

The effect of inherent uncertainty in time and space on the reliability of flood protection

J.K. Vrijling & P.H.A. J.M. van Gelder

Delft University of Technology, Faculty of Civil Engineering, Netherlands

ABSTRACT: When studying the probabilistic design of flood defences, it is important to develop a philosophy to discern between inherent uncertainty in time and in space. In the paper this philosophy will be described and applied on a fictitious example.

1 INTRODUCTION

When studying the probabilistic design of structures, such as flood defences like dikes or breakwaters, it is important to develop a philosophy to discern between inherent uncertainty in time and in space. Inherent uncertainty in time means that the realisations of the process in the future remain uncertain. For a dike design we have inherent uncertainty in time from the individual wave heights and water levels for instance. Unlimited data will not reduce this inherent uncertainty. Inherent uncertainty in space is from a different kind. The properties of the foundation and the strength of the dike have only one realisation per lifetime. An important aspect of this type of uncertainty is that without further investigations the knowledge of the foundation increases with time. During the life of the structure information will be gained as each storm exceeding the previous that is survived by the structure, pushes the lower limit of the strength upward (Bayesian updating of the strength). The above principles will be illustrated in this paper by calculating the probability distribution of the life time to failure of a structure with a resistance with inherent uncertainty in space subjected to a yearly maximal load with inherent uncertainty in time. From this distribution the conditional failure rate will be derived. Two examples will be discerned in which the standard deviation of the resistance will be varied against the standard deviation of the yearly maximal load. The consequences on the conditional failure rate will be analyzed.

The paper is organized as follows. First the two different kinds of uncertainties will be described in the sections on inherent uncertainty in time and inherent

uncertainty in space. The effect of both uncertainties on the lifetime reliability of flood protection will be described in the main part of the paper. Finally conclusions will be drawn.

2 INHERENT UNCERTAINTY IN TIME

Stochastic processes running in time (individual wave heights, significant wave heights, water levels, discharges, etc.) are examples of the class of inherent uncertainty. Unlimited data will not reduce this uncertainty. The realisations of the process in the future stay uncertain. The probability density function (p.d.f.) or the cumulative probability distribution function (c.d.f.) and the auto-correlation function describe the process.

In case of a periodic stationary process like a wave field the autocorrelation function will have a sinusoidal form and the spectrum as the Fourier-transform of the autocorrelation function gives an adequate description of the process. Attention should be paid to the fact that the well known wave energy spectra as Pierson-Moskowitz and Jonswap are not always able to represent the wave field at a site. In quite some practical cases swell and wind wave form a wave field together. The presence of two energy sources may be clearly reflected in the double peaked form of the wave energy spectrum.

An attractive aspect of the spectral approach is that the inherent uncertainty can be easily transferred through linear systems by means of transfer functions. By means of the linear wave theory the incoming wave spectrum can be transformed into the spectrum of wave loads on a flood defence structure. The p.d.f. of

wave loads can be derived from this wave load spectrum. Of course it is assumed here that no wave breaking takes place in the vicinity of the structure. In case of non-stationary processes, that are governed by meteorological and atmospheric cycles (sign. wave height, discharges) the p.d.f. and the autocorrelation function are needed. Here the autocorrelation function gives an impression of the persistence of the phenomenon. The persistence of rough and calm conditions is of utmost importance in workability and serviceability analyses.

If the interest is directed to the analysis of ultimate limit states e.g. sliding of the structure, the autocorrelation is eliminated by selecting only independent maxima for the statistical analysis. If this selection method does not guarantee a set of homogeneous and independent observations, physical or meteorological insights may be used to homogenise the dataset. For instance if the fetch in NW-direction is clearly maximal, the dataset of maximum significant wave height could be limited to NW-storms. If such insight fails, one could take only the observations exceeding a certain threshold (P.O.T.) into account hoping that this will lead to the desired result. In case of a clear yearly seasonal cycle the statistical analysis can be limited to the yearly maxima.

Special attention should be given to the joint occurrence of significant wave height H_s and spectral peak period T_p . A general description of the joint p.d.f. of H_s and T_p is not known. A practical solution for extreme conditions considers the significant wave height and the wave steepness as independent stochastic variables to describe the dependance. This is a conservative approach as extreme wave heights are more easily realised than extreme peak periods. For the practical description of daily conditions (SLS) the independence of s_p and T_p seems sometimes a better approximation. Also the dependance of water levels and significant wave height should be explored because the depth limitation to waves can be reduced by wind setup. Here the statistical analysis should be clearly supported by physical insight. Moreover it should not be forgotten that shoals could be eroded or accreted due to changes in current or wave regime induced by the construction of the flood defence structure.

3 INHERENT UNCERTAINTY IN SPACE

Soil properties can be described as stochastic processes in space. From a number of field tests the p.d.f. of the soil property and the (three-dimensional) autocorrelation function can be fixed for each homogeneous soil layer. Here the theory is further

developed than the practical knowledge. Numerous mathematical expressions are proposed in the literature to describe the autocorrelation. No clear preference has however emerged yet as to which functions describe the fluctuation pattern of the soil properties best. Moreover the correlation length (distance where correlation becomes zero) seems to be of the order of 30 to 100m while the spacing of traditional soil mechanical investigations for flood defence structures is of the order of 500m. So it seems that the intensity of the soil mechanical investigations has to be increased considerably if reliable estimates have to be made of the autocorrelation function.

The acquisition of more data has a different effect in case of stochastic processes in space than in time. As breakwater structures are immobile, there is only one single realisation of the field of soil properties. Therefore the soil properties at the location could be exactly known if sufficient soil investigations were done. Consequently the actual soil properties are fixed after construction, although not completely known to man. The uncertainty can be described by the distribution and the autocorrelation function, but it is in fact a case of lack of info.

An important aspect of this type of uncertainty is that without further investigations the knowledge of the foundation increases with time. During the life of the structure information will be gained as each storm exceeding the previous that is survived by the structure, pushes the lower limit of the strength upward (Bayesian updating of the strength).

4 EFFECT OF UNCERTAINTY ON LIFETIME RELIABILITY

From an engineering point of view the Bayesian approach that takes all uncertainties into account as p.d.f.'s reflects the designer's intuition very well. Keeping the physical structure equal an increase in uncertainty of any variable increases the formal probability of failure too.

From this point of view there is no difference between inherent, statistical and model uncertainty; all have to be incorporated in the probabilistic calculations. In the probabilistic calculations however a difference occurs between uncertainties that have many (e.g. yearly) realisations during the lifetime of the structure and those that have only one connected to the specific structure and the site. Every storm season shows an independent maximum H_s every year. The properties of the foundation and the strength of the flood defence structure have only one realisation per structure. Consequently the probability of failure is not solely a property of the structure but also a result of our

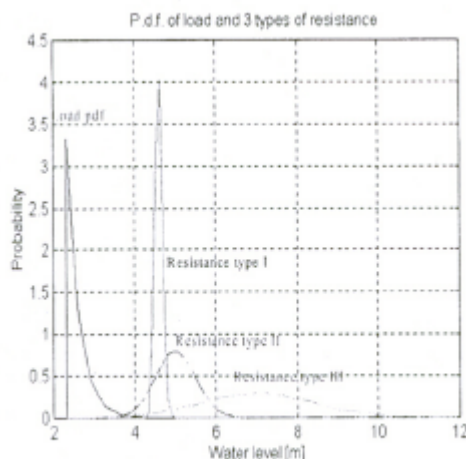


Figure 1. One type of load and three types of resistances.

lack of knowledge.

The effect of uncertainty on the lifetime reliability will be illustrated by the following hypothetical example of the probability of failure of a flood defence structure. In the example, one type of load function and three types of resistance functions are considered (see figure 1).

For the load function one can think of the exponential p.d.f. of the wave heights in front of the structure. For the three types of resistance functions one can think of the normal p.d.f.'s of the (quite certain; uncertain; very uncertain, resp.), crest height of the structure.

Failure is defined by $Z < 0$, in which $Z = R - S$ with R the resistance function and S the load function. A series of observations can be made.

4.1 First Observation

Let the failure at time i be defined as $\{Z_i < 0\}$. Consider the correlation in the reliability in two subsequent years i and $i+1$.

$$\rho(Z_i, Z_{i+1}) = \sigma_R^2 / (\sigma_R^2 + \sigma_S^2) \quad (1)$$

This result can be derived from the calculation of $\text{cov}(Z_i, Z_{i+1}) = E(((R - S_i) - (\mu_R - \mu_S))((R - S_{i+1}) - (\mu_R - \mu_S)))$. It can be noted from (1) that if σ_R increases, then ρ converges to one. In words: if the standard deviation of the yearly maximal wave load is large in relation to the standard deviation of the resistance, the dependence between failure in two subsequent years is low. If however, keeping the standard deviation of the reliability function equal, the opposite is true, the

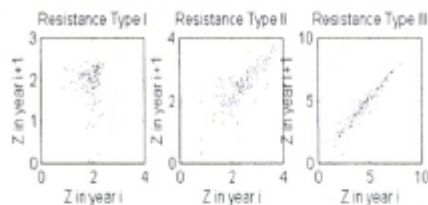


Figure 2. Monte Carlo simulation of the reliability in two subsequent years.

failure in subsequent years are dependent. With the load and resistance functions of figure 1 the following correlation coefficients are obtained: $\rho_1 = 0.0231$, $\rho_2 = 0.747$, and $\rho_3 = 0.964$ (see figure 2).

4.2 Second Observation

Consider the probability of failure at a certain moment (year number N), assuming that no failure occurred before that time. This probability is given by:

$$h(N) = f_L(N) / (1 - F_L(N)) \quad (2)$$

$h(N)$ is called the conditional failure rate (or hazard function). $F_L(N)$ the cdf of N (subscript L for lifetime).

It can be noted that if $\sigma_R = 0$, then $h(N)$ remains constant. If σ_R increases, then $h(N)$ converges to 0. This is illustrated in figure 3, where the probabilities of failure $F_L(N)$ for the three types of resistances are depicted (upper figure), together with the conditional failure rates $h(N)$ (lower figure). Conditional failure rates are usually decreasing functions in the beginning of the lifetime of the structure. This phenomenon is also known as "infant mortality"; i.e. early failure of the structure that is attributable to construction defects (see Hoeg, 1996 for some very interesting statistics of dam failures, occurring almost all in the beginning of their lifetimes (figure 5)). Conditional failure rates are increasing functions at the end of the lifetime of the structure, because of deterioration of the structure. Conditional failure rates therefore have a U-shaped (or bath tub curved) form (see also Langley, 1987).

4.3 Third Observation

It was already noted that if the standard deviation of the yearly maximal wave load is large in relation to the standard deviation of the resistance the dependence between failure in two subsequent years is low. Therefore, if the probability of failure is say p per year then the failure probability is approximately $N \cdot p$ during

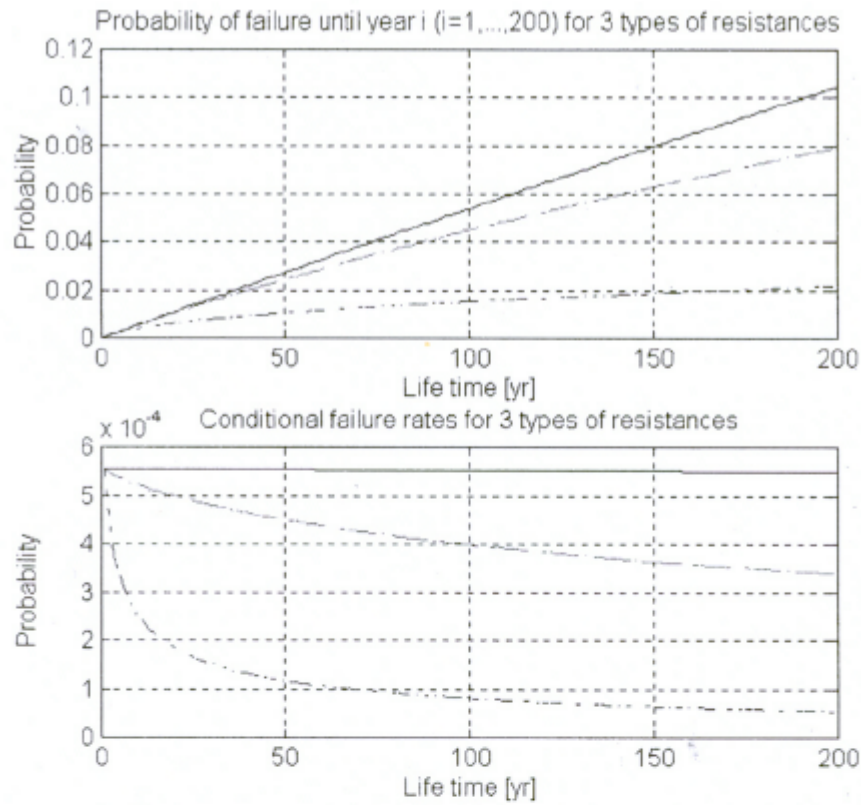


Figure 3 (top). Probabilities of failure during lifetime.

Figure 3 (down). Conditional failure rates.

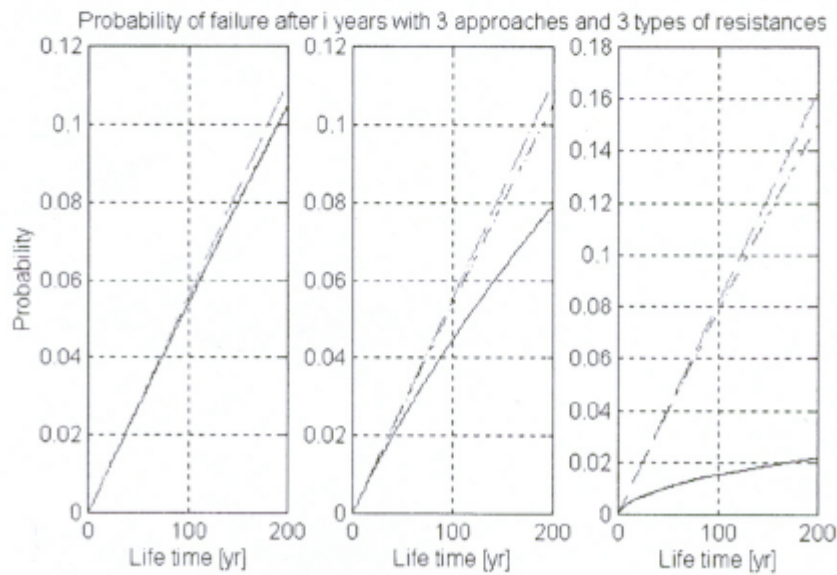


Figure 4. Comparison of three calculation methods (eqn. (3), (4) and (5)) for $F_L(N)$.

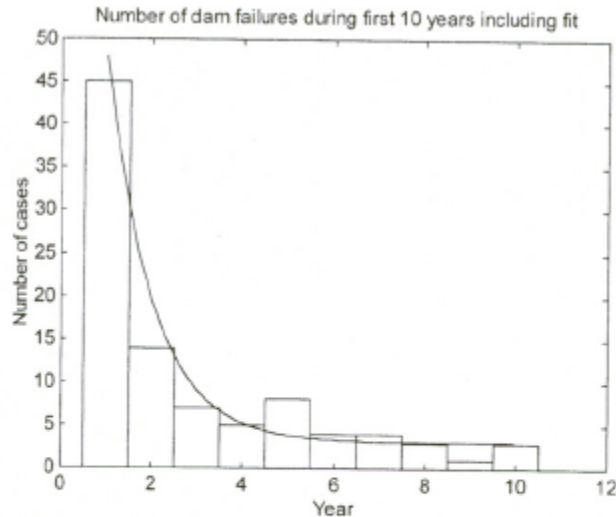


Figure 5. Example (after Hoeg, 1996).

the lifetime of N years. If however, keeping the standard deviation of the reliability function equal, the opposite is true, the failure in subsequent years are dependent and in that case the probability of failure in the first year is p and the probability of failure in the life time too. In the first case the conditional failure rate is constant over time and equal to p . In the second case however the conditional failure rate equals p in the first year and falls to zero afterwards.

In terms of formulae: for small σ_R , the expression

$$F_L(N) = \int F_S(x)^N f_R(x) dx \quad (3)$$

can be approximated by the simple form:

$$Np \quad (4)$$

or

$$1-(1-p)^N \quad (5)$$

in which p is the probability of failure in the first year.

In case of larger σ_R this approximation is not allowed and only use of the integral expression of $F_L(N)$ can be made (eqn. (3)). In figure 4, the integral expression and the 2 approximation formulae (eqn. (4) and (5)) are compared with each other.

5 CONCLUSIONS

In probabilistic calculations of structures a difference occurs between uncertainties that have many

realisations during the structure's lifetime (e.g. wave heights) and uncertainties that have only one realisation (e.g. soil parameters). This difference leads for example to the observation that every storm that was survived by the structure improves the knowledge of the p.d.f. of the resistance (pushing the lower tail to the right). Every year the knowledge of the owner of the structure grows and the probability of failure of the structure falls. In exactly the same way the failure probability of the structure can be improved by investigating e.g. the quality of the foundation assuming that this leaves the average unchanged and reduces the uncertainty. These ideas have been illustrated in this paper by a simple hypothetical example of a flood defence structure. An analysis of the effect of inherent uncertainty in time and space on the model has been given.

REFERENCES

- Ang, A.H-S., Tang, W.H., 1990, Probability concepts in engineering, planning and design, Volume II, Decision, risk and reliability.
- Feld, J., Construction failure, Wiley & sons, New York, 1997
- Høeg, K., 1996, Performance evaluation, safety assesment and risk analysis for dams, International journal on hydropower and dams, Issue 6, Vol. 3, November 1996.

Langley, R.S., 1987, Techniques for assessing the lifetime reliability of engineering structures subjected to stochastic loads, *Engineering structures*, Vol. 9, April 1997.