The probabilistic optimisation of the revetment on the dikes along the Frisian coast

M. Hussaarts, J.K. Vrijling, P.H.A.J.M. van Gelder
Subfaculty of Civil Engineering, Delft University of Technology, The Netherlands

H. de Looff
Ministry of Transport, Public Works and Water Management, Road and Hydraulic Engineering Division, The Netherlands

C. Blonk
Simtech BV, The Netherlands

ABSTRACT: For centuries the dikes in the Netherlands have been protected against wave attack by revetments constructed from pitched blocks. Based on experience a size was selected and, if a severe storm damaged the revetment, heavier blocks were applied. Recent research has however shown that the design storm surge would cause severe damage to the protection of the dike. A new design philosophy is developed that assesses the function of the revetment in the entire dike and optimises the thickness probabilistically.

1 INTRODUCTION

For centuries the dikes in the Netherlands have been protected against wave attack by revetments constructed from pitched blocks. In the old days these blocks took the form of relatively rounded rock (Vilvoordse steen, granite boulders) or basalt hexagones. Presently, rectangular concrete blocks are frequently used, but basalt is also still applied due to the high specific mass and the interesting visual appearance.

Until recently the dimension of the blocks was chosen empirically. Based on experience a size was selected and if a severe storm damaged the revetment heavier blocks were applied. When physical models verified with scale model tests were developed, it appeared that revetments were able to withstand e.g. a 1/10 year storm, but that the design storm surge would cause severe damage to the protection of the dike. Consequently, it is decided to replace the revetments on the Dutch dikes.

A new design philosophy has to be developed that centres on the function of the revetment in the task of the entire dike to protect the hinterland from inundation. In this paper a first effort is made to optimise the revetment size in this framework.

2 DESCRIPTION OF THE CASE

The Frisian coast is situated in the Northern part of the Netherlands, along the Wadden Sea.

Figure 1: The location of the study area

The storm surge level HW along the Frisian coast at the location is given by a Gumbel distribution with parameter values A= 2.91m and B = 0.36m. The design water level with an exceedance frequency of $2.5 \cdot 10^{-4}$ per year equals NAP+5.89m

The significant wave height in front of the dike is based on 22 storms, hindcasted with the wave growth model SWAN. The conditional mean of the significant wave height H_s is described by a linear function of the storm surge level:

$$E[H_s | HW] = 0.338 \cdot HW$$

The conditional distribution of H_s is assumed to be Normal with the standard deviation equal to 0.064m

The SWAN-calculations show that the wave steepness s_{0p}, defined on the peak period and the deep water wave length, is independent of the significant wave height. The steepness is described by a normal distribution with a mean of 0.036 and a standard deviation of 0.004.

Applying the relations given above the significant wave accompanying the design water level of NAP+5.89m is approximately equal to 2m with a steepness of 0.036 and a peak period of 6s.

3 THE STABILITY OF THE REVETMENT

The stability of a revetment block is derived considering the equilibrium of the water pressure under the revetment during wave run down and the weight of the block. The resulting formula reads:

$$\frac{H_s}{\Delta D} = \frac{3\cos \boldsymbol{a}}{x_{0p}}$$

In which:
H_s: Significant wave height;
Δ: Relative density of the blocks;
D: Thickness of the blocks;
$\boldsymbol{a}$: Angle of the slope of the revetment;
x: Iribarren parameter.

A definition sketch is given in figure 1.

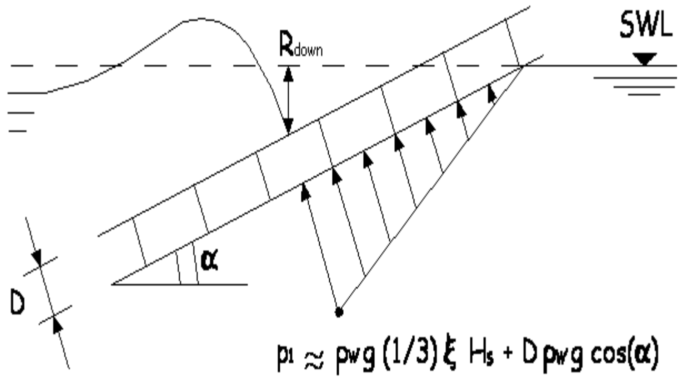

Figure 2: Definition sketch of revetment stability

In the analysis of the block stability the friction forces between the critical block and its neighbours is neglected, as well as the loss of water pressure due to leakage through the joints between the blocks. The derived formula must therefore be conservative. A comparison with scale model results confirms this. From a statistical analysis of 7 representative tests with irregular waves it appears that the factor 3 should be replaced by a factor M with a mean value of 4.06 and a standard deviation of 0.7. The distribution type cannot be inferred from the 7 data points but a normal distribution is not rejected. Rewriting the previous formula in basic variables results in:

$$\frac{H_s}{\Delta D} = \frac{M s_{0p} \cos^2 \boldsymbol{a}}{\sin \boldsymbol{a}}$$

It should be noted that from the scale model test performed with regular waves, a far higher value of M could be inferred. If one assumes that $H = H_s$ a factor M with a mean value of 10.9 and a standard deviation of 5.4 results. A difference that should be further studied.

For a revetment with a slope of 1:3 attacked by a wave field with a significant wave height of 2m and a steepness of 0.036 a revetment thickness of D = 0.55m is required according to the formula. This estimate does however not account for the uncertainties in the various variables.

4 RELIABILITY ANALYSIS OF THE REVETMENT UNDER WAVE ATTACK

The reliability of the revetment under wave attack is analysed using the following reliability function, that follows directly from the relations given above:

$$Z = \Delta D - H_s \frac{\sin \alpha}{M s_{0p} \cos^2 \alpha}$$

And after substitution of the relation gouverning H_s:

$$Z = \Delta D - \left(0.338 HW + fH_s\right) \frac{\sin \alpha}{M s_{0p} \cos^2 \alpha}$$

The following distributions and parameter values have been used in the analysis to account for the uncertainties

Table 1: Distribution type and parameter values of the basic variables

Variable	Distr. type	A	B	μ	σ
HW	Gumbel	2.91	0.36	2.05	0.30
s_{0p}	Normal	0.00	0.00	0.036	0.004
fHs	Normal	0.00	0.00	0.00	0.20
$\cot \alpha$	Normal	0.00	0.00	0.33	0.01
M	Normal	0.00	0.00	4.06	0.698
D	Normal	0.00	0.00	1.62	0.02
D	Normal	0.00	0.00	0.7	0.02

Probabilistic calculations give a relation between the thickness of the revetment blocks and the probability of failure.

Table 2 Probability of failure of the revetment

Revetment Thickness D (m)	Failure probability (1/year)
0.6	$5 \cdot 10^{-3}$
0.7	$8 \cdot 10^{-4}$
0.8	$5 \cdot 10^{-4}$
0.9	$1 \cdot 10^{-4}$

Failure of the revetment does however not immediately imply failure of the flood defence and inundation.

5 THE SEQUENCE OF EVENTS AFTER REVETMENT FAILURE

After revetment failure the clay layer behind the revetment is exposed to wave attack. Although clay has considerable resistance to the attack by clean water, the number of hours that it can withstand attack by waves armed with sand and gravel (originating from sandy shoals and the filter layer under the revetment respectively) is quite limited.

If the clay layer is damaged the erosion process progresses. Here the erosion process of the body of the dike is viewed as a dune erosion process as modelled by the DUROSTA-model. A breach in the dike is formed if the erosion destroys the cross section. The event tree (figure 2) depicts the sequence of events leading to inundation of the polder.

For high values of the storm surge level the probability of failure of the dike given revetment failure is approximately equal to 0.35. Applying this value for the conditional probability of a breach leads to the following refinement of table 2.

Table 3 Probability of failure of the revetment and of inundation

Revetment Thickness D (m)	Failure probability (1/year)	Probability of inundation (1/year)
0.6	$5 \cdot 10^{-3}$	$1.7 \cdot 10^{-3}$
0.7	$8 \cdot 10^{-4}$	$2.8 \cdot 10^{-4}$
0.8	$5 \cdot 10^{-4}$	$1.7 \cdot 10^{-4}$
0.9	$1 \cdot 10^{-4}$	$3.5 \cdot 10^{-5}$

6 ECONOMIC OPTIMISATION OF THE THICKNESS OF THE REVETMENT

In order to find the economically optimal size of the block revetment, the total cost consisting of the investment in the revetment on the dike and the present value of the risk is expressed as a function of the revetment thickness *D*.

The dike under study has an area *A* of 10,000 m^2 covered with blocks. The cost of constructing a block revetment is estimated at Dfl 500 per m^3 of concrete, neglecting the cost of design and mobilisation. The total variable component of the investment amounts to:

$$I(D) = 500AD = 5 \cdot 10^6 D$$

The damage in case of a breach in the dike consists of three componenents:
1. Repair of the failed revetment;
2. Closure of the breach in the dike;
3. Damage caused by the flooding of the polder.

The repair cost of the revetment exceeds the initial cost due to the smaller scale of the repair:

$$R(D) = 700AD = 7 \cdot 10^6 D$$

The closure of the breach is estimated at Dfl 10^5 per meter breach width. With a breach width of 50 m the cost of closing the breach becomes Dfl. 5 10^6.

The damage caused by flooding is an increasing function of the flooding depth. With a depth of only 2m the damage reaches its maximum value of Dfl $1.7 \cdot 10^8$.

From these numbers it becomes clear that the flood damage is by far the biggest component (97%) of the total damage *S*, that amounts to approximately Dfl 175 10^6.

The total cost of the revetment consisting of the construction cost and the present value of the risk over an infinite planning period, is derived as a function of *D*:

$$TC(D) = 5 \cdot 10^6 D + \frac{P_{f;inundation}(D) \cdot S}{r}$$

Where *r* denotes the rate of interest.

Using the values in table 3, the probability of inundation as a function of the block thickness can be approximated by:

$$P_{f;inundation}(D) = e^{-\frac{D-a}{b}} = e^{-\frac{D-0.113}{0.0767}}$$

Differentiating the total cost function *TC*(*D*) and equating the result to zero leads to the optimal probability of inundation (see also figure **).

$$P_{f;inundation}(D_{opt}) = \frac{5 \cdot 10^6 br}{S} = \ldots$$

$$\ldots = \frac{5 \cdot 10^6 \cdot 0.0767 \cdot 0.02}{175 \cdot 10^6} = 4.38 \cdot 10^{-5}$$

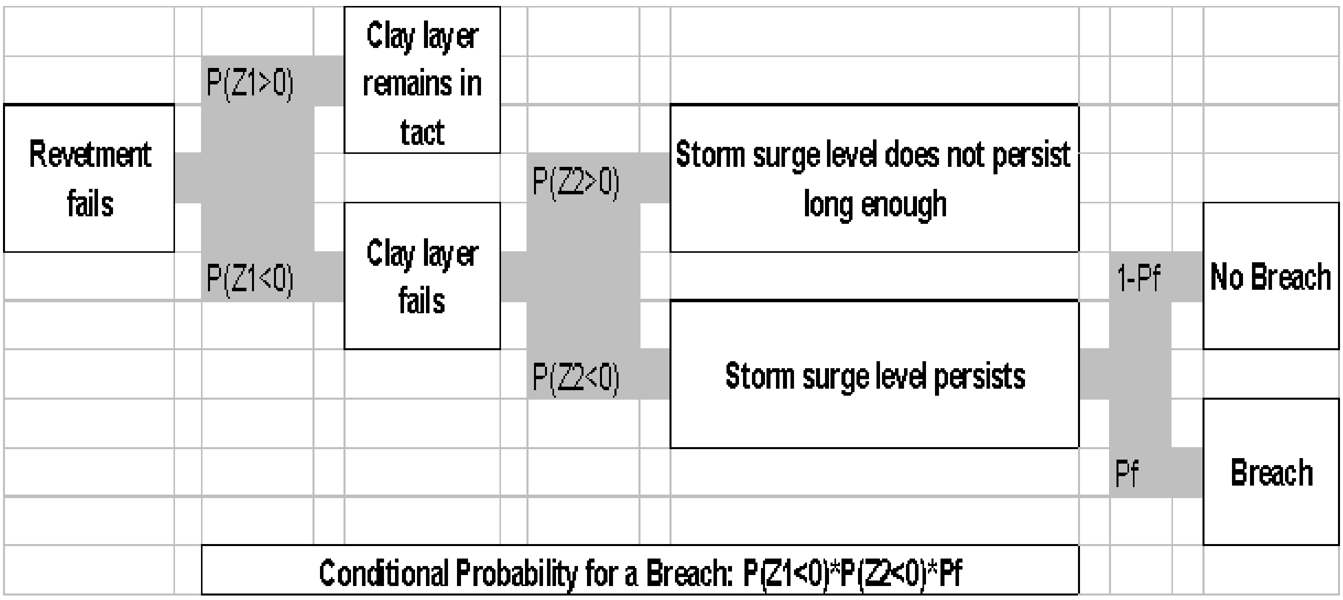

Figure 3: Event tree for inundation of a polder

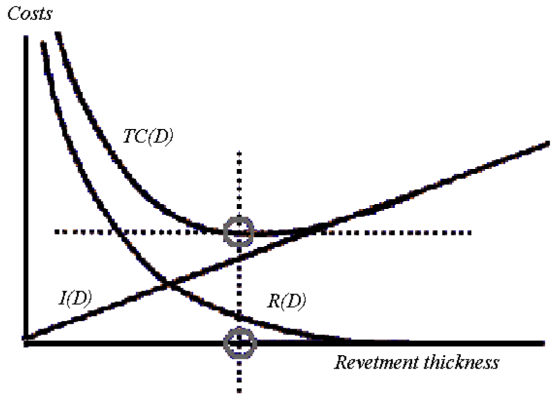

Figure 4: Economic optimisation of the revetment thickness

The optimal failure probability of the revetment is found using the conditional probability of a breach given revetment failure, which was found to be equal to 0.35.

$$P_{f;revetment}\left(D_{opt}\right) = \frac{P_{f;inundation}\left(D_{opt}\right)}{0.35} = 1.25 \cdot 10^{-4}$$

The optimal revetment thickness results from:

$$D_{opt} = -b \ln\left[P_{f;inundation}\left(D_{opt}\right)\right] + a = 0.88\text{m}$$

An optimal thickness of 0.88m exceeds the value 0.55m that was calculated previously. The extra thickness is needed to take account of the uncertainties in the variables other than the storm surge level *HW*.

7 CONCLUSIONS AND RECOMMENDATIONS

A simple model for the stability of revetment blocks based on the equilibrium of the water pressure under the revetment during wave run down and the weight of the block appears to be in good agreement with scale model tests. Due to block interaction and leakage the stability in the scale model exceeds the predicted value by 35%. However the scatter of the experimental results has a coefficient of variation of 17%.

Following the classical procedure for the design of dikes a revetment thickness of 0.55m is required. In this case the uncertainties of the significant wave height, the wave steepness, the design formula and the specific mass of the blocks are neglected.

Probabilistic calculations that take all uncertainties into account show that the probability of failure of the 0.55m thick revetment is approximately 9.5 10^{-3} per year.

To find the optimal probability of failure of the revetment, first it has been established how likely this leads to inundation via erosion of the clay layer and a breach in the dike. Consequently the investment in a thicker revetment is equated with the reduction of the risk of flooding. The risk of flooding is calculated as a fraction of the invested value in the polder and accounting for the probability of inundation as a function of the revetment thickness. The resulting optimal probability of inundation is 4.38 10^{-5} per year with a related thickness of the revetment of 0.88m.

The fundamental question to be answered regards the function of the revetment as a part of the dike. In the situation studied here it appeared to have an essential function in the sea defence. Failure led fairly likely to inundation of the polder. Thus it should be able to fully withstand the design conditions.

The effect of leakage through the joints and of block-block interaction (friction and moment) should be fully understood before *M*-values considerably exceeding 3 can be applied. Leakage has been studied and modelled (ANAMOS) quite pervasively but the study of the interactions is still at the start.

Requirements to guarantee the realisation of the beneficial effects of these phenomena during design and construction should be formulated as soon as sufficient insight has been gained. Findings on the phenomenon of block-block interaction could for instance lead to requirements regarding the number of blocks above the still water line.

The difference in stability of the blocks under regular and irregular wave attach should be further studied.

REFERENCES

Bakker,K.J., Meijers,P. (1988): *Stability against sliding of flexible revetments*, in: Modelling Soil-Water-Structure Interactions, ed. Kolkman et al., Balkema, Rotterdam

Klein Breteler,M., Bezuijen,A. (1988): *The permeability of closely placed blocks on gravel*, in Modelling Soil-Water-Structure Interactions, ed. Kolkman et alt., Balkema, Rotterdam

Burger,A.M. et alt. (1988): *Analytical design method for block revetments*, 21th International conference on Coastal Engineering (ICCE), Malaga