The Use of L-Moments in the Peak Over Threshold Approach for Estimating Extreme Quantiles of Wind Velocity

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ABSTRACT: An alternative approach to the estimation of the tail index of extreme value distributions in the POT approach is described. The proposed estimation method is based on sample L-moments. Quantile estimation of wind loading is presented as a case study.

1 INTRODUCTION

A fundamental theorem of extreme value theory states that sufficiently large values of independent identically distributed random variables generally follow one of the three extreme value distributions, namely, Frechet, Gumbel and reverse Weibull. The upper tail of Frechet and Gumbel distributions is infinite, whereas the reverse Weibull distribution has finite upper tail. In the estimation of extreme quantiles of wind loading, Frechet and Gumbel distributions are normally used. However, it has been shown that the Gumbel and Frechet models results in unrealistically high failure probability of wind sensitive structures due perhaps to the unbounded upper tail of the distribution (Ellingwood 1980).

The POT (peaks over threshold) approach for modelling the tail of a distribution has received considerable attention since it has been shown that the Pareto distribution (GPD) arises as the limiting distribution of peaks (or excesses) $X–u$ of a random variable X over a high threshold u (Pickands 1975). The central objective of this paper is to evaluate the effectiveness of the method of L-moments for estimating the GPD parameters, and compare its performance in quantile prediction against that of the de Haan method (1994). Monte Carlo simulations and actual wind velocity data collected at various stations in the United States have been utilized for comparison purposes.

The paper is organized as follows. The next section reviews basic concepts of the POT approach and the de Haan estimation method. The L-moment method is briefly outlined in Section 3. Section 4 presents numerical results using Monte Carlo simulations and actual wind speed data. The main conclusions and a list of references are presented in Section 5 and 6, respectively.

2 PEAKS OVER THRESHOLD AND DE HAAN

2.1 *Simiu and Heckert's studies*

In a recent study Simiu and Heckert (1996) applied the Peak Over Threshold (POT) approach to analyze the wind speed data and fit an appropriate extreme value distribution. The POT approach enables the analyst to use all the data exceeding a sufficiently high threshold, in contrast with the classical extreme value analysis which typically uses annual extreme values. Simiu and Heckert (1996) concluded that the reverse Weibull distribution is a more appropriate distribution than the Gumbel or Frechet. Their conclusion is also supported by a physical fact that non-tornadic extreme winds are expected to be bounded.

2.2 *POT*

The application of POT approach involves two main steps: selection of threshold and estimation of the tail index using order statistics above the threshold. Since the tail index is found to be very sensitive the threshold value, subsequent quantile estimates also become sensitive to the threshold. This is a major weakness of the POT approach, which is also confirmed from the analysis of Simiu and Heckert (1996). In fact, the fluctuation of quantiles of wind loading with threshold is so severe that no reliable design estimates could be recommended.

Let $X_1, X_2, \ldots, X_n$ be a series of independent random observations of a random variable X with the distribution function (DF) $F(x)$. To model the upper tail of $F(x)$, consider k exceedances of X over a threshold u and let $Y_1, Y_2, \ldots, Y_k$ denote the excesses (or peaks), i.e., $Y_i = X_i - u$. Pickands (1975) showed that, in some asymptotic sense, the conditional distribution of excesses follow the generalized Pareto distribution. Thus the DF of $Y_i = [(X_i - u)| X_i > u]$, $i = 1,2,\ldots, k$, is given as

$$G(y) = 1 - \left(1 + \frac{c(y-h)}{a}\right)^{-1/c} \quad (1)$$

where h, a and c denote the location, scale and shape parameters, respectively. Generally, the location parameter is taken as zero. The distribution is unbounded, i.e., $0 < y < \infty$ if $c \geq 0$ and bounded as $0 < y < a/c$ if $c < 0$. The exponential DF is a special case of eqn.(1) when $c = 0$. The GPD exhibits a unique threshold-stability property, i.e., if X follows GPD then the conditional distribution of excesses, i.e., $G(y)$, also follows GPD with the same shape parameter as that of X. It can also be shown that the distribution of maximum excesses, i.e., $Z = \max(Y_1, Y_2, \ldots, Y_k)$ follows the generalized extreme value distribution provided that exceedances over the threshold are generated from a Poisson process (Davison and Smith 1990). These elegant mathematical properties have motivated the use of the GPD model in extreme quantile estimation.

The calculation of a quantile value, x_R, corresponding to an R-year return period is based on the quantile of peaks, Y, corresponding to a return period of λR, where λ is the mean exceedance (or crossing) rate per year of X over u. Thus,

$$x_R = G^{-1}\left(1 - \frac{1}{\lambda R}\right) + u \quad (2)$$

where $G^{-1}(p)$ denotes the Pareto quantile function. If n denotes the number of samples collected over m years and k is the number of exceedances, then the mean crossing rate is estimated as $\lambda = k/m$. As the threshold is lowered to include more data in the inference, the crossing rate λ increases due to increasing values of k. An interesting trade-off is thus revealed from eqn.(2), i.e., the more the data considered, the farther in the tail region one has to go for quantile estimation due to increasing values of the effective return period λR. Conversely, by inclusion of additional data the accuracy of tail modelling must increase at a higher rate than the movement farther in the tail region. Otherwise, the accuracy of POT estimates would deteriorate. In this sense, it is expected that an optimal threshold might exists that would results in the minimum error (Caer and Maes 1998, Dougherty et al. 1999).

2.3 *Parameter estimation by De Haan*

De Haan (1994) proposed the estimation of scale and shape parameters using the order statistics of exceedances, $\{X_{n-k,n}, \ldots, X_{n,n}\}$, where $X_{n-k,n}$ is the smallest data point to exceed a given threshold. Based on an extensive mathematical analysis, the shape parameter was derived as

$c = c_1 + c_2$, where $c_1 = M_n^{(1)}$ and

$$c_2 = 1 - \frac{1}{2}\left(1 - \frac{(M_n^{(1)})^2}{M_n^{(2)}}\right)^{-1} \quad (3)$$

in terms of moments of excesses obtained from the log-transformed data:

$$M_n^{(r)} = \frac{1}{k}\sum_{i=1}^{k}[\log(X_{n-i+1,n}) - \log(X_{n-k,n})]^r \quad (4)$$

The scale parameter can be obtained as

$$a = u\frac{M_n^{(1)}}{\rho} \quad (5)$$

where $\rho = 1$ if $c \geq 0$, and $\rho = 1/(1-c)$ if $c < 0$.

Generally speaking, c_1 estimates the shape parameter of unbounded GPD, whereas c_2 corresponds to the bounded case (Reiss and Thomas 1997). The confidence bounds for an estimate of c were also derived based on the argument of asymptotic normality (de Haan 1994).

3 METHOD OF L-MOMENTS

3.1 *Order Statistics and L-Moments*

Using the density function of an r^{th} order statistic, $X_{r:n}$ (Kendall ad Stuart 1977), along with a transformation $u = F(x)$, its expectation can be expressed in terms of the quantile function, $x(u)$, as

$$E[X_{r:n}] = r\binom{n}{r}\int_0^1 x(u)u^{r-1}(1-u)^{n-r}\,du \quad (6)$$

Expectations of the maximum and minimum of a sample of size n can be easily obtained from eqn.(6) by setting $r = n$ and $r = 1$, respectively.

$$E[X_{n:n}] = n\int_0^1 x(u)u^{n-1}du, \quad \text{and}$$

$$E[X_{1:n}] = n\int_0^1 x(u)(1-u)^{n-1}du \quad (7)$$

The probability-weighted moment (PWM) of a random variable was formally defined by Greenwood et al. (1979) as

$$M_{i,j,k} = E[X^i u^j (1-u)^k] = \int_0^1 x(u)^i u^j (1-u)^k du \quad (8)$$

Certain linear combinations of PWMs, referred to as L-moments, are shown to be analogous to ordinary moments in a sense that they also provide measures of location, dispersion, skewness, kurtosis, and other aspects of the shape of probability distributions or data samples (Hosking 1990). An r^{th} order L-moment is mathematically defined as

$$l_r = \sum_{k=1}^{r} p^*_{r-1,k-1} b_k \quad (9)$$

where $p_{r,k}$ represents the coefficients of shifted Legendre polynomials (Hosking 1990). From an ordered random sample of size n, unbiased estimates b_k and a_k of β_k and α_k, respectively, can be obtained as (Hosking and Wallis 1997)

$$b_k = \frac{1}{n}\sum_{i=1}^{n}\binom{i-1}{k}X_i \Big/ \binom{n-1}{k} \quad \text{and}$$

$$a_k = \frac{1}{n}\sum_{i=1}^{n}\binom{n-i}{k}X_i \Big/ \binom{n-1}{k} \quad (10)$$

Normalized forms of higher order L-moments, $\tau_r = \lambda_r/\lambda_2$, ($r$ = 3, 4,. . .) are convenient to work with due to their bounded variation, i.e., $|\tau_r| < 1$. Extensive simulation-based studies and comparison with information-theoretic measures, e.g., cross-entropy and divergence, Pandey et al. (1999) have shown that L-moments are very effective in summarizing distribution properties and extreme quantile estimation.

3.2 *Parameter estimation*

Hosking and Wallis (1997) illustrated that L-moments are efficient in estimating parameters of a wide range of distributions from small samples. The main advantage of L-moments is that, being a linear combination of data, they are less influenced by outliers. In general, the bias of small sample estimates of higher-order L-moments is fairly small. Furthermore, the required computation is quite limited as compared with other traditional techniques, such as maximum likelihood and least-squares.

Figures 1 and 2 illustrate variations of the bias and root-mean-square error (RMSE), respectively, of sample estimates (size 50) of the first three GPD L-moments (λ_1, λ_2, τ_3) with respect to its shape parameter. The bias and RMSE estimates are normalized by exact values of respective L-moments. As seen from Figure 1, bias associated with λ_1 and λ_2 is almost zero and that of τ_3 is fairly small (≤3%). Figure 2 shows that the RMSE of λ_1 and λ_2 increases from 10% to 20% with increasing values of c. The RMSE of τ_3 remains somewhat insensitive to c, and is the order of 20%.

Using 3 sample L-moments, the location (h), scale (a) and shape (c) parameters of the GPD can be estimated as

$$c = \frac{(3t_3 - 1)}{(t_3+1)}, \quad a = (1-c)(2-c)\,l_2 \quad \text{and}$$

$$h = l_1 - (2-c)\,l_2 \quad (11)$$

In case that the location parameter is known, the first two L-moments can be used to estimate c and a as

$$c = 2 - \frac{(l_1 - h)}{l_2} \quad \text{and} \quad a = (1-c)(l_1 - h) \quad (12)$$

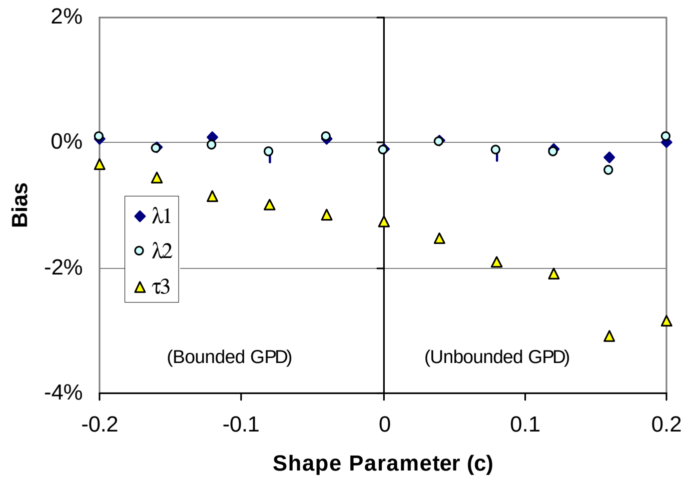

Figure 1: Normalized bias of sample estimates of L-moments (parent: GPD, sample size 50).

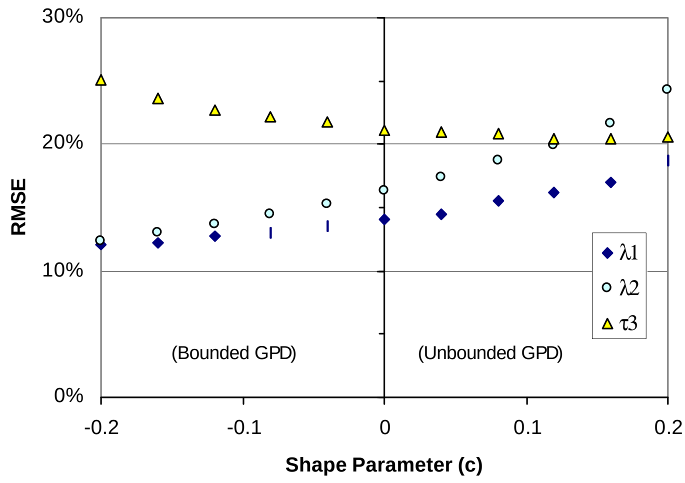

Figure 2: Normalized RMSE of sample estimates of L-moments (parent: GPD, sample size 50).

4 COMPARISON OF SHAPE ESTIMATES

4.1 *Three methods*

The paper presents an alternative approach to the estimation of tail index of extreme value distribution in the POT approach. The proposed estimation method is based on sample L-moments instead of order statistics. The L-moments can be considered as linear combinations of certain expectations of order statistics (Hosking 1990). In contrast with ordinary statistical moments, the L-moments of higher order can be reliably estimated from small samples. It is expected that L-moments could provide more reliable estimates of the tail index with a limited sensitivity to the threshold parameter.

In this study, three methods are considered for the estimation of *c*, namely,

(1) de Haan (DHN) - eqn.(3)
(2) 2 L-moments (2-LM) - eqn.(12)
(3) 3 L-moments (3-LM) - eqn.(11)

To compare the bias and RMSE of various estimates of *c*, a simulation experiment was designed in which samples, each of size of 1000, were generated from a GPD of prescribed parameters. Considering N_p top order statistics and corresponding sample of excesses, the three methods were applied to estimate the shape parameter. The simulation consisted of 10,000 cycles. Numerical results for N_p = 30 and 100 are summarized in Table 1 for various *c* values of the parent GPD.

As seen from Table 1, bias associated with all the three estimates is small (0-6%). In the first case of N_p = 30, the bias of DHN estimate is higher than those of L-moment estimates. In all the cases reported in Table 1, RMSE values of all the three estimates are almost identical. A general observation is that both bias and RMSE are almost insensitive to the shape parameter of the parent GPD used in the simulation. Considering these results, it is anticipated that L-moment estimates of the tail index (*c*) would at least be competitive to those obtained from the de Haan method. In contrast with the de Haan method, the L-moment method is more insightful since L-moments have intuitive meaning, such as Gini's mean difference (λ_2) and skewness. On the other hand, de Haan's formulae based on log-transformed data are of "black-box" nature. Since the distribution of L-moments ratios, λ_2/λ_1 and τ_3, is asymptotically normal (Hosking 1990), the confidence bounds on *c* can also be constructed in principle. This topic is an area of future research.

Table 1: Bias and RMSE the Pareto shape parameter estimated from POT samples (GPD parent, sample size 1000)

POT	Shape	Error Estimates					
Sample	Parameter	De Haan		2-L-Moments		3-L-Moments	
Size	*c*	Bias	RMSE	Bias	RMSE	Bias	RMSE
30	0.2	0.06	0.22	0.04	0.22	0.06	0.23
	0.1	0.06	0.22	0.03	0.22	0.04	0.22
	0.05	0.06	0.22	0.03	0.22	0.04	0.22
	-0.05	0.06	0.23	0.02	0.23	0.03	0.22
	-0.1	0.06	0.23	0.02	0.23	0.02	0.22
	-0.2	0.06	0.24	0.02	0.25	0.02	0.22
100	0.2	-0.02	0.11	0.02	0.12	0.02	0.13
	0.1	-0.02	0.11	0.01	0.12	0.01	0.12
	0.05	-0.02	0.11	0.01	0.12	0.01	0.12
	-0.05	-0.02	0.11	0.00	0.12	0.01	0.11
	-0.1	-0.01	0.12	0.01	0.12	0.01	0.11
	-0.2	-0.01	0.12	0.00	0.13	0.01	0.11

4.2 *Simulations*

To illustrate a trade-off associated with the selection of threshold in the POT method, two scenarios were analyzed via Monte Carlo simulations. In the first case, samples of size *N*=1000 were generated from the Gumbel distribution (*m*=50 mph, σ=6.25), and POT samples were constructed from upper N_p order statistics. Considering the original sample (*N*=1000) as a sample of annual maximum wind speeds, the exceedance rate was estimated as $\lambda = N_p/N$. Wind speed quantiles for return periods (*R*) of 100 to 5000 years were estimated from the POT sample by fitting the Pareto distribution using de Haan and L-moment methods. The simulation consisted of 10,000 cycles. The normalized bias and RMSE of quantile estimates are reported in Table 2 for various values of N_p and *R*. In all the cases, the bias is fairly small (< 3%) and RMSE varies in a narrow range from 2 - 10%, similar to results of Gross et al.(1994).

Table 2: Bias and RMSE of Gumbel quantile estimates (1000 years of simulated data).

POT	Crossing	Return	Quantile Error Estimates					
Sample	Rate/year	Period	De Haan		2-L-Moments		3-L-Moments	
Size	λ	(years)	Bias	RMSE	Bias	RMSE	Bias	RMSE
25	0.025	100	-0.002	0.019	-0.002	0.021	-0.002	0.021
		250	-0.006	0.028	-0.004	0.028	-0.003	0.028
		1000	-0.016	0.053	-0.004	0.048	-0.005	0.048
		5000	-0.025	0.101	0.007	0.104	0.003	0.099
50	0.05	100	-0.005	0.019	-0.002	0.019	-0.001	0.019
		250	-0.009	0.029	-0.003	0.026	-0.002	0.027
		1000	-0.016	0.055	-0.002	0.05	-0.002	0.048
		5000	-0.022	0.097	0.007	0.1	0.003	0.095
100	0.1	100	-0.009	0.02	-0.001	0.018	-0.001	0.018
		250	-0.013	0.031	-0.002	0.027	-0.002	0.026
		1000	-0.019	0.055	-0.002	0.051	-0.002	0.048
		5000	-0.024	0.09	0.001	0.093	0.001	0.087

Table 2 clearly suggests that the GPD representation of POT data is fairly accurate.

In the second case, a more realistic scenario was considered by assuming that N=1000 independent observations of wind speed were collected over m=25 years period such that the exceedance rate becomes $\lambda = N_p/m$ (e.g., Dougherty and Corotis 1998). In the simulation, samples were generated from the Gumbel distribution (m=30 mph, σ=6.5 mph) and quantile bias and RMSE were estimated for the same values of N_p and R as considered before. As seen from Table 3, the bias (20-40%) and RMSE (30-60%) have increased significantly in comparison to the previous case.

Table 3: Bias and RMSE of Gumbel quantile estimates from 25 years of simulated data consisting of 1000 observations.

POT Sample Size	Crossing Rate/year λ	Return Period (years)	Quantile Error Estimates: De Haan Bias	De Haan RMSE	2-L-Moments Bias	2-L-Moments RMSE	3-L-Moments Bias	3-L-Moments RMSE
25	1	100	-0.031	0.123	0.009	0.126	0.003	0.119
		250	-0.033	0.167	0.028	0.188	0.017	0.176
		1000	-0.029	0.249	0.078	0.333	0.056	0.308
		5000	-0.008	0.38	0.18	0.621	0.141	0.571
50	2	100	-0.03	0.119	0.01	0.121	0.006	0.115
		250	-0.03	0.154	0.022	0.17	0.016	0.161
		1000	-0.025	0.219	0.051	0.267	0.041	0.253
		5000	-0.009	0.316	0.105	0.431	0.089	0.41
100	4	100	-0.037	0.111	0.004	0.113	0.003	0.106
		250	-0.038	0.139	0.009	0.148	0.008	0.139
		1000	-0.036	0.187	0.021	0.214	0.019	0.199
		5000	-0.029	0.253	0.044	0.31	0.039	0.288

The increased error can be attributed to large values of the exceedance rate ($\lambda\approx$1-4), because the previous case corresponds to fairly small values of λ (λ<<1). A comparison of results presented in Tables 2 and 3 confirms our comments of Section 2.1 about a trade-off between the accuracy and amount of data in the POT method. In case of relatively short time series (20-50 years), the selection of right threshold is important because inclusion of unnecessary data could result in the loss of accuracy of quantile estimates. In short, hyper-sensitivity of quantile estimates to the threshold value is a major practical difficulty associated with the POT method.

Table 3 suggests de Haan method results in the least bias and RMSE, followed by the 3 L-Moment method. However, the differences in bias and RMSE associated with the three methods are limited (within 10%), and would change differently with variation in the POT sample size. Similar simulations can be performed for parent distribution other than the Gumbel, though the qualitative conclusion of the section is not expected to change.

4.3 *Case study*

To compare the performance of de Haan and L-moment methods in a more realistic setting, actual wind speed data collected at various stations the United States were analyzed. The method of L-moment was added to a computer program provided by Heckert and Simiu (1996), which generates an uncorrelated sample from the original wind data based on eight-day (four-day) interval and applies the de Haan method for GPD parameters estimation. Here, numerical results are presented for one station, namely, Helena (MT). This station is chosen for no particular reason other than the fact that it represents a typical long (35 years) term data in the available data-base of wind speeds.

Estimates of the tail shape parameter, c, obtained from Helena, is plotted against the threshold velocity in Figure 3.

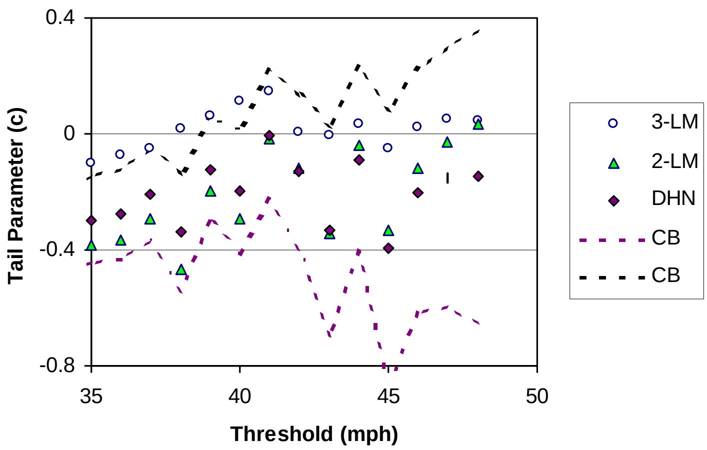

Figure 3: Tail parameter estimates using wind speed data from Helena, MT.

The 95%-confidence bounds on de Haan estimates obtained as $c\pm2s$, s being the standard deviation of c, are also shown in this figure. The formulae for s are already summarized by Heckert and Simiu (1996). Note that increasing values of the threshold velocity (u) on the X-axis implies decreasing size of the POT sample.

It is interesting to note that 3-LM estimates of c tend to be close to the upper confidence bound, whereas 2-LM estimates are fairly close to de Haan estimates (DHN). The variation of 3-LM estimates with the threshold exhibits more stable trend than those obtained from DHN and 2-LM methods. It is somewhat expected since the L-skewness, which determines c in 3-LM method, tends to be less sensitive to variations in the POT sample.

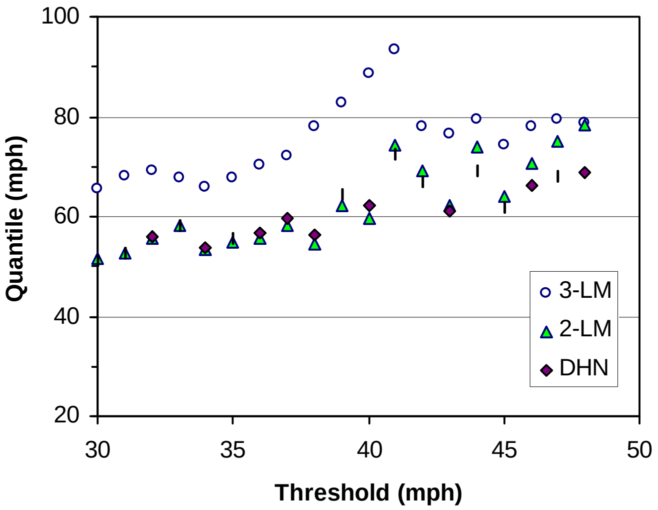

Figure 4: Quantile estimates using wind speed data from Helena, MT (*R*=1000 years)

In Figure 4, quantiles estimates for 1000 year return period are plotted against the threshold velocity. Note that 2-LM and DHN estimates are in very close agreement. 3-LM quantile estimates follow a smoother trend, which is helpful in identifying the representative threshold and quantile value. From Helena, a 3-LM quantile estimate corresponding to horizontal-stable part of the curves is roughly 80 mph.

5 CONCLUSIONS

An accurate estimation of parameters of the Pareto distribution (GPD) in the peaks-over-threshold approach is of considerable importance. The paper examines the method of L-Moments and compares its performance against a widely acceptable method of de Haan (1994). In the de Haan method, the first two moments of peaks (or excesses) of log-transformed data are used for parameter estimation, whereas the L-moment method utilizes linear combinations of expectations of order statistics of peaks in the original data. Two variations of the L-moment method are discussed, namely, 2-LM and 3-LM, which incorporate 2 and 3 L-moments, respectively, in the parameter estimation. Despite the procedural differences, it is interesting to note that the de Haan and 2-LM estimates of the tail shape parameter (c) are in fairly close agreement in simulated data (Tables 2 and 3) as well as in actual wind speed data collected in the United States (Figures 3-5). In this respect, an equivalence between the 2-LM and de Haan methods is a notable point of the paper. Furthermore, the L-moment method is more insightful because of intuitive meaning of L-moments, such as Gini's dispersion coefficient (λ_2) and skewness.

In the 3-LM method, higher order estimates of the shape parameter are obtained using the L-skewness of POT data. The analysis of actual wind speed data illustrates that 3-LM estimates of c tend to follow a more stable trend in the proximity of the upper bound estimates obtained from the de Haan method. Therefore, 3-LM method appears to be useful in identifying meaningful upper bounds of design quantiles.

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