Uncertainty Analysis of Water Levels on Lake IJssel in the Netherlands

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ABSTRACT: In this paper, the probability of inundation of the low-lying polders protected by dikes from the Dutch Lake IJssel (1200 km^2) is studied on the basis of a physical model. In particular the uncertainty in the probability of overtopping is analysed with a first order reliability methods (FORM). The FORM-calculations showed that the uncertainty in the wind speed and the uncertainty in the water level contributed most in the total uncertainty of the probability of overtopping of a dike with given height. A comparison has been made between the results of the FORM-calculations and the calculations where only intrinsic uncertainty has been assumed in the basic variables.

1 INTRODUCTION

Lake IJssel is situated in the northern part of the Netherlands (Fig. 1). It has an area of approximately 1200km^2. The lake is surrounded by dikes in order to protect the low-lying polders from flooding. The dikes are designed in a probabilistic manner (see for instance CUR, 1990, and Van Gelder et.al., 1995). The required safety against inundation of the polders is 1/4000 yr^{-1}.

In Westphal et.al. (1997), a physical model has been developed for the water levels of Lake IJssel. In this paper, the probability of inundation of the low-lying polders behind the Lake IJssel dikes will be studied on the basis of the physical model of Westphal et.al. (1997). In particular the uncertainty in the probability of inundation will be analysed with FORM. The following variables will be included in the FORM-analysis:

- Water level
- Wind speed
- Model uncertainties in:
  - Water level
  - Wind speed
  - Wind surge
  - Wave height
  - Wave steepness
  - Wave run-up
  - Lake oscillations

The outline of the paper is as follows. First the physical and reliability models will be briefly described. Some aspects of uncertainty modelling and the applied uncertanties in the physical model will be highlighted. The results of the FORM calculations and conclusions will end the paper.

Figure 1. Lake IJssel in frozen condition and the two locations of interest Enkhuizen and Rotterdamsche Hoek (Source: Remote sensing and Fotogrammetry, Delft).

2 PHYSICAL AND RELIABILITY MODEL

The physical model is described in Westphal et.al. (1997) and is based on WAQUA (a two-dimensional water flow model) and HISWA (a wave model).

In this paper the following reliability function will be used:

$$Z = K - M - \Delta - z_{\theta B}$$

in which:

Z: reliability function [m]
K: the crest height [m]
M: lake level (averaged over four locations on Lake IJssel) [m]
Δ: wind surge [m]
$z_{2\%}$: 2% wave run-up [m]

The wind surge is based on:

$$\Delta = \alpha \cdot \frac{W^2 \cdot F}{2 \cdot g \cdot D}$$

in which:

α: constant; $3.6.10^{-6}$ [-]
W: wind speed [m/s]
F: fetch length [m]
g: gravitation constant; 9.8 [m/s^2]
D: water depth [m]

The wave run-up is modelled with the Van der Meer formula (1997).

3 UNCERTAINTIES

Suppose that the true state of a variable is X. Prediction of X may be modeled by X*. As X* is a model of the variable X, imperfections may be expected; the resulting predictions will therefore contain errors and a multiplicative correction factor N may be applied. Consequentely, the true state of the variable may be represented as stated by Ang, (1973).

X = NX*.

If the state of the variable is random, the model X* is also a random variable. The intrinsic variability is described by the coefficient of variation (c.o.v.) of X*, given by σ(x*)/μ(x*).The necessary correction N may also be considered as a random variable, of which the mean value μ(N) represents the mean correction for systematic error in the predicted mean value, whereas the c.o.v. of N, given by σ(N)/μ(N), represents the random error in the predicted mean value.

It is reasonable to assume that N and X* are statistically independent. Therefore we can write the mean value of X as:

μ(X)=μ(N)μ(x*)

The total uncertainty in the prediction of X can be written as:

$Cov(X) = sqrt(Cov^2(N) + Cov^2(x^*))$

Beyond a multiplicative uncertainty modelling, also an additive model can be used:

X=X*+A

The necessary correction A may also be considered a random variable, of which the mean value μ(A) represents the mean correction for systematic error in the predicted mean value, whereas the c.o.v. of A, given by σ(A)/μ(A), represents the random error in the predicted mean value. As in the multiplicative case, it is reasonable to assume that A and X* are statistically independent. Therefore we can write the mean value of X as:

μ(X)=μ(A)+μ(X*)

The total uncertainty in the prediction of X is:

Var(X) = Var(A) + Var(X*)

4 UNCERTAINTIES IN THE PHYSICAL MODEL

4.1 *Intrinsic uncertainty in the lake level*

The annual maxima of the lake level can satisfactorily be modelled by a Gumbel distribution:

$$F_{\Phi_{1\ year}}(Mp) = e^{-\Omega^{-\frac{Mp - A_{1\ year}}{B_{1\ yearr}}}}$$

in which:

M_{1year}: the annual maximum of the lake level [m]
Mp: lake level [m]
A_{1year}: 0.02 [m]
B_{1year}: 0.11 [m]

4.2 *Intrinsic uncertainty in the wind speed*

In The Netherlands the hour-averaged wind speed from a direction sector ϕ can adequately be modelled by a 2-parameter Weibull distribution (Rijkoort, 1983, and Wieringa and Rijkoort, 1983):

$$F_{E}(w \mid \phi) = 1 - e^{-\left(\frac{}{\Upsilon_{\phi}}\right)^{k}}$$

where:

w: the wind speed [m/s]
a_{ϕ}: a constant dependent of the wind direction

4.3 *Statistical uncertainties*

The following statistical uncertainties are modelled by normal distributions:

Uncertainty in Lake Level *(add)*	Mean μ	Standard Deviation σ
fMp	0.0 m	0.1 m

Uncertainty in Wind *(add)*	Mean μ	Standard Deviation σ
fW	0.0 m/s	3 m/s

Uncertainty in Wind *(mult)*	Mean μ	Standard Deviation σ
fW	1	0.1

4.4 *Model uncertainties*

The following model uncertainties are modelled by normal distributions:

Uncertainty in surge *(add)*	Mean μ	Standard Deviation σ
fOpw	0.0 m	0.1 m

Uncertainty in Oscillations *(add)*	Mean μ	Standard Deviation σ
fOsc	0.1 m	0.05 m

Uncertainty in Significant Wave Height *(add)*	Mean μ	Standard Deviation σ
fHs	0.0 m	0.07 m

Uncertainty in Wave Steepness *(add)*	Mean μ	Standard Deviation σ
fsop	0	0.005

Uncertainty in Wave Run-up *(mult)*	Mean μ	Standard Deviation σ
fOplm	1	0.085

5 FORM CALCULATIONS

The probability of Z < 0 (overtopping of the Lake IJssel dikes) is calculated by a first order reliability method (FORM). A good overview of FORM is given in Thoft-Christensen and Baker (1982). Given the additive uncertainties in the physical model of section 3, the FORM calculations are presented for various crest heights for the location of Rotterdamsche Hoek in table 1 and for the location of Enkhuizen in table 2. The notation of the 29 variables is summarized in the list of symbols (attached to the last page of this paper).

Rotterdamsche Hoek is situated on the East-side of the lake, "facing" the strong winds from the West (most frequent direction in The Netherlands). Enkhuizen is situated on the West-side of the lake, and therefore more or less protected against the strong winds from the West. This observation also follows from the tables 1 and 2, in which it is seen that the crest heights at Rotterdamsche Hoek are appr. 1 m higher than the crest heights in Enkhuizen, given a fixed probability of overtopping.

In the case of multiplicative uncertainties, FORM calculations have been performed as well. The results were very similar to the results for the additive uncertainties, and therefore not shown in this paper. The reason is that the design wind speed is in the order of 30 m/s for which the multiplicative uncertainty is 30 x 0.1 = 3 m/s (the same as the assumed additive uncertainty).

The results of the FORM calculations can also be presented graphically (Figs. 2 and 3 for Rotterdamsche Hoek and Enkhuizen resp.). Notice the differences between the required crest heights for the three cases: intrinsic uncertainty, instrinsic + statistical uncertainty, and intrinsic + statistical + model uncertainty. These differences can be up to 1 metre. Also notice, that there is not much difference in the results for the choice of the type of uncertainty modelling (additive or multiplicative), for the reason as explained earlier.

Finally, the contributions of the various uncertainties in the overall uncertainty in the probability of overtopping are summarized in tables 3 and 4 (Rotterdamsche Hoek and Enkhuizen resp.). Notice that the ratio $\alpha^2_{fMp} / \alpha^2_{fW}$ is larger for Enkuizen, than for Rotterdamsche Hoek. From this, it follows that Enkhuizen is more "lake level dominant" and Rotterdamsche Hoek is more "wind dominant’.

6 CONCLUSIONS

Beyond intrinsic uncertainty, also statistical uncertainties and model uncertainties should be taken into account and are important quantities in the design philosophy of water defences. In addition to intrinsic uncertainty, the influences of the statistical and model uncertainties have been studied with FORM (tables 2 and 3), and it was observed that these were not negligable. Especially the uncertainty in the wind has a large contribution to the overall uncertainty in the probability of overtopping (about 20%).

Note that in this paper the probabilities of overtopping have been calculated. However, overtopping does not mean failure of the dikes. In order to determine the probability of inundation, the probability of overtopping should be multiplied by a so-called transition probability which describes the probability that a dike fails under the condition of overtopping.

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LIST OF SYMBOLS

K	=	crest height (m) (deterministic)
M_p	=	lake level (m)
$\alpha_{M_p}{}^2$	=	contribution of the lake level in the overall uncertainty
fMp	=	random variable of the uncertainty in the lake level
α^2_{fMp}	=	contribution of the uncertainty in the lake level in the overall uncertainty
W	=	wind speed (m/s)
$\alpha_W{}^2$	=	contribution of the wind speed in the overall uncertainty
fW	=	random variable of the uncertainty in the wind speed
α^2_{fW}	=	contribution of the uncertainty in the wind speed in the overall uncertainty
Δ	=	wind surge (m)
fOpw	=	random variable of the model uncertainty in the storm surge
α^2_{fOpw}	=	contribution of the model uncertainty in the storm surge in the overall uncertainty
fOsc	=	random variable of the model uncertainty in the lake level increase due to oscillations
α^2_{fOsc}	=	contribution of the model uncertainty in oscillations in the overall uncertainty
H_s	=	significant wave height (m)
fHs	=	random variable of the model uncertainty in H_s (m)
α^2_{fHs}	=	contribution of the model uncertainty in H_s in the overall uncertainty
$z_{2\%}$	=	2%- wave runup (m)
$fz_{2\%}$	=	random variable of the model uncertainty in the 2%- wave runup (m)
$\alpha^2_{fz2\%}$	=	contribution of the model uncertainty in the 2%- wave runup in the overall uncertainty
$s_{op} * 100$	=	wave steepness × 100
fs_{op}	=	model uncertainty in the wave steepness
$\alpha^2_{fs_{op}}$	=	contribution of the model uncertainty in the wave steepness in the overall uncertainty
β	=	reliability index
$P_{f\|\phi}$	=	probability of overtopping given wind from direction sector ϕ
P_f	=	probability of overtopping
T_p	=	peak wave period (s)
ξ_{op}	=	surf similarity parameter

Table 1. FORM results of Rotterdamsche Hoek.

K	Mp	α^2_{Mp}	fMp	α^2_{fMp}	W	α^2_W	fW	α^2_{fW}	Δ	fOpw	α^2_{fOpw}	fOsc	α^2_{fOsc}
1	2	3	4	5	6	7	8	9	10	11	12	13	14
4.1	-0.20	0.09	0.04	0.04	27.75	0.31	2.97	0.24	1.22	0.05	0.05	0.11	0.01
4.3	-0.19	0.08	0.04	0.03	28.56	0.35	3.39	0.24	1.32	0.05	0.05	0.11	0.01
4.5	-0.18	0.07	0.04	0.03	29.44	0.38	3.76	0.24	1.42	0.05	0.04	0.11	0.01
4.7	-0.18	0.06	0.04	0.02	30.38	0.42	4.10	0.24	1.53	0.05	0.04	0.11	0.01
4.9	-0.18	0.05	0.04	0.02	31.34	0.45	4.40	0.23	1.65	0.05	0.05	0.11	0.01

K	H_s	fHs	α^2_{fHs}	$z_{2\%}$	$fz_{2\%}$	$\alpha^2_{fz_{2\%}}$	s_{op} * 100	$f s_{op}$	$\alpha^2_{fs_{op}}$	β	$P_{f\|\phi}$	$P_f = P_{f\|\phi} * 0.101$	T_p	ξ_{op}
15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
4.1	2.12	0.00	0.00	3.04	1.06	0.13	3.18	0.00	0.13	2.00	$2.26*10^{-2}$	$2.291*10^{-3}$	6.54	1.00
4.3	2.16	0.00	0.00	3.13	1.07	0.12	3.15	0.00	0.12	2.29	$1.10*10^{-2}$	$1.111*10^{-3}$	6.63	1.00
4.5	2.20	0.00	0.00	3.22	1.07	0.11	3.12	0.00	0.12	2.56	$5.19*10^{-3}$	$5.245*10^{-4}$	6.72	1.01
4.7	2.24	0.00	0.00	3.30	1.08	0.11	3.10	0.00	0.11	2.82	$2.40*10^{-3}$	$2.431*10^{-4}$	6.81	1.02
4.9	2.28	0.00	0.00	3.39	1.08	0.10	3.07	0.00	0.11	3.06	$1.10*10^{-3}$	$1.113*10^{-4}$	6.89	1.02

Table 2. FORM results of Enkhuizen.

K	Mp	α^2_{Mp}	fMp	α^2_{fMp}	W	α^2_W	fW	α^2_{fW}	Δ	fOpw	α^2_{fOpw}	fOsc	α^2_{fOsc}
1	2	3	4	5	6	7	8	9	10	11	12	13	14
3.1	-0.19	0.12	0.04	0.05	16.38	0.23	1.07	0.15	0.33	0.04	0.05	0.11	0.01
3.3	-0.17	0.13	0.05	0.05	16.68	0.23	1.08	0.14	0.35	0.05	0.05	0.11	0.01
3.5	-0.14	0.14	0.05	0.04	16.98	0.23	1.09	0.13	0.37	0.05	0.05	0.11	0.01
3.7	-0.12	0.14	0.06	0.04	17.27	0.23	1.10	0.12	0.38	0.06	0.04	0.11	0.01
3.9	-0.09	0.15	0.06	0.04	17.54	0.22	1.10	0.11	0.40	0.06	0.04	0.11	0.01

K	H_s	fHs	α^2_{fHs}	$z_{2\%}$	$fz_{2\%}$	$\alpha^2_{fz_{2\%}}$	s_{op} * 100	$f s_{op}$	$\alpha^2_{fs_{op}}$	β	$P_{f\|\phi}$	$P_f = P_{f\|\phi} * 0.081$	T_p	ξ_{op}
15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
3.1	1.53	0.03	0.06	2.92	1.06	0.15	2.86	0.00	0.18	1.84	$3.323*10^{-2}$	$2.691*10^{-3}$	5.85	1.44
3.3	1.58	0.04	0.06	3.07	1.07	0.15	2.80	0.00	0.19	2.16	$1.540*10^{-2}$	$1.248*10^{-3}$	6.00	1.45
3.5	1.62	0.04	0.05	3.33	1.08	0.15	2.74	0.00	0.20	2.47	$6.478*10^{-3}$	$5.466*10^{-4}$	6.16	1.47
3.7	1.66	0.04	0.05	3.38	1.09	0.15	2.67	0.00	0.21	2.77	$2.818*10^{-3}$	$2.283*10^{-4}$	6.31	1.49
3.9	1.70	0.05	0.05	3.53	1.10	0.15	2.60	0.00	0.22	3.05	$1.130*10^{-3}$	$9.155*10^{-5}$	6.47	1.51

Table 3. Contributions of the uncertainties in the variance of $P_f \approx 1/4000$ 1/yr at Rotterdamsche Hoek.

Uncertainty: (percentages in variance)		Intrinsic	Intrinsic + statististical *additive*		Intrinsic + statististical *multiplicative*		Intrinsic + statististical + model *additive*		Intrinsic + statististical + model *multiplicative*	
Variable	Symbol									
Lake Level	Mp	13%	7%	69%	4.5%	71%	6%	48%	5%	55%
Wind	W	87%	62%		66.5%		42%		50%	
Uncertainty Lake Level	fMp	- -	2%	31%	2%	29%	2%	26%	2%	24%
Uncertainty Wind	fW	- -	29%		27%		24%		22%	
Other	fOpw, fOsc, fHs, $f_{z2\%}$, fs_{op}	- -	- -	- -	- -	- -	26%	26%	21%	21%
Total:		100%	100%	100%	100%	100%	100%	100%	100%	100%

Table 4. Contributions of the uncertainties in the variance of $P_f \approx 1/4000$ 1/yr at Enkhuizen.

Uncertainty: (percentages in variance)		Intrinsic	Intrinsic + statististical *additive*		Intrinsic + statististical *multiplicative*		Intrinsic + statististical + model *additive*		Intrinsic + statististical + model *multiplicative*	
Variable	Symbol									
Lake Level	Mp	55%	43%	60%	45%	78%	13%	25%	14%	36%
Wind	W	45%	17%		33%		12%		22%	
Uncertainty Lake Level	fMp	- -	6%	40%	6%	22%	4%	31%	4%	16%
Uncertainty Wind	fW	- -	34%		16%		27%		12%	
Other	fOpw, fOsc, fHs, $f_{z2\%}$, fs_{op}	- -	- -	- -	- -	- -	44%	44%	48%	48%
Total:		100%	100%	100%	100%	100%	100%	100%	100%	100%

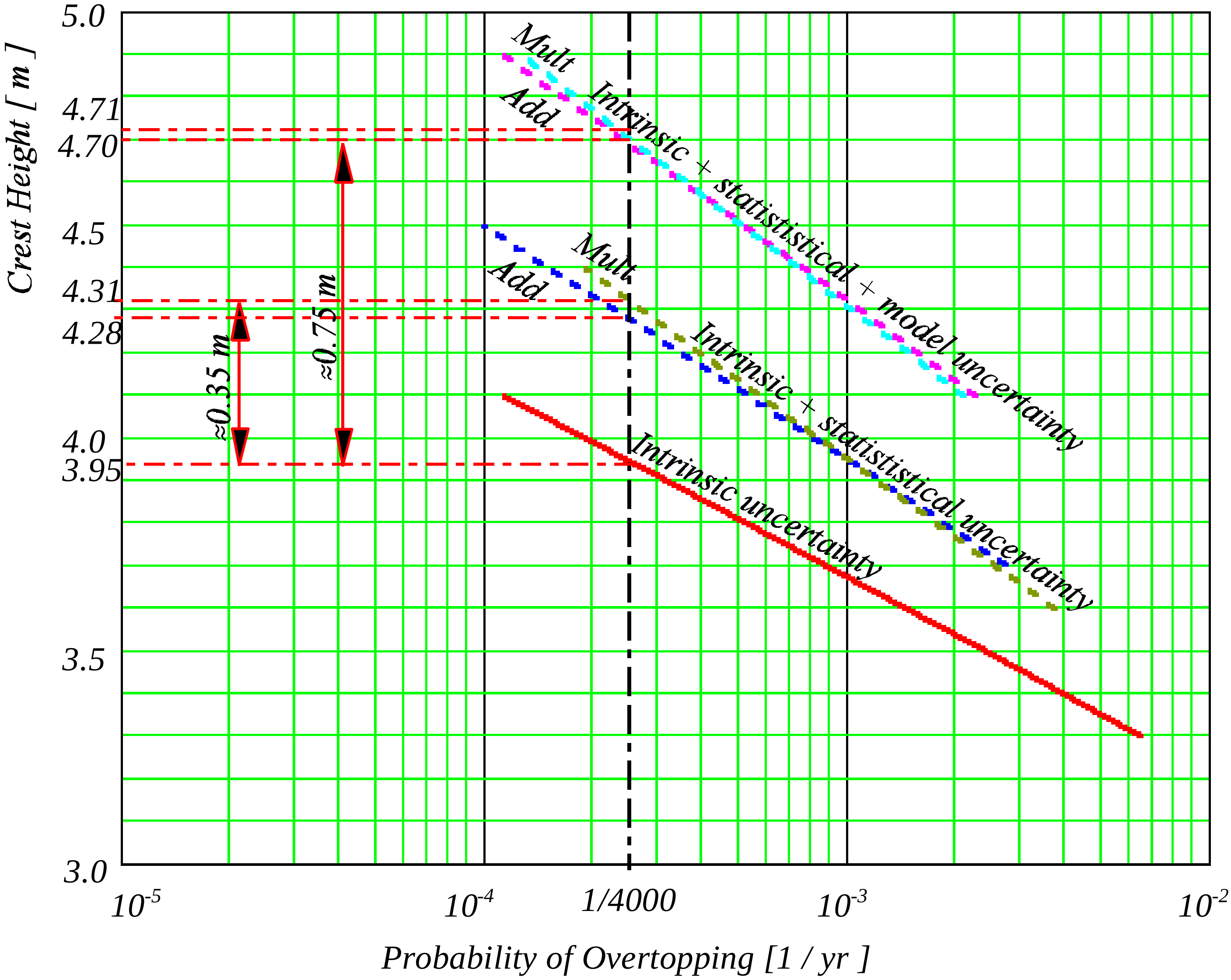

Figure 2. Crest height as a function of the probability of overtopping for Rotterdamsche Hoek.

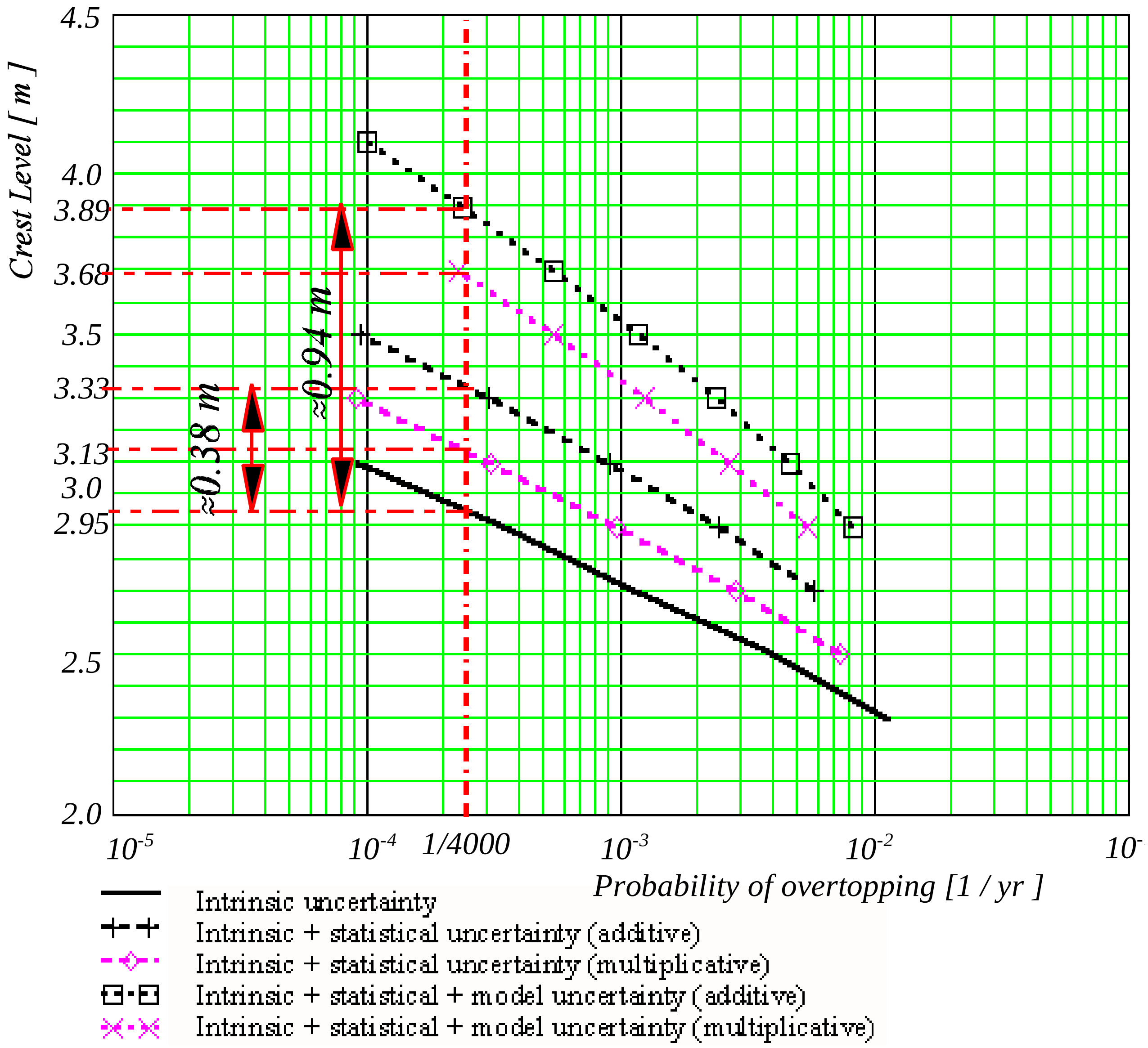

Figure 3. Crest height as a function of the probability of overtopping for Enkhuizen.