

Is the streamflow process chaotic?

Wen Wang^{1,2}, Pieter H.A.J.M. Van Gelder², J. K. Vrijling²

[1] *Faculty of Water Resources and Environment, Hohai University, Nanjing, 210098, China*

[2] *Faculty of Civil Engineering & Geosciences, Section of Hydraulic Engineering, Delft University of Technology, 2600 GA Delft, Netherlands*

ABSTRACT: In this paper, the existence of low-dimensional chaos is investigated for the streamflow series of the upper Yellow River and the lower Rhine River with the correlation dimension method. No finite correlation dimension, which is crucial for identifying a chaotic system, is found for the existence of low-dimensional chaos in the streamflow series under study. Because streamflow processes are inevitably influenced by measurement and dynamical noises, to investigate the impacts of dynamical noises on the identification of streamflow dynamics, we analyzed three known chaotic system corrupted with different levels of dynamic noise. It is found that when noise level is low, the chaotic attractor can still be well preserved and we can give basically correct estimate of correlation dimension. That indicates that even if we found the evidence of the existence of chaos in a time series, it does not necessarily mean determinism. A stochastic system could present chaotic behavior. Therefore, even if the streamflow process is chaotic, it does not necessarily mean determinism. On the other hand, in the presence of high-level dynamical noise, it is hard or even impossible to identify a chaotic system. Because the streamflow process usually suffers from strong natural and anthropogenic disturbances which are composed of both stochastic and deterministic components, consequently, it is not likely to identify the chaotic dynamics even if the streamflow process is indeed low-dimension chaotic process under ideal circumstances (i.e., without any or only with small enough stochastic disturbances).

1 INTRODUCTION

A major concern in many scientific disciplines is whether a given process should be modeled as linear or as nonlinear. Investigations on nonlinearity and applications of nonlinear models to streamflow series have received much attention in the past several decades (e.g., Amorocho, 1963; Amorocho & Brandstetter, 1971; Rogers, 1982; Rao & Yu, 1990; Chen & Rao, 2003). As a special case of nonlinearity, chaos is widely concerned in the last two decades, and chaotic mechanism of streamflows has been increasingly gaining interests of the hydrology community. Most of the research in literature confirms the presence of chaos in the hydrologic time series (e.g., Rodriguez-Iturbe et al., 1989; Jayawardena & Lai, 1994; Porporato & Ridolfi, 1997, 2003; Sivakumar, 2000; Elshorbagy et al., 2002). Consequently, some researchers (e.g., Sivakumar, 2000) believed that the dynamic structures of the seemingly complex hydrological processes, such as rainfall and runoff, might be better understood using nonlinear deterministic chaotic models than the stochastic ones. Meanwhile, some studies denied the existence of chaos in hydrologic processes (e.g., Wilcox et al., 1991; Koutsoyiannis & Pachakis, 1996; Pasternack, 1999; Khan et al., 2005), and there are many dis-

putes about the existence of low-dimensional chaos in hydrologic processes (e.g., Ghilardi & Rosso, 1990; Schertzer et al., 2002).

There are many methods available for detecting the existence of chaos, among which the correlation exponent method (e.g. Grassberger & Procaccia, 1983a), the Lyapunov exponent method (e.g. Wolf et al., 1985; Kantz., 1994), the Kolmogorov entropy method (e.g. Grassberger & Procaccia, 1983b), the nonlinear prediction method (e.g. Sugihara & May, 1990), and the surrogate data method (e.g., Theiler et al., 1992; Schreiber & Schmitz, 1996) are commonly used. However, many researchers (e.g. Sivakumar, 2000) have noticed that there is no reliable method to clearly distinct a chaotic system and a stochastic system. A finite correlation dimension may be observed not only for chaotic processes but also for a stochastic process (e.g. Osborne & Provenzale, 1989), and when examined by the Grassberger-Procaccia algorithm alone, filtered noise can mimic low-dimensional chaotic attractors (Rapp et al, 1993). The occurrence of a positive time-average Lyapunov exponent in a nonlinear system subject to noise, be additive or multiplicative, does not necessarily imply deterministic chaos (Van den Broeck & Nicolis, 1993), and a positive Lyapunov exponent may be

observed also for stochastic processes, such as ARMA processes (e.g. Jayawardena & Lai, 1994). Random noises with power law spectra may provide convergence of the Kolmogorov entropy, which implies that the observation of a finite or null value of the K2 entropy in the analysis of data is not enough to infer that the system is dominated by a deterministic process such as low-dimensional chaos. (e.g. Provenzale et al., 1991). A stochastic process, be it linear or nonlinear, can also produce accurate short-term prediction but not long-term predictability, which is a typical characteristic of a chaotic process. Surrogate data method, which has a typical null hypothesis of Gaussian linear stochastic process, is not specifically designed for detecting chaos. In addition, phase-randomized surrogates may produce spurious identifications of non-random structure (e.g. Rapp et al., 1994). Consequently, none of the above methods can provide a conclusive resolution of whether a given data set is chaotic. What makes it worse for testing for chaos is the inevitable presence of noise in the observed data sets (e.g., Schertzer, 1997), such as hydrological time series.

While we cannot prove the existence of chaos with the above methods, we can conclude the absence of chaos if there is no finite correlation dimension for the data set of interest, because it is well recognized that chaotic processes should have finite correlation dimension, provided that the data set is of sufficient size. Therefore, it is crucial to estimate correlation dimension for detecting the presence or absence of chaos. For this reason, correlation exponent method is used by almost all the researchers for detecting chaos in hydrological processes (e.g., Jayawardena & Lai, 1994; Porporato & Ridolfi, 1997; Pasternack, 1999; Bordignon & Lisi, 2000; Elshorbagy et al., 2002, Khan et al., 2005).

In this study, we will investigate whether or not chaos exists in streamflow processes with correlation exponent method, and whether it is possible to identify a chaotic system in the presence of strong dynamical noises. The paper is organized as follows. After a brief description of the data used in this study in Section 2, correlation exponent method will be applied to test for the presence of chaos in the streamflow series of two rivers in Section 3. In Section 4, the influence of dynamical noises on streamflow dynamics will be discussed. Finally, the paper ends with a conclusion in Section 5.

2 DATA USED

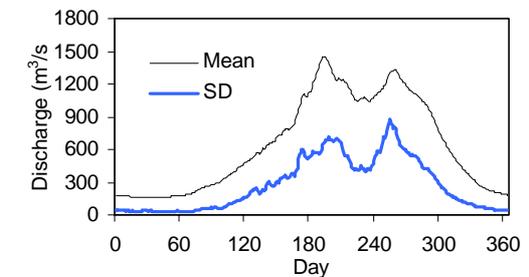
Streamflow series of two rivers, i.e., the Yellow River in China, the Rhine River in Europe are analyzed in this study.

The streamflow of the Yellow River is gauged at Tangnaihai. The gauging station Tangnaihai has a 133,650 km² drainage basin in the northeastern Tibet

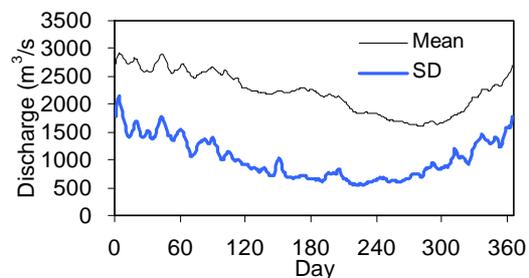
Plateau, including an permanently snow-covered area of 192 km². The length of main channel in this watershed is over 1500 km. Most of the watershed is 3000 ~ 6000 meters above sea level. Snowmelt water composes about 5% of total runoff. Because the watershed is partly permanently snow-covered and sparsely populated, without any large-scale hydraulic works, the streamflow process is fairly pristine.

The streamflow of the Rhine River is gauged at Lobith, the Netherlands. The gauging station Lobith is located at the lower reaches of the Rhine, near German-Dutch border, with a drainage area is about 160,800 km². Due to favorable distribution of precipitation over the catchment area, the Rhine has a rather equal discharge. The data are provided by the Global Runoff Data Centre (GRDC) in Germany (<http://grdc.bafg.de/>).

The sizes of daily average discharge records used in this study are 45 years for the Yellow River and 96 years for the Rhine River. The mean daily discharges and standard deviations of these streamflow series are plotted in Figure 1.



(a) Yellow River at Tangnaihai



(b) Rhine River at Lobith

Figure 1. Variation in daily mean and standard deviation of streamflow processes

3 TEST FOR CHAOS IN STREAMFLOW PROCESSES WITH CORRELATION EXPONENT METHOD

Correlation exponent method is most frequently employed to investigate the existence of chaos. The basis of this method is multi-dimension state space reconstruction. The most commonly used method for reconstructing the state space is the time-delay coordinate method proposed by Packard et al. (1980) and Takens (1981). In the time delay coordinate method, a scalar time series $\{x_1, x_2, \dots, x_N\}$ is converted to

state vectors $X_t = (x_t, x_{t-\tau}, \dots, x_{t-(m-1)\tau})$ after determining two state space parameters: the embedding dimension m and delay time τ . To check whether chaos exists, the correlation exponent values are calculated against the corresponding embedding dimension values. If the correlation exponent leads to a finite value as embedding dimension increases, then the process under investigation is considered as being dominated by deterministic dynamics. Otherwise, the process is considered as stochastic.

To calculate the correlation exponent, the delay time τ should be determined first of all. Therefore, the selection of delay time is discussed first in the following section, followed by the estimation of correlation dimension.

3.1 Selection of delay time

The delay time t is commonly selected by using the autocorrelation function (ACF) method where ACF first attains zeros or below a small value (e.g., 0.2 or 0.1), or the mutual information (MI) method (Fraser & Swinney, 1986) where the MI first attains a minimum.

ACF and MI of flow series of the Yellow River are shown in Figure 2a. Because of strong seasonality, ACF first attains zeros at the lag time of about 1/4 period, namely, 91 days. The MI method gives similar estimates for τ to the ACF method, about approximately 1/4 annual period. We therefore select $\tau = 91$ for estimating correlation dimension for the streamflow series of the Yellow River. But for the Rhine River, the seasonality is not as obvious as that of the Yellow River. We cannot find the seasonal pattern in the ACF and MI of daily flow series of the Rhine River, as shown in Figure 2b. If we determine the delay time according to the lags where ACF attains 0 or MI attains its minimum for the Rhine River, the lags would be about 200 days, which seems to be too large and would possibly make the successive elements of the state vectors in the embedded multi-dimensional state space almost independent. Therefore we select the delay time equal to the lags before ACF attains 0.1, namely, $\tau = 92$.

3.2 Estimation of correlation dimension

The most commonly used algorithm for computing correlation dimension is Grassberger - Procaccia algorithm (Grassberger & Procaccia, 1983a), modified by Theiler (1986). For a m -dimension phase-space, the modified correlation integral $C(r)$ is defined by (Theiler, 1986)

$$C(r) = \frac{2}{(M+1-w)(M-w)} \sum_{i=1}^M \sum_{j=i+w+1}^{M-i} H(r - \|X_i - X_j\|) \quad (1)$$

where $M = N - (m - 1)\tau$ is the number of embedded points in m -dimensional space; r the radius of a

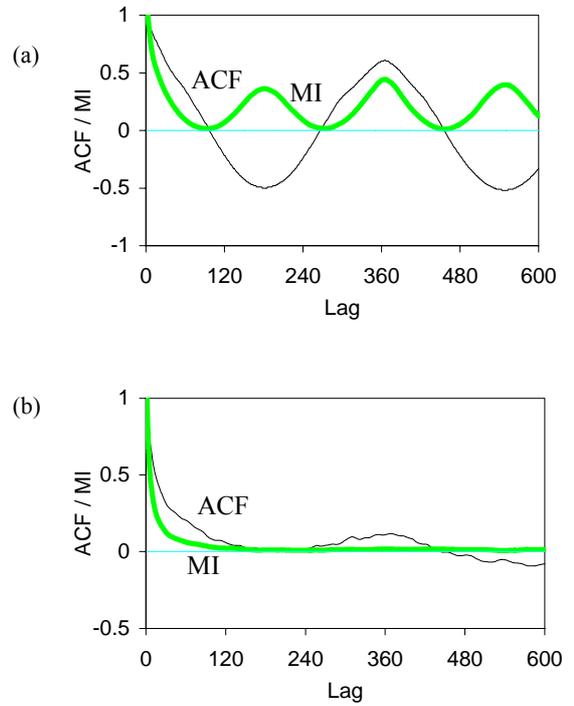
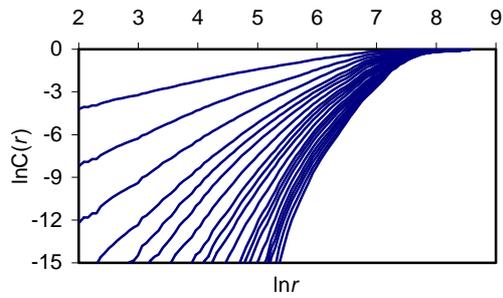


Figure 2 ACF and MI of daily river flow of (a) the Yellow River and (b) the Rhine River

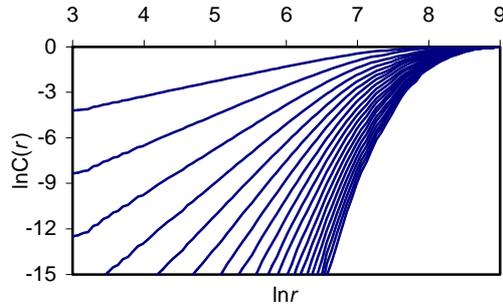
sphere centered on X_i ; $H(u)$ is the Heaviside step function, with $H(u) = 1$ for $u > 0$, and $H(u) = 0$ for $u \leq 0$; $\|\cdot\|$ denotes the sup-norm; $w (\geq 1)$ is the Theiler window to exclude those points which are temporally correlated. In this study, w is set as about half a year, namely, 182 days.

For a finite dataset, there is a radius r below which there are no pairs of points, whereas at the other extreme, when the radius approaches the diameter of the cloud of points, the number of pairs will increase no further as the radius increases (saturation). The scaling region would be found somewhere between depopulation and saturation. When $\ln C(r)$ versus $\ln r$ is plotted for a given embedding dimension m , the range of $\ln r$ where the slope of the curve is approximately constant is the scaling region where fractal geometry is indicated. In this region $C(r)$ increase as a power of r , with the scaling exponent being the correlation dimension D . If the scaling region vanishes as m increases, then finite value of correlation dimension cannot be obtained, and the system under investigation is considered as stochastic. Local slopes of $\ln C(r)$ versus $\ln r$ plot can show scaling region clearly when it exists. Because the local slopes of $\ln C(r)$ versus $\ln r$ plot often fluctuate dramatically, to identify the scaling region more clearly, we can use Takens-Theiler estimator or smooth Gaussian kernel estimator to estimate correlation dimension (Hegger et al., 1999).

The $\ln C(r)$ versus $\ln r$ plots of daily streamflow series of the two rivers are displayed in Figure 3a and 3b, and the Takens-Theiler estimates (D_{TT}) of correlation dimension are displayed in Figure 4a and 4b.

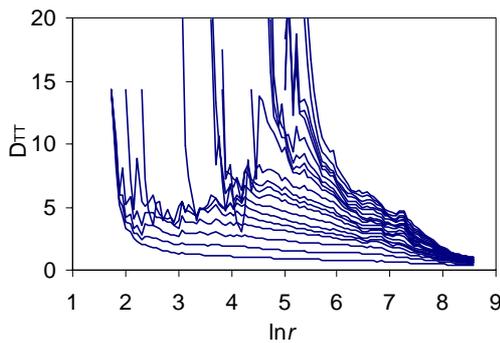


(a) Yellow River

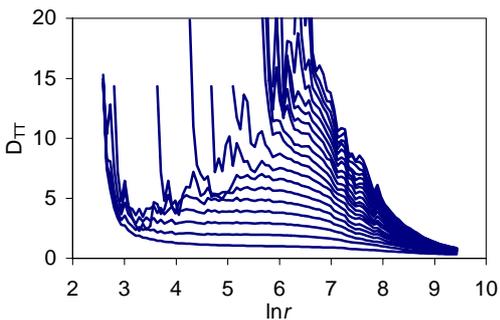


(b) Rhine River

Figure. 3 $\ln C(r)$ versus $\ln r$ plot for daily streamflow processes



(a) Yellow River



(b) Rhine River

Figure. 4 Takens-Theiler estimates of correlation dimension for daily streamflow processes

We cannot find any obvious scaling region from Figure 4a and 4b. Take the Yellow River for instance, an ambiguous $\ln r$ region could be identified as scaling region is around $\ln r = 7 \sim 7.5$ for the three flow series of different timescales. But in this region, as shown in Figure 5, the DTT increases with the increment of the embedding dimension, which indicates that the system under investigation is stochastic.

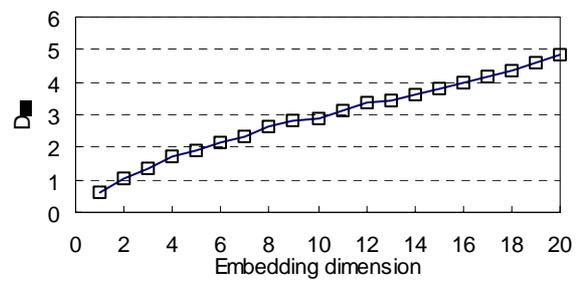


Figure. 5 Relationship between DTT and embedding dimension for streamflow of the Yellow River

3.3 Two issues about estimation of correlation dimension

Two issues regarding the estimation of correlation dimension should be noticed.

First, about scaling region. Some authors do not provide scaling plot (like Figure 4) when investigating the existence of chaos (e.g., Jayawardena & Lai, 1994; Sivakumar, 1999; Elshorbagy et al., 2002), whereas some other authors provide scaling plot, but give no clear scaling region (e.g., Porporato & Ridolfi 1997, Figure 7). However, a clearly discernible scaling region is crucial to make a convincing and reliable estimate of correlation dimension (Kantz & Schreiber, 2003, pp 82-87).

Second, about temporally related points for computing $C(r)$. To exclude temporally related points from the computation of $C(r)$, the Theiler window as in Equation (1) is indispensable. Grassberger (1990) remarked that when estimating the dimension of an attractor from a time sequence, one has to make sure that there exist no dynamical correlations between data points, so that all correlations are due to the geometry of the attractor rather than due to short-time correlations. He urged the reader to be very generous with the Theiler window parameter. Because streamflow series is highly temporally related, especially for daily flow, therefore, without setting Theiler window (i.e., $w = 1$), we would find a spurious scaling region in the plot of D_{TT} versus $\ln r$ which gives an incorrect estimate of correlation dimension. This problem has been pointed out by Wilcox et al. (1991) a decade ago, however, some authors ignored this (e.g., Elshorbagy et al., 2002), and some others take a very small Theiler window, which is maybe not large enough to exclude temporal correlations between the points (for example, Porporato & Ridolfi (1997) take $w = 5$ for daily flow series). Figure 6 shows the Takens-Theiler's estimate for daily streamflow series of the two rivers with w set to be 1. It is clear that with $w = 1$, we would find spurious scaling regions in both plots. Furthermore, comparing the plots for the daily streamflow of the Rhine river with different values of w , namely, Figure 6b, Figure 7a and 7b, we can further find that the smaller

4 THE EFFECTS OF DYNAMICAL NOISES ON THE IDENTIFICATION OF CHAOTIC SYSTEMS

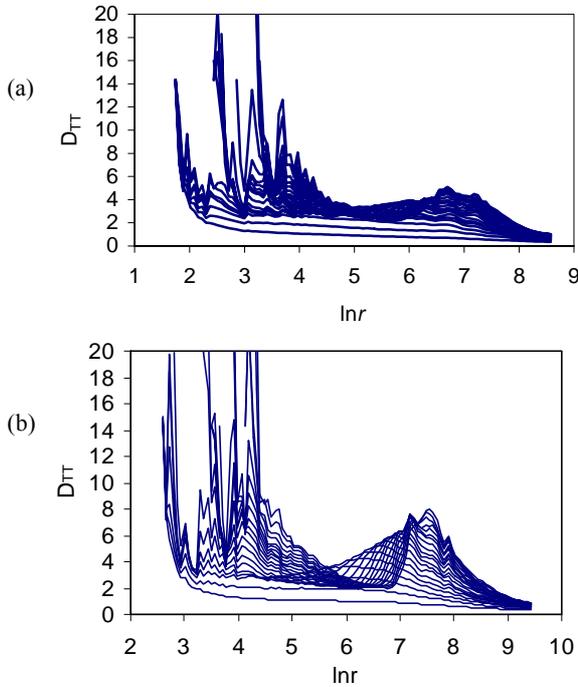


Figure. 6 Takens-Theiler estimates of (a) the Yellow River and (b) the Rhine River without considering Theiler window for daily streamflow

the value of w , the lower the estimated correlation dimension. According to these plots, when $w = 1$, the correlation dimension D is less than 4; when $w = 5$, D is less than 8, and when $w = 15$, D is less than 10. Therefore, the dimension estimate could be seriously too low if temporal coherence in the time series is mistaken for geometrical structure (Kantz & Schreiber, 2003, pp 87-91).

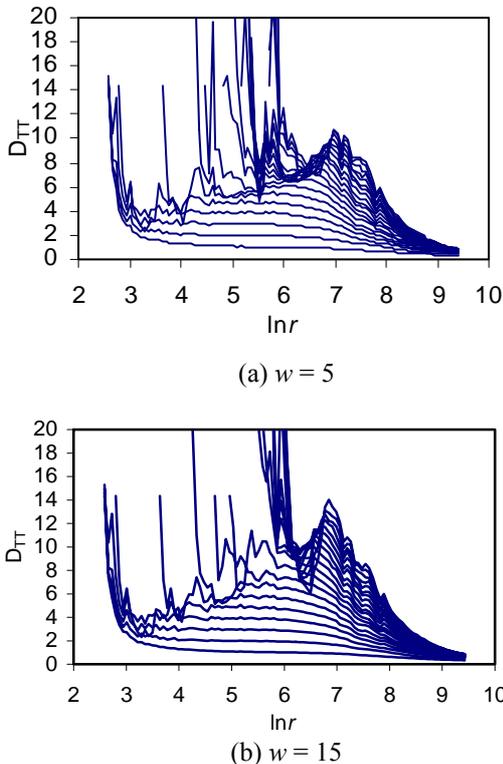


Figure 7 Takens-Theiler estimates with small Theiler window for daily streamflow of the Rhine River

When analyzing the chaos properties in observational time series, we cannot avoid the problem of noise. There are two distinct types of noise: (1) measurement noise, which refers to the corruption of observations by errors that does not influence the evolution of the dynamical system; and (2) dynamical noise, which perturbs the system more or less at each time step. In the presence of dynamical noises, the time series is not a simple superposition of signal plus noise, but rather a signal modulated by the noise. With regard to an observed hydrologic series, its dynamics is inevitably contaminated not only by measurement noise, but also more significantly by dynamical noise, such as the disturbance of storm rain. What are the effects of dynamical noise on the estimation of characteristic quantities of chaotic systems? We will discuss this issue through experiments with three well-known chaotic systems: (1) Henon map (Henon, 1976), which has one attractor with an attraction basin nearly touching the attractor in several places; (2) Ikeda map (Ikeda, 1979), which has one chaotic attractor with a small attraction basin and a non-chaotic attractor with much larger attraction basin; (3) Mackey–Glass flow (Mackey & Glass, 1977), which has one attractor with unbounded attraction basin.

To analyze the effects of dynamical noises, we add different levels of dynamical noises to the known chaotic systems by adding a noise item to the equations, and then check the phase state portraits and correlation dimensional estimates. The noise we add to the chaotic series are independently identically distributed Gaussian noise, with zero mean and variance of 2%, 5%, 10% and 100% of the original pure chaotic series. Besides the Gaussian noise, autoregressive (AR) dynamical noises of AR(1) structure have also been tested. Results show that, due to the impacts of the serial dependence of AR noises, the impacts of the AR noise on the dynamics of chaotic systems are slightly stronger than, but similar to, those of i.i.d. noises. To save space, the results are not displayed here.

4.1 Impacts of dynamical noises on Henon map

First, we consider the Henon map (Henon, 1976) of the form

$$\begin{cases} x_{n+1} = 1 - ax_n^2 + by_n \\ y_{n+1} = x_n \end{cases}$$

With $a = 1.4$ and $b = 0.3$, for initials such as $x = 0$, $y = 0.9$, Henon map has one strange chaotic attractor (Fig. 8a). Experiments show Henon map is sensitive to the disturbance of dynamical noises. Even 1% dynamical noise can lead the Henon map to infinity.

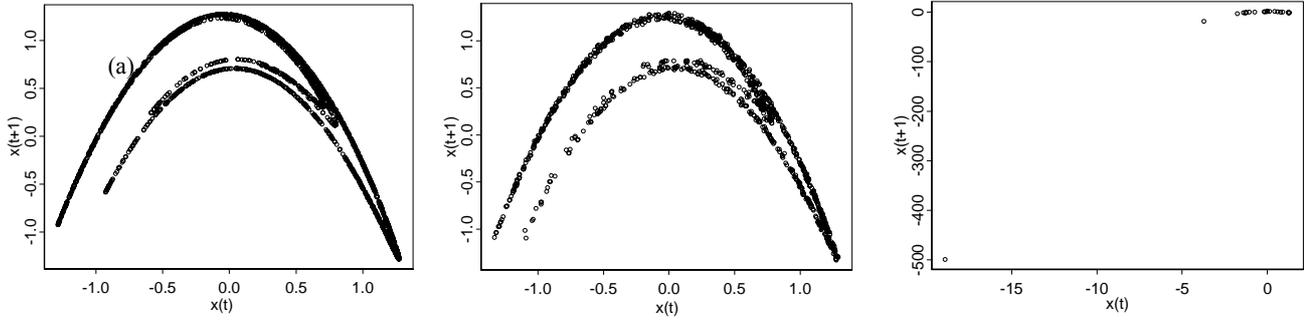


Figure 8 Henon map with (a) 0%; (b) 2% and (c) 10% Gaussian dynamical noise

Noises above 2% could easily push the orbit outside the basin of attraction, and the series goes to minus infinity exponentially. Figure 8b shows the portrait of a comparatively short Henon map series (1000 points) with 2% noise. It resembles the pure Henon map series in the appearance. However, with the evolution of the system, namely, the increase of the iterations of Henon map (e.g., >2000), it will surely go to infinity. The reason that such a low level as 2% of noise may lead the Henon map to infinity is that the boundary of the attraction basin of the Henon map nearly touches the attractor in several places, at these places, very small disturbances will push the trajectory outside the basin. Therefore, in the presence of dynamical noise, only for short series, the series can stay in the attractor. For example, with 2% noise, the Henon map rarely remains in the attractor after 1000 iterations. But as the noisy Henon map remains in the chaotic attractor, we can identify a clear scaling region on the Takens-Theiler estimate D_{TT} versus $\ln r$ plot (Fig. 9), and give a finite correlation dimension estimate about 1.25 (for noise-free series, about 1.22). With 10% dynamical noise, the Henon map series usually start to grow exponentially to infinity within 20 iterations (as shown in Figure 8c). With 100% dynamical noise, the exponential growth starts within 10 steps. With such short series, low-dimension chaos cannot be identified.

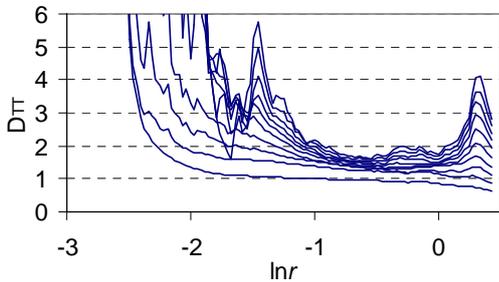


Figure 9 Correlation dimension estimate of Henon map series with 2% dynamical noise

4.2 Impacts of dynamical noises on Ikeda map

Secondly, we consider Ikeda map (Ikeda, 1979) of the form

$$z_{n+1} = \gamma + \mu \exp\left(i\beta - \frac{i\alpha}{1 + |z_n|^2}\right) z_n$$

where the z_n are complex variables. This map can be written as a two-dimensional system in the following form

$$\begin{cases} x_{n+1} = \gamma + \mu(x_n \cos \phi - y_n \sin \phi) \\ y_{n+1} = \mu(x_n \sin \phi + y_n \cos \phi) \end{cases}$$

where $\phi = \beta - \alpha / (1 + x_n^2 + y_n^2)$. With $\alpha = 6$, $\beta = 0.4$, $\gamma = 1$ and $\mu = 0.9$, for an initial such as $x = 0$ and $y = 0$, the map gives chaos (Fig. 10a). It is known that Ikeda map has a chaotic attractor with a complex attraction basin and one stable point attractor centered at (2.9721316, 4.145946). With 2% dynamical noise, the system will usually stay in the chaotic attractor first (Fig. 10b), then move to point attractor as the system evolves. With the increase of noise level, the chance of staying in the chaotic attractors decrease, meanwhile the chance of moving to the point attractor increase. When noise level reaches 10%, the system only stays in chaotic attractor for short time (usually less than 1000 iterations), then move to the point attractor (Fig. 10c). When the noise level is as high as 100%, the system usually escapes the chaotic attractor within 10 steps. Even for small noise level (e.g., 2% noise), with the evolution of the system (e.g., iterate the map more than 20000 times), the system will finally be trapped into the point attractor.

We also calculate the Takens-Theiler estimate D_{TT} of correlation dimension for Ikeda map series with 2% and 10% dynamical noise when the series stays in the chaotic attraction basin, and plot D_{TT} versus $\ln r$ in Figure 11. We can clearly identify the scaling region in Figure 11a, and estimate the correlation dimension about 1.85 (for pure series, about 1.69). But the scaling region in Figure 11b is not clearly discernable. From the vague region around $\ln r = -0.5$, the correlation dimension estimate for Ikeda series with 10% noise is about 2.12.

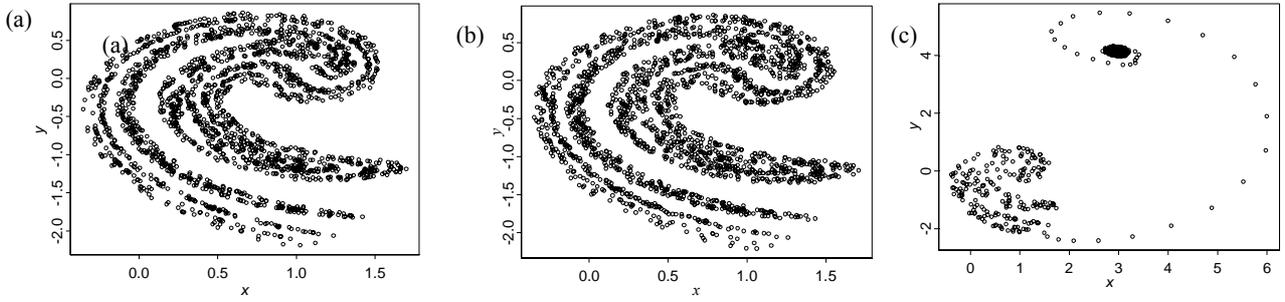


Figure 10 Ikeda map with (a) 0%; (b) 2% and (c) 10% Gaussian dynamical noise

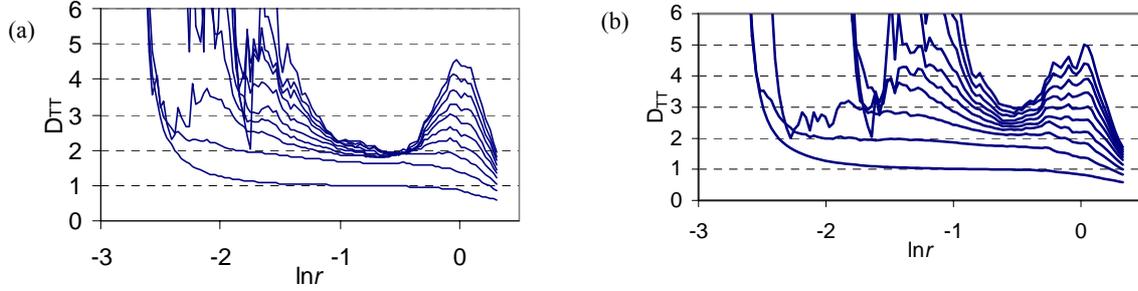


Figure 11 Correlation dimension estimate of Ikeda map with (a) 2% and (b) 10% dynamical noises

4.3 Impacts of dynamical noises on discretized Mackey-Glass flow

Finally, we consider Mackey-Glass delay differential equation (Mackey & Glass, 1977) of the form

$$\frac{dx}{dt} = \frac{ax_{t-\tau}}{1+x_{t-\tau}^c} - bx_t$$

where $x_{t-\tau}$ is the value of x at time $t-\tau$. It can be written as an approximate $m+1$ dimensional map in delay coordinates:

$$x_{n+1} = \frac{2m-b\tau}{2m+b\tau}x_n + \frac{a\tau}{2m+b\tau} \left(\frac{x_{n-m}}{1+x_{n-m}^c} + \frac{x_{n-m+1}}{1+x_{n-m+1}^c} \right)$$

With $a = 0.2$, $b = 0.1$ and $c = 10$, this map can generate time series with chaotic attractors of different dimension for $\tau > 16.8$. We choose $m = 30$, $\tau = 30$. The noise-free series and the series with 2%

and 10% dynamical noise are plotted in Figure 12a, 12b and 12c.

From Figure 12b, we see that the chaotic attractor is very clear with 2% Gaussian noise. Because the attraction basin of Mackey-Glass flow is unbounded, even for high level (e.g., 10%) of noises, the attractor still can be vaguely discerned (Fig. 12c). But the attractor is not discernable anymore with the noise of 100% level.

We calculate the D_{TT} for discretized Mackey-Glass flow contaminated with 2% and 10% dynamical noise. The results are plotted in Figure 13. With 2% noise, we can get an estimate of correlation dimension about 2.5 (correct correlation dimension is about 2.45). Although we can discern the attractor vaguely with noises as high as 10% as shown in Figure 12c, it is hard to define the scaling region in Figure 13b and hard to estimate the correlation dimension correctly. With the noise level as high as 100%, the scaling region is totally lost.

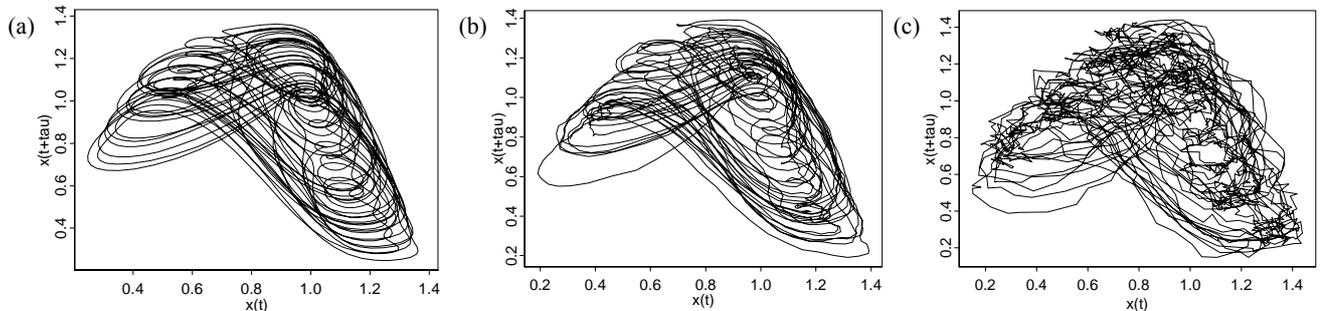
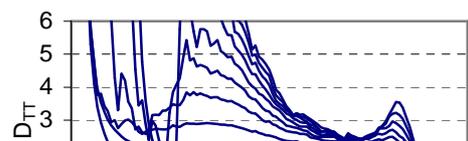


Figure 12 Mackey-Glass flow with (a) 0%; (b) 2% and (c) 10% Gaussian dynamical noise



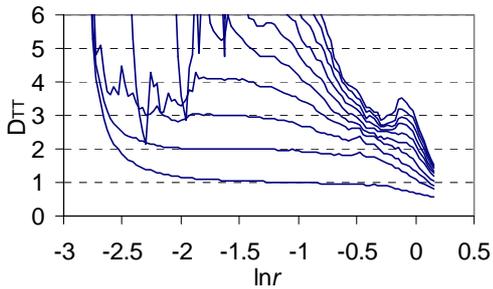


Figure 13 Estimate of correlation dimension for Mackey-Glass flow series with (a) 2% and (b) 10% dynamical noise

4.4 Analysis of the results for the cases of known chaotic processes

According to the above analyses, we have some remarks on the following two aspects:

(1) About the identification of chaotic system: from possible to impossible

Although the presence of noise limits the performance of many techniques of identification and prediction of chaotic systems [e.g., Schreiber & Kantz, 1996], with low level (e.g., 2%) Gaussian noise, the chaotic attractor can still be well preserved and basically correct estimate of correlation dimension can be made. However, in the presence of dynamical noises, the estimate is biased to a higher value, and the higher the noise level, the larger the bias. When the level of dynamic noise is very high (e.g., 10%), it is hard to identify the systems analyzed above correctly, let alone in the presence of 100% level noise.

(2) About the property of chaotic system: from deterministic to stochastic

Although chaotic systems are widely considered as deterministic, in the presence of dynamical noise, the system may still present chaotic behavior. Because a chaotic system with dynamic noise has a stochastic component and the system turns out to be stochastic instead of being deterministic, that means, a stochastic may present chaotic behavior. In the presence of dynamical noise, whether or not the chaotic system remains in the chaotic attractor depends on the intensity of stochastic disturbances. If the disturbance is so strong as to push the orbit outside the chaotic attraction basin, then the system may go to infinity, or fall into neighboring non-chaotic attractors, or just lost the geometry of the chaotic attractor, and the system becomes non-chaotic.

Two factors affect whether the trajectories would escape the chaotic attractor of a chaotic sys-

tem with stochastic noise. (1) The distance DB between the boundary of attraction basin and that of the attractor. The larger the distance, the harder to push the trajectory away from the attractor. If the attraction basin is unbounded, which means infinity DB , then the system will never escape. Systems with chaotic attractors nearly touch the boundary of their attraction basins in some places (namely, have very small DB in these places, such as the Henon map), may easily be pushed outside the chaotic attractors. (2) The distribution of the stochastic noise. For uniform distributed noise, if the maximum value of disturbance is less than minimum DB , then the stochastic chaotic system will never escape the chaotic attractor. For ununiformly distributed noise, when the attraction basin is bounded, the dynamical systems would probably ultimately escape the attractor basin and go to infinity or fall into other neighboring attractors as the evolution of the system goes to infinity.

4.5 The case of streamflow processes

With regard to an observed hydrologic series, its dynamics is inevitably contaminated by not only measurement noise, but also dynamical noise. For instance, a streamflow process, which is the major output of a watershed system, may be influenced by many factors, including external inputs (e.g., precipitation, temperature, solar radiation), other outputs (e.g., evaporation, transpiration), and various human interventions. These factors are generally composed of both deterministic components and stochastic components. Among all the factors, the precipitation is the dominant one which may disturb the streamflow process most significantly. In a flood event, it is very common that the flow generated by storm rainfall makes up over 50% of the total discharge. Assuming that base flow of a streamflow process is the noise-free time series, that 50% streamflow is generated by rainfall means that the dynamical noise is 100% of the original series. Even if the streamflow process is chaotic, according to the experiments we made with the known chaotic systems, it is impossible to detect the chaotic characters with such intense disturbances.

One may argue that dynamical noise may be a higher dimensional part of the system dynamics. If viewing it in this way, then we may say, the streamflow process may be chaotic, but definitely not a low-dimensional chaotic process, because as a major output of watershed system, it is influenced by not only many dynamical inputs and other outputs of the watershed system, but also dynamical variations of many internal factors, such as the spatial-temporal variability of soil infiltration capacity. That means, if the streamflow process is chaotic, it would be a very high dimensional cha-

otic process, rather than a low-dimensional process as claimed by some researchers. Although it is possible that collective behavior of a huge number of external and internal degrees of freedom may lead to low-dimensional dynamics, it seems not the case for streamflow processes because we cannot observe finite correlation dimension in the streamflow processes we studied.

The objective of detecting chaos should be giving a better understanding of the hydrologic time series, rather than just trying to give evidences of the presence of chaos. After the tide of detecting the existence of chaos in hydrologic time series, some researchers have tried to make some physical explanation to the chaos they claimed. For example, Porporato & Ridolfi (2003) state that the climate dynamics that produces the input of the rainfall-runoff transformation is the first source of possible determinism, and the strong low-pass filtering action of the basin, while smoothing out some of the space-time complexity of rainfall, could make more evident the low-dimensional deterministic components originating from both climate and rainfall-runoff transformation. This is a doubtful statement. On one hand, whether climate dynamics is deterministic is questionable, and consequently, whether the inputs of watershed system are deterministic is questionable. On the other hand, watershed system should be a damping system, which damps down the disturbance of the space-time complexity of rainfall as well as other dynamical inputs, rather than smoothes out the high-frequency space-time complexity with so-called strong low-pass filtering action. While the disturbance is being damped down, it will surely cause space-time variations of internal variables. In fact, the focus of the enormous efforts made by the hydrology community of developing physically-based distributed models (e.g., MIKE-SHE model) is to describe the rainfall-runoff via considering the spatial-temporal heterogeneity rather than searching for a mechanism which can smooth out the space-time complexity of rainfall or other variables.

All in all, on one hand, due to high level of dynamic disturbances, it is not possible to accurately identify the chaos in streamflow processes even if it exists; on the other hand, the existence of chaotic characteristics does not necessarily mean determinism, consequently, even if we chaos exhibits in a streamflow process, we cannot conclude that the streamflow process is deterministic. As pointed out by Schertzer et al. (2002), it is a questionable attempt to reduce complex systems to their low-dimensional caricatures, and there is no obvious reason that processes should be run by deterministic equations rather than by stochastic equations, since the former are merely particular cases of the latter.

5 CONCLUSIONS

Streamflow processes may be governed by various nonlinear mechanisms acting on different temporal and spatial scales. However, it is not clear what kind of nonlinearity is acting underlying the streamflow processes. One related interesting subject is whether the streamflow processes is a deterministic low-dimensional chaotic process.

There are many methods available for detecting chaos in time series. However, no method so far can provide conclusive evidences of the existence of chaos for real-world physical time series. In this study, correlation dimension method is applied to two daily streamflow series because define a finite correlation dimension is crucial for identifying chaotic systems. It is found that there is no finite correlation dimension for both streamflow series of interest. Consequently, it implies that the streamflow processes may not be chaotic. When testing for chaos in streamflow processes with correlation dimension method, some cares must be taken. For instance, providing convincing scaling plots with clearly discernible scaling regions is imperative for identifying the finite correlation dimension. In the computation of correlation dimension for serially dependent hydrological series, the serially dependent points must be excluded by using Theilor window, because temporal coherence could be mistaken for geometrical structure if temporally correlated points are not excluded for calculating correlation integrals.

The dynamics of observed hydrologic series is inevitably contaminated by not only measurement noise, but also dynamical noise. Experiments with three well-known chaotic systems (i.e., Henon map, Ikeda map, discretized Mackey-Glass flow) show that, with low-level (e.g., 2% or less) Gaussian noise, especially low-level uniformly distributed noise, the chaotic attractors may still be well preserved and we can give basically correct estimate of correlation dimension. That indicates that even if we found clear evidences of the existence of chaos in a time series, it does not necessarily mean determinism. A chaotic system with stochastic components, which turns out to be a stochastic system, could present chaotic behavior. It behaves similar to a noise-free system when the stochastic disturbances are not strong. Therefore, even if we find undoubtable evidences that a streamflow process is chaotic, it does not necessarily mean determinism. On the other hand, when the noise level is high (e.g., 10%), it is hard to correctly identify the chaotic system, let alone 100% level noise. Consequently, because the streamflow process usually suffers from strong natural and anthropogenic disturbances (could be of a level of up to 100% or higher) that are by themselves composed of both stochastic and deterministic components, it is not

likely to correctly identify the chaotic dynamics even if the streamflow process is indeed low-dimension chaotic process under ideal circumstances (i.e., without any or only with small enough stochastic disturbances).

REFERENCES

- Amorocho, J. 1963. Measures of the linearity of hydrologic system. *J. Geophys. Res.* 68(8), 2237–2249.
- Amorocho, J. & Brandstetter, A. 1971. Determination of nonlinear functional response functions in rainfall–runoff processes. *Water Resour. Res.* 7(5), 1087–1101.
- Bordignon S, & Lisi F. 2000. Nonlinear analysis and prediction of river flow time series. *Environmetrics* 11, 463–477.
- Chen, H.L., & Rao, A.R. 2003. Linearity analysis on stationary segments of hydrologic time series. *J. Hydrol.* 277, 89–99.
- Elshorbagy, A., Simonovic, S.P., & Panu, U.S. 2002. Estimation of missing stream flow data using principles of chaos theory. *J. Hydrol.* 255, 125–133.
- Fraser, A. M., & Swinney, H. L. 1986. Independent coordinates for strange attractors from mutual information. *Phys. Rev. A* 33(2), 1134–1140.
- Ghilardi, P., & Rosso, R. 1990. Comment on chaos in rainfall. *Water Resour. Res.* 26(8), 1837–1839.
- Grassberger, P., & Procaccia, I. 1983a. Measuring the strangeness of strange attractors. *Physica D* 9, 189–208.
- Grassberger, P., & Procaccia, I. 1983b. Estimation of the Kolmogorov entropy from a chaotic signal. *Phys. Rev. A* 28, 2591–2593.
- Grassberger, P. 1990. An optimized box-assisted algorithm for fractal dimensions. *Phys. Lett. A* 148, 63–68
- Hammel S M, Jones C K R T & Moloney J.V. 1985. Global dynamical behavior of the optical field in a ring cavity. *J. Opt. Soc. Am. B* 2, 552–564
- Hegger, R., Kantz, H. & Schreiber, T. 1999. Practical implementation of nonlinear time series methods: The TISEAN package. *Chaos* 9, 413–435.
- Henon, M. 1976. A two-dimensional mapping with a strange attractor. *Commun. Math. Phys.* 50(1), 69–77
- Ikeda, K. 1979. Multiple-valued stationary state and its instability of the transmitted light by a ring cavity system. *Opt. Commun.* 30, 257–261.
- Jayawardena, A. W., Lai, F. 1994. Analysis and prediction of chaos in rainfall and stream flow time series. *J. Hydrol.* 153, 23–52.
- Kantz, H. 1994. A robust method to estimate the maximal Lyapunov exponent of a time series, *Phys. Lett. A* 185, 77–87.
- Kantz, H., & Schreiber, T., 2003. *Nonlinear Time Series Analysis* (2nd edition), Cambridge: Cambridge University Press.
- Khan S., A. R. Ganguly, & Saigal, S. 2005. Detection and predictive modeling of chaos in finite hydrological time series. *Nonlinear Processes in Geophysics* 12, 41–53.
- Koutsoyiannis, D., & Pachakis, D. 1996. Deterministic chaos versus stochasticity in analysis and modeling of point rainfall series. *J. Geophys. Res.* 101 (D21), 26 441–26 451.
- Mackey, M. C. & Glass, L. 1977. Oscillations and chaos in physiological control systems. *Science* 197: 287–289.
- Osborne, A.R., & Provenzale, A. 1989. Finite correlation dimension for stochastic systems with power-law spectra. *Physica D* 35, 357–381.
- Packard, N.H., Crutchfield, J.P., Farmer, J.D., & Shaw, R.S. 1980. Geometry from a time series, *Phys. Rev. Lett.* 45(9), 712–716.
- Pasternack, G.B. 1999. Does the river run wild? Assessing chaos in hydrological systems. *Advances in Water Resources* 23, 253–260.
- Porporato, A., & Ridolfi, L. 1997. Nonlinear analysis of river flow time sequences. *Water Resour. Res.* 33(6), 1353–1367.
- Porporato, A., & Ridolfi, L. 2003. Detecting determinism and nonlinearity in riverflow time series. *Hydrol. Sci. J.* 48(5), 763–780
- Provenzale, A., Osborne, A.R., & Soj, R. 1991. Convergence of the K_2 entropy for random noises with power law spectra. *Physica D* 47, 361–372.
- Rao, A.R., & Yu, G.-H. 1990. Gaussianity and linearity tests of hydrologic time series. *Stochast. Hydrol. Hydraul.* 4, 121–134.
- Rapp, P.E., Albano, A.M., Schmah, T.I., & Farwell, L.A. 1993. Filtered noise can mimic low-dimensional chaotic attractors. *Phys. Rev. E* 47, 2289–2297.
- Rapp, R.E., Albano, A.M., Zimmerman, I.D., & Jimenez-Montano, M.A. 1994. Phase-randomised surrogates can produce spurious identifications of non-randomstructure. *Phys. Lett. A* 192, 27–33.
- Rodriguez-Iturbe, I., De Power, F.B., Sharifi, M.B. & Georgakakos, K.P. 1989. Chaos in rainfall. *Water Resour. Res.*, 25(7), 1667–1675.
- Rogers, W.F. 1982. Some characteristics and implications of drainage basin linearity and nonlinearity. *J. Hydrol.* 55, 247–265.
- Sauer, T. 1992. A noise reduction method for signals from nonlinear systems. *Physica D* 58, 193–201
- Sauer, T., Yorke, J.A. 1993. How many delay coordinates do you need? *Int. J. Bif. Chaos* 3, 737–744.
- Schreiber, T., Schmitz, A. 1996. Improved surrogate data for nonlinearity tests. *Phys. Rev. Lett.* 77(4), 635–638.
- Schreiber, T. 1997. Influence of Gaussian noise on the correlation exponent. *Phys. Rev. E* 56(1), 274–277.
- Schertzer D, Tchiguirinskaia I, Lovejoy S, Hubert P, Bendjoudi H. 2002. Discussion on “Evidence of chaos in the rainfall-runoff process”: Which chaos in the rainfall-runoff process? *Hydrol Sci J.* 47(1), 139–147.
- Sivakumar, B. 2000. Chaos theory in hydrology: important issues and interpretations. *J. Hydrol.* 227, 1–20.
- Sugihara, G., & May, R.M. 1990. Nonlinear forecasting as a way of distinguishing chaos from measurement error in time series. *Nature* 344, 734–741.
- Takens, F. 1981. Detecting strange attractors in turbulence. In *Lecture Notes in Mathematics vol.898, Dynamical systems and turbulence*. Berlin: Springer, 366–381.

- Theiler, J. 1986. Spurious dimension from correlation algorithms applied to limited time-series data, *Phys. Rev. A* 34(3), 2427–2432.
- Theiler, J., Eubank, S., Longtin, A., Galdrikian, B., & Farmer, J. D. 1992. Testing for nonlinearity in time series: the method of surrogate data. *Physica D* 58, 77–94.
- Van den Broeck C., & Nicolis G. 1993. Noise-induced sensitivity to the initial conditions in stochastic dynamical systems. *Phys. Rev. E* 48, 4845–4846.
- Wilcox, B.P., Seyfried, M.S., & Matison, T.H. 1991. Searching for chaotic dynamics in snowmelt runoff. *Water Resour. Res.* 27(6), 1005-1010.
- Wolf, A., Swift, J. B., Swinney, H. L., & Vastano, A. 1985. Determining Lyapunov exponents from a time series, *Physica D* 16, 285–317.