Tongguan

For the streamflow at Tangnaihai, *CE* value remains greater than 0 even for unrealistic long time steps, which indicates its deficiency for evaluating seasonal processes, while *ACE* value reaches 0 at step 51 for daily streamflow at Tangnaihai, therefore we determine the predictable time length as 50 days.

For the streamflow at Tongguan, *CE* value also remains greater than 0 for unrealistic long time steps, whereas *ACE* decays to 0 at step 14, indicating the predictable time length is 13 days, much shorter than that of the streamflow at Tangnaihai.

If we fit AR(1) models to the logarithmized and deseasonalized daily streamflow series at Tangnaihai and the detrended, logarithmized and deseasonalized daily streamflow series at Tongguan, the first order autoregressive coefficient is about 0.983 and 0.928 respectively. According to what we learn from simulated processes discussed in the last section, AR process of such high autoregressive coefficient usually should have long predictable time length longer than 30. But only the daily streamflow at Tangnaihai has a long predictable time length, whereas the predictable time length of the daily streamflow at Tongguan is only 13. The reason probably lies in the difference of noise level which can be reflected by the coefficient of variation, which is calculated as $CV_m = \frac{SD_m}{\overline{x}_m}$, where m = 0,… S-1, representing the season (e.g., day or month); SD_m is standard deviation of each season; $\overline{x}_m$ is the mean value of each season. From Figure 4, we can see that the streamflow at Tonggauan has much higher variation which indicates that it suffers much more strongly from noise disturbance than the streamflow at Tangnaihai. That results in the short predictability of the streamflow at Tongguan.

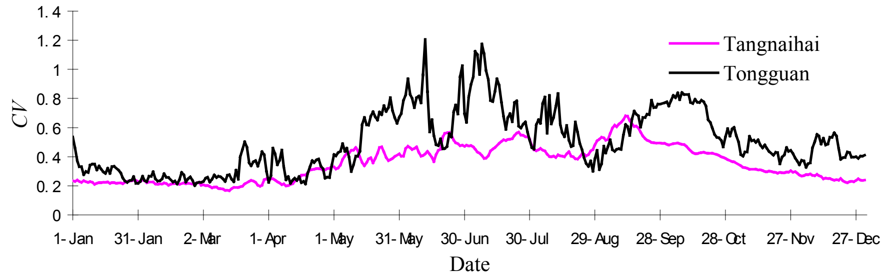

Fig.4 Variation in coefficient of variation of the daily streamflow of the Yellow River at Tangnaihai and Tongguan

CONCLUSION

Predictability of a process may be broken down into two categories: (1) model predictability, which mainly focus on the accuracy of the forecasting method; and (2) potential predictability, which mainly focus on the autocorrelation and relationship with exogenous variables. To quantify the predictability for a k-step-ahead prediction of a time series, we need some measures. Granger and Newbold (1978) and Bhansali (1992) proposed a definition of predictability for covariance stationary series based on the ratio of the variance of the optimal prediction to the variance of the original time series. In fact, this is essentially the same as the coefficient of efficiency (*CE*) which is proposed by Nash and Sutcliffe (1970) and widely used as model fitting criterion in hydrology community.

CE is a good criterion for evaluating model performance or model predictability for stationary processes, however this criterion could be misleading about the model performance when being applied to seasonal processes. Therefore, in this paper, an adjusted *CE* (*ACE*) is proposed to make it more suitable for evaluating seasonal processes.

We define the predictable time length as the future steps for which the prediction is no better than the mean value for a stationary process or the seasonal mean value for a seasonal process. Therefore, for stationary series, we define the predictable time length as the step in the future just before the *CE* value of the prediction attains 0 for the first time; and for seasonal processes, we define the predictable time length as the step just before the *ACE* value of the prediction reaches 0 for the first time.

The predictability of several simulated AR(1) processes with different first order autoregressive coefficients and the daily streamflow of the Yellow River at Tangnaihai and Tongguan are studied. It is found that there is an exponential or high order polynomial relationship between the autoregressive coefficients and the average predictable time length. Although both daily streamflow processes at Tangnaihai and Tongguan have high first order autoregressive coefficients, only the streamflow at Tangnaihai has a long predictable time length, 50 days, whereas the streamflow at Tongguan has a much shorter predictable time length, 13 days. The main reason causing this difference is that the streamflow at Tonggauan has much larger variation which indicates that the streamflow process at Tonggauan suffers much more strongly from noise disturbance than the streamflow at Tangnaihai.

In present research, only model predictability based on linear models is considered. Future study will take nonlinear models into consideration. In addition, the potential predictability study of the streamflow of the Yellow River is ongoing, in which streamflow and/or atmospheric data from upstream locations are taken into account.

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