Risk based pricing in Civil Engineering

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Abstract

During the last decades the introduction of probabilistic engineering has had a lot of impact in the field of civil engineering. In several design methodologies these techniques were incorporated, for instance in structural and hydraulic engineering but also in (economic) risks evaluation of civil projects. In this paper the risk-based approach is taken one step further and a probabilistic framework is presented to examine the collective probability of economic loss based on individual project risks. The diversification principle forms the foundation of this framework based upon which large companies operating in the field of civil engineering can determine their strategy for project acquisition and risk mitigation.

1 Introduction

Since the introduction of statistical mathematics in civil engineering several decades ago, applications have been developed in diverse areas of this profession. For instance the structural safety can nowadays be evaluated with statistically founded safety factors and the liability of entire structures can be analysed probabilistically. Hydraulic engineers use probabilistic methods to analyse wave fields and their resulting forces on structures and in traffic engineering the travel time and capacity are probabilistically assessed.

Besides in pure engineering probabilistic methods are also applied in economic risk management and risk assessment of civil projects. Based on the calculated individual risk a fair pricing can be determined, which is effectively a reward for the risk a company is taking by accepting the assignment. However by doing so, the risk might be overestimated, and a non-competitive price will arise (Ahmed 1990). A more realistic approach is to focus on the collective probability of economic loss. The foundation for this collective risk analysis actually was laid out in 1952 by the world famous economist H. Markowitz (1952) for which he eventually earned his Nobel Prize in Economic Science in 1990. In his article he described how portfolios are to be constructed for investors who 'consider expected return a desirable thing and variance of return an undesirable thing'. Apart from portfolio managers trading in stocks this mechanism can (and should) also be applied to large companies whose continuance depends on the revenues generated by (a limited amount of) projects they execute (Vergara 1977; Vergara and Boyer 1977; Kangari and Boyer 1981). In both systems the mathematics of diversification maintains the rate of return steady, while reducing the volatility and hence the probability of default. In this paper the mathematics of risk-based pricing in civil engineering is analysed under different circumstances assuming an acceptable default probability. The individual risk per project, combined with the mitigating portfolio effects as described in this paper, should assist large companies like contractors and engineering agencies in their determination of a sound strategy for project acquisition and risk pricing.

2 Single project analysis

During the regular tender process several contractors are invited to tender for the project, once the detailed design has been completed. These contractors analyse the design, the prescribed building technique, the available building time, etc.(Han and Diekmann 2001) and calculate for what price they are willing to build the structure. To be more accurate, the tender price is based upon an estimation of the actual cost price (operational and financial) of the construction and this cost price can therefore be depicted as a stochastic variable or a probability distribution function.

In this paper we chose to model the costs price by K·O in which O is normally distributed (~ N(μ(O),σ ^{2}(O))) and K is a deterministic value[1]. In order to reduce the probability of loss on an individual project an extra fee is incorporated in the tender price, which is ultimately a reward for the risk the entrepreneur is taking. This raise is modelled as the product of a factor representing the risk aversion of the contractor and the risk present in the project.

This results in the following equation to calculate the tender price:

$$\mu(O) \cdot K + k \cdot \sigma(O) \cdot K \;{}^{2} \qquad (1)$$

The profit can be calculated by subtracting the final cost price (a realisation of the probability distribution function) from the initially calculated tender price. Due to the fact that we do not know what the final costs are going to be, the profit is uncertain. In our example it is normally distributed with mean and standard deviation given by:

$$\mathbf{Mean}_{\text{profit}} = \mu_{\text{contract price}} - \mu_{\text{project costs}} = k \cdot \sigma(O) \cdot K \qquad (2)$$

$$\mathbf{Standard\ deviation}_{\text{profit}} = \sigma(O) \cdot K \qquad (3)$$

When excluding the benefits of diversification, this profit surcharge should be incorporated in the tender price to cover for the risk run by the company. Given the distribution of the profit it is possible to

1. By applying this equation indirectly a constant variance coefficient V(O) is assumed; as described in the introduction the individual risk should be used in the determination of the risk based pricing, however in our analysis we have taken project magnitude as the only driver for risk; despite the fact that this is crude approximation it seems plausible and suffices for the analysis present in this paper.
2. By applying this calculation method the probability of loss on this project is reduced to p, in which $k = -\Phi^{-1}(p)$.

determine the probability of loss on this individual project. This can be done by calculating the probability that the profit is less than zero, mathematically depicted by:

$$\mathbf{p = \Phi(- \mu_{profit} / \sigma_{profit}) = \Phi(- k \cdot \sigma(O) \cdot K / \sigma(O) \cdot K) = \Phi(- k)} \text{......................................(4)}$$

Apart from modeling O by a normal distribution, also other types of distributions could be applied here. For instance by the triangular distribution or the Beta distribution, in case a lower- and upper bound of the uncertain costs can be identified easily, or by an extreme value distribution if the upper tail of the costs is much longer than the lower tail (Van Gelder 2000).

3 Portfolios with independent equally-sized projects

Although the previously presented analysis provides us with insight in the risk associated with an individual project, large companies rarely deploy all their assets in just one project. It is more credible that a company is executing several projects at the same time. Besides the fact that this induces a more flexible workflow, also the financial risk is mitigated. To prove how the 'mathematics of diversification' (Markowitz 1952) works for a company executing several projects (like civil engineering agencies and contractors) we could assume these projects to be equal of size. The individual project costs are distributed as described in section 2, and the amount of projects the company is executing, is N.

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2. By applying this calculation method the probability of loss on this project is reduced to p, in which $k = - \Phi^{-1}(p)$.

When the profit is analysed, it also appears to be normally distributed with mean and standard deviation of:

$$\textbf{Mean}_{\text{profit}} = N \cdot \{ \mu_{\text{contract price}} - \mu_{\text{project costs}} \} = N \cdot k \cdot \sigma(O) \cdot K \quad (5)$$

$$\textbf{Standard deviation}_{\text{profit}} = \sqrt{N} \cdot \sigma(O) \cdot K \quad (6)$$

Again the probability of a loss for the entire company can be calculated by means of the following equation:

$$p = \Phi(- \mu_{\text{profit}} / \sigma_{\text{profit}}) = \Phi(- N \cdot k \cdot \sigma(O) \cdot K / \sqrt{N} \cdot \sigma(O) \cdot K) = \Phi(- \sqrt{N} \cdot k) \quad (7)$$

Assuming a maximum risk or probability of default a company is willing to run, one can calculate the minimum surcharge needed to meet this constraint given an amount of projects a company is executing simultaneously. If this probability is restricted to 15%, then $\sqrt{N} \cdot k \approx 1$. In other words, k (the profit surcharge) should be taken approximately $1/\sqrt{N}$ in order to keep the probability of loss on the portfolio less than 15%. This shows that under these circumstances diversification initiates a decrease in individual profit surcharge of $1/\sqrt{N}$.

4 Portfolios with independent different-sized projects

When the portfolio of a contractor is examined it appears it not only exists of several projects simultaneously, but also that these projects are not equal of size. In this section the portfolio analysis is altered accordingly. This is accomplished by changing the project costs calculation into $K_i \cdot O$ for i = 1 to N and in which $O \sim N(\mu(O),\sigma^2(O))$. By applying this equation the contract price will be given by:

1. In which K is seen as a random variable of which K_i (i=1,…,N) are N realisations of K and $\mathbf{E(K^2) = 1/N \cdot \Sigma K_i^2}$
2. In which $\boldsymbol{\mu_K = E(K)}$ and $\boldsymbol{\sigma_K = \sqrt{\{E(K^2) - (E(K))^2\}}}$

$$\mu(O) \cdot K_i + k \cdot \sigma(O) \cdot K_i \text{ ....} (8)$$

and the profit of this portfolio is again normally distributed with the following mean and standard deviation:

$$\textbf{Mean}_{\text{profit}} = k \cdot \sigma(O) \cdot \Sigma K_i = k \cdot \sigma(O) \cdot N \cdot E(K) \quad ^{1} \text{..........} (9)$$

$$\textbf{Standard deviation}_{\text{profit}} = \sqrt{\{\ \Sigma\ (\sigma(O) \cdot K_i)^2\ \}} = \sqrt{\{\ N \cdot \sigma(O)^2 \cdot E(K^2)\ \}} \quad ^{1} \text{..........} (10)$$

If K is interpreted as a random variable with K_i as a realisation, $E(K^2)$ is equal to $E^2(K) + Var(K)$. Hence the probability of loss for this portfolio can be written as:

$$\begin{aligned} p = \Phi(-\mu_{\text{profit}} / \sigma_{\text{profit}}) &= \Phi(-k \cdot \sigma(O) \cdot N \cdot E(K) / \sqrt{\{N \cdot \sigma(O)^2 \cdot E(K^2)\}}) \\ &= \Phi(-\sqrt{N} \cdot k \cdot E(K) / \sqrt{E(K^2)}) \\ &= \Phi(-\sqrt{N} \cdot k \cdot \sqrt{\{\mu_K^{\ 2} / (\sigma_K^{\ 2} + \mu_K^{\ 2})\}}) \quad ^{2} \text{..........} (11) \end{aligned}$$

As described in the previous section based upon this equation it is possible to calculate the minimum surcharge needed to meet a probability of default given the amount of projects a company is executing simultaneously. If this probability is again restricted to 15%, then k should be taken as $1/ \{\sqrt{N} \cdot \sqrt{(\mu_K^{2}+\sigma_K^{\ 2}) / \mu_K^{\ 2}}\ \}$. This multiplication factor should be applied to each project.

Note that if all K_i's are equal, then $Var(K) = 0$ resulting in equation (7). If the K_i's are exponentially distributed, then $\mu=\sigma$, which results in $k = \sqrt{(2/N)}$. Other types of distributions for K can be considered here and resulting expressions for k can be derived.

1. In which K is seen as a random variable of which K_i (i=1,…,N) are N realisations of K and $\mathbf{E(K^2) = 1/N \cdot \Sigma K_i^2}$
2. In which $\boldsymbol{\mu_K} = \mathbf{E(K)}$ and $\boldsymbol{\sigma_K} = \sqrt{\{E(K^2) - (E(K))^2\}}$

5 Portfolios with dependent different-sized projects

In the previous analyses the costs for all projects were assumed independently distributed. However this assumption is not very realistic, for instance due to the fact that wages are a common factor in each project (Vrijling 1995; Minato and Ashley 1998). Therefore a change in the collective loan agreement will cause an increase in all project costs. To incorporate a partial dependency in the analysis a simple way of correlation is introduced. If we decompose the project costs K as

K = A + M in which,

A are the wages and M the material costs (both of them are assumed independently). The project costs K are simulated by adding a dependant part and an independent part. The correlation between (the dependent parts) of two projects is taken as:

$$\mathbf{corr(K_i,K_j) = \alpha^2} \quad ^{1} \tag{12}$$

For the portfolio with partial correlation, the mean and standard deviation can be calculated with the equations given below:

$$\mathbf{Mean_{profit} = k \cdot \sigma (O) \cdot N \cdot E (K)} \quad ^{2} \tag{13}$$

$$\mathbf{Standard\ deviation_{profit} = \sqrt{ \{ (\alpha \cdot \sigma(O) \cdot N \cdot \mu_K)^2 + (1 - \alpha^2) \cdot \sigma(O)^2 \cdot N \cdot (\mu_K^{\ 2} + \sigma_K^{\ 2}) \} }}\ ^{2} \tag{14}$$

Based upon these equations the probability of a loss can be calculated with $\mathbf{\Phi(- \sqrt{ (k^2\sigma^2 (O) N^2E^2 (K) / \{(\alpha\sigma(O)N\mu_K)^2 + (1 - \alpha^2) \sigma(O)^2 N (\mu_K^{\ 2}+\sigma_K^{\ 2})\} }))}$, which can be simplified to:

$$\mathbf{p = \Phi(- \mu_{profit} / \sigma_{profit}) = \Phi(- \sqrt{ \{ k^2 \cdot E^2(K) / (\alpha^2 \cdot \sigma(O) \cdot N \cdot \mu_K^2}}$$

1. In which **for i≠j, i,j=1,…,N**
2. In which $\mathbf{E (K^2) = 1/N \cdot \Sigma K_i^2}$, $\mathbf{\mu_K = E(K)}$ and $\mathbf{\sigma_K = \sqrt{E(K^2)}}$
3. Based on the previously presented assumption, profit will be normally distributed (see appendix 2)

$$+ (1 - \alpha^2) \cdot \sigma(O) \cdot (\mu_K{}^2 + \sigma_K{}^2) \}) \quad ^3 \text{..................} (15)$$

6 Case study

To examine the previously presented equations, in this section a corporate example will be reviewed. Based on several information sources, a plausible project portfolio for a hypothetical construction company was drawn up. This company, active in two different fields of civil construction, possesses a project portfolio consisting of 1500 projects, in a cost range of EUR 10 thousand to EUR 35 million, resulting in a portfolio of EUR 1 billion.

Independent equally sized projects

Based on the portfolio (EUR 1 billion) and the amount of projects (1500) an average cost price of EUR 666 thousand can be calculated. Assuming a standard deviation of 10% and an individual probability loss of 5%, the required profit surcharge is 16.5%. However assuming these conditions for all projects in the portfolio, the collective probability of loss is negligible. Even under the profit surcharge 2%, which is normally used in this market the collective probability of loss is improbably small. This shows that a lower profit surcharge is justifiable if, instead of using individual risk, the collective loss distribution is used (preferably including all influencing factors as presented in the following sections).

1. In which **for** $i \neq j$, $i,j=1,\ldots,N$
2. In which $E(K^2) = 1/N \cdot \Sigma K_i^2$, $\mu_K = E(K)$ and $\sigma_K = \sqrt{E(K^2)}$
3. Based on the previously presented assumption, profit will be normally distributed (see appendix 2)

Independent different sized projects

As described in section 4, a portfolio rarely consists of equally sized projects but rather of projects in a certain cost spread. For our analysis we assumed the project costs to be between EUR 10 thousand and EUR 35 million. A probability distribution has been constructed by adding two portfolios, one for each line of business, of which a simplified distribution has been depicted in figure 1.

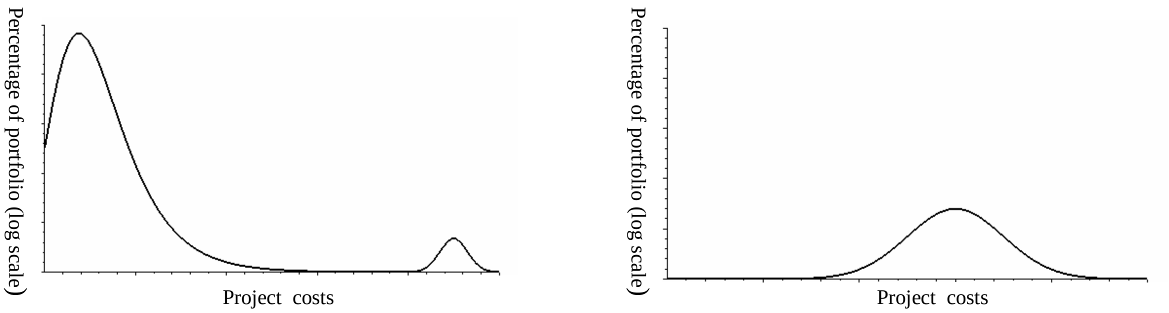

Fig. 1. Project portfolio per trade

The first graph could be interpreted as a portfolio consisting of a lot of small projects and several high priced projects. The second distribution is a portfolio normally distributed on a log scale around a certain average construction price. The graph below depicts the company portfolio on a consolidated level.

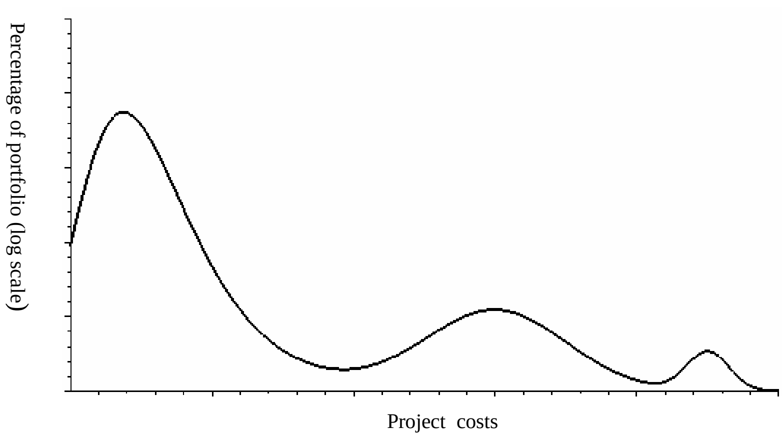

Fig. 2. Corporate portfolio

Based on this portfolio the probability of loss has been calculated for different levels of correlation. Again we have assumed the individual projects to be normally distributed with a standard deviation of 10% and a profit level of 2% for each project. In figure 3 the results of this calculation is shown. A conservative level of correlation would be 30% (technical and economical correlation), resulting in a probability of loss of 36%. This is a significant increase when compared to the negligible probability of loss we found in the uncorrelated example with equally sized projects. Assuming these conditions the average profit level should have been 9% in order to come to an acceptable probability of loss (5%). This surcharge is still considerably smaller than the original 16.5% profit per individual project. For the different levels of correlation we have calculated the required average level of profit needed to limit the probability of loss to 5%. The results have been presented in figure 3 (right graph), and evidence the previously described facts (both the independent portfolio combined with a profit of 2%, and the dependent portfolio (α = 30%) combined with a profit of 9% results in a probability of loss of 5%).

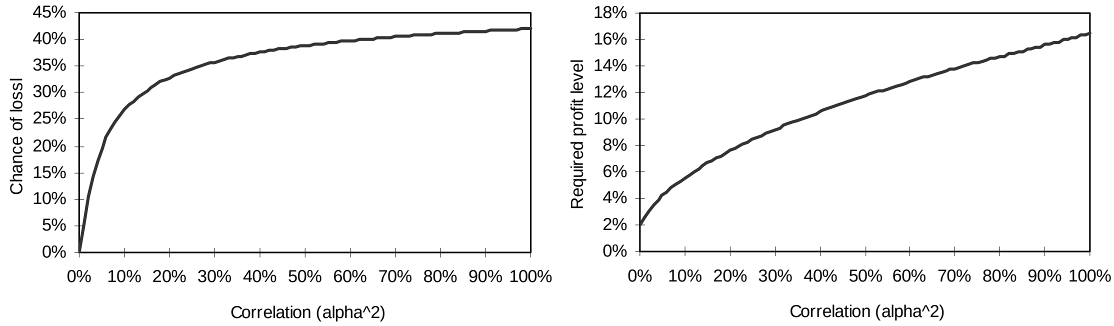

Fig. 3. Probability of loss for different correlation coefficients (left) and required profit level for a 5% probability of loss (right)

Besides the analysis of the average profit level it is also possible to calculate the required profit level of a new project. Based on the portfolio, a correlation of 30% and an average profit level of 9% per project (resulting in a probability of loss of 5%), for a new project ad EUR 25 million the required profit level comes to 8%. In figure 4 an overview is given for the profit level needed assuming different project sizes.

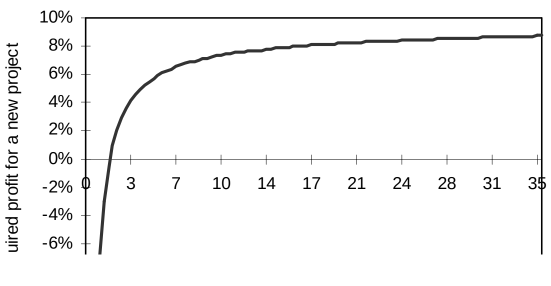

Fig. 4. Required profit level for a new project with a given project size

One of the basic assumptions of the example treated in this paper is a constant variance coefficient V(O). One might disagree with the assumption that there is a direct relationship between the projects magnitude and the amount of risk involved for instance because risk diversification not only works between but also 'inside' a project. The law of large numbers would therefore induce a risk reduction relative to the projects size. Although this is unmistakably true, there are several phenomena that initiate an increase of risk with an expanded project. First of all due to correlation (Vrijling 1995) the decrease is not fully squared but has a more linear tendency. Secondly because of an increased complexity the risk is increased even further 'justifying' a one to one relationship between magnitude of risk and project size.

7 Portfolio based pricing

In this paper a framework for portfolio based risk pricing in civil engineering (assuming a restricted probability of loss) has been presented. Based on a risk aversion factor k, a first approach would be to determine a profit hurdle per project of $k \cdot \sigma \cdot K$ above the calculated cost price ($\mu \cdot K$). However this approach disregards the positive effects of diversification, which should also be incorporated to achieve a competitive tender price. The (qualitative) effects of portfolio diversification that have been presented in this paper are not (or should not be) new to 'project driven' companies, because they can be observed in ordinary life. Most companies are well aware that 'putting all of their eggs in one basket' is a risky business, so they try to build a well spread and qualitatively strong portfolio. Additionally several companies recognise the negative influence of correlation and have developed activities in different geographical and technical fields (i.e. several construction companies provide services throughout the world, ranging from dredging and hydraulic engineering to real estate development and environmental conservation advise).

Despite the fact that the effects have been known (qualitatively), the major benefit of the mathematical description given in this paper is that it provides a crude quantitative description of risk given an amount of projects, a distribution of cost prices and the mutual correlation. With the presented equations (1) to (15) contractors and engineering agencies can determine a strategy for project acquisition with an adequate risk spread and a founded profit hurdle. From the previous examples it can be derived that diversification

of uncertainties (or risks) is most effective when the portfolio of building-projects consists of a large number of projects that are equally large and that are independent from each other. If a company intends to limit the probability of loss *for its portfolio* to p, a reduction to k can be applied. For a portfolio of independent equally sized projects k is calculated by $\Phi^{-1}(p)/\sqrt{N}$. If the projects are not equally sized (but still independent), then diversification is less effective compared to a portfolio consisting of equally sized projects; a multiplication factor of $1/\sqrt{(\mu^2/(\sigma^2+\mu^2))}$ should be added to k. If the projects are unequal of sized and furthermore a dependency is present (with correlation α^2), the effectivity decreases even further. The multiplication factor changes to $\sqrt{\{\alpha^2 N+(1-\alpha^2)(\sigma^2+\mu^2)/\mu^2\}}$. Notice that if N tends to infinity that k tends to $\Phi^{-1}(p)\alpha$.

Despite the fact that the presented equations are rather crude, the presented case study shows they do offer insight in the relation between portfolio size, correlation, profit level and the probability of loss for project driven companies. Obviously a more accurate profit estimation can be achieved by refining the model (i.e. including time variance, different profit levels and different correlation levels) possibly requiring a numerical model in stead of the analytical model presented in this paper.

1. It has to be noted that predicting the required cost of equity is a very difficult task depending on the expectations of the investors. This is confirmed by the low P/E raio for contractors currently traded at the Amsterdam Stock Exchange as opposed to their underlying values (referring to the dredging-war several months ago).

Qualitatively it can finally be noted that the equations provide mathematical evidence of the fact that by diversifying a project portfolio the profit hurdle needed to limit the possibility of loss is reduced. Because the revenues are stabilised, it becomes easier to budget financial costs and acquire the necessary funding. Also the risk profile of the company decreases, which might result in a reduction of the equity costs (in accordance to the Capital Asset Pricing Model[1]; Brealy et al. 1998).

As a conclusion it can therefore be said that despite the presented framework for risk based pricing is rather crude, it does comply to the requirements for sound economic business decisions. Hence the framework can quantitatively assist large contractors and engineering agencies in their project acquisition and pricing policies, leading to a mitigated company risk profile and a balanced financial management.

8 Recommendation and contemplation

To accurately calculate the required profit level, the framework assumes a company to be aware of its acceptable collective probability of loss. Although by simply assuming a loss percentage the framework does provide insight in the relation between risk, correlation and pricing, in light of a shareholder value approach (Copeland et al. 2000; Hull 2000) the framework could be adjusted by easing the limited loss percentage requirement. Based on the individual risk embedded in a project, the necessary equity requirements and for a project could be calculated. When the expected revenues are reduced by the expected losses and the costs (operational and financial; ergo assuming the individual finance structure is known) the risk adjusted return is obtained. This should be equal to the expected value of the individual profit probability distribution function. Based on this return a transactional 'risk adjusted return on risk adjusted capital' (RARORAC) can be calculated by dividing the risk adjusted return by the capital requirements (ensuing from individual risk reduced by portfolio effects). This RARORAC can than be compared to the required return on equity (RRE) as was promised to the shareholders and the transactional added shareholder value can be obtained by multiplying the spread (difference between RARORAC and RRE) with the capital employed. Since the results should (qualitatively) be equal to the analysis presented in this paper, and to keep the analysis comprehensible, a more elaborate treatment of this risk-pricing framework based on added shareholder value has been omitted.

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