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Bivariate description of offshore wave conditions with physics-based extreme value statistics

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Abstract

This paper aims to find a model for the bivariate description of extreme wave heights and wave periods representing relatively deep-water sea states. To achieve this, four statistical models taken from literature and a model based on the physical behaviour of waves have been tested on wave data at two sites. The models have been compared with regard to their ability to correctly describe the data, their behaviour when used for extrapolation to extreme events, and the transparency of the methods.

Keywords: Bivariate offshore environment; Extreme value statistics; Threshold exceedance; Bivariate log-normal; North Sea; Indian Ocean; Contour plots

1. Introduction

In the design of coastal structures, a probabilistic approach is often adopted. In such an approach, the failure modes of the structure are described in the form of limit states. The input parameters of the limit state function are the stochastic load and strength parameters corresponding with the failure mode, like wave heights, wave periods, soil characteristics, etc.

Well-known statistical methods are available to derive the marginal distributions of all stochastic wave parameters. However, in view of the dependence structure between wave heights and wave periods, marginal analysis is in itself insufficient to come to an accurate description of the long-term wave climate. This is recognised in earlier research by several authors. Research effort of several groups around the world has led to a large number of methods to deal with the problem of bivariate statistical analysis in wave climate studies and other areas (see reference list).

It appears that, despite the large number of methods in existence, they are not very well known in engineering practice. This might be caused by the fact that the criteria based on which a choice should be made are not very clear.

This paper presents the results of a comparison of five bivariate models. The first two methods, i.e. the bivariate log-normal distribution and the Fang and Hogben distribution, contain two marginal distributions for the description of the significant wave height (H_s) and the wave period (zero-upcrossing period (T_z) or peak period (T_p)). Both models are based on a linear relation between H_s and T_p. According to Fang and Hogben [2], their model is a modified version of the bivariate log-normal model. A disadvantage of these two models is that they are committed to the marginal log-normal distribution. In the case of the following three models, for the marginal probability density functions (PDFs), also other distributions like for instance the Weibull, Gumbel and Frechet distribution, which are known as the extreme value distributions, can be used. The third model consists of a marginal PDF for H_s and a conditional distribution for T_p. In this third model, the dependence of T_p on H_s is described by the location, scale (and shape) parameter of the conditional PDF, which are

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defined as a function of H_s and T_p. The fourth model is a relatively complicated model that has been introduced by Morton and Bowers [6]. It contains two marginals for H_s and T_p. In this method, the marginal PDFs are transformed into the so-called Frechet space. Furthermore, the dependence between T_p and H_s is described by a complex statistical function. (For a more detailed description of the model, see Morton and Bowers [6]). The fifth model is an attempt to obtain a model that includes the physical limitations of waves. This model, which has been introduced by Vrijling and Bruinsma [5], contains a marginal PDF for H_s and a marginal PDF for wave steepness (s).

All the methods are tested on wave data at two sites, representing relatively deep-water wave fields. The study is focused on the description of *extreme* wave data.

2. Overview of the methods

The following methods for the bivariate description of wave height and wave period have been studied:

- A bivariate log-normal model (Ochi [1], 1978)
- A bivariate log-normal with correction for skewness (Fang and Hogben [2], 1982)
- A marginal distribution for the significant wave height and a conditional distribution for the peak period (Mathiesen et al. [3], 1993; Haver [4], 1985)
- A model based on marginal distributions of significant wave height and wave steepness, combined with a function describing the peak period as a function of wave height and steepness (Vrijling and Bruinsma [5], 1980)
- The bivariate model of Morton and Bowers [6] (1997).

2.1. A bivariate log-normal model (Ochi [1], 1978)

Ochi introduced the use of a bivariate log-normal distribution for the joint distribution of the significant wave height (H_s) and the peak period (T_p) (or zero-upcrossing period (T_z)). The joint probability density function (PDF) of this model may be written as

$$f(H_s,T_p)=\frac{0.5}{H_sT_p\pi\delta_{H_s}\delta_{T_p}\sqrt{1-\rho^2}}\times\exp\left\{-\frac{0.5}{1-\rho^2}\left[\frac{(\log T_p-\lambda_{T_p})^2}{\delta_{T_p}^2}-\frac{2\rho(\log T_p-\lambda_{T_p})(\log H_s-\lambda_{H_s})}{\delta_{H_s}\delta_{T_p}}+\frac{(\log H_s-\lambda_{H_s})^2}{\delta_{H_s}^2}\right]\right\} \quad (1)$$

in which λ_{T_p}, λ_{H_s}, δ_{T_p} and δ_{H_S} are the location and scale parameters of the marginal PDF of T_p and H_s, respectively.

The parameter ρ forms a linear correlation coefficient between the two variables, and may be written as

$$\rho=\frac{\mathrm{Cov}(\log(T_p),\log(H_s))}{\delta_{T_p}\delta_{H_s}} \quad (2)$$

2.2. A bivariate log-normal with correction for skewness (Fang and Hogben [2], 1982)

An attempt to improve the bivariate log-normal model has been made by Fang and Hogben. They included a measure of skewness in a term modifying the log-normal form of the marginal distribution of H_s. The PDF of this bivariate model, also called the Fang and Hogben distribution, is given by

$$f(H_s,T_p)=\frac{0.5}{T_pH_s\pi\delta_{T_p}\delta_{H_s}\sqrt{1-\rho^2}}\times\exp\left\{-\frac{0.5}{1-\rho^2}\left[\frac{(\log T_p-\lambda_{T_p})^2}{\delta_{T_p}^2}-\frac{2\rho(\log T_p-\lambda_{T_p})(\log H_s-\lambda_{H_s})}{\delta_{T_p}\delta_{H_s}}+\frac{(\log H_s-\lambda_{H_s})^2}{\delta_{H_s}^2}\right]\right\}\times\left(1-\frac{\kappa_{H_s}}{6}[3(\log H_s-\delta_{H_s})-(\log H_s-\delta_{H_s})^3]\right) \quad (3)$$

where κ_{H_s} is the coefficient of skewness for log H_s. (The remaining parameters are similar to those of the bivariate log-normal distribution.)

2.3. A marginal distribution for the significant wave height and a conditional distribution for the peak period (Mathiesen et al. [3], 1993; Haver [4], 1985)

This model consists of a marginal PDF for the significant wave height and a conditional PDF for the peak period. It is based on the expression

$$f(H_s,T_p)=f(H_s)f(T_p|H_s) \quad (4)$$

in which,

$f(H_s, T_p)$ the bivariate PDF of H_s and T_p
$f(H_s)$ the marginal distribution of H_s
$f(T_p|H_s)$ the conditional distribution of T_p.

As mentioned in Section 1, in the case of this third and the following fourth and fifth model, various distributions can be used for the marginal (and conditional) distribution. The most obvious procedure is to perform a marginal analysis in order to select the marginal distributions which give the most accurate description of the data.

The conditional distribution is a marginal distribution of T_p, which is related to H_s. Its location, scale (and shape) parameter are defined as a function of H_s and T_p. In agreement with earlier studies (Mathiesen et al. [3], Haver [4]), in the present study, these parameters are modelled by purely empirical regression functions. The following equations have been used:

$$\begin{aligned} g(H_s)_i &= aH_s + b \\ g(H_s)_i &= a[H_s]^2 + bH_s + c \\ g(H_s)_i &= a\exp(bH_s) + c\exp(dH_s) \end{aligned} \tag{5}$$

in which

$g(H_s)_i$ estimators for the parameters of the conditional distribution as a function of H_s ($i=1$, scale parameter; $i=2$, location parameter; $i=3$, shape parameter);
a, b, c, d parameters of the regression functions.

2.4. A model based on marginal distributions of significant wave height and wave steepness, combined with a function describing the peak period as a function of wave height and steepness (Vrijling and Bruinsma [5], 1980)

This method has been proposed by Vrijling and Bruinsma [5]. It is based on the assumption that the significant wave height (H_s) and the wave steepness (s) are independent.

Assuming a deep-water wave field and starting from the linear wave theory, the wave steepness based on the peak period is defined as

$$s_p = \frac{H_s}{\left[\frac{gT_p^2}{2\pi}\right]} \tag{6}$$

in which g is the acceleration of gravity. The joint PDF of the significant wave height and peak period can be derived by transforming the joint PDF of significant wave height and wave steepness. The bivariate model of significant wave height and peak period is then given by

$$f(H_s, T_p) = f(H_s, s_p)|J| = f(H_s, s_p)\frac{4\pi H_s}{g\left[\frac{2\pi H_s}{g s_p}\right]^{3/2}} \tag{7}$$

where J is the Jacobian of the transformation.

Since the wave parameters H_s and s are assumed to be independent, the joint PDF can be obtained by simply multiplying the marginal distributions of H_s and s. Similar with the third and fifth model, various distributions can be used for the marginals. Formulae (6) can be used to calculate the wave steepness values on basis of the wave height and wave period data.

2.5. The bivariate model of Morton and Bowers [6] (1997)

Morton and Bowers [6] have developed a relatively complicated bivariate model.

The joint distribution is given by

$$\frac{\partial^2}{\partial x_1 \partial x_2} P(X_1 > x_1, X_2 > x_2) \tag{8}$$

where

$$P(X_1 > x_1, X_2 > x_2) = \exp(-V(z))$$

Therefore,

$$\frac{\partial^2}{\partial x_1 \partial x_2} P(X_1 > x_1, X_2 > x_2) = \left(V_1(z)V_2(z) - V_{12}(z)\right)\frac{\partial z_1}{\partial x_1}\frac{\partial z_2}{\partial x_2}\exp(-V(z))$$

where

$$V(z) = \left(\left(\frac{1}{z_1}\right)^{\varphi} + \left(\frac{1}{z_2}\right)\right)^{1/\varphi}$$

and

$$V_1(z) = \frac{\partial V}{\partial z_1} = (-z_1^{-\varphi-1})\left(\left(\frac{1}{z_1}\right)^{\varphi} + \left(\frac{1}{z_2}\right)^{\varphi}\right)^{(1/\varphi)-1}$$

$$V_2(z) = \frac{\partial V}{\partial z_2} = (-z_2^{-\varphi-1})\left(\left(\frac{1}{z_1}\right)^{\varphi} + \left(\frac{1}{z_2}\right)^{\varphi}\right)^{(1/\varphi)-1}$$

$$V_{12}(z) = \frac{\partial^2 V}{\partial z_1 \partial z_2} = (-z_1^{-\varphi-1})(-z_2^{-\varphi-1})(1-\varphi) \times \left(\left(\frac{1}{z_1}\right)^{\varphi} + \left(\frac{1}{z_2}\right)^{\varphi}\right)^{(1/\varphi)-2}$$

In the above equations, X_1 and X_2 are the significant wave height and the peak period, respectively. P is the joint exceedance probability distribution function of T_p and H_s.

The parameters z_1 and z_2 represent the significant wave height and the peak period transformed into the Frechet space. Further ϕ is a parameter describing the dependence between the significant wave height and the peak period.

A more detailed description of the model (i.e. the definition of parameter ϕ, the transformation into the so-called Frechet space) can be found in the paper of Morton and Bowers [6].

3. Case studies

The different methods have been used to analyse deep-water wave data from the North Sea and the Indian Ocean.

3.1. Description of the North Sea data

The North Sea data has been measured at the wave station of the Euro platform (by the Dutch Ministry of Water Management, RIKZ, The Hague), which is situated along the Dutch coast near Hoek van Holland. In this region, the average daily wave field contains swell formed in the Atlantic Ocean and wind waves generated by a South Westerly wind. The local extreme sea states are mainly caused by South Westerly and North Westerly storms. The most extreme sea states at this site are most likely to be generated by North Westerly winds, due to the bathymetry of the North Sea.

The selection of data for the extreme wave analysis consisted of three steps. In the first step, the homogeneity of the data has been considered. On basis of physical arguments, from the initial data set, that consisted of 37,591 three-hourly observations of significant wave height and *zero-upcrossing* period for 1979–1991, the data of swell have been censored. To be able to distinguish wind waves and swell, waves with a wave steepness (based on the zero-upcrossing period) smaller than 5.5% have been assumed to be swell. (It must noted that using 5.5% instead of for example 5, or 6% is rather subjective. To be able to make clear distinction between wind waves and swell, the underlying physical process should be examined in more detail, for instance by analysing the energy-density spectrum of the local wave field (see Vrijling and Bruinsma [5])).

In the second step, from the remaining data set (swell has been removed), data points representing (independent) maxima of separate storms have been selected. In order to guarantee the independence of the selected storm data, the minimum time interval between successive data points has been taken into account, considering the minimum duration of a storm in the North Sea. It has been assumed that the minimum duration of a storm is approximately 25 h.

In the third step, with the Peak Over Threshold (POT) method, three sets of extreme observations have been composed, using the threshold levels $H_s = 2.00$ m (971 observations), 4.50 m (59 observations), and 5.00 m (22 observations).

3.2. Description of the Indian data

The Indian data have been measured on the South–West coast of India, near Karwar. The local wave climate is characterised by monsoon periods.

Each year during the months June, July and August, the South Westerly monsoon is blowing, causing a wave field with an average significant wave height of approximately 2 m. During this period, the wind waves generated by the South West monsoon grow on swell formed in the South of the Indian Ocean (‘roaring forties’).

During the other months of the year, the sea is very calm. Further, on the average, one time a year the coast is being hit by a hurricane.

Two sets of wave data have been analysed. The first data set consisted of three-hourly observations of significant wave height and *peak* period measured during the months June and July of 1988. The sample contained 167 observations above the threshold level $H_s = 1.95$ m. It must noted that in this case the swell has not been removed from the wind waves. It appeared to be very difficult to distinguish both wave types. This is due to the fact that during the South–West monsoon the direction of swell corresponds with the wind direction of the monsoon.

The second data set consisted of 25 hindcasted significant wave heights and *peak* periods of hurricanes.

3.3. Marginal statistical analysis of the wave climates

To construct the bivariate models, first a marginal analysis has been performed. As marginal distributions for the significant wave height, the zero-upcrossing or peak period, and the wave steepness (s), the Exponential, the Gumbel, the three-parameter Frechet, the three-parameter Weibull, and the two-parameter log-normal distribution have been tested. As parameter estimation method the method of moments (MOM), the linear least squares method (Lin LS), the non-linear least squares method (N Lin LS) and the maximum likelihood method (MAX) have been used. Various combinations of distribution type and parameter estimation method have been tried in order to find marginal models that closely fit to the data. The goodness-of-fit of the various combinations has been judged visually. (The equations of the marginal distributions and a description of the parameter estimation methods can be found in Van Gelder [10]).

3.4. Bivariate statistical analysis of the wave climates

On basis of the outcome of the marginal analysis, for each data set of extreme observations, a number of bivariate models have been composed. Table 1 shows the tested bivariate models, including the corresponding marginal distributions and the estimation methods for the parameters of the marginal models. The bivariate models, which gave a relatively accurate representation of the data, have been marked in grey.

4. Results

4.1. Data selection

The data selection procedure has found to be a crucial aspect of the statistical analysis. To obtain a homogeneous data set, it appeared to be necessary to examine the data on its physical origin. In the case of the Karwar study, the data set of the South–West monsoon period contained both wind waves and swell. On basis of physical arguments, this data set should be considered as inhomogeneous. Probably

Table 1
Overview of the tested bivariate models

Bivariate model	Marginal distribution of H_s	Parameter estimation method	Marginal distribution of T_z (1)/ s_z (2)	Parameter estimation method
North Sea data. Treshold level H_s = 2.00 m (971 observations)				
Bivariate log-normal	Log-normal	MAX	Log-normal[1]	MAX
T_z conditional on Hs	Gumbel	MOM/MAX/LinLS	Log-normal[1]	MAX
Wave steeepness model	Gumbel	MOM/MAX/LinLS	Weibull[2]	Lin LS
Model of Morton & Bowers	Gumbel	MOM/MAX/LinLS	Log-normal[1]	MAX
Model of Morton & Bowers	Gumbel	MOM/MAX/LinLS	Weibull[1]	MAX
North Sea data. Treshold level H_s = 4.50 m (59 observations)				
Bivariate log-normal	Log-normal	MAX	Log-normal[1]	MAX
T_z conditional on Hs	Weibull	Lin LS/ N Lin LS	Log-normal[1]	MAX
T_z conditional on Hs	Exponential	MAX/ Lin LS	Log-normal[1]	MAX
Wave steeepness model	Weibull	Lin LS/N Lin LS	Weibull[2]	Lin LS
Wave steeepness model	Gumbel	Lin Ls/ N Lin Ls	Weibull[2]	Lin Ls
Model of Morton & Bowers	Weibull	Lin Ls/N Lin LS	Log-normal[1]	MAX
North Sea data. Treshold level H_s = 5.00 m (22 observations)				
Bivariate log-normal	Log-normal	MAX	Log-normal[1]	MAX
Wave steeepness model	Weibull	MOM/MAX/(N) Lin LS	Weibull[2]	Lin LS
Wave steeepness model	Gumbel	MOM/MAX/(N) Lin LS	Weibull[2]	Lin Ls
Model of Morton & Bowers	Weibull	MOM/MAX/(N) Lin LS	Weibull[1]	MAX
Indian data. South-west monsoon period. Treshold level H_s = 1.95 m (167 observations)				
Bivariate log-normal	Log-normal	MOM/MAX	Log-normal[1]	MOM/MAX
T_z conditional on Hs	Gumbel	MOM/MAX/(N)Lin LS	Log-normal[1]	MOM/MAX
T_z conditional on Hs	Weibull	MOM/MAX/(N)Lin LS	Log-normal[1]	MOM/MAX
Wave steeepness model	Weibull	MOM/MAX/(N Lin LS	Gumbel[2]	MOM/MAX
Model of Morton & Bowers	Weibull	MOM/MAX/(N)Lin LS	Log-normal[1]	MOM/MAX
Indian data. 25 hindcasted hurricanes				
Bivariate log-normal	Log-normal	MOM/ MAX	Log-normal[1]	MOM/ MAX
Wave steeepness model	Weibull	MOM/MAX/(N) Lin LS	Gumbel[2]	MOM/ MAX
Model of Morton & Bowers	Weibull	MOM/MAX/(N) Lin LS	Log-normal[1]	MOM/ MAX

as a result of the inhomogenity of the data, it appeared to be difficult to obtain a close fit to the data. In the case of the North Sea data, an attempt to improve the homogeneity of the data has been made by censoring the data of swell. After the censoring of swell, it was found that the remaining data (wind waves) could much better be described by the statistical models. Though, an even better fit would probably have been reached, if the remaining data set would have been split up in two sub sets: the first subset containing waves generated by South Westerly storms, the second subset containing waves generated by North Westerly storms. Such an approach might be interesting for further investigations.

Furthermore, the fit of the models to the data appeared to be sensitive to the threshold level selected. In general, it seemed to improve as the threshold level was raised, especially in the extreme region. When a relatively low threshold level is used, the data set not only includes extreme observations, but also measurements of daily waves. As a result, the models tended to overpredict the extreme sea states. By raising the threshold level, i.e. censoring the daily wave data, the influence of the extreme observations increases, which leads to an improved modelling of the extremes.

4.2. Marginal analysis

It appeared that, in general, the quality of fit of the bivariate models to the data strongly depends on the fit of the corresponding marginal distributions (which seems to be obvious). In case that the marginal distributions poorly fit to the data, the performance of the bivariate models is also poor.

Fig. 1a–c show a selection of the marginal analysis of the North Sea data. The data shown have been selected with a threshold level of $H_s = 4.50$ m (59 observations).

In the case of the marginal analysis of the significant wave height, The Weibull and Exponential distribution provided the best results. As can be seen in the figures, the the Gumbel and log-normal distribution tended to under predict the extreme sea states, whereas the Frechet distribution appeared to over predict the extreme storm events. The bad performance of the Gumbel distribution is quite unusual, in most other case studies (see for instance Mathiesen et al. [3]) the distribution closely fits to the extreme data. However, the poor fit of the log-normal distribution to the significant wave height data have been recognised earlier (Mathiesen et al. [3], Fang and Hogben [2]). Starting from the extreme value theory, the Weibull, Gumbel, and Frechet distributions should provide the best results, since they are the limit distributions in extreme value theory.

With regard to the wave period (zero-upcrossing or peak period), the marginal log-normal distribution gave the closest fit to the data. For the marginal description of the wave steepness, the Weibull distribution is preferred. Due to the inclusion of the shape parameter, this distribution could be fitted relatively close to the steepness data.

4.3. Bivariate analysis

It appeared to be difficult to judge the goodness-of-fit of the bivariate models. In the literature, for marginal analysis, a lot of goodness-of-fit tests are available. However, for bivariate analysis, the availability is much lower. In the present study, the models have been judged visually, taking the physical limitations of waves into account. It seems to be obvious that the models should not predict waves, which cannot occur. To be able to check the models, the minimum and maximum steepness of waves should be considered. In general, the steepness of *extreme* waves varies between 5 and 8% (starting from a wave steepness defined on basis of the zero-upcrossing period T_z).

As mentioned by Tucker [9], it is nearly always found, that the area containing the observations, is bounded on the upper side by a line of constant wave steepness. Further Tucker recognised that the range of periods narrows at the higher values of H_s. It seems to be interesting to study whether the models do take these physical features of waves into account.

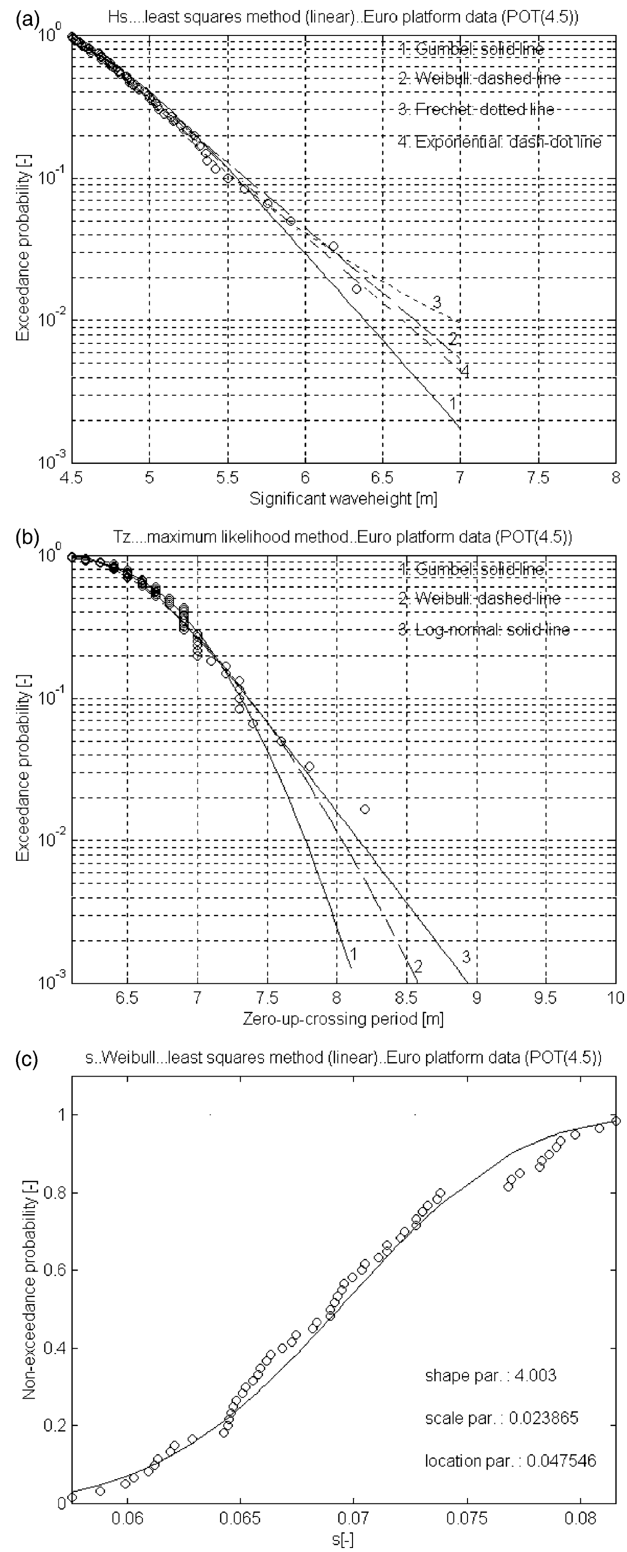

Fig. 1. (a–c) Selection of the marginal analysis of the North Sea data above $H_s = 4.50$ m.

The bivariate log-normal distribution tended to under predict the upper sea states. As mentioned above, this is probably caused by the fact that the marginal log-normal distribution of the significant wave height tended to under predict the extreme region.

The bivariate log-normal model with correction for skewness has previously been proposed by Fang and Hogben [2] as a modified version of the bivariate log-normal model. In the present study, the model appeared to be almost identical to the 'regular' bivariate log-normal method; no improvements were found in the fit of the extreme sea states.

The method based on a conditional distribution for the wave period appeared to be only suitable when large data sets are used. As described previously, in this method, empirical regression functions describe the parameters of the conditional distribution of the wave period as a function of the significant wave height. To model the regression functions, it is necessary to split up the wave period data set in a number of subsets, and to estimate for each subset the parameters of the corresponding conditional distribution. It is obvious that this method can only be followed when a large data set (approximately 100 data points or more) is available. The problem is however that, in case of an extreme wave analysis, the available number of storm observations is usually relatively small.

The method of Morton and Bowers and the bivariate wave steepness model provided the best results. Figs. 2a,b and 3a,b show the fit of these models to the North Sea data above the threshold levels $H_s = 4.50$ and 5 m. The contour lines shown represent points of equal probability of occurrence, see Table 2. Further the figure shows lines representing 50-year return period values of significant wave height and zero-upcrossing period. Line 1 corresponds with the model based on a maximum likelihood fit of the marginals, line 2 and 3 with the models based on a linear and non-linear fit of the marginals, respectively.

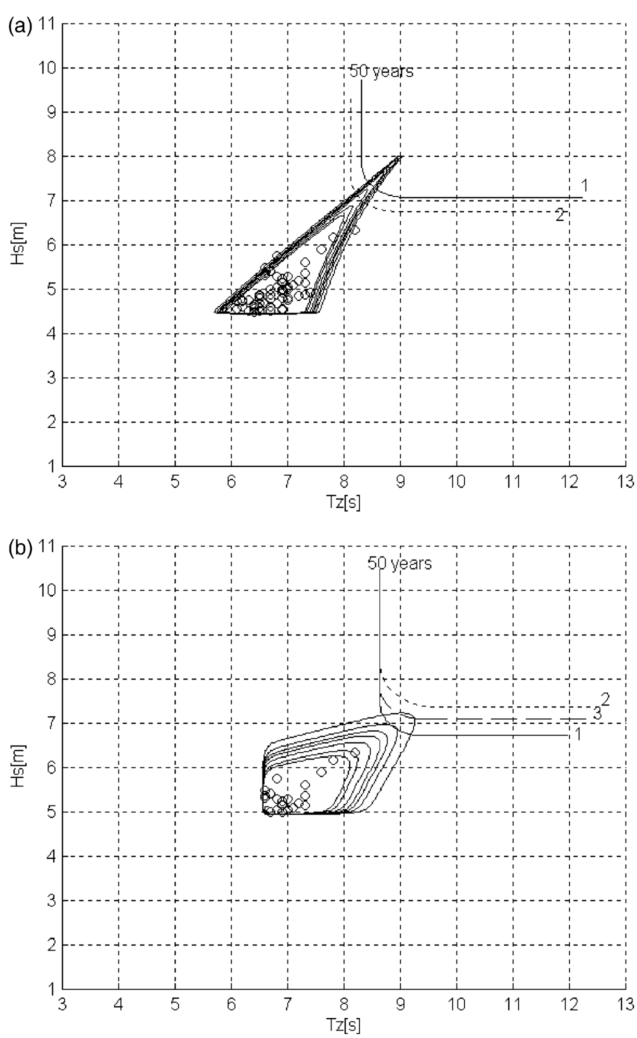

Fig. 2. (a) and (b) Fit of the bivariate model of Morton and Bowers to the North Sea data above $H_s = 4.50$ and 5.00 m. The bivariate model fitted to the data above 4.5 m is based on a Weibull and a log-normal distribution for the marginal description of H_s and T_z, respectively. In case of the bivariate model fitted to the data above 5 m both marginals are described by a Weibull distribution.

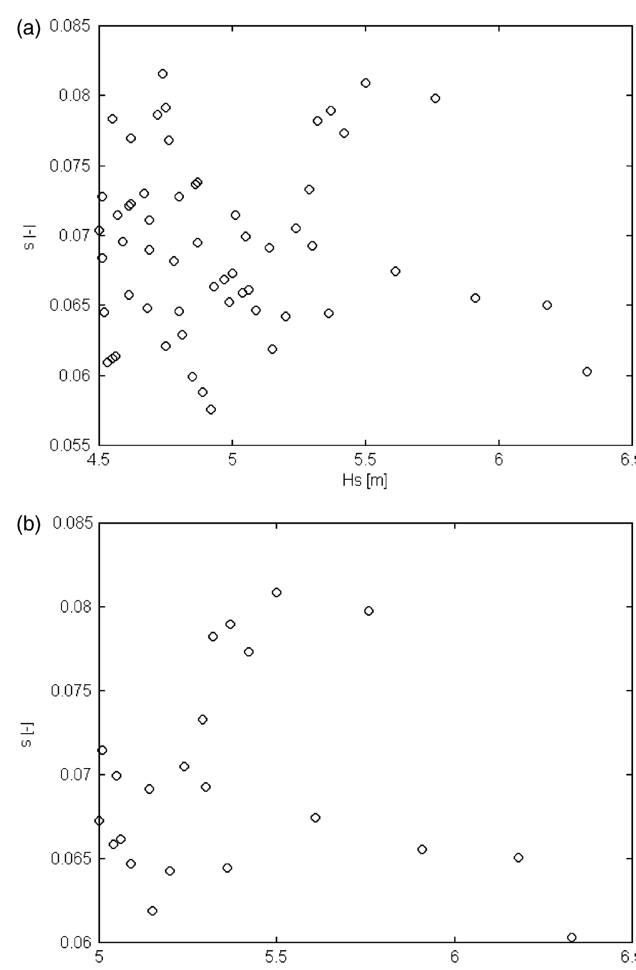

Fig. 3. (a) and (b) The wave steepness data of the North Sea (above $H_s = 4.5$ and 5.0 m) as a function of the corresponding significant wave height data.

Table 2
Definition of the contour lines shown in the illustrations of the bivariate models (Figs. 2a,b and 4a,b)

Levels	Value of probability density (1/s m)
1 (outer line)	0.005
2	0.010
3	0.015
4	0.020
5	0.030
6	0.050
7 (inner line)	0.070

4.4. Return period curves

In the case of bivariate models, the return period R, can be described as

$$R = 1/[kF(H_s T_p)] \tag{9}$$

in which

R the return period [years]
k the mean number of extreme wave events per year (based on the data set)
$F(H_s\ T_p)$ the bivariate exceedance probability function of H_s and T_p.

From Eq. (9) it follows that

$$F(H_s T_p) = 1/[kR] \tag{10}$$

Example. Mean number of storm events per year = 5. Starting from a return period of 50 years, the corresponding 50-years return curve represents values of H_s and T_p with a constant exceedance probability of $1/(5\times 50) = 4\times 10^{-3}$.

As can be seen in Fig. 2a and b, the shape of the contour lines of the bivariate model of Morton and Bowers is almost triangular. Apparently, the model predicts that the range of periods narrows as the conditions get more extreme, which seems to correspond with the earlier mentioned observations of Tucker [9]. However, it should be noted that the extreme area of the model narrows relatively strongly, especially in the case when the model has been fitted to the observations above $H_s = 4.50$ m.

A more realistic approach of the prediction of extreme sea states is probably be given by the bivariate wave steepness model. As can be seen in Fig. 4a and b, the shape of the contour lines of this model seems to correspond with the upper part of an ellipse. Further it can be shown that the contour lines are bounded on the upper side by a line representing the minimum wave steepness value of the data. Apparently, due to the inclusion of the wave steepness distribution, the model seems to take the physical features of waves into account.

The model of Morton and Bowers might be too complicated for use in daily practice. It is most likely that civil engineers will consider this sophisticated mathematical model as a black box.

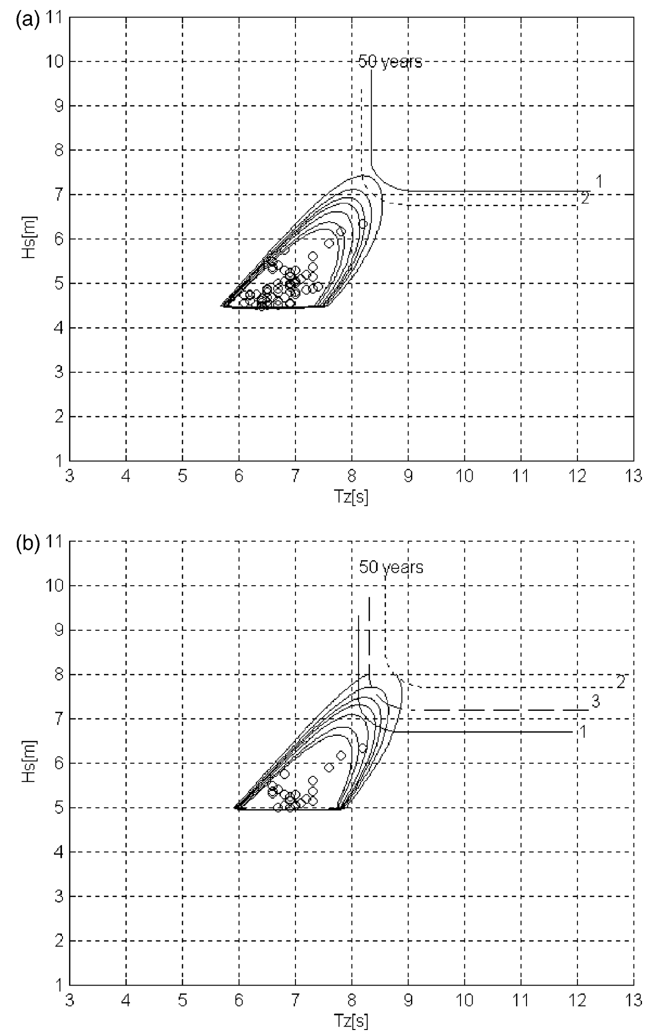

Fig. 4. (a) and (b) Fit of the wave steepness model to the North Sea data above $H_s = 4.50$ and 5.00 m. In the bivariate models, the significant wave height and the wave steepness are both described by a Weibull distribution.

The wave steepness model is based on the assumption that significant wave height and wave steepness are independent. Fig. 3a and b show that this approach seems to be justified (starting from visual judgement, no correlation test has been performed).

Mitsuyasu [7] proposed a dependence model between wave height and wave period that shows decreasing wave steepness as conditions get more extreme. This is not supported by the data in this study. Further research is needed to explain this discrepancy for which Webbers et al. [8] show some preliminary results.

5. Conclusions

The aim of this paper was to find a model to be used for the bivariate description of extreme wave heights and wave periods representing relatively deep-water sea states. To achieve this, four statistical models taken from literature and a model based on the physical behaviour of waves have been tested on wave data at two sites. The models have been compared with regard to their ability to correctly describe the data, their behaviour when used for extrapolation to extreme events, and the transparency of the methods.

In the case studies, the model based on physical relations gave the best results and is therefore recommended. It is based on the marginal distribution of significant wave height and the marginal distribution of wave steepness. In contrast to the other models, the model gave a relatively close fit to the data and included the physical limitations of waves. Furthermore the model is relatively easy to use in daily practice.

Besides the performance of the bivariate models, the study also showed that data selection forms a crucial aspect of a statistical analysis. In case of an extreme wave analysis, it is necessary to use a homogeneous data set that contains observations representing maxima of separate storm events. To obtain a homogeneous data set, it is advised to examine the data on its physical origin.

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References

[1] Ochi MK. Wave statistics for ships and structures. Proceedings of SNAME conference, New York; 1978.

[2] Fang ZS, Hogben N. Analysis and prediction of long-term probability distributions of wave height and periods, Technical Report of the National Maritime Institute, London; 1982.

[3] Mathiesen M. et al. Intercomparison of extreme wave analysis: a comparative study, Proceedings of WAVES '93, New Orleans; 1993.

[4] Haver S. Wave climate off northern Norway. Appl Ocean Res 1985;7: 85–92.

[5] Vrijling JK, Bruinsma J. Hydraulic boundary conditions. In: Paape A, Stuip J, Venis WA, editors. Proceedings on hydraulic aspects of coastal structures; developments in hydraulic engineering related to the design of the Oosterschelde storm surge barrier in the Netherlands. Delft University Press;90-6275-0540; 1980. p. 109–33.

[6] Morton ID, Bowers J. Extreme values in a multivariate offshore environments. Appl Ocean Res 1997;18:303–17.

[7] Mitsuyasu H. Observations of the directional spectrum of ocean waves using a cloverleaf buoy. J Phys Ocean 1975;5:750–60.

[8] Webbers PB, Voortman HG, van Gelder PHAJM, Vrijling JK. Multivariate statistics of hydraulic boundary conditions for the Rotterdam harbour extension. 28th International Conference on Coastal Engineering (ICCE 2002), Solving Coastal Conundrums, Cardiff, 07 July 2002–12 July 2002.

[9] Tucker MJ. Waves in ocean engineering, measurement, analysis, interpretation. London: Ellis Horwood; 1991.

[10] van Gelder PHAJM, Statistical methods for the risk-based design of civil structures, Delft University of Technology, ISBN 90-9013452-3; 1999.