THE USE OF L-KURTOSIS IN THE ESTIMATION OF EXTREME FLOODS

P.H.A.J.M. van Gelder[1] , M.D. Pandey[2], and J.K. Vrijling[1],
[1]Delft University of Technology, Subfaculty of Civil Engineering, P.O. Box 5048, NL-2600 GA Delft, The Netherlands (http://hydr.ct.tudelft.nl/wbk)
[2]University of Waterloo, Department of Civil Engineering, 200 University Avenue West, Waterloo, Ontario, Canada N2L 3G1 (http://sunburn.uwaterloo.ca/~mdpandey)

Abstract

In this paper, the use of the L-Kurtosis measure in the probability model selection for extreme river discharges is studied.

Introduction

Today flash floods are the number one weather-related killer in the United States, claiming an average of 150 lives each year, according to the National Weather Service. Flooding takes various forms: ice jams, when river ice breaks up in the spring floes can get caught on bridges or other obstacles, damming the water upstream and causing it to over flow the banks (type A in Fig. 1), river flooding, as the result of days or weeks of continuous rain rivers can overflow their banks (type B), dam bursts as the result of extreme rainfall, design defects (type C), mudflow, when loose soil on a slope become so heavily saturated by intense precipitation that a spontaneous slide results (type D). The most dangerous type of flooding is a flash flood (type E), which usually occurs within minutes or hours of a tremendous rainfall. Flash floods can sweep away everyone and everything in their path. They are so strong they can toss boulders around like pebbles, rip out trees and knock down buildings and bridges. Water may reach heights of 10 metres or more, and the rains that cause them can also trigger dangerous mud slides.

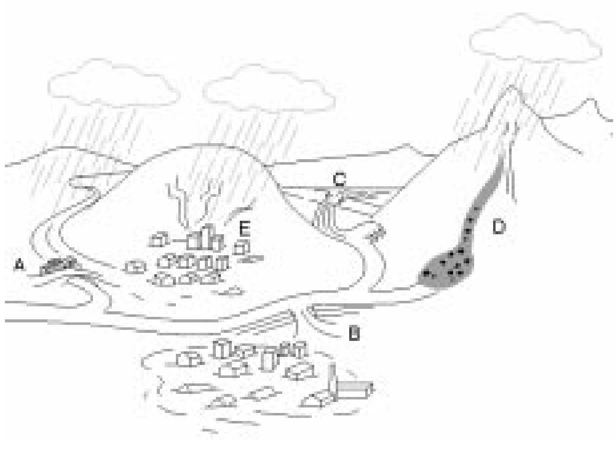

Figure 1: Various types of floods (Swiss RE, 1999).

The probability of occurence of extreme floods of each of the above given types can be modelled with PDF's (Probability Distribution Functions). The selection of a certain PDF is a difficult issue and depends on the criteria which are used to define the optimal selection. Many people have looked into this problem, of which are mentioned Yamaguchi (1997), Van Gelder et.al. (1997), and Burcharth et.al. (1994). In this paper it is proposed to use the so-called L-kurtosis measure in the selection of a PDF to describe the occurence probabilities of extreme floods. The paper is organized as follows. First a short overview is given of the theory of L-Moments and in particular the L-kurtosis. Then the results of some simulation experiments will be briefly presented. Finally a short case study will be described to find the optimal PDF for the river Meuse.

L-Moments and L-Kurtosis

L-moments are summary statistics for probability distributions and data samples. They are analogous to ordinary moments -- they provide measures of location, dispersion, skewness, kurtosis, and other aspects of the shape of probability distributions or data samples -- but are computed from linear combinations of the ordered data values (hence the prefix L). Hosking and Wallis (1997) give an excellent overview on the whole theory of L-Moments.

Kurtosis is based on the size of a distribution's tails. Distributions with relatively large tails are called "leptokurtic"; those with small tails are called "platykurtic." A distribution with the same kurtosis as the normal distribution is called "mesokurtic." The following formula can be used to calculate kurtosis:

$$\text{kurtosis} = \frac{\Sigma(X-\mu)^4}{\sigma^4} - 3$$

where σ is the standard deviation. The kurtosis of a normal distribution is 0. The following two distributions have the same variance, approximately the same skew, but differ markedly in kurtosis.

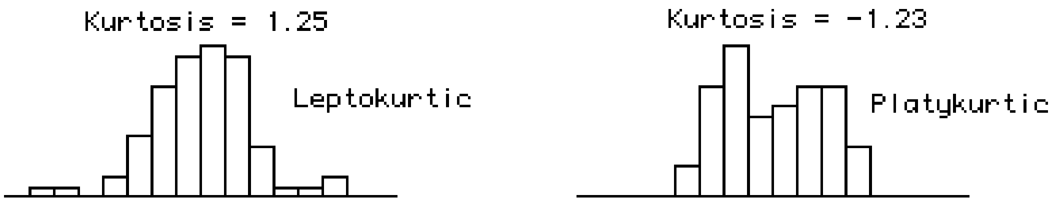

Figure 2: Examples of different kurtosis values.

Note that in the above formula to calculate the kurtosis the observations are powered to the order 4. Small deviations in the observations cause a large error in the kurtosis. However, this is not the case if the L-kurtosis is considered. The L-kurtosis is calculated with formulae of the type:

$$b_r = n^{-1} \sum_{j=r+1}^{n} \frac{(j-1)(j-2)\ldots(j-r)}{(n-1)(n-2)\ldots(n-r)} X_j$$

in which X_j are the ordered observations and only linear combinations are considered.

Simulation Experiments

In Pandey et.al. (1999), various Monte Carlo simulations have been performed. They considered samples drawn from a known distribution (e.g., kappa distribution), and a set of several candidate distributions were fitted to the data based on matching L-moments. The PDF closest to the parent can be identified by computing divergence or probabilistic distance of each of the candidate distributions from the parent. Obviously, the distribution with the least divergence would be closest to the parent PDF. With this line of thinking, the experiments were designed to evaluate systematically the effectiveness of the L-Kurtosis criterion (the minimum difference of L-kurtosis of fitted PDFs from that of the sample) against the divergence criterion for estimating extreme quantiles. Results of the simulation experiments indicated that quantile estimates obtained from the L-kurtosis criterion are in fairly close agreement with those obtained from the minimum divergence criterion. In this respect, it can be concluded that L-kurtosis is a reliable indicator of distribution shape and its use in quantile estimation is very effective. Remarkable simplicity of computation makes the L-Kurtosis criterion an attractive tool for distribution fitting.

Case Study River Meuse

In this case study the frequency distribution of the river Meuse (experiencing extreme high water in the winters of 1993 and 1995) is investigated in detail. A data set is given of size 86 consisting of annual maxima of river discharges. The analysis results are as follows:

	L-Moments
1	0.1000E+01
2	0.1950E+00
3	0.9681E-01
4	0.1643E+00

For six distribution functions (Generalized Pareto, Generalized Extreme Value, Pearson type III, Generalized Log-Normal, Gumbel and Generalized Logistic), the following L-moments are obtained:

	L-Moments			
	1	2	3	4
1 GPA	0.1000E+01	0.1950E+00	0.9681E-01	0.2819E-01
2 GEV	0.1000E+01	0.1950E+00	0.9681E-01	0.1260E+00
3 PE3	0.1000E+01	0.1950E+00	0.9681E-01	0.1254E+00
4 GNO	0.1000E+01	0.1950E+00	0.9681E-01	0.1300E+00
5 GUM	0.1000E+01	0.1950E+00	0.1699E+00	0.1504E+00
6 GLO	0.1000E+01	0.1950E+00	0.9681E-01	0.1745E+00

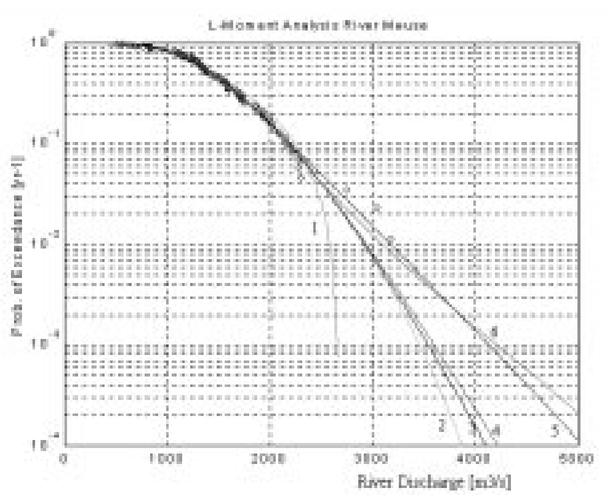

Figure 3: Case study results

Note that the GLO (line 6) with L-Kurtosis of 0.1745 gives the best fit (in L-Kurtosis sense) to the sample data. The 1/1250 years quantile is given by 4200m^3/s.

Conclusions

The occurence frequency models of extreme floods has been a subject of discussion during the last decades and will probably still be in the new millenium. The purpose of this paper was to introduce a new PDF-selection criterion, based on the L-Kurtosis measure. This criterion has been evaluated with a large Monte Carlo simulation scheme in a previous paper (Pandey et.al., 1999) and was applied in this paper on the extreme river flows of the River Meuse.

Acknowledgements

The authors would like to acknowledge Mr H Chbab from the Dutch Ministry of Public Works, Transport and Water Management for providing the Meuse data.

References

Swiss RE, (1999). Floods: an insurable risk, *Swiss RE – publication.*

M.D. Pandey, P.H.A.J.M. van Gelder and J.K. Vrijling, (1999). Application of Information Theory to the Assessment of Effectiveness of L-Kurtosis Criterion for Quantile Estimation, Submitted to*: Journal of Hydrologic Engineering*, ASCE.

Yamaguchi, M. (1997). Intercomparison of parameter estimation methods in extremal wave analysis. In B.L. Edge, editor, *25th International Conference on Coastal Engineering, Orlando, Florida, U.S.A., 1996*, pages 900-913, New York: American Society of Civil Engineers (ASCE).

Burcharth, H.F., and Liu, Z. (1994). On the extreme wave height analysis, In Port and Harbour Research Institute, Ministry of Transport, editor, *International Conference on Hydro-Technical Engineering for Port and Harbor Construction (Hydro-Port), Yokosuka, Japan, 1994*, pages 123-142, Yokosuka: Coastal Development Institute of Technology.

Hosking, J.R.M. and Wallis, J.R. (1997). *Regional Frequency Analysis: An Approach based on L-Moments*. Cambridge University Press, Cambridge, UK.

P.H.A.J.M. van Gelder and J.K. Vrijling, (1997). A comparative study of different parameter estimation methods for statistical distribution functions in civil engineering applications, *Proceedings of ICOSSAR'97*, pp.665-668, 7th International Conference on Structural Safety and Reliability, Kyoto.