ENVIRONMETRICS
Environmetrics 2007; **18**: 621–631
Published online 26 February 2007 in Wiley InterScience
(www.interscience.wiley.com) DOI: 10.1002/env.840

Estimating joint tail probabilities of river discharges through the logistic copula

D. J. de Waal[1], P. H. A. J. M. van Gelder[2*,†] and A. Nel[1]

[1] *University of the Free State, Bloemfontein, Free State, South Africa*
[2] *Technical University of Delft, Delft, The Netherlands*

SUMMARY

In flood analysis, apart from extreme precipitation or sudden snow melt, also the duration or persistence of high water levels needs proper description. The purpose of this paper is to estimate tail probabilities from the joint distribution of variables such as one-day annual maximum river discharges and its aggregate seven-day annual maximum discharges. An application will be shown to the river Rhine in The Netherlands. The marginal distributions of the annual maxima (AM) exceeding certain thresholds are assumed to be bounded Strict Pareto and the logistic copula is used for the joint distribution. The main methodological issue discussed is the fitting of the logistic copula within a Bayesian framework. The estimation of parameters are obtained through a Markov Chain Monte Carlo simulation. In this respect, the paper provides a method for selecting the univariate and bivariate thresholds. Copyright © 2007 John Wiley & Sons, Ltd.

KEY WORDS: flood risk; persistence; joint probabilities; copula; Pareto

1. INTRODUCTION

In flood analysis, apart from extreme precipitation or sudden snow melt, the duration or persistence of high water levels also needs proper description.

In 1993, the Mississippi River basin in the Midwestern United States experienced anomalously high rainfall. Record flooding resulted from an abnormally persistent atmospheric weather pattern. Moist, unstable air flowing northward from the Gulf of Mexico converged with unseasonally cool, dry air moving southward from Canada, and this pattern was very consistent.

Walker *et al.* (1994) analysed the monthly mean discharges of 1993 and compared them with long-term climatological values of minimum, mean, and maximum discharges, compiled from data collected over the past 63 years. This data analysis reveals that monthly mean discharges were above normal during the first 9 months of 1993. Monthly mean discharges of April and May 1993 were approximately 50% higher than their long-term monthly mean values. August and September 1993 discharges exceeded those of the previous 63 years. Discharge in August 1993 exceeded that of an average month during the annual spring flood (Figure 1).

*Correspondence to: P. H. A. J. M. van Gelder, Technical University of Delft, Postbus 5048, 2600 GA Delft, The Netherlands.
†E-mail: p.h.a.j.m.vangelder@tudelft.nl

Received 9 May 2006
Accepted 2 January 2007

As a result of extreme rain during July 1997, Poland was affected by a devastating flood, the worst experienced in the past 200 years. Areas in seven voivodships in the upper and middle Oder river basin and upper Vistula river basin were flooded over 25% of their territory causing a flood damage of approximately 3 billion US dollars. Also for this flood event, the main cause had to be found in the abnormally persistent atmospheric weather pattern above middle Europe in the summer of 1997. A statistical analysis of the flood occurrence was reported by Van Gelder *et al.* (1999).

In this paper, the persistence of the river Rhine will be studied. The Rhine is one of the largest and most important rivers of Europe, with its origin in Grisons, Switzerland and its mouth in the North Sea. The basin countries are Switzerland, Italy, Liechtenstein, Austria, Germany, France, Luxembourg, and finally The Netherlands. Its length is1320 km and source elevation Vorderrhein: approximately 2600 m. The average discharge at Basel: 1060 m^3/s, at Strasbourg: 1080 m^3/s, at Cologne: 2090 m^3/s, and at the Dutch border (location Lobith): 2260 m^3/s. The total watershed area is 185 000 km^2. The Rhine gave high water levels in 1993 and 1995, which caused significant damage in the upstream countries, and caused concern in The Netherlands. For safety 250 000 people in low lying areas along the Dutch part of the Rhine were evacuated.

In The Netherlands, flood defences such as river dikes are designed and constructed with probabilistic engineering. Annual maxima (AM) and peaks over threshold river discharges are analysed with statistical models. The Dutch Ministry of Water Management and Public Works prescribes the use of an exponential distribution for the river Rhine (Van de Langemheen and Berger, 2001):

$$Q = a * \ln(T) + b \text{ with } a = 1517.78 \text{ and } b = 5964.63, \text{ valid for } 2 < T < 25 \text{ and } Q$$
$$= a * \ln(T) + b \text{ with } a = 1316.44 \text{ and } b = 6612.61, \text{ valid for } 25 < T < 10.000$$

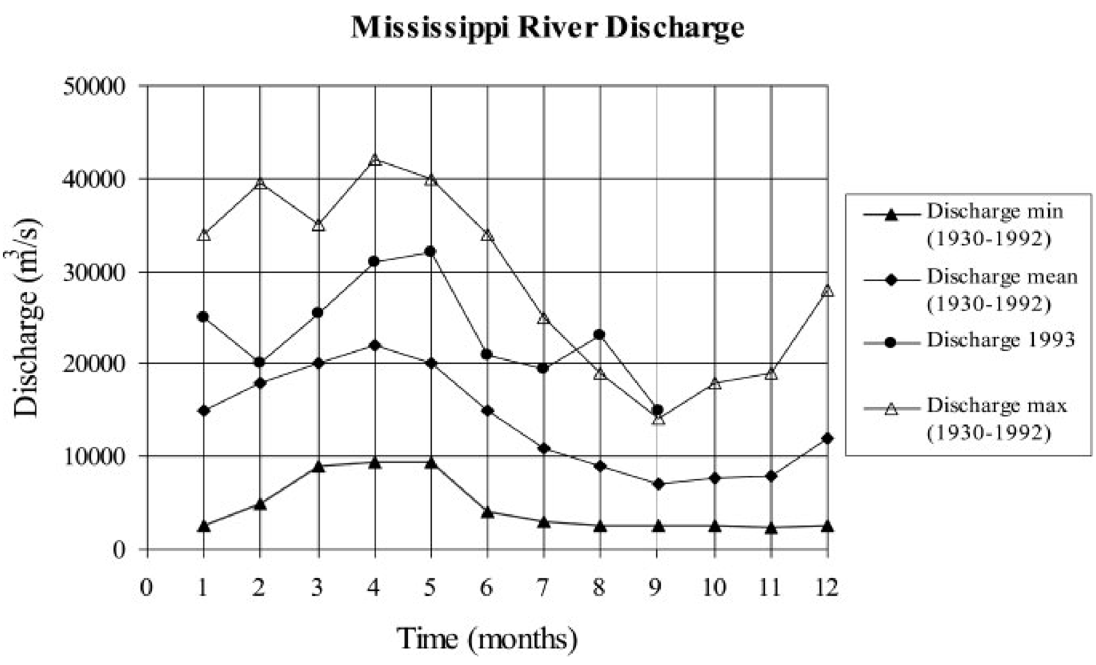

Figure 1. Mississippi River discharge for January through September 1993 in comparison with long-term monthly means, maxima, and minima from 1930–1992 (Walker *et al.*, 1994)

DOI: 10.1002/env

in which Q the Rhine discharge at Lobith (in m^3/s) and T the recurrence interval in years, which is equal to one over the exceedance probability. The distribution is plotted in Figure 2. Notice that the threshold level is taken as 7000 m^3/s (with a recurrence interval of 2 years). This is exactly the moment when the water levels exceed the summer dikes and discharge water in the winter bed. The moment, when water levels exceed the winter dikes, occurs with a recurrence interval of 1250 years (according to Dutch law for dike construction).

Persistence of high river discharges receive not so much attention in research than peak discharge analysis. For the river Rhine, some persistence analysis has been carried out by Buishand and Lenderink (2004) and Wojcik *et al.* (2004). The current paper will investigate the persistence of the river Rhine by developing a model for 7-day aggregate AM. The proposed model will be a joint probability distribution with the 1-day AM discharge. The so-called logistic copula will be introduced in Section 2, followed by an investigation of the threshold selection for the 7-day and 1-day AMs. An negative differential entropy (NDE) approach is followed for that purpose. In Section 4, the parameters of the logistic copula are estimated with Bayesian methods, using MCMC. In Section 5, finally, a proposal for threshold selection of bivariate data is elaborated.

2. LOGISTIC COPULA

The dataset consists of the daily discharges of the Rhine river measured in m^3/s at Lobith from 1 January 1901 till 31 December 1996. We consider the annual 1-day maximum X as the one variable and

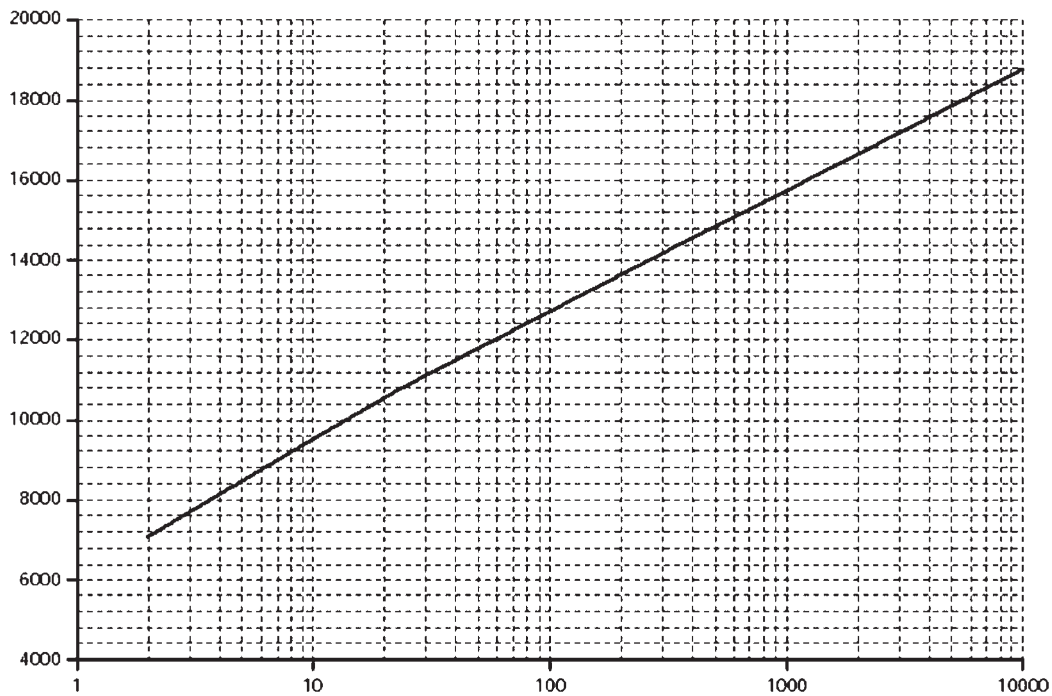

Figure 2. Discharge quantiles (in m^3/s) as a function of recurrence interval (in years) for the river Rhine at the location Lobith (border between The Netherlands and Germany)

DOI: 10.1002/env

the 7-day aggregate annual maximum Y as the other variable. The purpose of this paper is to consider the joint distribution of X and Y through the logistic copula (Tawn, 1988), although other copulas could also be applied. Many of the currently known models for bivariate (multivariate) extreme value distributions are restrictive. Nadarajah (1999a) introduced a new model based on polynomial terms with a high simplicity and flexibility. The model is also suitable for simulation of bivariate extremes as shown in Nadarajah (1999b). A list of explicit algebraic expressions is given for ready implementation of the methodology. A generalisation of the methodology to multivariate extremes is also outlined in Nadarajah (1999b). The distribution function of the logistic copula, to be discussed in this paper, is given by

$$G(u,v) = \exp\left\{-\left(u^{-\phi} + v^{-\phi}\right)^{1/\phi}\right\}, u, v >, \phi > 1 \tag{1}$$

where the random variables U and V have Fréchet marginals (Nadarajah *et al.*, 1999), namely

$$U = \frac{-1}{\log F_x(x|t_x)}, V = \frac{-1}{\log F_y\left(y|t_y\right)} \tag{2}$$

It is assumed that the marginal distributions of X and Y bounded by the thresholds t_x and t_y, respectively, are Strict Pareto (Beirlant *et al.*, 2004) with the distribution function of X for instance given by

$$F_x(x|t_x) = 1 - \left(\frac{x}{t_x}\right)^{-1/\gamma_x}, \quad x > t_x \tag{3}$$

γ_x is the extreme value index (EVI) for the daily maximum and similarly will γ_y be the EVI for the 7-day aggregate maxima. The EVI is a parameter indicating an EVI in modelling the extremes through a Pareto model. If the EVI is close to zero, then a model from the Gumbel class may be appropriate to consider. Since $X \geq Y$ and Y is a 7-day aggregate variable, we can state *a priori* that $\gamma_x \geq \gamma_y$ To take care of this constraint, γ_x and γ_y will have to be estimated jointly. Figure 3 shows the 1-day AM plotted against the 7-day aggregate AM with thresholds $t_x = 7750\,\mathrm{m}^3/\mathrm{s}$ and $t_y = 49\,845\,\mathrm{m}^3/\mathrm{s}$, respectively. The logistic copula is also suitable for analysing n-day aggregate maxima with m-day aggregate maxima to study the quantile behaviour for pairs such as $(n, m) = (1, 14), (1, 28), (7, 14), (7, 28)$.

The parameter $\phi \geq 1$ in Equation (1) is a dependence parameter indicating independence if $\phi \geq 1$. The estimation of ϕ will be done using the mode of the posterior distribution of ϕ with prior

$$\pi(\phi) \propto \exp\{E \log g(U,V)\} \tag{4}$$

(referred to as the MDI prior, Zellner (1977)) where the joint density of U and V from Equation (1) is given by

$$g(u,v) = G[u^{-\phi} + v^{-\phi}]^{\frac{1}{\phi}-2} v^{-\phi-1} u^{-\phi-1}\left[(u^{-\phi} + v^{-\phi})^{\frac{1}{\phi}} + (\phi - 1)\right] \tag{5}$$

The sample space is discretised at m^2 points (u_i, v_j) such that $u_i \in (0,7)$ and, $v_j \in (0,7), i = 1,\ldots,m, j = 1,\ldots,m$ for large m. Taking $E \log g(U,V) = \sum_{i=1}^{m}\sum_{j=1}^{m} \log g\left(u_i, v_j\right) g\left(u_i, v_j\right)$, the prior for ϕ is calculated and turns out to be close to a uniform distribution.

DOI: 10.1002/env

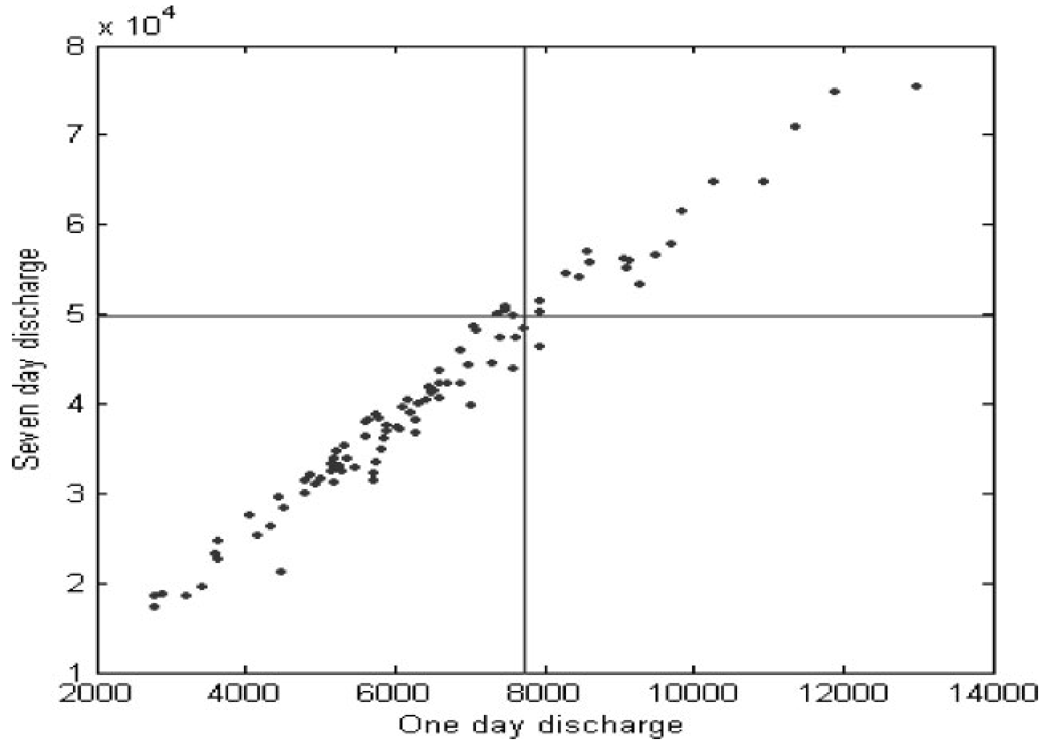

Figure 3. One-day annual maxima discharges, x, on the Rhine and 7-day aggregate annual maxima y with thresholds $t_x = 7750\,\mathrm{m^3/s}$ and $t_y = 49\,845\,\mathrm{m^3/s}$

3. CHOOSING THE THRESHOLDS

It is assumed that the 1-day annual maximum X and the aggregate 7-day annual maximum Y are distributed bounded Strict Pareto, respectively, with, for example, the distribution function of X given by Equation (3).

An approach to select the threshold is through the entropy of the Dirichlet distribution. Suppose the true distribution function of X conditional on t_x is $F^*(x|t_x)$ and we assume $F_x(x|t_x)$ given in Equation (3) as our best belief to describe $F^*(x|t_x)$. According to the ordered partition $x_1, \ldots, x_k$ of the sample space on exceedances of $t_x, F^*(x|t_x)$ can be described through a Dirichlet process with parameters $F_x(x|t_x)$ and β (Ferguson, 1973, 1983; Lo, 1984; Mazzuchi, 2002). The Dirichlet process is a non-parametric process where the spacings in the probabilities of occurrences of the observations are assumed to be Dirichlet distributed. The advantage is that a new approach through the entropy of the Dirichlet distribution for selecting the threshold is considered. The concentration parameter β is chosen as $\beta = k + 1$ which will be explained. Let $\nu_1 = \beta F_x(x_1|t_x), \nu_2 = \beta\{F_x(x_2|t_x) - F_x(x_1|t_x)\}, \ldots, \nu_{k+1} = \beta\{1 - F_x(x_k|t_x)\}$ and $Y_i, i = 1, \ldots, k+1$ as the random variables $Y_1 = F^*(x_1|t_x), Y_2 = F^*(x_2|t_x) - F^*(x_1|t_x), \ldots, Y_{k+1} = 1 - F^*(x_k|t_x)$, then the Dirichlet process defines $Y_1, \ldots, Y_{k+1}$ to be Dirichlet distributed with parameters $\nu_1, \ldots, \nu_{k+1}$. Denote the joint density of $Y_1, \ldots, Y_k$ by $y(y_1, \ldots, y_{k+1})$ stating $Y_1, \ldots, Y_{k+1}\tilde{D}(\nu_1, \ldots, \nu_{k+1})$.

The selection of the threshold t_x above will be done by considering the entropy of the Dirichlet distribution. The entropy captures the information from the data and the idea is to look at the threshold

DOI: 10.1002/env

where the information from the data is optimal. This is equivalent to minimise the NDE relative to the lower bound. Honkela (2001) showed that the NDE for $y(Y_1, \ldots, Y_{k+1})$ is given by

$$J_{k+1} = E \log y(Y_1, \ldots, Y_{k+1}) = \log \Gamma(\nu_0) - \sum_{i=1}^{k+1} \log \Gamma(\nu_i) + \sum_{i=1}^{k+1} (\nu_i - 1)\{\psi(\nu_i) - \psi(\nu_0)\} \qquad (6)$$

where $\nu_0 = \sum_{i=1}^{k+1} \nu_i$ and $\psi(\nu) = \frac{d}{d\nu} \log \Gamma(\nu)$, the digamma function.

From Equation (6) follows that the lower bound for J_{k+1} is reached when $\nu_i = 1, i = 1, \ldots, k+1$, namely $\log \Gamma(k+1)$. This implies that $\beta = k+1$ and the spacings on the data are such that the information of the data to $F_x(x|t_x)$ is maximal.

The aim is, given the dataset $x_1, \ldots, x_k$, to select k (or the threshold t_x) as large as possible such that the NDE is close to $\log \Gamma(k+1)$.

Applying this to the 1-day AM data with distribution function (3), we obtain $t_x = 7750$ m^3/s. Figure 4 shows graphs of the estimated negative differential entropies (NDE) with minimums at the largest 19 and 22, respectively, exceedances over the thresholds after smoothing the entropies with second degree polynomials.

The threshold for the 7-day aggregate AM is estimated as $t_y = 49\,845$ at $k = 22$ exceedances. The extreme value indices are estimated as $\hat{\gamma}_x = 0.2046$ and $\hat{\gamma}_y = 0.1486$ for X and Y, respectively. The estimates are obtained as the respective posterior means of the posterior distributions shown in Figure 5. For instance, the posterior distribution of γ_x is given by

$$\pi_x(\gamma_x|x_1, \ldots, x_k) \propto + \pi_x(\gamma_x) \prod_{i=1}^{k_p} \left(\frac{x_i}{t_x}\right)^{-1/\gamma_x - 1} \qquad (7)$$

and the MDI prior $\pi_x(\gamma_x) \propto \exp\{E \log f_x(X|t_x)\}$ is given by

$$\pi_x(\gamma_x) \propto \frac{1}{\gamma_x} e^{-\gamma_x} \qquad (8)$$

k_p above refers to the number of paired observations (x_i, y_i)uch that $(x_i > t_x \Lambda y_i > t_y)$. It is calculated as $k_p = 18$.

Graphs of the posteriors of γ_x and γ_y are shown in Figure 5.

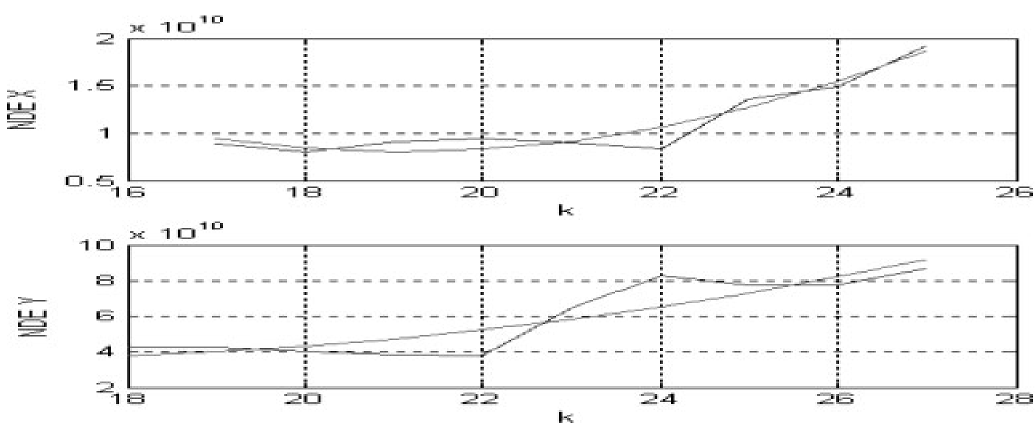

Figure 4. Estimated negative differential entropies on the 1-day annual maxima and 7-day aggregate annual maxima

DOI: 10.1002/env

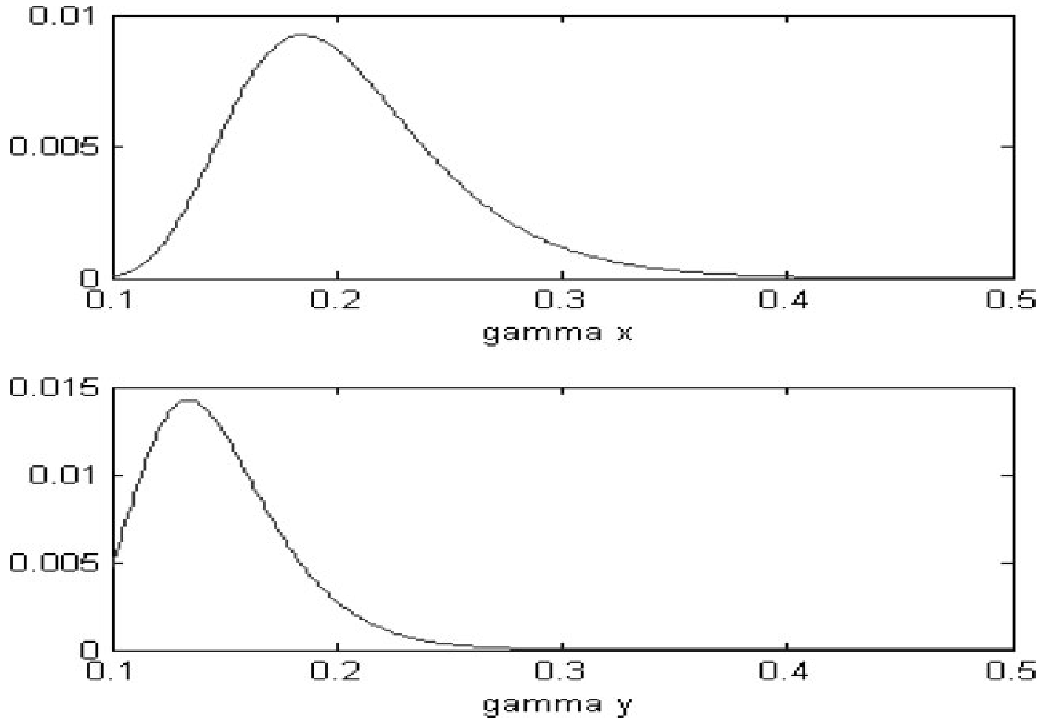

Figure 5. Posterior distributions and γ_x and γ_y

The maximum of each posterior distribution shown in Figure 5 are the estimates of γ_x and γ_y, respectively. Substituting the estimates for γ_x and γ_y in the distribution functions in Equations (2), the corresponding $(u_i v_i)$ values can be calculated for each pair of observations . A comparison of the NDE method for selecting the threshold with the method in Beirlant *et al*. (2004) to minimise the asymptotic mean square error (AMSE) of the Hill estimate of γ (Equation 4.22, page 125) shows some differences, namely k(Beirlant) = 29 compared to k(NDE) = 19 on the X data. For the Y data it was found that k(Beirlant) = 28 and k(NDE) = 22. These values compare fairly well taking into account that the formula in Beirlant *et al*. (2004) are based on parameter free methods.

4. FITTING THE LOGISTIC COPULA

The three parameters γ_x,γ_y and can be estimated jointly through MCMC using the Metropolis algorithme. The likelihood function given the observations $(x_i, y_i), i = 1, \ldots, k_p$ exceeding the thresholds t_x and t_y jointly, is given by like

$$(\gamma_x, \gamma_y, \phi|(x_i, y_i), i = 1, \ldots, k)$$
$$= \prod_{i=1}^{k} G(u_i, u_i) u_i^{-\phi-1} v_i^{-\phi-1} \left(u_i^{-\phi} + v_i^{-\phi}\right)^{\frac{1}{\phi}-2} \left\{u_i^{-\phi} + v_i^{-\phi}\right\}^{\frac{1}{\phi}} + (\phi - 1)\left(u_i^{-\phi} + v_i^{-\phi}\right) \quad (9)$$

where u_i and v_i are calculated using Equation (2).

DOI: 10.1002/env

The joint prior is taken as

$$\pi(\gamma_x, \gamma_y, \phi) \propto (\gamma_x \gamma_y)^{-1} e^{-(\gamma_x+\gamma_y)} \pi(\phi), \ \gamma_x \geq \gamma_y \tag{10}$$

where $\gamma_x^{-1} e^{-\gamma_x}$ is the MDI prior of γ_x for the Strict Pareto given in Equation (3). $\gamma_y^{-1} e^{-\gamma_y}$ is the corresponding MDI prior for γ_y and 1 is the uniform prior of ϕ.

Using $\gamma_x = 0.3$, $\gamma_y = 0.23$ and $\phi = 7$ as starting values in the Metropolis algorithm, the simulated posterior distributions after 10 000 iterations are shown in Figure 6.

From Figure 6 the values of the posterior distributions for each parameter seems to converge to a certain value. The mean value to which each posterior converges is assumed to be the estimated value for each parameter. The estimates are $\hat{\gamma}_x = 0.3022$, $\hat{\gamma}_y = 0.2255$ and$\phi = 6.9011$. The estimates obtained in Section 3 were estimated as $\hat{\gamma}_x = 0.2046$ and $\hat{\gamma}_y = 0.1486$, in Section 4 the three parameters were estimated simultaneously as $\hat{\gamma}_x = 0.3022$, $\hat{\gamma}_y = 0.2255$ and $\phi = 6.9011$. The estimated parameter values in Section 3 differ slightly from those in Section 4, therefore the dependence parameter ϕ influences the other parameters. A contour of the logistic copula in terms of the original data is shown in Figure 7 with the $k_p = 18$ observations.

From Figure 7 the model seems to give a good fit to data. One would like to have a more technical measure for goodness of fit, but it is not explored here, except the graphical presentation above. Goodness of fits of the model with special emphasis on the joint upper tails can be found for instance in Klugman and Parsa (1999), who examined various processes in casualty insurances involving high correlated pairs of variables, such as the loss and allocated loss adjustment expenses on single claims.

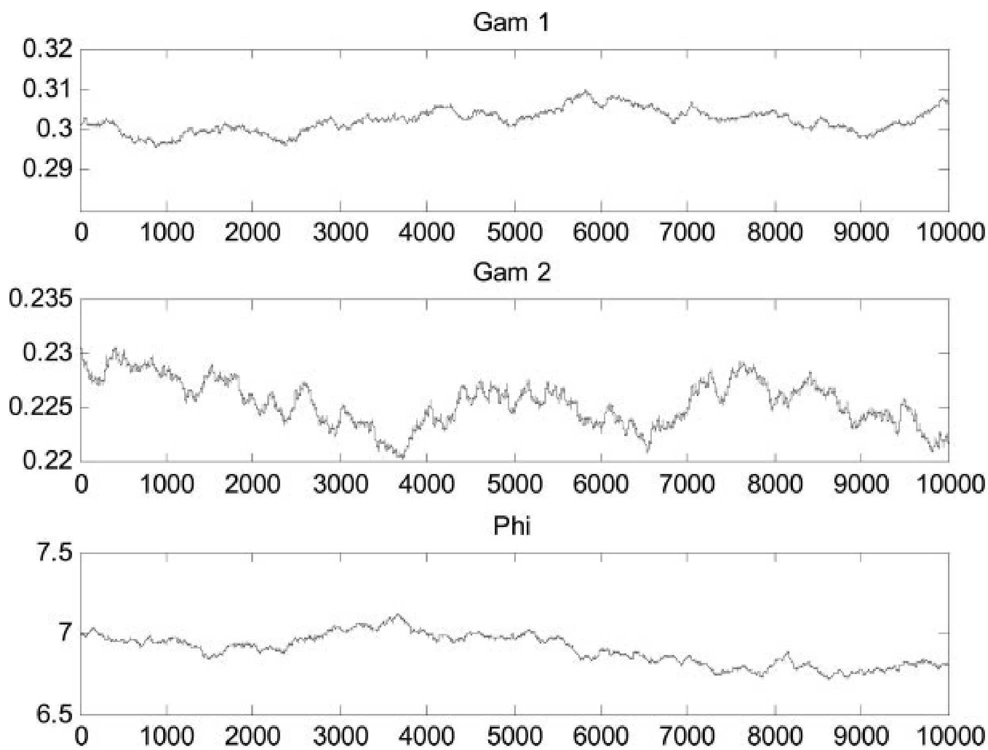

Figure 6. Simulation results of and φ

DOI: 10.1002/env

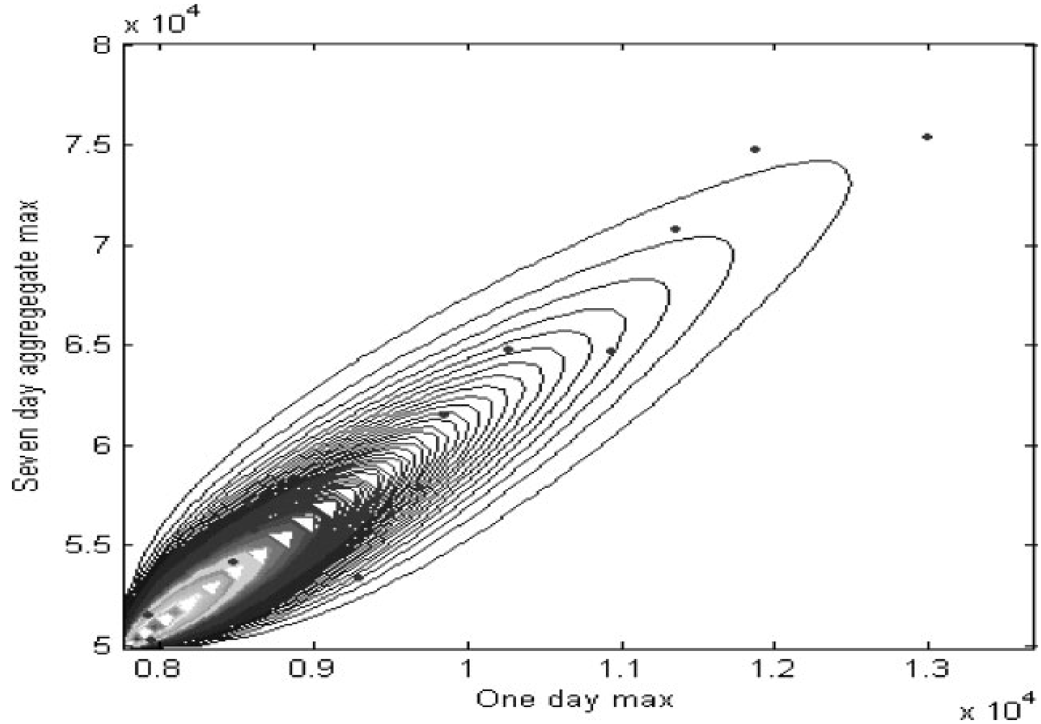

Figure 7. Contours of the logistic copula density with data indicated by $\cdot$s

Also Genest *et al.* (2006) address the goodness of fit procedures for copula models (based on probability integral transformations).

Suppose we have to consider a tail probability for joint events such as $P(X > 1100, Y > 6.4 \times 10^4)$. Through the logistic copula, probabilities such as the following can be addressed:

$$\begin{aligned} P(X > 1100, Y > 6.4 \times 10^4) &= P(U > 2.9017,\ V > 2.3382) \\ &= 1 + G(2.9017, 2.3382) - G(2.9017, \infty) - G(\infty, 2.3382) \end{aligned}$$

where $G(\infty, v) = \exp(-v^{-1})$ and $G(u, \infty) = exp(-u^{-1})$.

5. SELECTING THE THRESHOLD ON BIVARIATE DATA

To select the threshold on the bivariate data, we can follow the approach for selecting the threshold in the univariate case described in Section 3. The benefit of choosing a single threshold is based on the joint behaviour of the variables which may be an advantage. The method proposed here has not been investigated in all its consequences, but does provide a calculated single threshold instead of selecting it on the basis of the minimum exceedances as given in Section 3, namely min(19,22) = 19 exceedances. We propose to use the distance of the observations from the origin as the variable, namely

DOI: 10.1002/env

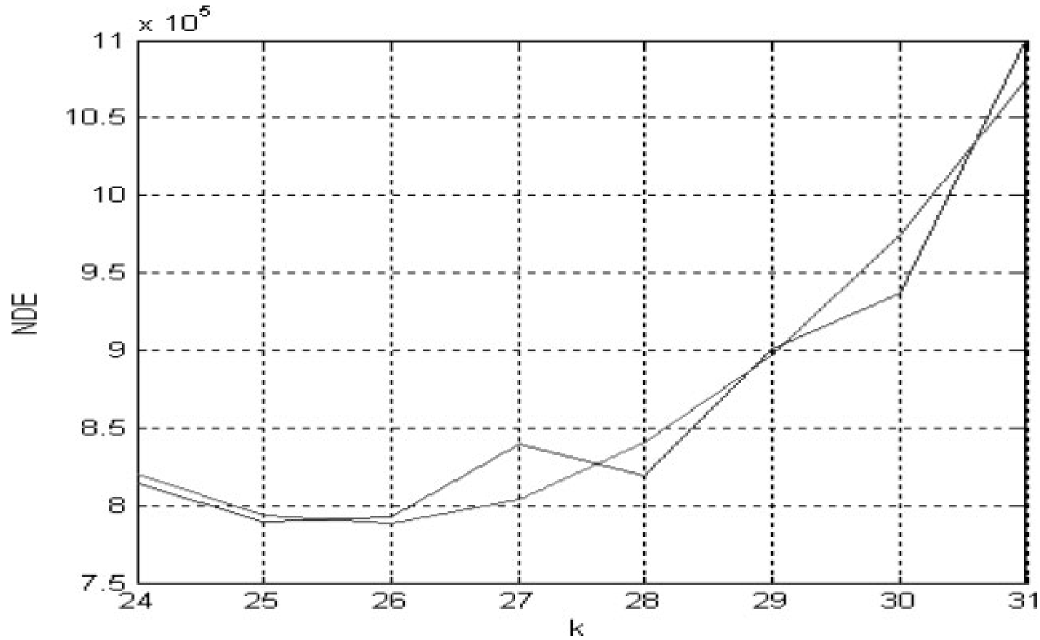

Figure 8. NDE on distances from the origin with a smoothed quadratic function

$Z = \sqrt{X^2 + Y^2}$. Let $Z \sim \text{SPareto}(\gamma)$ conditional on $Z > t$. If $F(z|t)$ is the true unknown distribution function of Z and $G(z|t)$ the SPareto(γ) proposed model, then $F(z|t)$ can be described through the Dirichlet process with parameters $(G(z|t), k + 1)$ for the largest $k \quad z's$. The estimate of the NDE J_{k+1} is calculated and is shown in Figure 8 with a qandratic smoothing. Taking the minimum on the smoothed curve, $k = 26$ is chosen for selecting the threshold $t = 4.8691 \times 10^4$. The estimate for γ (taken as the posterior mean), is given by $\hat{\gamma} = 0.1584$ based on the largest $k = 26$ observations. Using Beirlant's *et al.* (2004) min (AMSE) method for selecting the threshold, $k = 28$ against $k = 26$ through the NDE was obtained.

The selection of k differs from the $k = 19$ selected through selecting the thresholds for each variable seperately as in Section 3. The use of the distance measure Z can be criticised since two points (x_1, y_1) and (x_2, y_2) can have the same extreme z value, but x_1 and x_2(or y, and y_2) are not extremes. Suppose $x_1 = 0$, then the point (x_1, y_1) will not be considered an extreme point according to the method used in Section 3, while using the distance, it will be an extreme point.

For the Rhine data considered here where the 7-day aggregates are at least equal to the 1-day discharges, the distance measure for selecting a threshold will be valid.

6. CONCLUSION

This paper has discussed the joint modelling of annual maximum river discharges and its aggregate 7-day annual maximum discharges. The environmental issue discussed is the importance of the duration or persistence of high water levels. The case study used to illustrate the methodology is the river Rhine in The Netherlands. The logistic copula is applied to estimate small tail probabilities on joint events. Bounded strict Pareto marginals are assumed although other appropriate marginals can be

DOI: 10.1002/env

assumed. The logistic copula seems to fit well to this data. It is interesting to see that the threshold selection for 1-day AM with the minimisation based on NDE almost leads to the same result as a physical-based threshold selection. At 7000 m^3/s the winterbed of the river Rhine starts filling up. The EVI estimation of the marginal distributions differ quite much from the EVI estimation of the joint probability density function, mainly caused by the dependency between the EVIs and the phi parameter of the logistic copula.

REFERENCES

Beirlant J, Goegebeur Y, Seger J, Teugels J. 2004. *Statistics of Extremes, Theory and Applications*. 1st Edition. John Wiley & Sons Ltd: Canada.

Buishand TA, Lenderink G. 2004. Estimation of future discharges of the river Rhine in the SWURVE project, *Technical Report TR-273*, KNMI.

Ferguson TS. 1973. A Bayesian analysis of some nonparametric problems. *Annals of Statistics* **1**: 209–230.

Ferguson TS. 1983. Bayesian density estimation by mixtures of normal distributions. In *Recent Advances in Statistics*, Rizvi H, Rustagi J (eds). Academic Press: New York; 287–302.

Genest C, Quessy JF, Remillard B. 2006. Goodness-of-fit procedures for copula models based on the probability integral transformation. *Scandinavian Journal of Statistics* **33**(2): 337–366.

Honkela A. 2001. Dirichlet distribution. http://www.cis.hat.fi/ahonkela/dippa/node95:html

Klugman SA, Parsa R. 1999. Fitting bivariate loss distributions with copulas. *Insurance Mathematics & Economics* **24**(1–2): 139–148.

Lo AY. 1984. *On a class of Bayesian non-paramatric estimates: I Density estimates.Annals of Statistics* **12**: 351–357.

Mazzuchi T. 2002. Bayes estimate and inferences for entropy and information index of fit. *SANPAD workshop proceedings*, TU Delft. http://www.waterbouw.tudelft.nl/public/gelder/sanpadworkshop2002pdf

Nadarajah S. 1999a. A polynomial model for bivariate extreme value distributions. *Statistics & Probability Letters* **42**(1): 15–25.

Nadarajah S. 1999b. Simulation of multivariate extreme values. *Journal of Statistical Computation and Simulation* **62**(4): 395–410.

Nadarajah S, Anderson CW, Tawn JA. 1999. Ordered multivariate extremes. *Journal of Royal Statistical Society B* **60**(Part 2): 473–496.

Tawn JA. 1988. Bivariate extreme value theory: models and estimation. *Biometrika* **75**: 397–415.

Van de Langemheen W, Berger HEJ. 2001. Hydraulische randvoorwaarden 2001: maatgevende afvoeren Rijn en Maas. *RIZA Rapport* 2002.014, ISBN 9036954355, RIZA, Arnhem, Oktober 2001.

Van Gelder PHAJM, Van Noortwijk JM, Duits M. 1999. Selection of Probability Distributions with a Case Study on Extreme Oder River Discharges. *Safety and Reliability* **2**: 1475–1480.

Walker Nan D, Rouse Lawrence J Jr, Fargion Giulietta S, Biggs Douglas C. 1994. The great flood of summer 1993: Mississippi River discharge studied. *Earth in Space* **7**(3): 11–14.

Wojcik Rafal, Beersma Jules J, Buishand T Adri. 2004. Rainfall generator for the Rhine basin, multi-site generation of weather variables for the entire drainage area. *KNMI Publication* 186-IV.

Zellner A. 1977. *Bayesian analysis in Econometrics and Statistics*. Edward Elgar: Lyme US.