Estimation techniques for inhomogeneous hydraulic boundary conditions along coast lines with a case study to the Dutch Petten Sea Dike

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ABSTRACT: In this paper, estimation techniques to determine the hydraulic boundary conditions along coast lines, such as water levels, wave heights, wave periods and wind set-up, will be described and applied on a case study of the Petten Sea Dike along the coast of the Netherlands. Estimation based on regional frequency techniques, usually applied in hydrology, appears also to perform well within the area of coastal engineering.

1 INTRODUCTION

The Netherlands is a low-lying country which has to protect itself against flooding from the sea and its rivers. Reliable flood defenses are essential for the safety of the country. The sea dikes are designed to withstand floods with a mean frequency of exceedance of 1/10,000 per year. This height is used to be calculated using statistics on the measured hydraulic variables (such as sea levels, wave heights, wave periods, wave set-up and wave direction). For instance, sea levels are measured along the Dutch coast since 1880. Annual maxima and peaks over threshold data are used as input data (Van Gelder, 1996). Gumbel and Exponential models are most commonly applied together with Maximum Likelihood and Least Squares parameter estimation methods.

Hydraulic boundary conditions at the toe of the water retaining structure are needed for the reliability analysis. However, the hydraulic conditions are in these regions heavily influenced by the local water depth. Therefore, it is not possible to derive reliable statistics of water levels, wave heights and wave periods directly at the structure's location. The simultaneous distribution function of these three parameters is derived on relatively deep water instead. Physical modelling is used to transform the conditions at relatively deep water to the conditions at the toe of the structure.

In order to estimate the risk that a given flood will be exceeded during the design life of a structure, flood frequency analysis is needed to relate the rarity of the flood to its magnitude. The objective of flood frequency analysis is to estimate the flood magnitude corresponding to any required return period of occurrence through the use of probability distributions. However, estimating the frequencies of extreme environmental events such as floods is difficult because extreme events are by definition rare and the relevant data record is often short. Regional Frequency Analysis (RFA) developed by Hosking and Wallis (1997) can resolve this problem by trading space for time. It does so by using data from several sites, which are judged to have frequency distributions similar to the site of interest, in estimating event frequencies at that site.

The RFA on sea level data along the Dutch coast has been performed by Van Gelder et.al. (1998). The RFA on wave height data along the Dutch coast has been performed by Van Gelder et.al. (1999). The results of these RFA's will be used in this paper to determine the hydraulic boundary conditions of a certain location along the Dutch coast, namely the Petten location (Fig. 1 and 2). It will be shown how the other two hydraulic boundary conditions wave period and wave steepness can be determined and how the transition of these boundary conditions from 20m-deep to the toe of the dike can be performed. The

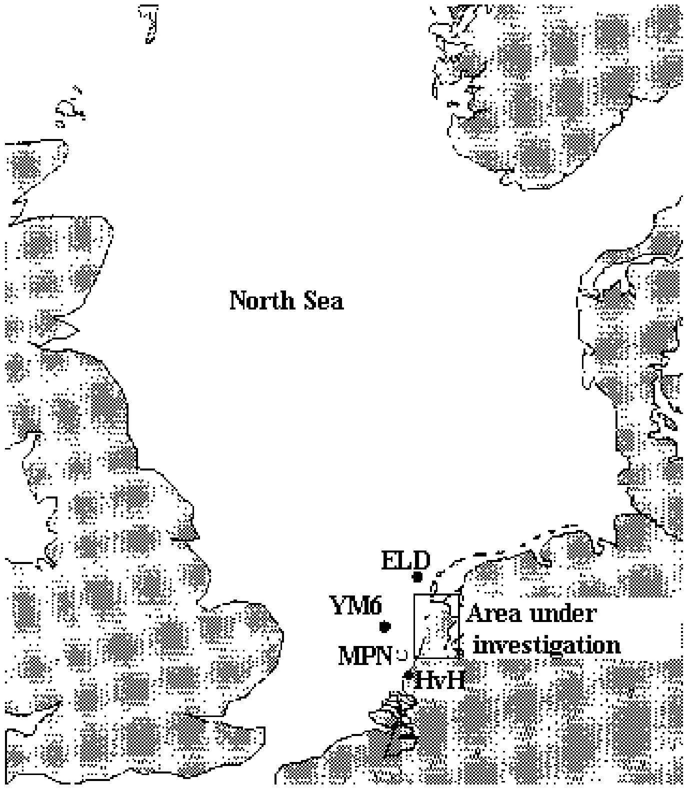

Figure 1. North Sea and the area under investigation.

Figure 2. Coast line of Northern Holland.

paper is organized as follows: First the general approach of a RFA will be described. Then the results
of the RFA's on the extreme sea levels and the extreme wave heights (on deep water) will be presented. For the case study of Petten, it will be described how the wave period, wind set-up and wave steepness were determined and how the transition of the boundary conditions from deep water to the toe of the dike is treated. The paper will end with some conclusions.

2 REGIONAL FREQUENCY APPROACH

RFA resolves the data scarcity problem by 'trading space for time': it does so by using data from several sites, which are judged to have frequency distributions similar to the site of interest, in estimating event frequencies at that site. L-moments are a recent development within statistics (Hosking and Wallis, 1997). They form the basis of an elegant mathematical theory in their own right, and can be used to facilitate the estimation process in regional frequency analysis. L-moment methods are demonstrably superior to those that have been used previously, and are now being adopted by major organizations worldwide (Hosking and Wallis, 1996).

Analysis of sea levels (such as mean levels, tides, extremes and currents) and wave heights is currently restricted to data from a relatively sparse network of coastal gauges (containing long records of up to 100 years) and a few offshore gauges (containing short records of up to 25 years). Recent increases in theoretical models have enabled significant improvements in predicting occurence frequencies of the extremes. For instance the so-called max-stable extreme value models (Coles, 1992) offer a suitable basis for analysing extremal characteristics of environmental processes with a spatial dimension. However, these models will require a considerable extension and development in form to handle (i) the high spatial dimensionality of the problem, (ii) knowledge of covariates and directionality, (iii) trends and seasonality of marginal and spatial (such as changes in occurrence of storm types extreme values. Therefore, in this paper the aim was to use extreme value models based on other methods. A simple regional frequency approach based on L-moments, including as much of the available data as possible, is proposed to analyze the extreme value distributions of the coastal data.

The main stages of the RFA procedure are: (i) screening of the data; (ii) identification of homogeneous regions; (iii) choice of a regional frequency distribution; (iv) estimation of the regional frequency distribution. In this paper the results of the RFA procedure are presented for the extreme water levels and the extreme wave heights along the Dutch coast.

L-moments are summary statistics for probability distributions and data samples. They are analogous to ordinary moments -- they provide measures of location, dispersion, skewness, kurtosis, and other aspects of the shape of probability distributions or data samples -- but are computed from linear combinations of the ordered data values (hence the prefix L).

Probability weighted moments, defined by Greenwood et al. (1979), are precursors of L-moments. Sample probability weighted moments, computed from data values X_1, X_2, ... X_n, arranged in increasing order, are given by:

$$b_0 = n^{-1}\sum_{j=1}^{n} X_j$$

$$b_r = n^{-1}\sum_{j=r+1}^{n} \frac{(j-1)(j-2)\ldots(j-r)}{(n-1)(n-2)\ldots(n-r)} X_j$$

L-moments are certain linear combinations of probability weighted moments that have simple interpretations as measures of the location, dispersion and shape of the data sample. The first few L-moments are defined by :

$$l_1 = b_0$$
$$l_2 = 2b_1 - b_0$$
$$l_3 = 6b_2 - 6b_1 + b_0$$
$$l_4 = 20b_3 - 30b_2 + 12b_1 - b_0$$

(the coefficients are those of the "shifted Legendre polynomials").
The first L-moment is the sample mean, a measure of location. The second L-moment is (a multiple of) Gini's mean difference statistic, a measure of the dispersion of the data values about their mean. Hosking and Wallis (1997) give an excellent overview on the whole theory of L-Moments.

3 ERGODICITY AND HETEROGENITY

An *ergodic* process (Papoulis, 1991) is one where all orders of statistical parameters are interchangeable. Thus, any statistic of the random process can be determined from the sample function of that process. The mean of an ergodic process is given by:

$$E(x(t)) = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} x(t)dt$$

A simple example for an ergodic process is a sequence of i.i.d. (independent and identically distributed) random variables. The datasets of annual maxima for the sea levels and wave heights are assumed i.i.d. They therefore form an ergodic process.

A *heterogeneity measure* should be able to estimate the degree of heterogeneity in a group of sites and to assess whether the sites might reasonably be treated as a homogeneous region. A formal definition of the heterogeneity measure is given in Hosking and Wallis (1993, 1997). Basically the heterogeneity measure compares the between-site variations in sample L moments for the group of sites with what would be expected for a homogeneous region. What “would be expected” is evaluated through Monte Carlo simulation from the four parameter Kappa distribution. Hosking and Wallis (1997) suggested that regions can be classified as "acceptably homogeneous" if $H<1$, "possibly heterogeneous" if $1= H<2$, and "definitely heterogeneous" if $H\geq 2$. (The H(2) and H(3) statistics based on V2 and V3 respectively lack power to discriminate between homogeneous and heterogeneous regions according Hosking and Wallis (1997)).

4 EXTREME WATER LEVELS

Peaks over thresholds (POT) of water level data from several gauging stations located along the Dutch coast are stored at the RIKZ (Ministry of Water Management) in the Hague, the Netherlands. Their POT data sets contain a total of 6818 water level observations with sample sizes varying from 53 to 104 years. In Van Gelder et.al. (1998), a RFA was performed on these data sets. With L-Moments parameter estimation techniques, they concluded that the Generalized Pareto distribution (GPA) was the best probability distribution function to describe the extreme water levels along the Dutch coast. For five sites along the Dutch coast they also provided the parameters of the GPA’s (Fig. 3).

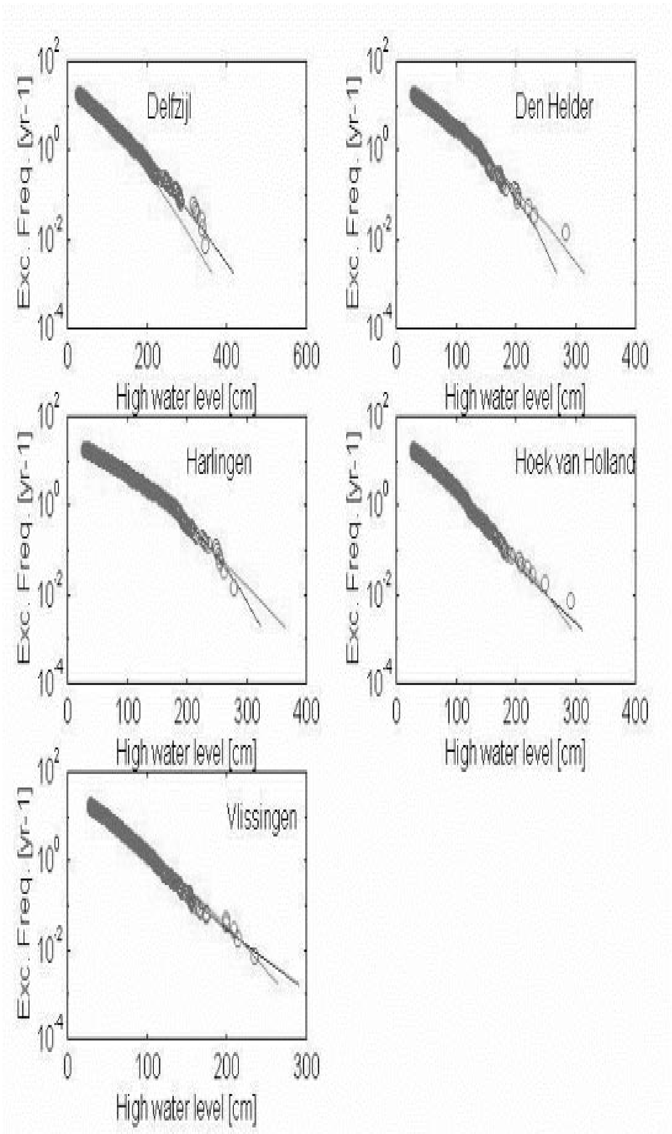

Figure 3. Results of the RFA for the extreme water levels along the Dutch coast; the light lines give the regional estimates; the dark lines give the at-site estimates; all of them are GPA's (source: Van Gelder et.al., 1998).

The Generalized Pareto (GPA) distribution is given by the expression:

$$f(x) = \alpha^{-1} e^{-(1-k)y}, \text{ with}$$
$$y = \begin{cases} -k^{-1}\log\{1-k(x-\xi)/\alpha\}, & k \neq 0 \\ (x-\xi)/\alpha, & k = 0 \end{cases}$$

So if $k \neq 0$, the PDF can also be written in the more familiar expression:

$$f(x) = 1/\alpha\ (1-k(x-\xi)/\alpha)^{-1+1/k})$$

However, we prefer to use the earlier given expression, since the value k=0 follows in a continuous way by taking the limit for $k \rightarrow 0$ and resulting in the exponential distribution. The value k=1 results in a uniform distribution.

The CDF of the Generalized Pareto distribution is given by:

$$F(x) = 1 - e^{-y}$$

and the inverse distribution by:

$$x(F) = \begin{cases} \xi + \alpha\{1-(1-F)^k\}/k, & k \neq 0 \\ \xi - \alpha \cdot \log(1-F), & k = 0 \end{cases}$$

With a L-Moment parameter estimation method the three parameters of the GPA are calculated very easily with the following formulae:

$$k = (1-3\tau_3)/(1+\tau_3),$$
$$\alpha = (1+k)(2+k)\lambda_2,$$
$$\xi = \lambda_1 - (2+k)\lambda_2.$$

It was observed by Van Gelder et.al. (1998) that the sites along the Dutch coast do not form a homogeneous region. However, Hosking and Wallis (1997) stated that even in regions with heterogeneity, the results of a RFA give valuable information; especially the distribution type that may be used in the region. The RFA resulted in a GPA as being the best distribution for the extreme sea levels along the Dutch coast (despite of its inhomogeneity). In the sequence of this paper we will use the results of the 10^{-2} and
10^{-4} quantiles at the location of Den Helder (Fig. 2) which are given by the RFA by 2.8m and 3.8m respectively (Fig. 3). The astronomical tide has to be added to these values (RIKZ, 1994). For Den Helder this value is 58cm and for Petten it is given by 81cm.

5 EXTREME WAVE HEIGHTS

A RFA of the extreme wave heights along the Dutch coast has been performed using POT (Peaks over Threshold) data at nine sites along the Dutch coast in Van Gelder et.al. (1999). The data was again obtained from the RIKZ. The objective was to identify regional probability distributions for the extreme wave heights. Each site contained about 50 observations over the period 1979-1993 and is shown in Fig. 4. Also here, a Generalized Pareto Distribution appears to be the optimal regional fit for the extreme wave heights on the North Sea.

It was observed by Van Gelder et.al. (1999) that the locations of wave height measurements along the Dutch coast form a homogeneous region (expect for the one location MPN (Fig. 1)).

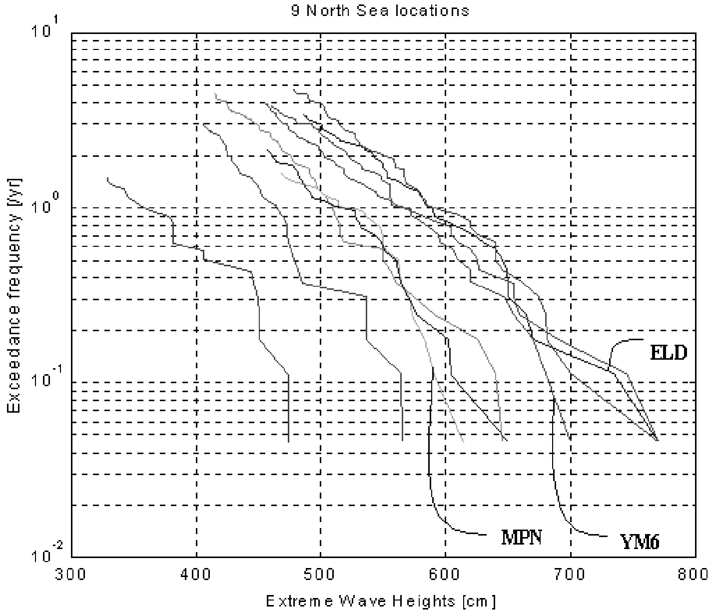

Figure 4. The frequency exceedance curves for nine locations (unnormalized); source: Van Gelder et.al. (1999).

The parameter estimations for the GPA following from the RFA are given by:

.863 .170 .244 (shift, shape and scale).

The following quantile estimations are the result:

p	.010	.020	.050	.100	.200	.500	.900	.950	.990	.999
x_p	.865	.867	.872	.881	.900	.972	1.163	1.225	1.333	1.431

These quantile estimates have to be multiplied with the mean POT-values at the location of interest and corrected with the ratio between the total number of observations and the number of years over which the wave heights have been measured.

At the location of YM6 (Fig. 1), 61 POT-observations over a period of 15 years with an average value of 523cm were measured. At the location of ELD (Fig. 1), 52 POT-observations over a period of 15 years with an average value of 564cm were given. With this information, at the location of YM6, the 10^{-2} and 10^{-4} quantiles are given by 7.40m and 8.35m. At the location of ELD, the 10^{-2} and 10^{-4} quantiles are given by 8.00m and 8.95m.

6 TRANSITION OF BOUNDARY CONDITIONS FROM DEEP TO SHALLOW WATER

When going from relatively deep water to the coast, the hydraulic conditions are more and more influenced by the local water depth. Generally, the decreasing depth leads to an increasing wind setup and a decreasing wave height. In the reliability analysis of a coastal water retaining structure, one has to account for these effects.

The most straight-forward way of dealing with the physical behaviour of the boundary conditions is by using parametric models derived from linear wave theory (Dean and Dalrymple, 1984).

On a more advanced level, several numerical models are available. The most simple models apply a parametric frequency spectrum, of which the parameters change under the influence of the local water depth. Similarly, also the directional spreading of the wave field, described by the directional spectrum, may be of a parametric form.

The most advanced of numerical models adopt a full spectral description of the wave field, in the frequency domain as well as in the directional domain. Solving the energy balance equation on a rectangular or curved grid provides the frequency and directional spectra of the wave field. In these models the spectral shape is completely free. This might be advantageous, especially if wave conditions are to be derived for a complex bathymetry.

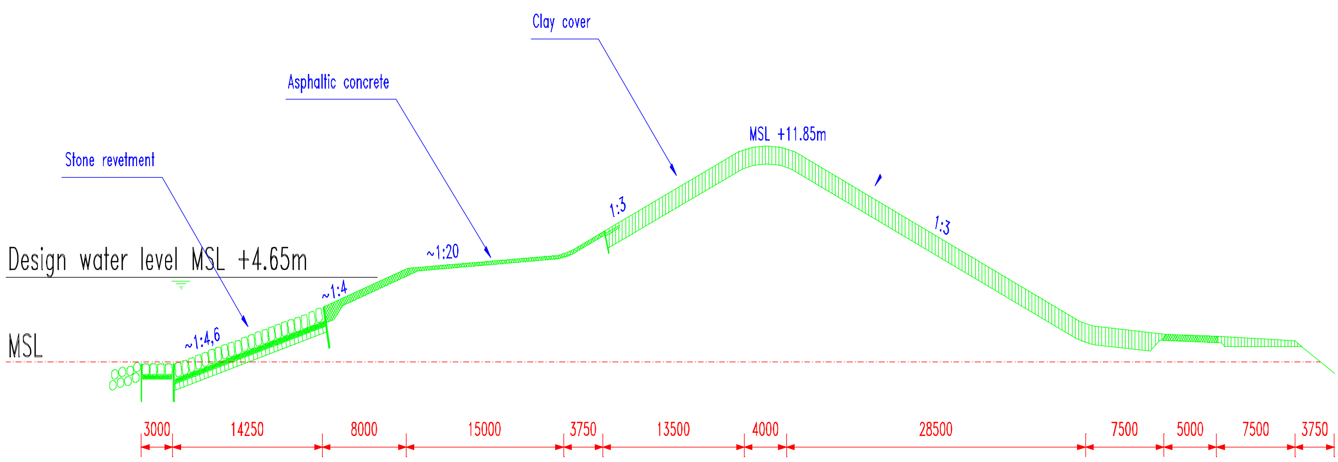

Figure 5: Cross section of the Petten Sea Dike

7 CASE STUDY PETTEN SEA DIKE

The Dutch North Sea coast is generally protected by dunes. However, in the North a section of dunes of about 7 km is missing. Here, in the past a river mouth was present, which prevented the formation of dunes. The first dike in this location was already built as early as the 17th century. This dike is known as the Petten sea dike, after the name of town directly behind the dike.

The design of the Petten sea dike may be called a typical Dutch design (Fig. 5). The basis of the profile is trapezoidal. On the sea side, a berm is placed approximately at design water level. The berm reduces wave run-up considerably (v.d. Meer and Janssen, 1994).

The main body of the dike consists of a sand core. In order to protect the sand core from wave attack, on the sea side a revetment is placed. The lower part of the revetment consists of a placed stone revetment. This is an open layer, which prevents water pressure build-up in the sand core. The zone that is most heavily attacked under extreme conditions is covered by a layer of asphaltic concrete. Above this zone, the dike is mainly loaded by running water as a consequence of wave run-up. This zone, as well as the back-side of the dike are therefore covered by a clay layer, again to prevent erosion of the sand core.

Experience from a major flooding disaster in 1953 showed that in several cases a dike collapses due to erosion of the backside as a consequence of overtopping waves. Therefore, limiting the number of overtopping waves, as well as reducing the overtopping volume of water is considered one of the major issues in dike design. In the later part of this case study, attention will be addressed to this failure mode.

An extensive study of the wave climate at the Petten location including numerical modelling has been carried out (Voortman and Vriezekolk, 1994). In this paper, it will be assumed that the physical relations between deep water conditions and local conditions, derived by numerical modelling, also hold for the new deep water conditions that result from RFA.

The wave climate analysis is performed using the numerical model HISWA (Holthuijsen et.al., 1993). Calculations are made for the exceedance frequencies 10^{-2} per year and 10^{-4} per year, using the original water level, wave height and wave period input. From the results a crude parametric model has been estimated, describing the wave height at the Petten location as a function of the water level and the deep water wave height.

A phyisical relation between significant wave height and peak period may be found on the basis of wave steepness. In this case the steepness is defined as:

$$s_{0p} = \frac{H_s}{\frac{g}{2\pi} T_p^2}$$

In which:
H_s: Significant wave height (m);
T_p: Peak period (s);
g: Accelaration of gravity (m/s^2).

Analysis of measurements from the North Sea indicates that the significant wave height and the wave steepness are uncorrelated for extreme conditions. The marginal distributions of wave height and wave steepness may be derived from measurements. Theoretically, numerical models are able to calculate changes of the wave period due to wave transformation on shallow water. However, in this study a safe estimate based on linear wave theory is used, which comes down to assuming that the wave period is not influenced by wave transformation. For the Petten location it appears that it is most relevant to use the period derived at the station ELD. Analysis of measurements from this station indicates that the mean steepness amounts to 0.049.

An overview of the hydraulic boundary conditions at the Petten location is given in the tables below.

Table 1: Overview of water levels

Exc. freq (1/yr)	Model	Water level Den Helder (m+ MSL)	Water level Petten (m+ MSL)
10^{-2}	Trad.	3.33	3.41
10^{-4}	Trad.	4.40	4.65
10^{-2}	RFA	3.38	3.61
10^{-4}	RFA	4.38	4.61

Table 2: Overview of wave heights

Exc. freq (1/year)	Model	Wave height YM6 (m)	Wave height ELD (m)	Wave height Petten (m)
10^{-2}	Trad.	7.64	8.37	2.70
10^{-4}	Trad.	9.10	10.0	3.65
10^{-2}	RFA	7.40	8.00	2.63
10^{-4}	RFA	8.35	8.95	3.26

Table 3: Overview of spectral peak periods

Exc. freq (1/year)	Model	Wave period ELD (s)	Wave period Petten (s)
10^{-2}	Trad.	12.2	12.2
10^{-4}	Trad.	13.3	13.3
10^{-2}	RFA	11.9	11.9
10^{-4}	RFA	12.6	12.6

Generally, it appears that the estimates of the hydraulic boundary conditions are less conservative when RFA is used. This effect is most prominent for the wave conditions. The water levels are not so much influenced by the estimation method.

The results show that the estimation method influences the boundary conditions for dike design. However, the number of points calculated in this study is too small to use for definite conclusions.

The shape and orientation of the North Sea causes North-Westerly winds to be the most dangerous to the coast of the Netherlands. This is caused by the fact that these wind directions cause higher water level setup and larger wave heights and associated with that longer wave periods. The influence of wind direction may be important, especially for dikes not oriented towards the North-West. Therefore, the estimation of hydraulic boundary condtions is now moving into the direction of including the influence of wind direction as well.

As all are caused by wind fields, there is a strong dependency of water level, wave height and wave period. In a probabilistic description of the boundary conditions, this dependency has to be accounted for. A description on the basis of physical behaviour of water levels and waves is under study.

8 CONCLUSIONS

In this paper it is shown how the Regional Frequency Approach can be used in the determination of the hydraulic boundary conditions which are needed in the design of sea dikes. For decision making it is better to rely on the regional quantile estimates than on the at-site quantiles, as is proven in Hosking and Wallis (1997).

In the Petten case study it appeared that the estimates of the hydraulic boundary conditions are slightly less conservative when RFA is used. This effect was most prominent for the wave conditions. The water levels were not so much influenced by the estimation method. However, the number of points calculated in this study is too small to use for definite conclusions.

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