The principles of catastrophic flood forecasting

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Abstract It is well known that there are a lot of causes in changes of hydrological processes, such as climate changes, human activities, exogenous processes, etc. These factors lead to a deviation in the statistical estimation of model parameters and statistical models may become biased. Therefore, problems such as extreme flood forecasting and the computation of expected damage on flooded areas cannot be solved by traditional methods. In order to provide the forecast certainty and to have a possibility of GIS-based presentation of inundated zones, other principles of forecasting should be chosen (based on robustness: a model is robust if the small mistakes of parameters do not lead to large errors in forecasts).

In this paper the method of model selection for catastrophe cases and a clear tool of forecasting of extreme floods are shown. A case study of flood prediction of the river Rhine is presented.

Introduction

The problem of extreme flood forecasting is very popular under hydrologists. A lot of original methods exist and new ways appear regularly (Kachroo and Liang, 1992). But only few tools can provide so-called forecast certainty in catastrophe cases. The unevenness of precipitations, the limitation of the model describing the high number of runoff generated factors, the changes of parameter prototypes, the natural fluctuations of hydrological characteristics, sparse and incorrect data, difficulties in the model selection are the main reasons of bad forecasts (Kartvelishvili, Galaktionov, 1976).

It is necessary to introduce new kind of methods of forecasting which are based on principles such as:

1. Forecasting model must be robust in relation to parameters and input data;
2. Calibration of parameters and verification of the model must be produced in real-time regime;
3. Forecasting model must reflect stochastic fluctuations of its “input” despite of its limitation. Secondary and unaccounted factors must be taken into consideration by the forecasting method automatically;
4. The less preliminary information is used, the better it is.

In general from a practical point of view, we could add to this list one principle more: the GIS-based presentation of forecast results (Noman and Nelson, 1999). Certainly, this is not quite “the principle” in the broad meaning of the word. However GIS may help to solve the problem of evacuation of inhabitants more efficiently. A clear picture showing the flooded area is the best decision tool for any officer or supervisor.

Thus, our main task may be formulated as the elaboration of an universal method for extreme flood forecasting which would be suitable for utilization in many situations (Kuzmin, 1997).

Method of model selection

In order to provide a forecast certainty, the chosen model must be F-robust. The term F-robustness means that the n-parameter relative index F_K^n of a given model is the smallest out of a set of n-parameter relative indices of other competitive models. The index F_K^n may be obtained by an analysis of forecast results in various conditions as:

$$F_K^n = \frac{\int_{a_1}^{A_1} \dots \int_{a_n}^{A_n} D(P)dP_1 \dots dP_n + D_{lim}(P)\left(\prod_{i=1}^{i=n}(P_{i,max} - P_{i,min}) - \prod_{i=1}^{i=n}(A_i - a_i)\right)}{D_{lim}(P)\prod_{i=1}^{i=n}(P_{i,max} - P_{i,min})}, \quad (1)$$

where $D(P)$ is the minimized criterion (for example, RMSE) depending on parameters P. A_i and a_i are the maximum and the minimum possible values of parameter P_i. $P_{i,max}$ and $P_{i,min}$ are the maximum and the minimum values of P, providing acceptable values of $D(P)$, $D_{lim}(P)$ - extremely acceptable value of $D(P)$. For example, in Russia $D = S/s$ (S is RMSE of the forecasting errors, and s is RMSE so-called natural forecasting). This index shows how the accuracy of the model depends on a mistake of the parameter. The smaller F_K^n and the wider the limits of acceptable values of P_i, the better the model is.

Available data is to be used for selection of the model, not for the determination of its parameters. As our experience evidences, the methods which calculate the mode of the distribution are the best.

Elements of the theory of stochastic modeling

All known ways of the flood forecasting are based on the inertia properties of the given natural object. The length of forecast of extreme runoff depends on inertia of flood-generating factors. The procedure of calibration of the traditional model assumes that a number of parameters are constant and the their conformity to natural laws is established. The generation of extreme flood differs from the usual floods. Secondary and unmeasured factors suddenly become more important, that is why they must be accounted by the forecast model. These factors have various inertias; consequently the procedure of preliminary calibration is useless. As for the verification, this procedure must concentrate upon the structure of the model and not upon parameters or coefficients. So, statistical and empirical models are to be refused, and fundamental differential equations are too limited. Where is the exit?

Let's start *ab ovo*. Any natural process, including runoff Q may be exhaustively described by the probability curve. In its turn the curve of the forecasted distribution of runoff $p(Q,t)$ is expressed by a stochastic equation:

$$\frac{\partial p(Q,t)}{\partial t} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \frac{\partial^n}{\partial Q^n} \left[K_n(Q) p(Q,t) \right], \tag{2}$$

where $K_n = \lim\limits_{t \to 0} (m_n(Q)/t)$, is the average velocity of the systematic changing of co-ordinate $Q(t)$, or the limit of the corresponding initial moment, t is current time, t is the length of forecast.

In the first approach, we can find the mode of distribution, i. e. $\max\limits_{[0;\infty]}(p(Q,t+t))$ (where $p(Q,t+t) = p(Q,t) + \sum\limits_{n=1}^{\infty} \frac{(-1)^n}{n!} \frac{\partial^n}{\partial Q^n} \left[K_n(Q)\, p(Q,t) \right] t$).

It is too difficult to determine the coefficients K_n directly. Even the equation written at $n = 2$ is to be solved very cumbersomely, and the forecasting scheme looks awkwardly.

The polynom $\sum\limits_{n=1}^{\infty} \frac{(-1)^n}{n!} \frac{\partial^n}{\partial Q^n} \left[K_n(Q) p(Q,t) \right] t$ can be found more easily or indirectly to a certain degree.

There are many methods of construction of probability distribution curves, available from the past and from the present. In general a shape of curve depends on our knowledge of the process. Having absolutely exact instruments, we could construct the distribution as a δ-function. Mistakes in measurements and current fluctuations of runoff increase dispersion. Moreover, no method can decrease dispersion and the most essential part of dispersion is to be born by the forecast method. We must mark that the dispersion changes a scale and the limits of *the transfer function* (Fig.1) and does not influence the coordinate of the mode.

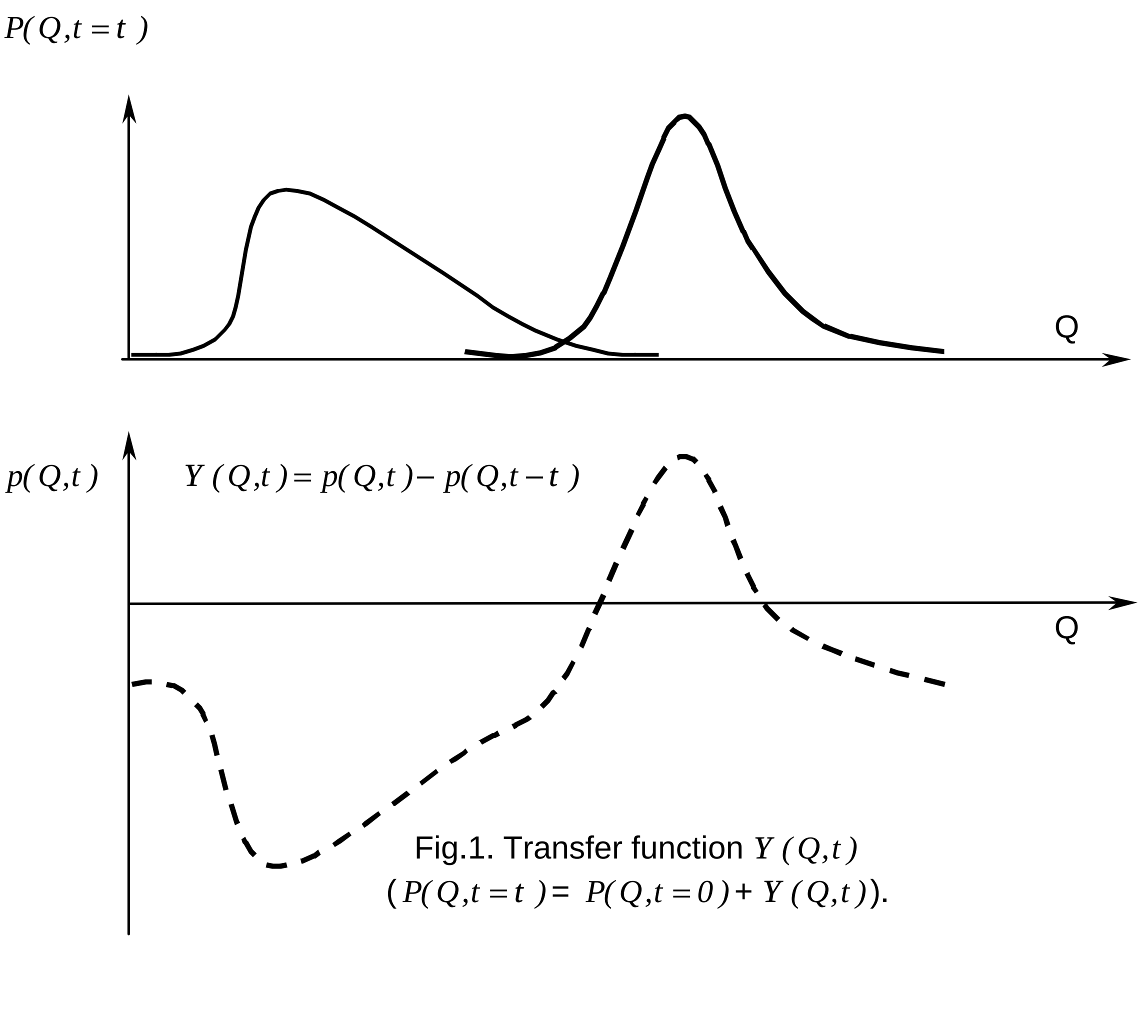

Fig.1. Transfer function $Y(Q,t)$
($P(Q,t=t) = P(Q,t=0) + Y(Q,t)$).

Let us consider two distributions, $p(Q,t=0)$ and $p(Q,t=t)$, and recall the equation (2). The first addendum of the polynom is $-\partial\left[K_1(Q)p(Q,t)\right]/\partial Q$, which determines a distance of distribution transference. As it was shown above, the second addendum $\partial^2\left[K_2(Q)p(Q,t)\right]/2\partial Q^2$ is not interesting for us. On the contrary, the third one $-\partial^3\left[K_3(Q)p(Q,t)\right]/6\partial Q^3$ is most important for us, as it is the main cause of changes of a mode. From a practical point of view, the fourth addendum $\partial^4\left[K_4(Q)p(Q,t)\right]/24\partial Q^4$ may be accounted for the evaluation of the forecast certainty; like the second one, it does not influence the mode coordinate. So, the even addendums determine a probability of every discharge including a mode, and the odd addendums determine the coordinate of the forecasted mode. Therefore our task is simplified and we shall find $\max\limits_{[0;\infty]}(\tilde{p}(Q,t+t))$, where

$$\tilde{p}(Q,t+t) = p(Q,t) - \partial\left[K_1(Q)\,p(Q,t)\right]t\,/\partial Q - \partial^3\left[K_3(Q)p\,(Q,t)\right]t\,/6\partial Q^3 - \ldots \quad (3)$$

Higher addendums are to be determined very difficult, and in further consideration this equation may be limited to $n=3$. In general the initial distribution $p(Q,t=0)$ depends on the way of measurement, it is symmetric usually or has inessential asymmetry, which may be ignored. Finally we have got the fundamentals of the algorithm for extreme flood forecasting as follows:

1. Observed discharge has a meaning of the mode of the initial distribution;
2. Transference of the distribution is to be determined by the basic differential equation using measurable “input” and parameters only; *in extreme cases the utilization of statistic or empiric equations is not to be allowed*;
3. The goal of the forecaste is the mode of the transferred distribution.

As the algorithm based on the classic equation (2) is too awkward and it is not providing a successful result, it is possible to reach this goal by different indirect ways. The following two methods are recommended:

1. Method of main components (MMC) or similar procedures (for example SSA);
2. Offered by the authors a simplified stochastic self-training procedure (SSTP).

Let us consider the simplified SSTP in details.

Stochastic self-training procedure (SSTP)

Consider the separated forecast. Not depending on the type of forecast, the following expression can be written:

$$\hat{Q} - \hat{Q}_{mod} = \hat{m}_3/2\hat{m}_2 + d \quad (3^a)$$

or

$$\hat{m}_1 + d - \hat{Q}_{mod} = \hat{m}_3/2\hat{m}_2 + d = D\,, \quad (3^b)$$

where $\hat{m}_1$ is the mathematic expectation of the forecasted distribution, $\hat{Q} = \hat{m}_1 + d$ is the discharge obtained by basic differential equation, $\hat{Q}_{mod}$ is the mode and the observed discharge simultaneously, $\hat{m}_2$ and $\hat{m}_3$ are the second and the third central moments of the probability distribution, D is the error of forecast. It is possible to use any suitable differential equation for the computation of $\hat{Q}$ as follows:

1. Equations of balance (including the artificial neural networks);
2. Equation of continuity (for example, the model of kinematic wave or the diffusion equation);
3. Equation of motion, etc.

Note that equations such as the tendency equation, regression and empirical equations cannot be used in extreme cases.

Being the consequence of secondary factors action, the difference d between $\hat{m}_1$ and $\hat{Q}$ is an inertial value; it is "almost" constant. Therefore d may be obtained from (3^b):

$$d(t) = D(t) - \hat{m}_1(t) + \hat{Q}_{mod}(t), \qquad (4^a)$$

or $d(t) = \hat{m}_3(t) / 2\hat{m}_2(t) - D(t).$ (4^b)

Let us write the forecasting equation:

$$\hat{Q}_{mod}(t+t) = \hat{m}_1(t+t) + f(d(t)), \qquad (5)$$

where $f(d(t))$ in the simplest variant equals to $d(t)$ and may be computed by (4^a) or (4^b). These equations are the subject of all further investigations (Aitken, 1973).

The equation (4^a) is more suitable in absence of preliminary data and in cases when the analysis of data or of time series is impossible. Other equations may be used in stable situations, when the second and the third central moments, m_2 and m_3, changes in time smoothly (Kuzmin, 1998). In the same case methods such as MMC or SSA may be very useful.

Case study River Rhine

The Rhine is one of the largest rivers in Europe. At Lobith (the village where the Rhine enters the Netherlands), the average discharge is about 2200m^3/s. However peak discharges of 10,000m^3/s can occur with a realistic probability of occurence. In 1988 a maximum discharge of 10,160m^3/s was measured (Fig. 2).

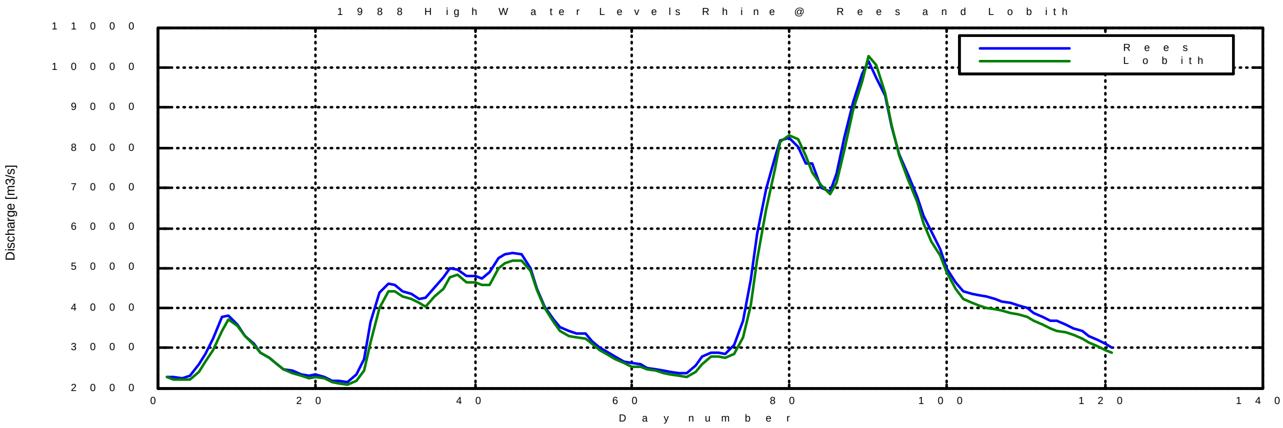

Fig. 2: Daily discharge first 120 days in 1988.

The period of forecasting is from January 1, 1988 till April 30, 1988. Length of the forecasting is 1 day. As a predictor the data for Rees (30 km stream upwards from Lobith) and Koeln (120 km upwards from Lobith) was shown in 2 variants:

1. Using the MKW in which the parameters were defined in field conditions,

$$Q_{Lobith}^{t+1} = (1-C)Q_{Lobith}^{t} - CQ_{\mathrm{Re}es}^{t},$$

where $C = 0.347$ (= L/T = 30000m / 86400sec). The ratio of S/s was found equal to 0.865. The SSTM decreased the ratio of S/s two times to 0.432.

2. Using the MKW in which the parameters were determined statistically,

$$Q_{Lobith}^{t+1} = 0.32Q_{Lobith}^{t} - 0.70Q_{Koe\ln}^{t}.$$

The ratio of S/s was found equal to 0.426. The SSTM decreased the ratio of S/s two times to 0.207. The index F_K^1 of MKW was equal to 1.2.

Table 1: Unrobustness of MKW versus SSTP.

C	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
S/s MKW	0.849	0.853	0.860	0.871	0.885	0.903	0.923	0.946	0.972	1.000
S/s SSTP	0.390	0.405	0.423	0.441	0.462	0.484	0.506	0.530	0.555	0.581

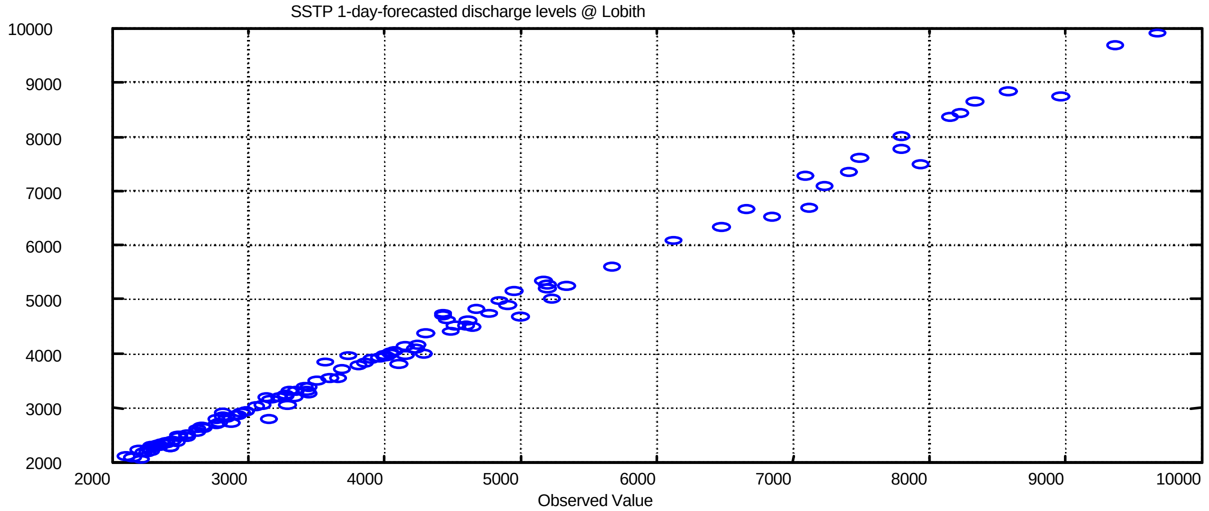

Fig. 3: Comparison between computed and observed discharges

Summary of other case studies

Apart from the application of the described principles of catastrophic flood forecasting to the river Rhine as described above, the method has also been used in the forecasting of 90 very hard-predictable floods occurring in rivers such as Oka, Volga, North Dvina, Pechora, Don, Aldan, Klyazma (Russia), Missouri, Mississippi (USA), and Fylios (Turkey).

Kuzmin (1997) analyzed the Oka River (location Gorbatov, Russia) with a length of forecasting of 4 days. Using the MKW, the ratio of S/s was found equal to 0,60 and $F_K^1 = 0.86$. The SSTP decreased the ratio of S/s to 0,38 and F_K^n to 0.55.

Kuzmin et al. (2000) analyzed the Fylios River (location Derecikviran, Turkey) with a length of the forecasting of 1 day. Using the regression equation, the ratio of S/s was found equal to 0.80. The SSTM decreased the ratio of S/s with 14% to 0.699.

Results are summarized as given in the following Table 2.

Table 2: Results of the utilization of SSTP in short-term flood forecasting

No.	River	Station	Country	Years	S/s_{Δ}	$((S_{SSTP}-S_{BASIC})/S_{BASIC})\cdot 100\%$
1	Oka	Gorbatov	Russia	1966	0.405	18.5
2				1968	0.353	26.5
3				1970	0.265	25.1
4				1981	0.256	37.9
5				1982	0.406	44.6
6				1983	0.480	34.9
7				1984	0.522	27.8
8				1985	0.392	29.1
9				1986	0.358	39.9
10				1987	0.294	41.0
11				1988	0.368	46.3
12				1989	0.672	22.8
13				1990	0.430	22.9
14				1991	0.486	21.4
15				1992	0.410	49.6
16				1993	0.285	45.0
17				1994	0.418	26.9
18	Volga	Nizhni Novgorod	Russia	1980	0.237	22.2
19				1992	0.358	19.8
20				1994	0.377	14.8
21	Missouri	Kansas City	USA	1991	0.661	11.7
22				1992	0.589	19.9
23				1993	0.498	21.4
24				1994	0.520	16.6
25				1995	0.632	10.0
26	Don	Peskovatka	Russia	1960	0.774	8.9
27	Irtysh	Tobolsk	Russia	1966	0.423	7.5
28				1968	0.498	11.8
29				1970	0.473	10.4
30				1972	0.601	8.8
31	Pechora	Pechora	Russia	1966	0.225	39.5
32				1968	0.299	32.5
33				1970	0.230	31.4
34				1972	0.333	24.7
35	Filyos	Derecikviran	Turkey	1979	0.699	14.3
36	North Dvina	Verkhnaya Toyma	Russia	1966	0.545	47.8
37				1968	0.508	49.0
38				1970	0.478	35.1
39				1972	0.421	36.1
40				1980	0.563	40.8

Traditional methods of forecast gave worse results in all case studies. SSTP appears to be very suitable in regions with sparse data, in areas where there are no gauge stations, in changed basins, in expeditions, etc. Furthermore, the offered principles of the model selection and of extreme flood forecasting may be used for application in GIS-technologies.

Conclusions

The stochastic self-training method (SSTM) is a suitable method in extreme flood forecasting. The SSTP foresees a combination of calibration, verification and forecasting. It consists of two parts:

1. Main equation for the first initial moment of the forecasted probability distribution, for example, the model of kinematic wave (MKW), artificial neural net, equation of continuity, equation of motion, “rainfall – runoff”, water balance equation, etc;
2. Correction of the forecast produced by stochastic analysis of previous errors and accounting of evolution of second and third central moments.

As numerical experiments showed, the SSTM improves the accuracy essentially. A case study of floods on the Rhine River was presented. As a main model, the MKW was used. The ratio of $\boldsymbol{S}/\acute{o}_{\ddot{A}}$ was decreased significantly.

The fluctuations of runoff are reflected better by the mode than by other statistics. The suggested procedure is very useful in the absence of preliminary data, in regions with sparse data, in areas where there are no stations for measurements at all, in changed basins, in expeditions, etc. The method showed good results in different geographic zones (Table 2).

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