PERFORMANCE OF PARAMETER ESTIMATION TECHNIQUES WITH INHOMOGENEOUS DATASETS OF EXTREME WATER LEVELS ALONG THE DUTCH COAST.

P.H.A.J.M. VAN GELDER

TU Delft, Faculty of Civil Engineering,
Stevinweg 1, 2628CN Delft, The Netherlands
Tel:+31-15-2786544
Fax:+31-15-2785124
E-mail: p.vangelder@ct.tudelft.nl

ABSTRACT

Hosking and Wallis (1997) have introduced a regional statistical method (called L-moments) for probability distribution functions in a multivariate setting (when data is given about a particular quantity at more than one site). Generally, L-moments are linear combinations of ordered observations, which are unbiased regardless of the parent population. In this paper the L-moments technique will be analyzed for univariate datasets. With help of Monte Carlo simulations, results will be presented that the L-moments technique perform better than the ML- and LS-techniques, even if the data is contaminated or the model assumptions (such as homogeneity) are slightly violated.

A Case Study is presented on a database of extreme water levels at various locations along the Dutch coast. The Netherlands is a low lying country which has to protect itself by sea- and river dikes (Van Gelder et.al., 1995). An accurate determination of the flood quantiles is extremely important in the design of the dikes. In this paper, the L-moment technique will be applied to the database in order to determine the quantiles. If the model assumption of homogeneity of the database is violated the L-moment technique yields better results than the traditional estimation techniques.

Keywords: L-moments, inhomogeneity, sea dikes, extreme events, quantiles

INTRODUCTION

Perhaps the most important paper with relevance to frequency analysis in recent time is that published by Hosking in 1990. In this work, L-moments were introduced. They have become popular tools for solving various problems related to parameter estimation, distribution identification, and regionalization. It can be shown that L-moments are linear functions of probability weighted moments (PWM's); (Hosking, 1986). However, L-moments have significant advantages over PWM's, notably their ability to summarize a statistical distribution in a more meaningful way. Since L-moment estimators are linear functions of the sample values, they are virtually unbiased and have relatively small sampling variance. L-moment ratio estimators also have small bias and variance, especially in comparison with the classical coefficients of skewness and kurtosis. Moreover, estimators of L-moments are relatively insensitive to outliers. These often-heard arguments in favour of estimation of distribution parameters by L-moments (or PWM's) should, nevertheless, not be accepted blindly. In a frequency analysis, the interest is the estimation of a given quantile, not in the L-moments themselves. Although the latter may have desirable sampling properties, the same does not necessarily apply to a function of them, such as a quantile estimator. As compared with for example the classical method of moments, the robustness towards sample outliers is clearly a characteristic of L-moment estimators.

In this paper first the theory of L-Moments will be briefly described, followed with an extensive overview of papers with applications of L-moments. The literature review has shown that the theory of L-moments have mostly been applied in a regionalized setting combining data from more than one site. However, in univariate settings the method of L-moments has not been investigated so much. Therefore in this paper a Monte Carlo experiment is designed in a univariate setting in order to compare the L-moments method with other parameter estimation methods, such as Maximum Likelihood (ML), Least Squares (LS) and Classical Moments (CM). The performance of these methods will in particularly be analyzed w.r.t. inhomogeneous data. Finally in this paper a case study is presented on a database of extreme water levels at various locations along the Dutch coast, followed by conclusions and recommendations for future work.

L-MOMENTS

L-moments are summary statistics for probability distributions and data samples. They are analogous to ordinary moments -- they provide measures of location, dispersion, skewness, kurtosis, and other aspects of the shape of probability distributions or data samples -- but are computed from linear combinations of the ordered data values (hence the prefix L). Hosking and Wallis (1997) give an excellent overview on the whole theory of L-Moments.

L-MOMENTS FOR DATA SAMPLES

Probability weighted moments, defined by J. A. Greenwood et al (1979), are precursors of L-moments. Sample probability weighted moments, computed from

$$b_0 = n^{-1} \sum_{j=1}^{n} X_j \,,$$

$$b_r = n^{-1} \sum_{j=r+1}^{n} \frac{(j-1)(j-2)\dots(j-r)}{(n-1)(n-2)\dots(n-r)} X_j \,.$$

data values X_1, X_2, ... X_n, arranged in increasing order, are given by

L-moments are certain linear combinations of probability weighted moments that have simple interpretations as measures of the location, dispersion and shape of the data sample. The first few L-moments are defined by :

$$\ell_1 = b_0 \,,$$

$$\ell_2 = 2b_1 - b_0 \,,$$

$$\ell_3 = 6b_2 - 6b_1 + b_0 \,,$$

$$\ell_4 = 20b_3 - 30b_2 + 12b_1 - b_0$$

(the coefficients are those of the "shifted Legendre polynomials").

The first L-moment is the sample mean, a measure of location. The second L-moment is (a multiple of) Gini's mean difference statistic, a measure of the dispersion of the data values about their mean.

By dividing the higher-order L-moments by the dispersion measure, we obtain the L-moment ratios,

$$t_r = \ell_r / \ell_2 .$$

These are dimensionless quantities, independent of the units of measurement of the data. t3 is a measure of skewness and t4 is a measure of kurtosis -- these are respectively the L-skewness and L-kurtosis. They take values between -1 and +1 (exception: some even-order L-moment ratios computed from very small samples can be less than -1).

The L-moment analogue of the coefficient of variation (standard deviation divided by the mean), is the L-CV, defined by

$$t_r = \ell_r / \ell_2 .$$

It takes values between 0 and 1.

L-MOMENTS FOR PROBABILITY DISTRIBUTIONS

For a probability distribution with cumulative distribution function F(x), probability weighted moments are defined by

$$\beta_r = \int x \{F(x)\}^r \, dF(x) , \qquad r = 0, 1, 2, \ldots .$$

L-moments are defined in terms of probability weighted moments, analogously to the sample L-moments:

$$\lambda_1 = \beta_0 ,$$
$$\lambda_2 = 2\beta_1 - \beta_0 ,$$
$$\lambda_3 = 6\beta_2 - 6\beta_1 + \beta_0 ,$$
$$\lambda_4 = 20\beta_3 - 30\beta_2 + 12\beta_1 - \beta_0 .$$

L-moment ratios are defined by

$$\tau_r = \lambda_r / \lambda_2 .$$

Examples:

Uniform (rectangular) distribution on (0,1):

$$\lambda_1 = 1/2, \quad \lambda_2 = 1/6, \quad \tau_3 = 0, \quad \tau_4 = 0.$$

Normal distribution with mean 0 and variance 1:

$$\lambda_1 = 0, \quad \lambda_2 = 1/\sqrt{\pi}, \quad \tau_3 = 0, \quad \tau_4 \approx 0.123.$$

The theory of L-moments has been applied in numerous papers. The following work is worth to mention: Rao, A Ramachandra, Hamed, Khaled H (1997), Duan, Jinfan, Selker, John, Grant, Gordon E (1998), Ben-Zvi, Arie, Azmon, Benjamin (1997), Van Gelder, Pieter, Neykov, Neyko, (1998), Demuth, Siegfried, Kuells, Christoph, (1997), Pearson, C P. McKerchar, A I. Woods, R A. (1991) Ruprecht, J K. Karafilis, D W. (1994), François Anctil, Nicolas Martel and Van Diem Hoang, (1998), Lin, Bingzhang. Vogel, John L. (19930 Gingras, Denis, Adamowski, Kaz. (1994).

GENERATION OF INHOMOGENEOUS DATASETS

There are several ways to generate inhomogeneous data. One can draw data from distribution 1 with certain probability and from distribution 2 with complementary probability and combine the data. However, in this paper the approach will be followed in which the parameters of the distribution function are considered uncertain. The parameters are drawn from a Normal distribution with a fixed coefficient of variation (for instance 5% or 20%; this is called the inhomogeneity factor). The realisations are substituted in the distribution function and a realisation from this distribution function is drawn.

Consider the exponential distribution with 2 parameters > (location) and ∀ (scale). The L-moments are given by:
8_1=>+∀; 8_2=0.5∀; ϑ_3=1/3; ϑ_4=1/6.

Therefore the L-moment estimators for the parameters > and ∀ are given by:
>=8_1- 2$8_2$; ∀=2$8_2$.

Data can be generated with x_i = >-∀log(rand) in which rand is a uniform distributed random variable between 0 and 1. The parameters > and ∀ themselves are also considered as random variables (normally distributed). In this way an inhomogeneous dataset can be generated.

The performance of the estimation methods is judged on two measures as was suggested also in Van Gelder and Vrijling, (1997). These are the relative bias of the quantile estimation and the root mean squared error (RMSE) of the quantile estimation. Various sample sizes are investigated varying between 10 and 100. The Monte Carlo simulation is repeated 2000 times in order to ensure convergence in the results. The results of a typical simulation are summarized in the next section.

RESULTS MONTE CARLO SIMULATIONS

The results for one typical simulation (sample size = 10) will be given in table 1. Each variable (∀, >, F(0.99), F(0.999), and F(0.9999)) will be summarized in the table by its mean value and standard deviation. Note that the following mean values were given as input parameters in the simulations:
∀ = 0.5, > = 1. For this choice the quantiles are F(0.99) = 3.30, F(0.999) = 4.45, and F(0.9999)=5.60.

Table 1 (n=10);

Inhomogeneity 0%

	$∀_{mean}$	$∀_{std}$	$>_{mean}$	$>_{std}$	$F(0.99)_{mean}$	$F(0.99)_{std}$
ML	0.4462	0.1527	1.0476	0.0498	3.1025	0.7045
LM	0.4968	0.1878	0.9970	0.0995	3.2847	0.8174
CM	0.4596	0.1899	1.0342	0.1216	3.1508	0.8145
LS	0.6554	0.2784	0.9121	0.1659	3.9305	1.1596

Inhomogeneity 0%

	$∀_{mean}$	$∀_{std}$	$>_{mean}$	$>_{std}$	$F(0.999)_{mean}$	$F(0.999)_{std}$
ML	0.4516	0.1528	1.0490	0.0479	4.1686	1.0531
LM	0.4996	0.1919	1.0010	0.1034	4.4522	1.2690
CM	0.4623	0.2014	1.0383	0.1322	4.2319	1.3122
LS	0.6612	0.3015	0.9139	0.1885	5.4811	1.9318

Inhomogeneity 0%

	$∀_{mean}$	$∀_{std}$	$>_{mean}$	$>_{std}$	$F(0.9999)_{mean}$	$F(0.9999)_{std}$
ML	0.4547	0.1502	1.0502	0.0488	5.2383	1.3838
LM	0.5060	0.1899	0.9989	0.1045	5.6594	1.6930

CM	0.4667	0.1926	1.0382	0.1250	5.3363	1.7048
LS	0.6647	0.2822	0.9150	0.1696	7.0369	2.4682

Inhomogeneity 5%

	$\forall_{mean}$	$\forall_{std}$	$>_{mean}$	$>_{std}$	$F(0.99)_{mean}$	$F(0.99)_{std}$
ML	0.4642	0.1518	1.0384	0.0691	3.1761	0.6974
LM	0.5049	0.1883	0.9977	0.1039	3.3230	0.8191
CM	0.4658	0.1928	1.0368	0.1265	3.1819	0.8248
LS	0.6646	0.2858	0.9128	0.1745	3.9734	1.1865

Inhomogeneity 5%

	$\forall_{mean}$	$\forall_{std}$	$>_{mean}$	$>_{std}$	$F(0.999)_{mean}$	$F(0.999)_{std}$
ML	0.4665	0.1523	1.0300	0.0641	4.2526	1.0456
LM	0.5055	0.1885	0.9910	0.1029	4.4830	1.2486
CM	0.4676	0.1964	1.0289	0.1304	4.2592	1.2824
LS	0.6688	0.2941	0.9030	0.1839	5.5227	1.8869

Inhomogeneity 5%

	$\forall_{mean}$	$\forall_{std}$	$>_{mean}$	$>_{std}$	$F(0.9999)_{mean}$	$F(0.9999)_{std}$
ML	0.4644	0.1517	1.0369	0.0703	5.3138	1.3904
LM	0.5015	0.1873	0.9998	0.1066	5.6185	1.6711
CM	0.4621	0.1935	1.0392	0.1289	5.2954	1.7110
LS	0.6593	0.2873	0.9162	0.1776	6.9884	2.5090

Inhomogeneity 20%

	$\forall_{mean}$	$\forall_{std}$	$>_{mean}$	$>_{std}$	$F(0.99)_{mean}$	$F(0.99)_{std}$
ML	0.6044	0.1876	0.9054	0.1504	3.6889	0.7986
LM	0.5737	0.1971	0.9361	0.1377	3.5780	0.8435
CM	0.5210	0.2008	0.9887	0.1488	3.3883	0.8511
LS	0.7456	0.2951	0.8481	0.1933	4.2818	1.2193

Inhomogeneity 50%

	$\forall_{mean}$	$\forall_{std}$	$>_{mean}$	$>_{std}$	$F(0.99)_{mean}$	$F(0.99)_{std}$
ML	1.0242	0.3201	0.4802	0.3230	5.1966	1.2737
LM	0.8594	0.2706	0.6450	0.2496	4.6025	1.1353
CM	0.7640	0.2774	0.7404	0.2551	4.2586	1.1577
LS	1.1057	0.4096	0.5231	0.3135	5.6150	1.6723

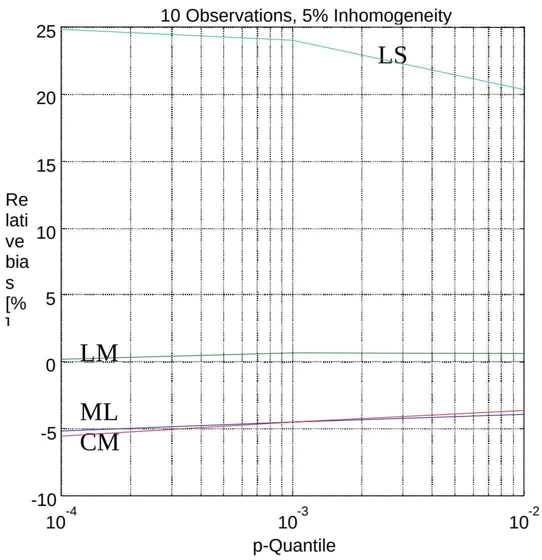

figure 1

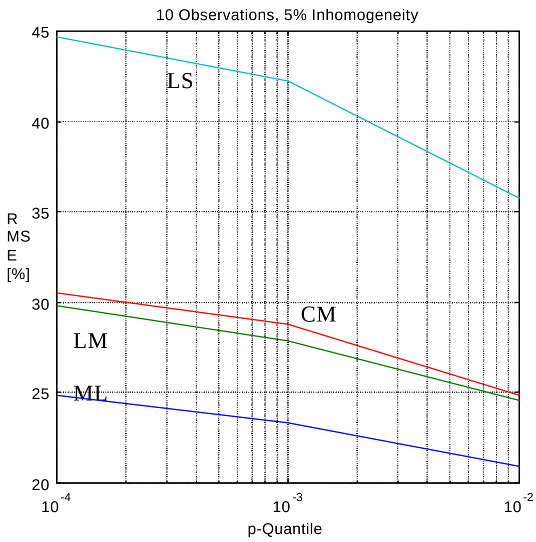

figure 2

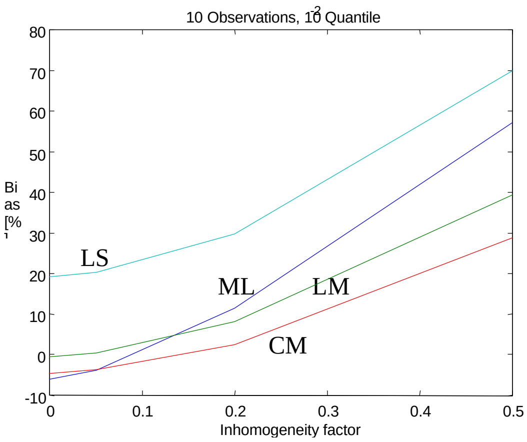

figure 3

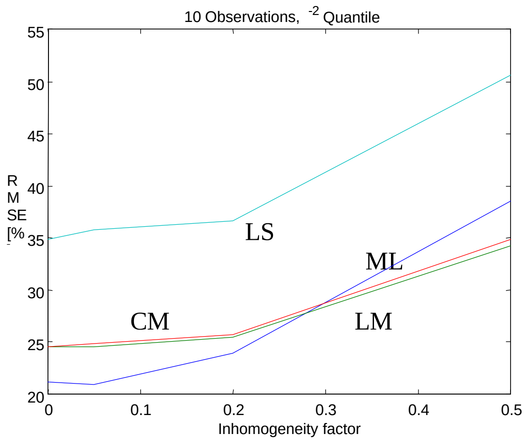

figure 4

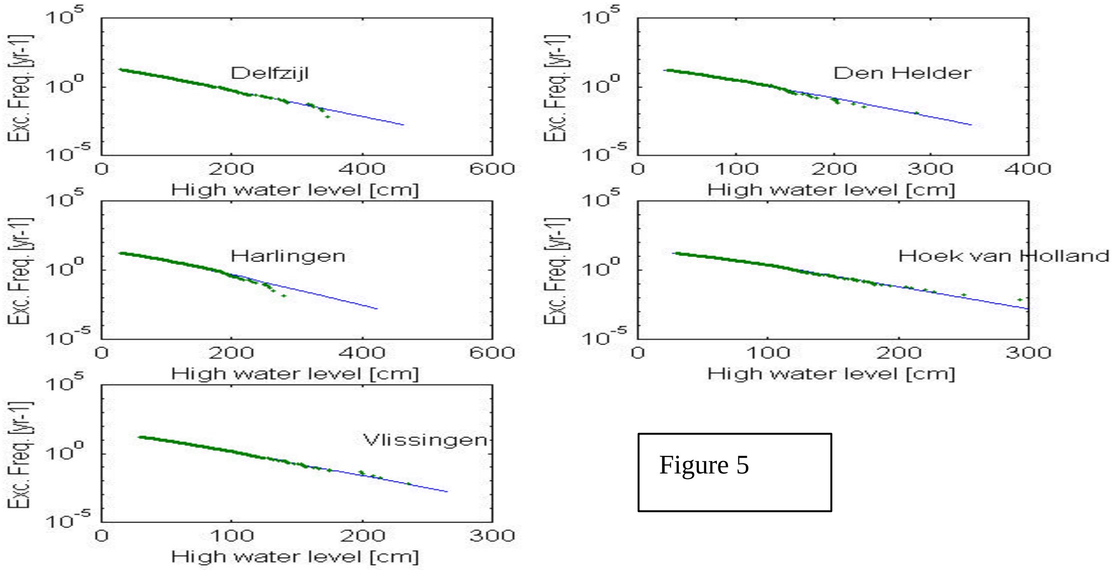

Figure 5

Note that the relative bias in the quantiles with the L-moments method is the lowest of the 4 methods in the above simulation. Also the RMSE is quite low. The performance of the 4 methods given 5% inhomogeneous data and 10 datapoints is presented in figures 1 and 2 showing the bias and RMSE of the 10^{-2} , 10^{-3} and 10^{-4} quantiles. LM give the smallest bias of about 1% over the whole range of p-quantile levels. The smallest RMSE is obtained with the ML method. The LS-method give the highest bias and RMSE (although the estimations are overestimations of the quantiles; i.e. safe values for engineers, whereas CM and ML are underestimations). The performance of the 4 methods given inhomogeneous data and 10 datapoints is presented in figures 3 and 4 showing the bias and RMSE of the 10^{-2} quantile given 0%, 5%, 20% and 50% inhomogeneity. Note that the bias in the quantiles is the smallest with the LM-method up to about 10% inhomogeneity. Above 10%, the CM-method give the smallest bias. The RMSE of LM and CM are approximately the same and are lower than the RMSE of ML for data with an inhomogeneity factor of 30% and higher.

CASE STUDY ON EXTREME SEA LEVELS ALONG THE DUTCH COAST

Peaks over thresholds of water level data from a number of gauging stations located along the Dutch coast were collected from the RIKZ in the Hague, the Netherlands. The data set contains a total of 6818 water level observations with sample sizes at the 5 sites varying from 53 to 104 years. The 5 sites are (from north to south): Delfzijl, Den Helder, Harlingen, Hoek van Holland and Vlissingen.

The data is corrected for sea level rise and a threshold value of 30cm filters the data. Although this will homogenize the data, there is still some inhomogeneity suspected. Therefore the L-Moments methods will be used for the paramater estimation. A Generalized Pareto distribution is selected to fit the data. Figure 5 shows the quantiles for the 5 sites. A comparison with other estimation methods is difficult to make, because it is unknown what the exact inhomogeneity level is of the database. Only Monte Carlo simulations (or an analytical study) are suitable in comparing estimation methods with eachother.

CONCLUSION

i) The L-moment method shows its power in case of inhomogeneous datasets. The quantile estimation with the L-moment method gives the smallest bias and RMSE compared with other methods (in case of inhomogeneous data from an exponential distribution and for all sample sizes).

ii) Conclusion i) is expected to hold also for other distribution functions; this will be verified in follow-up simulations.

iii) For homogeneous datasets it is better to use other estimation techniques such as the Maximum Likelihood for example. In practical situations however, it is unknown if the dataset can be considered homogeneous.

REFERENCES

Hosking, J.R.M., and Wallis, J.R., Regional Frequency Analysis, An Approach Based on L-moments, Cambridge University Press, 1997.

Van Gelder, P.H.A.J.M., Roos, A., and Tonneijck, M.R., On the probabilistic design of dikes in the Netherlands. Applications of Statistics and Probability, ICASP7, Volume 3, pp.1505-1508, A.A. Balkema, July 1995.

Rao, A Ramachandra. Hamed, Khaled H., Regional frequency analysis of Wabash River flood data by L-moments, Journal of Hydrologic Engineering. v 2 n 4 Oct 1997. p 169-179.

Duan, Jinfan. Selker, John. Grant, Gordon E., Evaluation of probability density functions in precipitation models for the Pacific Northwest, Journal of the American Water Resources Association. v 34 n 3 Jun 1998. p 617-627.

Ben-Zvi, Arie. Azmon, Benjamin., Joint use of L-moment diagram and goodness-of-fit test: A case study of diverse series, Journal of Hydrology. v 198 n 1-4 Nov 1997. p 245-259.

Demuth, Siegfried. Kuells, Christoph. Probability analysis and regional aspects of droughts in southern Germany. Symp 1: Sustainability of Water Resources under Increasing Uncertainty IAHS Publication (International Association of Hydrological Sciences). n 240 1997.

Ruprecht, J K. Karafilis, D W. Regional flood frequency - caution needed Proceedings of the Water Down Under 1994 Conference. Part 3 (of 3). Adelaide, Aust. 19941121-19941125.

Gingras, Denis. Adamowski, Kaz. Performance of L-moments and nonparametric flood frequency analysis. Canadian Journal of Civil Engineering. v 21 n 5 Oct 1994. p 856-862.

François Anctil, Nicolas Martel et Van Diem Hoang. Analyse régionale des crues journalières de la province de Québec. Pages 360-369 Volume 25, Number 2, April 1998 ISSN 1208-6029.

Lin, Bingzhang. Vogel, John L. Comparison of L-moments with method of moments Proceedings of the Symposium on Engineering Hydrology Proc Symp Eng Hydrol. Publ by ASCE, New York, NY, USA. 1993. p 443-448.

P.H.A.J.M. van Gelder, N. Neykov, Regional frequency analysis of extreme water levels along the Dutch coast using L-moments, International Scientific Conference on Stochastic models of hydrological processes and their applications to problems of environmental preservation, pp.14-20, 23-27 November 1998, Moscow, Russia.

Pearson, C P. McKerchar, A I. Woods, R A. Regional flood frequency analysis of Western Australian data using L-moments. International Hydrology and Water Resources Symposium 1991 Part 2. v 2 n 91 pt 22. p 631-632.

Hosking, J.R.M., The theory of probability weighted moments, Research Rep. RC 12210, 160 pp., IBM Research Division, 1986.

Hosking, J.R.M., L-moments: Analysis and estimation of distributions using linear combinations of order statistics, J. R. Statist. Soc., Ser. B, 52(1), 105-124, 1990.

P.van Gelder and J.K. Vrijling, A comparative study of different parameter estimation methods for statistical distribution functions in civil engineering applications, Proceedings of ICOSSAR'97, pp.665-668, 1997.