SENSITIVITY ANALYSIS OF RELIABILITY-BASED OPTIMIZATION IN SEA DIKE DESIGNS

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ABSTRACT

The reliability concept is now widely accepted in civil engineering as a tool to find the most appropriate structural system in terms of safety and economy. Design engineers can determine the optimum degree of safety for a structure to satisfy the safety demands and economical concerns of owners or users. One of the difficulties however in applying a reliability-based optimization lies in the lack of supporting data that can be used in the associated probability-based decision models. Also the uncertainty in the economic parameters of structural systems can cause difficulties in the optimization procedures. In this paper a sensitivity analysis will be performed in the reliability-based optimization of a sea dike design in the Netherlands. The Netherlands is a low-lying country which has to protect itself against flooding from the sea and its rivers. Reliable flood defenses are essential for the safety of the country. In a sea dike design one has to take account for a lot of uncertainties. These uncertainties are caused on one hand by the lack of sufficient data of extreme water levels (leading to a frequency distribution with stochastic parameters) and on the other hand by the uncertainty in construction costs and failure costs. The influence of these uncertainties causes the reliability-based optimal dike height to rise quite substantially.

1 INTRODUCTION

Probabilistic design of flood protection has become quite accepted nowadays. Already in the late 1950's a very extensive research project was initiated in the Netherlands by the Delta Committee on the probabilistic analyses of flood defences [1,2]. Since then many research projects on the field of risk-based decision-making in flood protection followed of which [3,4] give good overviews. In the design of dikes, many uncertainties play an important role. However, the influence of uncertainties in for example construction costs, damage costs and statistical uncertainty hasn't been investigated in a probabilistic framework so far. Statistical uncertainty due to a limited amount of data can be approached very well by Bayesian statistics. Van Gelder [5] showed how to deal with statistical uncertainty for sea level data along the Dutch coast. In this paper we will discuss the reliability-based optimal model for dike design in section 2. In sections 3 and 4 the statistical-, construction- and damage uncertainty on the economical dike design will be investigated.

2 RELIABILITY-BASED OPTIMAL MODEL

In this section the economic-mathematical model of Van Dantzig [1] will be discussed for the construction of dikes. Assume that an existing dike has a height of H_0. The dike will be heightened to an optimal height H. The costs involved with this heightening are a function of X where $X = H - H_0$. These costs can be assumed linearly with X by the relation: $I = I_0 + I'X$ in which:
I_0 are the mobilisation costs and
I' are the costs per meter dike heightening.

In Van Dantzig's model, the only failure mechanism of dikes that is considered is overtopping. If the waterlevel h is higher than the dike height H, then inundation takes place with a total damage of W. The probability of inundation can be modeled in many different ways and is still a very controversial subject [6]. Van Dantzig choose the exponential distribution function for modeling the probability of inundation at Dutch coast near Hook of Holland:

$$F(h)=1-e^{-(h-A)/B} \quad (h>A) \tag{1}$$

The expectation of the damage per year is now given by F(h)W and the expectation of damage over the lifetime of the dike is $\Sigma_i F(h)W/(1+r)^i$ with i = 1..∞, which can be simplified to F(h)W/r with r the discount factor.
The total costs are therefore given by the expression:

$$K(X) = I + R = I_0 + I'X + \frac{W.F(H_0+X)}{r} \tag{2}$$

which is the summation of construction costs and discounted damage expectation.
The optimal dike heightening X_{opt} can be found by solving the equation $dK(X)/dX = 0$.

3 SENSITIVITY ANALYSIS OF THE STATISTICAL UNCERTAINTY

The probability of inundation is determined on a dataset of annual maxima of water levels at the location of Hook of Holland from the period of 1887 till 1996. In [5], it is described how model and statistical uncertainties can be determined with Bayesian methods and it is applied on the Hook of Holland dike. The results are summarized in the next table:

Table 1: Influence of statistical and model uncertainties on the reliability-based optimal dike height

	Opt. dike height[m]	Prob. of inundation	Costs [gld]	Opt. dike height[m]	Prob. of inundation	Costs [gld]
Exponential Model				Gumbel Model		
Without stat. uncertainty	5.88	7.52×10^{-6}	157×10^{6}	5.51	6.43×10^{-6}	141×10^{6}
With stat. uncertainty	6.00	8.38×10^{-6}	164×10^{6}	5.67	7.24×10^{-6}	148×10^{6}

We observe a higher optimal dike height if we include statistical uncertainty in the economic optimization procedure both for the exponential and the Gumbel model. Consequently also the probabilities of inundation increase as well as the total costs increase as uncertainty is taken into account.

4 SENSITIVITY ANALYSIS OF CONSTRUCTION COSTS AND DAMAGE COSTS UNCERTAINTY

Apart from statistical uncertainty there is also uncertainty in the costs of dike heightening (i.e. the parameters I_0 and I’), and in the damage costs W. In this section the influence of the uncertainty in the parameters I_0, I’ and W will be analyzed. We consider these parameters as random variables with a normal distribution. Formula (2) becomes:

$$\underline{K}(X) = \underline{I} + \underline{R} = \underline{I_0} + \underline{I}' X + \frac{p\underline{W}}{r} \tag{3}$$

Rather than optimizing μ(K), as is done in the Van Dantzig calculation, we will optimize μ(K)+kσ(K), in which k is the risk aversion index [7]. Risk averse designers will tend to chose a high k (k towards 2) and invest more in a design to be sure of a safe structure. The influence of the uncertainty in the cost - and damage variables and the choice of the risk aversion index on the optimal dike height will be analyzed next. Costs of dike heightening can be quite well estimated on before hand. Therefore we assign to I_0 and I’ a variation coefficient of 10%. Costs of damage caused by inundation is more difficult to estimate, leading to a variation coefficient for W of 30%.

In figure 1, we have plotted the mean cost function added with k times the standard deviation of the costs (k=0,½,1 and 1½). In table 2, the results of the optimal dike height are shown.We observe that a higher risk aversion leads to a higher optimal dike height, which gives a safer construction (the probability of inundation decreases), but at the same time higher construction costs. For example if we add one standard deviation in the optimization procedure, the total costs increase 50% but the safety increases 2000%.

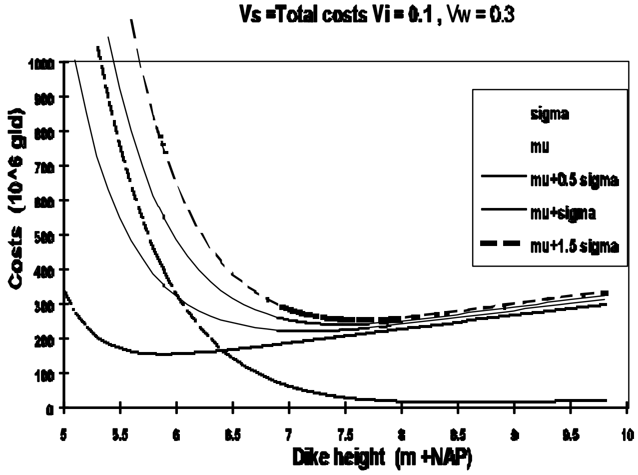

Figure 1: μ(K)+kσ(K) for different values of k

Table 2: Results of optimization

Risk avers index	Minimal Costs, [10^6gld].	Opt. dike height [m]	Prob. of inundation
k=0	157	5,88	$7{,}52.10^{-6}$
k=½	221	7,13	$1{,}18.10^{-7}$
k=1	241	7,48	$3{,}70.10^{-7}$
k=1½	255	7,65	$2{,}10.10^{-8}$

The influence of the value of the variation coefficient has been investigated. It appears that V_W has a very low influence on the optimal dike height. The reason for this can be explained if we look at the variance of the total costs which contains a factor $\sigma^2(W) + (1-p)\mu^2(W)$. This can be approximated almost by the second term solely because $\mu^2(W) \gg \sigma^2(W)$. In the optimization procedure of μ(K)+kσ(K), there is therefore negligable influence of σ(W). The influence of the variance of I' is however much larger. The higher the variation coefficient of I', the lower the optimal dike height, but the higher the total costs. From a coefficient of 10% to 50%, we observe an increase of about 20% in the total costs, and a decrease in optimal dike height of about 30cm.

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