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Stochastic simulation of episodic soft coastal cliff recession

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Abstract

Probabilistic methods provide a means of demonstrating the potential variability in predictions of coastal cliff recession. They form the basis for risk-based land use planning, cliff management and engineering decision-making. A range of probabilistic methods for predicting soft coastal cliff recession has now been developed, including statistical techniques, process-based simulation and structured use of expert judgement. A new episodic stochastic simulation model is introduced, which models the duration between cliff falls as a gamma process and fall size as a log-normal distribution. The method is applied to cliff recession data from a coastal site in the UK using maximum likelihood and Bayesian parameter estimation techniques. The Bayesian parameter estimation method enables expert geomorphological assessment of the local landslide characteristics and measurements of individual cliff falls to be combined with sparse historic records of cliff recession. An episodic simulation model is often preferable to conventional regression models, which are based on assumptions that are seldom consistent with the physical process of cliff recession.

Keywords: Coastal erosion; Stochastic simulation; Risk analysis

1. Introduction

Recession of soft coastal cliffs and bluffs presents significant risks to land use and development (Lee, 1995, 1997a; Lee and Clark, 2002). Sunamura (1992) reports that chronic problems of average cliff recession rates in excess of 1 m per year are experienced at coastal sites in Denmark, Germany, Russia, Japan, New Zealand, Canada, the UK and the USA. Generally, this will involve small-scale failures, but occasionally large-scale events do occur, as at Holbeck Hall near Scarborough on the East Coast of the UK when around 90 m of land was lost almost overnight (Clark and Guest, 1994). Often, problems have arisen as a result of a lack of coordination between land use planning and coastal defence strategy. Many coastlines suffer from an inheritance of communities built on eroding cliff tops and bluffs.

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An awareness of the possible cliff position at some future date is fundamental to coastal planning and shoreline management. Dependable predictions of future recession rates are needed to support the formulation of land use planning policies that seek to avoid development in vulnerable areas. In those situations where coast protection works or improvements may be required, future recession rates are needed to evaluate scheme options and to test their cost effectiveness.

Approaches to predicting cliff recession range from extrapolation of historic recession data to methods which rely on understanding of the cliff recession process, for example, methods that relate incident wave energy, water levels and cliff strength to recession rate (Gelinas and Quigley, 1973; Thornton et al., 1987; Kamphuis, 1987; Mano and Suziki, 1999; Kirk et al., 2000). Bray and Hooke (1997) presented a variety of empirical-based methods, including the modified Bruun Rule (Bruun, 1962) for predicting recession rates with accelerating sea-level rise. On the whole, these approaches provide deterministic predictions of cliff recession, but can be criticised on the grounds that they do not reflect the potential uncertainty and variability in the cliff recession process. Cliff recession predictions are uncertain for several fundamental reasons.

- Historic data on cliff recession is usually scarce and of questionable quality.
- Cliff recession is driven by storm (wave and rainfall) events, which in the long term can only be predicted in statistical terms.
- Cliff recession is influenced by coastal changes on a local and macro scale. Future conditions may vary from historical conditions.
- The cliff recession process is complex. Relevant parameters (for example, ground water levels and soil shear strength) can vary within the cliff and with time. They may be difficult and costly to sample.

2. Probabilistic prediction of coastal cliff recession

Increasingly, the uncertainty in recession predictions is being expressed in probabilistic terms. Probabilistic predictions form the basis for risk-based cliff management. Methods of risk-based economic appraisal of coast protection schemes that make use of probabilistic prediction of cliff recession have been proposed for use within the UK regulatory framework (Hall et al., 2000a; MAFF, 2000). This move towards probabilistic prediction of coastal cliff recession is consistent with established probabilistic methods for risk assessment and design of flood defences (Bakker

Table 1
Summary of applicability of probabilistic recession prediction methods

	Historic record	Recession process	Stability of conditions	Complexity/effort of analysis
Regression analysis	Based on historic recession data	Reasonably continuous erosion	Future conditions must resemble past conditions	Routine statistical method, which is incorporated in most spreadsheet packages.
Stochastic simulation of episodic recession	Based on historic recession data	Multiple recession episodes	Future conditions must resemble past conditions	Requires Monte Carlo simulation. Maximum likelihood parameter estimation is computer-intensive. Bayesian parameter estimation uses geomorphological analysis to select prior parameter distributions.
Process simulation models	Requires good historic recession data for calibration	Any mechanism provided it can be described in mathematical terms	Can simulate changing future conditions	Considerable expertise and information (cliff and beach geometry and material properties, loading conditions, etc.) needed.
Structured use of expert judgement	Informed by historic evidence when available	Used for all types of cliff but predominates at sites where only the next landslide is of interest	Can be used for interpretation of changing future conditions	Varies. Requires special skills to elicit dependable expert judgements.

and Vrijling, 1980; CUR/TAW, 1990; Vrijling, 1993; Reeve, 1998) and breakwaters (Van der Meer, 1988; CIRIA/CUR, 1991; Burcharth, 1992, Vrijling and Van Gelder, 1996; Burcharth, 1997; Vrijling et al., 1999) and probabilistic prediction of beach erosion (Vrijling and Meijer, 1992; Reeve and Fleming, 1997; Dong and Chen, 1999).

The range of probabilistic cliff recession prediction methods that is now available is summarised in Table 1. The methods can be thought of a ranging from data-oriented approaches, which rely primarily on statistical analysis of past cliff recession data (Dolan et al., 1991a,b), to process-oriented approaches, which rely on expert interpretation and modelling of the physical process of cliff recession (Lee, 1998; Lee et al., 2001). While these two approaches have tended to predominate in the past, best use is made of scarce information about cliff recession by combining the two approaches. Thus, stochastic analysis should draw on an understanding of the geomorphology of the shore and cliff, termed 'cliff behaviour' (Lee, 1997b; Lee and Clark, 2002). Intermediate classes of prediction method are therefore emerging. Advances in computing power and numerical model descriptions of the cliff recession process mean that numerical models of the cliff recession process can now be run in a Monte Carlo mode to develop ensemble predictions of cliff recession (Meadowcroft et al., 1999; Hall et al., 2000b; Walkden et al., 2001).

The focus of this paper is another intermediate class of models that are strongly data-based, addressing the cliff recession process at a more abstract level than detailed process-based simulation models, while still having a clear physical interpretation. First, regression analysis for generating probabilistic predictions from historic data is reviewed before introducing a stochastic method for simulating episodic cliff recession.

3. Simple linear regression analysis

The most straightforward approach to predicting cliff recession using historic data is a continuous linear model (Crowell et al., 1997; Amin and Davidson-Arnott, 1997):

$$X_t = \beta_0 + \beta_1 t + \varepsilon$$

where X_t is the recession distance at time t and ε is a random variable that has a Gaussian distribution with zero mean and variance v. Hence, the distribution of X_t will be Gaussian with mean $\beta_0 + \beta_1 t$ and variance v. The maximum likelihood estimators for β_0 and β_1 can be found from linear regression theory. Dolan et al. (1991a,b) compare a number of linear methods of characterising shoreline rate of change. Milheiro-Oliveira and Meadowcroft (2001) suggest a similar model based on a Wiener type dynamical process (Singpurwalla, 1995). Crowell et al. (1997) also examined quadratic and cubic recession models but found that they were not preferable to linear regression and can be extremely inaccurate.

Besides assuming that the regression of X_t is a linear function of t, there are three further assumptions about the joint distribution of X_{t_i}, the realisations of X_t at time t_i, where $i = 1, \ldots, n$.

(i) Each variable X_{t_i} is normally distributed.
(ii) The variables $X_{t_1}, \ldots, X_{t_n}$ are independent.
(iii) The variables $X_{t_1}, \ldots, X_{t_n}$ have the same variance v.

The normality assumption is difficult to justify since the cliff position changes monotonically, i.e. recession cannot be recovered. The episodic method proposed in the following section does not permit the possibility of negative recession so is preferable to regression analysis based on a Gaussian distribution of residuals.

There is likely to be some correlation between successive observations, depending particularly on the relationship between the sampling interval and the characteristic time between landslides at the site in question. A large sampling interval will generate data that will more closely resemble a series of independent observations, whereas if the sampling interval is small or the characteristic cliff-forming interval is large, there may be very significant correlation between consecutive measurements. However, more often than not, the historic record of coastal cliff recession is short in statistical terms, and a large sampling interval will yield a very small population for statistical analysis. Linear regression is therefore most appropriate in situations where the recession is dominated by an ongoing removal of cliff material or frequent landslides.

4. Stochastic simulation of episodic cliff recession processes

The assumptions of a linear regression model are most difficult to sustain at sites that are characterised by strongly episodic recession processes. A more attractive approach can be developed by postulating cliff recession as an episodic random process. In other words, cliff recession is assumed to proceed by means of a series of discrete landslides, the size and frequency of which are modelled as random variables. The episodic process is complex and does not conform directly to the models of random processes used widely in stochastic hydraulics. Landsliding is not an inevitable consequence of the arrival of a storm that removes material from the cliff toe or raises groundwater levels in the cliff. In order to fail, the cliff must already be in a state of deteriorating stability, which makes it prone to the effects of an initiating storm. Of particular significance are

- the triggering and preparatory factors (i.e. the causes),
- the size and type of recession events (i.e. the retrogression potential) and
- the timing and sequence of recession events (i.e. the recurrence interval).

A discrete model for the cliff recession X_t during duration t that includes these issues is

$$X_t = \sum_{i=1}^{N} C_i \tag{1}$$

where N is a random variable representing the number of cliff falls that occur during duration t and C_i is a random variable representing the magnitude of the ith recession event.

This model can be used to simulate synthetic time series of recession data by sampling from two distributions: the landslide timing distribution, which represents the duration between recession events, and the landslide size distribution. Three typical realisations of the model are shown in Fig. 1. The time series are stepped reflecting the episodic nature of the cliff recession process. Probabilistic predictions of either recession distance X_t in a given duration t, or time T_x to recess a given distance x can be extracted from multiple realisation of the simulation model.

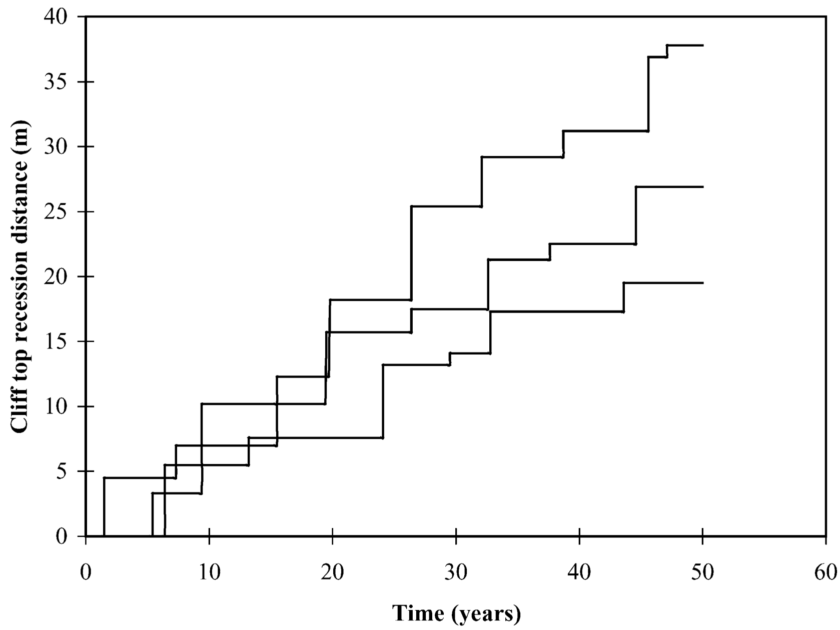

Fig. 1. Typical realisations of the two-distribution simulation model.

4.1. The landslide timing distribution

The duration between successive coastal landslides is modelled as a gamma distribution. The model incorporates understanding of the cliff recession process by representing the role which storms have in destabilising cliffs. The approach has links to renewal theory (Cox, 1962) inasmuch as the cliff is considered to be progressively weakened by the arrival of storms. The arrival of damaging storms is assumed to conform to a Poisson process (Borgman, 1963; Shane and Lynn, 1964), i.e. successive storms are assumed to be independent incidents with a constant average rate of occurrence but random actual intervals between storms. This is a reasonable assumption provided the threshold for storm damage is sufficiently high. The times T_i, $i=1, 2, \ldots, k$, between storms conform to an exponential distribution. After a number of storms of sufficient severity, landslide occurs. Suppose that it takes k storms of given severity to cause a recession event. The time to the kth storm is the sum $T_1+T_2+\ldots+T_k$. The time between successive landslides is therefore gamma distributed with density function:

$$f_T(t \mid k, \lambda) = \frac{\lambda^k t^{k-1}}{\Gamma(k)} \mathrm{e}^{-\lambda t} \tag{2}$$

where $\Gamma(k)$ is the gamma function. The shape of this distribution is defined by a scaling parameter λ (the reciprocal of the return period of the significant storm event) and a shape parameter k (the average number of storms above a certain threshold which cause damage to the toe of the cliff that is sufficiently severe to trigger failure). The model therefore has the ability to differentiate between high and low sensitivity cliffs by representing the number and magnitude of storms needed to initiate a landslide. The mean of the gamma distribution is k/λ and the variance is k/λ^2.

An analogous approach has been adopted in investigations of the process of rock displacement on berm breakwaters with a modified generalised gamma process (Van Noortwijk and Van Gelder, 1996) based on the probability of breaches in the armour layer and, given a breach has occurred, the uncertainty in the rate of longshore rock transport. Vrijling and Van Gelder (1996) showed the very high sensitivity in the formulae describing longshore rock transport of berm breakwaters to uncertain parameters of the wave climate, an issue that is addressed presently in this paper in the analysis of parameter uncertainty.

4.2. The landslide size distribution

The landslide size distribution is the distribution of cliff top recession distance in each landslide, measured perpendicular to the shore. Rarely is there sufficient site data to conclusively identify a distribution type, though modern high resolution monitoring of cliff recession (Balson and Tragheim, 1999) may mean that landslide size distributions are available for some sites in future. In the absence of field information, experimental studies of marine erosion of cliffs have been used to establish the form of the landslide size distribution. Damgaard and Peet (1999) conducted wave basin tests on a model cliff made from damp sand. The three experiments yielded populations of 86, 35 and 103 landslides, the size of which was subsequently measured from video records of the tests. A variety of distributions were optimally fitted to the experimental data. The best fit was obtained from a log-normal distribution:

$$f(x \mid \mu, \sigma) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left[\frac{-(\ln x - \mu)^2}{2\sigma^2}\right] \tag{3}$$

The mean of this distribution is $\exp(\mu+0.5\sigma^2)$ and the variance is $\exp(2\mu+\sigma^2)[\exp(\sigma^2)-1]$. The log-normal hypothesis was not rejected in chi-squared tests at significance values of 10%, 5% and 1%, respectively, for the three sets of experimental test results. The log-normal model also conforms to the physical constraint that it should be nonnegative, corresponds to the nonexistence of negative cliff recession.

4.3. Empirical density estimation

Monte Carlo simulation is used to generate multiple realisations of the recession process over the duration of interest (Eq. (1)). A kernel density method is used to generate a smooth probability density function either at given recession distance x or duration t from multiple simulated time series of recession. Suppose predicted recession distances $x_1, \ldots, x_s$ have been obtained from s realisations of the simulation model over t years, then the kernel density estimate at recession distance x (given time t) is given by

$$\hat{f}(x \mid T=t) = \frac{1}{sh}\sum_{i=1}^{s} K\frac{x-x_i}{h} \tag{4}$$

where K is the kernel function and h is the smoothing parameter. It can be shown (Silverman, 1986) that the kernel that in general minimises the approximate mean integral square error in the density estimate is Epanechnikov kernel $K_e(t)$

$$K_e(t) = \begin{cases} \frac{3}{4\sqrt{5}}\left(1-\frac{1}{5}t^2\right) & \text{for} -\sqrt{5}\leq t\leq\sqrt{5} \\ 0 & \text{otherwise} \end{cases}. \tag{5}$$

An estimate of the optimal smoothing parameter h_{opt} (which is reasonably robust to skewness in the underlying distribution) is given by

$$h_{opt} = 0.9\min\left[\sigma_e, \frac{r}{1.34}\right]s^{-1/5} \tag{6}$$

where σ_e is the standard deviation and r is the interquartile range of the distribution (Silverman, 1986).

The steps in the proposed simulation methodology are summarised in Fig. 2. The simulation model is characterised by four parameters, λ and k from the gamma distribution, and μ and σ from the log-normal distribution. These parameters have to be estimated based on information from the cliff site in question. Two parameter estimation methods are proposed. In the following section, the method of maximum likelihood is used to estimate the model parameters based only on scarce measurements of past recession at an example site. In Section 4.5, a Bayesian method is proposed which uses the past recession data to condition prior parameter estimates based on expert assessment of the cliff recession process at the site.

4.4. Maximum likelihood estimation of model parameters

A method of maximum likelihood for parameter fitting is illustrated schematically in Fig. 3 and demonstrated here using historical recession data for 20-m high cliffs at Cliff End in Sussex on the South Coast of England. The position of the cliff top was obtained from 1:2500 scale historical maps dating from 1907, 1929, 1936, 1962 and 1991. Cliff top locations were extracted at eight positions along the coast, covering a total length of about 400 m. There are therefore 32 recession distances $x_{1,1}, \ldots, x_{i,j}, \ldots, x_{8,4}$ covering the eight positions $i=1, \ldots, 8$ and four durations $j=1, 2, 3, 4$ (Table 2), a fairly typical quantity of data for a coastal cliff site. Note that for each time interval, there were some sections that showed no recession at all, indicating that there had been no landslide episodes in that time interval. The samples at eight positions $i=1, \ldots, 8$ are regarded as independent and identically distributed. For further discussion of spatial autocorrelation between shoreline erosion measurements and guidance on optimal sample distances, see Dolan et al. (1991a,b).

If the simulation model parameters λ, k, μ and σ are written as a vector Θ, the sample likelihood is given by

$$L(\Theta \mid x_{1,1},\ldots,x_{i,j},\ldots,x_{8,4}) = \prod_{j=1}^{4}\prod_{i=1}^{8} f_{XT}(x_{i,j} \mid \Theta)$$

where $f_{XT}(x_{i,j}|\Theta)$ is the probability density at cliff section i and duration j, estimated using the simulation and kernel density method outlined above (Fig. 2).

The maximum likelihood estimate of the model parameters is

$$\max_{\Theta} L(\Theta \mid x_{1,1},\ldots,x_{i,j},\ldots,x_{8,4}) \quad \text{subject to } \Theta\geq 0.$$

The vector Θ that maximises likelihood identifies the statistical model from the family of models of given form, which is most likely to have generated the measured data. Because it is a product of 32 density estimates, the likelihood function is rather sensitive to small variations in the density estimate. A less sensitive likelihood estimate can be obtained using a log-likelihood function

$$\log[L(\Theta \mid x_{1,1},\ldots,x_{i,j},\ldots,x_{8,4})] = \sum_{j=1}^{4}\sum_{i=1}^{8}\log[f_{XT}(x_{i,j} \mid \Theta)]. \tag{7}$$

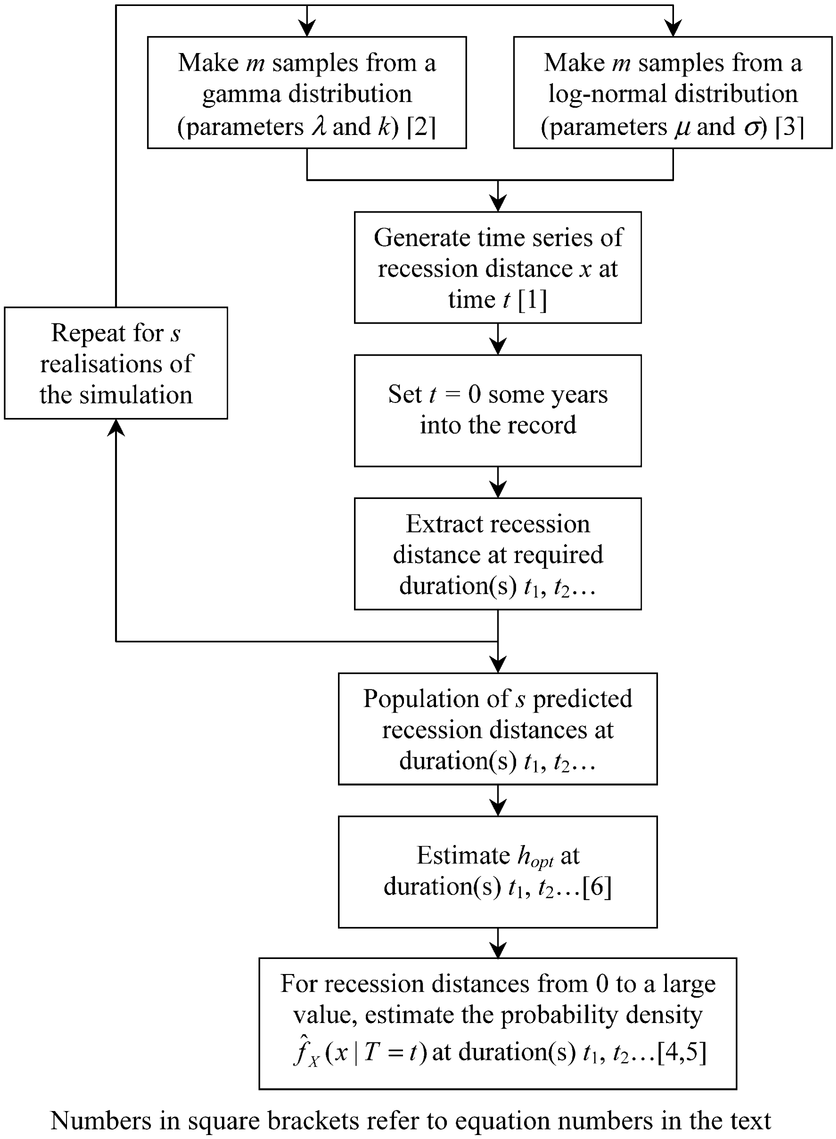

Fig. 2. Steps in generating simulation predictions of cliff recession.

An assessment of the form of this function for each of the parameters in Θ was conducted by randomly sampling sets of parameter values and calculating the corresponding simple likelihood (Fig. 3). Multiple test of the log-likelihood projected onto the model parameters (transformed to represent recognisable dimensions) is shown in Fig. 4. The simulation approach to locating the maximum likelihood parameter estimates is more robust than endeavouring to maximise the likelihood function using numerical optimisation techniques. Note from Fig. 4 that the likelihood surface has local maxima, which could prove to be problematic for numerical optimisation routines.

The maximum likelihood parameter estimates for the data set in Table 2 are listed in Table 3. The parameters of the gamma distribution correspond to a mean time between landslides of 51 years and a standard deviation of 4 years. Note from Fig. 4 that

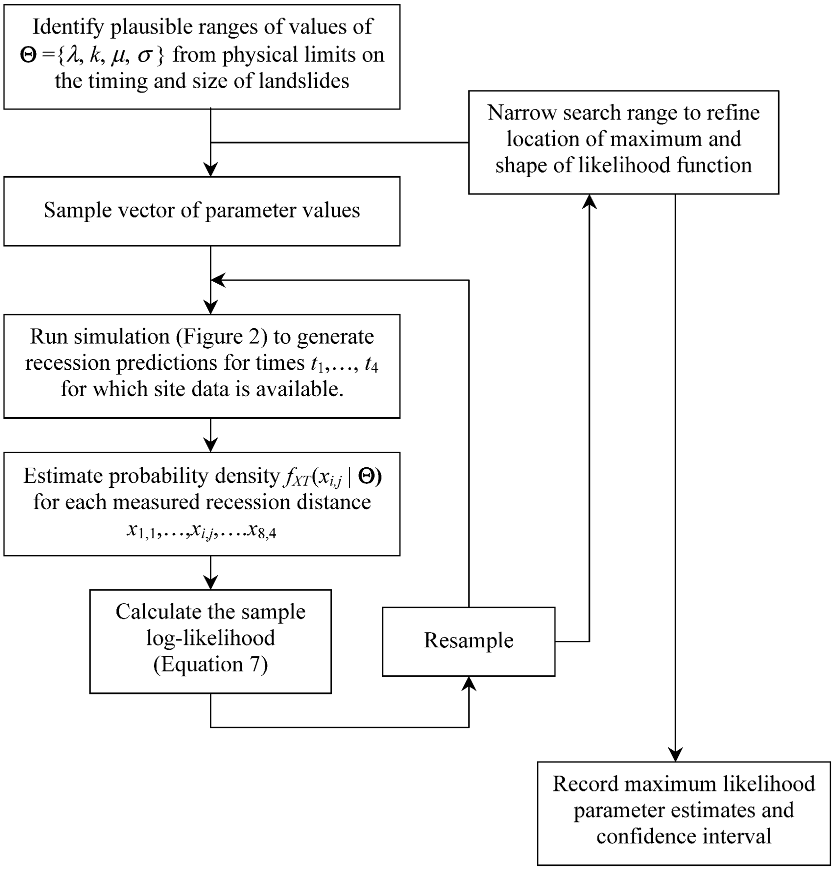

Fig. 3. Steps in maximum likelihood estimation of simulation model parameters.

this rather low standard deviation of 4 years is close to the lower limit of the likely parameter range. The parameters of the log-normal distribution correspond to a mean landside size of 27 m and a standard deviation of 13 m.

Table 2
Cumulative recession distances $x_{i,j}$ (in m) at four different durations since 1907 and at eight cliff sections

Cliff section i	1929 $t=22, j=1$	1936 $t=29, j=2$	1962 $t=55, j=3$	1991 $t=84, j=4$
1	2	12	47	47
2	46	60	68	70
3	36	40	40	50
4	20	24	30	39
5	14	16	24	32
6	0	2	6	16
7	2	2	16	21
8	28	28	42	42
Mean	18.5	23.0	34.1	39.6

The predicted distributions of recession distance over the four durations in the calibration data set, based on the maximum likelihood parameter estimates (Table 3), are illustrated in Fig. 5. Note how the bimodal predictions reflect the possibility that no landslides will take place, perhaps over very long durations.

By drawing the upper envelope of the projections of the likelihood function on the model parameters $\Theta=(\theta_1, \ldots, \theta_n)$, it is possible to estimate their confidence limits. Write $l(\Theta)=\log[L(\Theta)]$. If $l(\theta_i)$ is the upper envelope of the projection of $l(\Theta)$ on θ_i, i.e.

$$l(\theta_i) = \max_{\theta-i} l(\theta_i, \theta_{-i}) \tag{8}$$

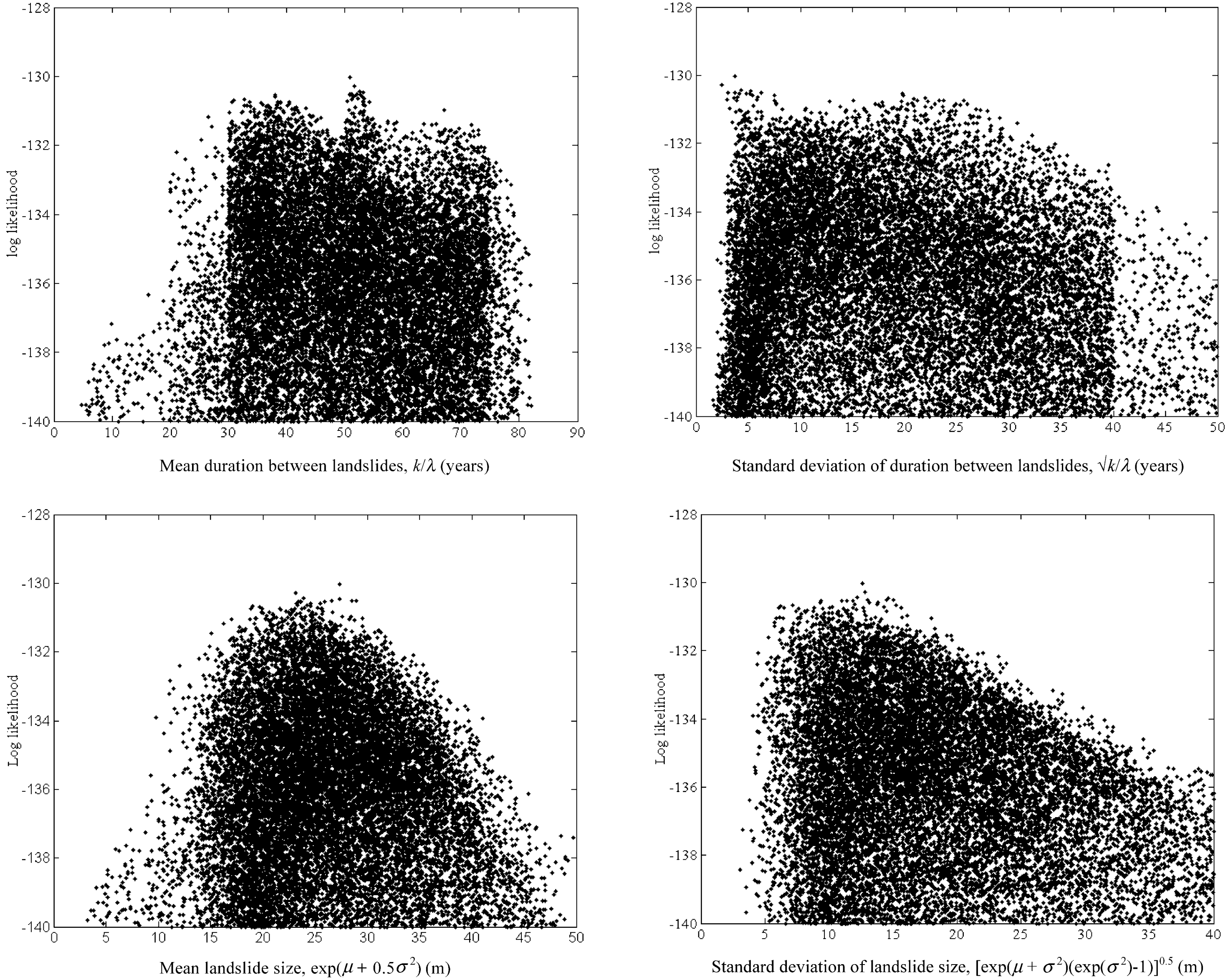

Fig. 4. Log-likelihood estimates for multiple sampled parameter sets.

Table 3
Maximum likelihood parameter estimates and confidence intervals

Distribution	Parameter	Maximum likelihood parameter estimate	95% confidence interval
Gamma	λ	3.64	[0.03, 12.5]
Gamma	k	186	[1.0, 645]
Log-normal	μ	3.21	[2.6, 3.4]
Log-normal	σ	0.44	[0.3, 0.7]

and $\hat{\theta}_i$ is the maximum likelihood estimate of θ_i, then $2[l(\hat{\theta}_i)\text{-}l\hat{\theta}_i] \sim \chi_1^{\,2}$ (Cox and Snell, 1989; Coles, 2001). The confidence interval is therefore obtained from the 95% quantile ($c_{1,0.05} = 3.84$) of the $\chi_1^{\,2}$ distribution, so the 95% confidence interval is obtained at $l(\hat{\theta}_i) - 0.5 \times c_{1,0.05}$. These confidence intervals are listed in Table 3. Note how the parameters of the log-normal distribution are quite precisely identified, while the parameters of the gamma distribution have somewhat larger confidence intervals.

Fig. 6 illustrates how the variance (due to random fluctuations in the density estimate) of the log-likelihood estimator (Eq. (7)) reduces with increasing number, s, of realisations of the simulation model. Observe that at 10^5 realisations of the time series, the variance on the log-likelihood function is only 0.1, which does not represent a significant inaccuracy in the parameter confidence intervals. Fig. 6 also illustrates how the optimum smoothing parameter in the kernel density estimator (Eq. (6)) reduces with s.

4.5. Bayesian parameter estimation

In Bayesian analysis (Box and Tiao, 1973), parameters (in this case Θ) are treated as random variables, i.e. they are assigned probability distributions. This contrasts with likelihood approach described above where it was implicitly assumed that the parameters were constant, albeit unknown. Furthermore, in Bayesian analysis, it is accepted that beliefs about Θ as well as data can be used to estimate Θ. Expert beliefs about Θ are encoded in a prior distribution of Θ, which is then combined with measured data using Bayes' theorem. Bayesian approaches therefore provide the opportunity to merge expert knowledge on the cliff recession process with statistical evidence. The statistical simulation method introduced in this paper lends itself to the application of a Bayesian approach because the model parameters have a clear physical interpretation in terms of the size and timing of cliff falls. Thus, geomorphological analysis of the cliff form and process, informed perhaps by recent observations of cliff falls, can be used to generate

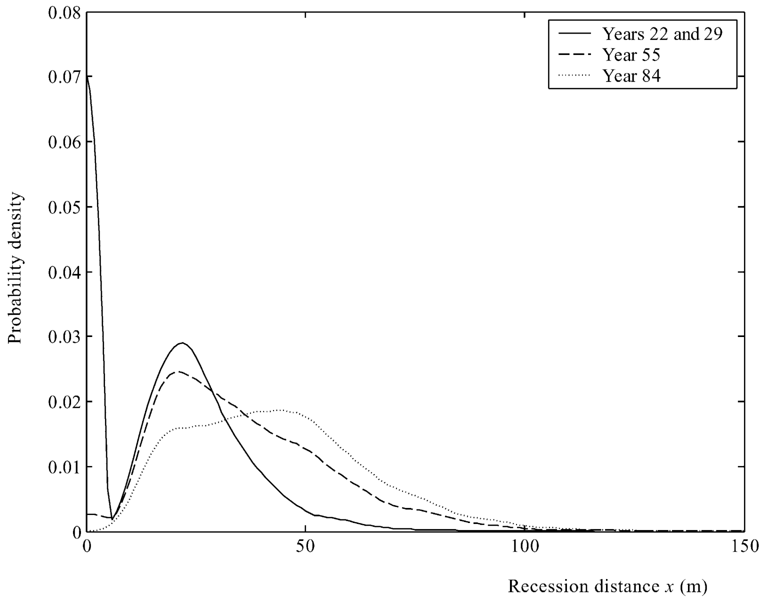

Fig. 5. Recession predictions from simulation model using maximum likelihood parameter estimates ($\lambda = 3.64$, $k = 186$, $\mu = 3.21$, $\sigma = 0.44$).

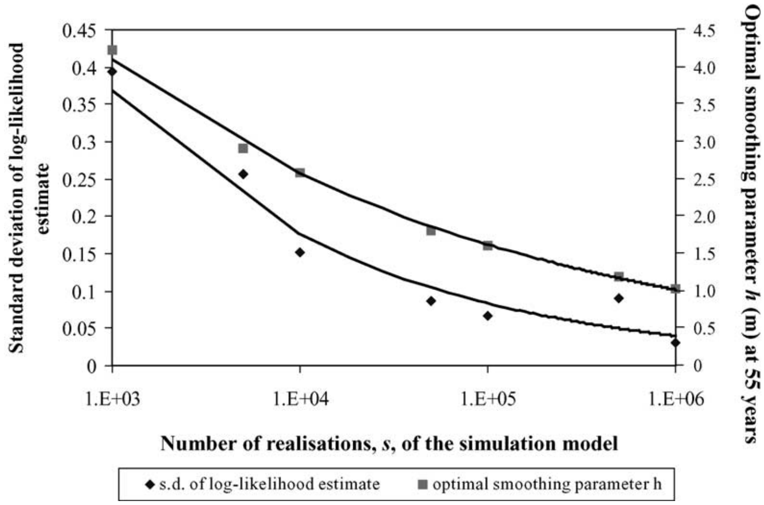

Fig. 6. Example of the relationship between variance in the log-likelihood estimator and the number of realisations of the simulation model.

prior estimates of the mean size and variance of the cliff falls at the site, and the frequency and severity of storms required to initiate cliff falls. Moreover, the historic data set (Table 2) contains no information on the size and frequency of individual cliff falls or on the associated storm conditions, but some information on individual cliff falls and storm loading will often be available from more intensive recent monitoring. The Bayesian approach therefore enables the sparse long-term records of cliff recession to be combined with the higher resolution short-term measurements of individual falls. If the prior parameter estimates are generated primarily from expert judgement, information on the 'calibration' of the experts, which may be assessed from their performance in tests of judgement, can be used to estimate the variance on the prior parameter estimates (Yates, 1990; Cooke, 1991).

Let $f(\Theta|\xi)$ denote the prior density of the model parameters Θ, where ξ is a vector of the parameters of this prior distribution. The joint posterior parameter distribution, given the observations in Table 2, is

$$f(\Theta \mid x_{1,1},\ldots,x_{i,j},\ldots,x_{8,4}) = C\cdot f(\Theta \mid \xi)\cdot L(\Theta \mid x_{1,1},\ldots,x_{i,j},\ldots,x_{8,4}) \quad (9)$$

where C is a normalising constant. Having estimated the posterior parameter distribution, the final stage in Bayesian analysis is to calculate the predictive distribution of cliff recession at time t, which can be thought of as a combination of model predictions based on all parameter values, weighted by the posterior probability of these values. The predictive distribution of cliff recession at time t is

$$f(x \mid T=t) = \int_{\Theta} f(x \mid \Theta, T=t) \times f(\Theta \mid x_{1,1},\ldots,x_{i,j},\ldots,x_{8,4})\mathrm{d}\Theta \quad (10)$$

At the site in question, recent observations and expert geomorphological assessment indicated that cliff falls would be expected to be caused, on average, by two storms with a return period of 10 years, i.e. $\lambda=0.1$ and $k=2$. Recent observations suggested a mean fall size of 3 m and a standard deviation of 3 m, i.e. $\mu=0.75$ and $\sigma=0.83$. The parameters λ, k and σ cannot be negative so were assigned log-normal prior distributions, while μ was assigned a normal prior distribution. The prior distributions were assumed to be independent so

$$f(\Theta \mid \xi) = \prod_{i=1}^{4} f_i(\theta_i \mid \xi). \quad (11)$$

Note that these estimates differ somewhat from the maximum likelihood parameter estimates (Table 3). The Bayesian approach provides a mechanism for combining and reconciling the information provided by the scarce data set with uncertain expert judgements and complementary measurements. The prior and posterior marginal parameter distributions are illustrated in Fig. 7. Note that the parameters λ and k from the gamma distribution were not precisely defined from the sample likelihood. The prior distribution has therefore not been greatly influenced by conditioning on the data. The parameter μ of the log-normal distribution has, meanwhile, been significantly shifted by the Bayesian conditioning. The prior estimate of σ was close to the maximum likelihood estimate so has remained stable. While independence was assumed for the prior parameter estimates, the posterior estimates shown in Fig. 7 are marginals of a joint parameter distribution and are not independent. The predictive distributions (Eq. (9)) for cliff recession distance over 22, 29, 55 and 84 years are illustrated in Fig. 8 and can be compared with the predicted distributions obtained using the maximum likelihood parameter estimates (Fig. 5). The Bayes predictions can be thought of as a probability-weighted combination of the predictions based on many parameter sets and, consequently, can be observed to be rather smoother than the predictions based on maximum likelihood parameter estimates. The mean predicted recession distances from the

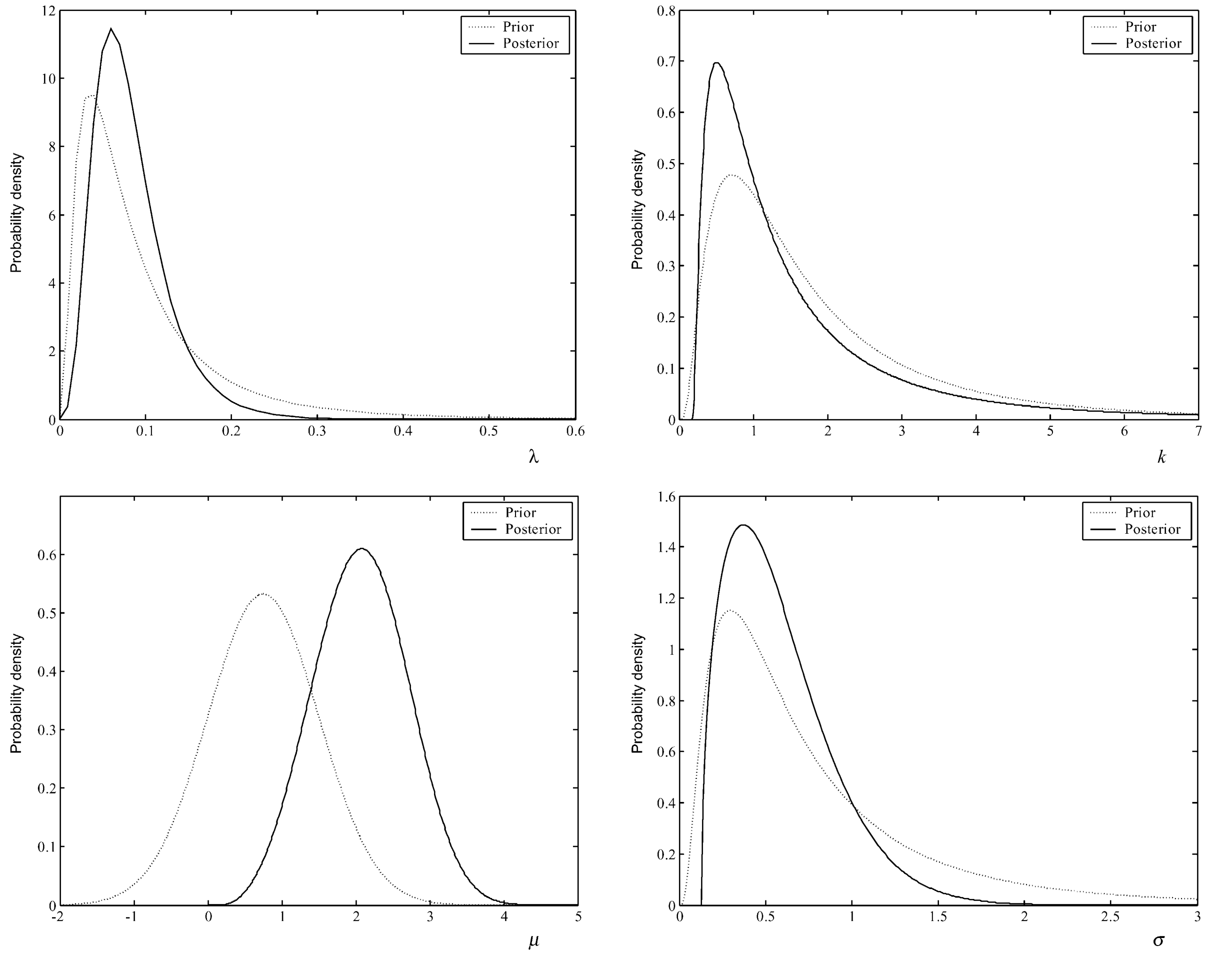

Fig. 7. Prior and posterior marginal parameter distributions for λ, k, μ and σ.

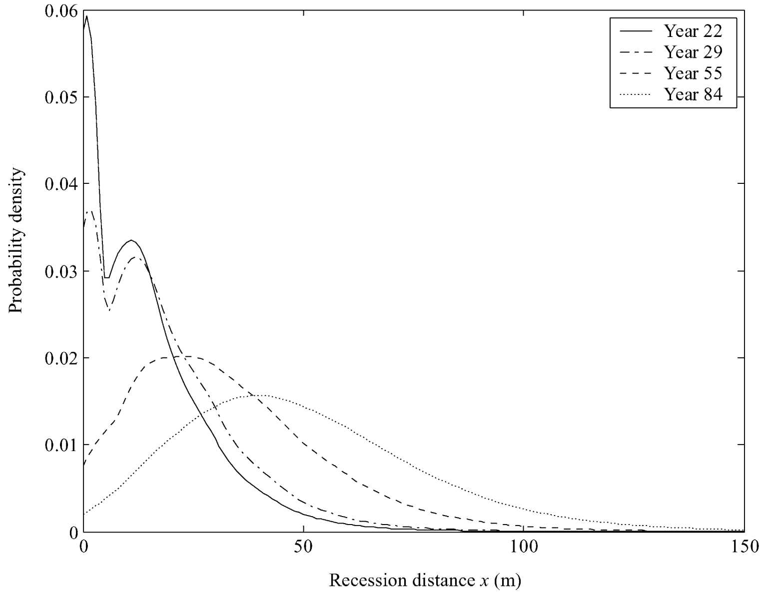

Fig. 8. Predictive distribution of cliff recession obtained from Bayesian method.

Bayesian method and from the maximum likelihood parameter estimates are compared in Table 4.

5. Stochastic recession prediction methods in context

The episodic stochastic simulation method introduced in this paper is but one of the probabilistic coastal cliff recession prediction methods reviewed in Table 1. The methods in Table 1 are classified according to

1. the nature of available information on cliff recession;
2. the cliff recession process at the site in question;
3. whether future conditions at the site are expected to resemble past conditions;
4. the amount of investigation and analysis which can be justified.

Table 4
Predicted mean recession distances from maximum likelihood and Bayesian methods

Year	Maximum likelihood predicted mean recession distance (m)	Bayes predictor mean recession distance (m)
22	20.0	15.1
29	20.0	19.0
55	36.2	33.3
84	43.1	49.6

The nature of the cliff recession process influences the type of information that is available, so the first two determinants are closely related. For example, at a site characterised by very rare landslides followed by long periods of stasis, the historic record, which may include only one landslide or perhaps none at all, will be of little value in statistical terms. Under those circumstances, the recession prediction will be guided by information about the recession potential at the site and similar sites, and will include an element of expert judgement.

At coastal cliff sites where future conditions are not expected to resemble past conditions, due to significant natural or man-induced change in the physical processes, historic records alone cannot be used for recession prediction even if, as in the case of the method proposed in this paper, the prediction method has a physical interpretation. Under changing conditions (most notably due to planned engineering intervention to control erosion), deterministic methods combined with engineering judgement have tended to dominate in the past. However, it is under changing conditions that the process simulation models of Meadowcroft et al. (1999), Hall et al. (2000b) and Walkden et al. (2001) come into their own, justifying the considerable investment in data and analysis that they involve. Even under changing conditions, a statistical prediction based on an assumption of stationary long-term average recession rate can often be used to provide upper or lower bounds on future recession.

The amount of investigation and analysis that can justifiably be invested in the prediction is a function of the stage in the decision-making process (e.g. strategy planning, feasibility study, detailed design of erosion control measures, etc.) and the level of risk at the site. Coastal zone management planning decisions will need to be supported by a general indication of probable future trends while engineering of scheme options will require more detailed analysis. Detailed probabilistic predictions are likely to be best suited to situations where there is a clear but uncertain risk to property or public safety.

The stochastic simulation method proposed in this paper can be applied in situations where there is a relatively small quantity of recession data, provided the recession process is stationary in statistical terms. At sites where there is more frequent (for example, annual) recession data, it may be possible to discern individual landslides or to separate 'event flux' of larger landslides from 'base flux' of progressive recession, for example, due to rainfall eroding the exposed cliff face. This is the case on the Holderness coast, north of the Humber estuary in the UK, where annual measurements of cliff recession have been made on 70 cross-sections since 1951 (Pethick, 1996). The data set reveals cycles of erosion with a periodicity of 6 or 7 years, though a much longer-period cycle is also postulated. The short-term periodicity would not be identified in the coarse, but also more typical, sample data shown in Table 2 and analysed in this paper. A detailed data set of the type available on the Holderness coast lends itself to analysis as a two-dimensional (longshore distance × time) random field (Vanmarcke, 1983).

Wherever possible, more than one method should be adopted to provide an indication of the robustness of the predictions. In many situations, it will be appropriate to undertake progressively more detailed analysis, beginning with an assessment of average past recession rates and rates of change, and progressing to more detailed statistical or process-based analysis.

6. Conclusions

A range of probabilistic prediction methods, which aim to reflect the uncertainties in the cliff recession process, have been reviewed. Uncertainties in prediction of cliff recession will often be large due to scarcity of monitoring data and complexity of the recession process. It has been argued that scarce monitoring data and expert interpretation of the recession process should be combined in the design of statistical prediction methods that are constrained by the physical nature of the cliff recession process and represent, at some level of abstraction, the main influences on cliff behaviour.

Simple linear regression can be used to generate probabilistic predictions but is based on assumptions of independence and Gaussian distribution of residuals that are hard to justify at many cliff sites. More strongly process-based models have the capacity to represent and predict the impacts of coastal change but require considerable data to establish and calibrate.

A new stochastic simulation method has been described and demonstrated, which reflects the episodic nature of cliff recession at many coastal sites, whereby cliff recession proceeds primarily via occasional landslide episodes followed by periods of relative inactivity. This is very different to the continuous process that has been implicit in many previous approaches to predicting cliff recession. The stochastic simulation model combines two probability distributions. A gamma distribution models the interval between landslide events, reflecting the cumulative damage to the cliff caused by the random arrival of storms. A log-normal distribution was chosen, based on evidence from experimental tests on modelled cliffs, to model the size of the landslide. The model therefore effectively captures the triggering and preparatory factors (i.e. the causes of cliff recession), the size and type of recession events (i.e. the retrogression potential) and the timing and sequence of recession events (i.e. the recurrence interval).

The model has been demonstrated using data from a cliff site in the UK, using both maximum likelihood and Bayesian parameter estimation methods. The proposed model is characterised by four parameters, so is not particularly parsimonious and, consequently, both maximum likelihood and Bayesian parameter estimation methods are computer-intensive. However, the computational cost is to be weighed against the benefits of improved representation in the model of the episodic nature of cliff recession and clear physical interpretation of the model parameters. The Bayesian method makes use of this physical interpretation, enabling expert judgement and measurements of individual cliff falls to be combined with records of past cliff recession distance. Despite the computer-intensive nature of parameter estimation, the method is straightforward to implement and is widely applicable to coastal cliff or bluff sites that are characterised by an episodic recession process, have some measurements of past recession and where the long-term average recession rate is reasonably steady.

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