Formula sheet: Mathematics - v2023

Trigonometry

Pythagorean identity

$$\cos^2(x) + \sin^2(x) = 1 \tag{1}$$

Angle sum and difference identities

$$\cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta) \tag{2}$$

$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta) \tag{3}$$

$$\sin(\alpha - \beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta) \tag{4}$$

$$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta) \tag{5}$$

$$\tan(\alpha - \beta) = \frac{\tan(\alpha) - \tan(\beta)}{1 + \tan(\alpha)\tan(\beta)} \tag{6}$$

$$\tan(\alpha + \beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha)\tan(\beta)} \tag{7}$$

Double-angle formulae

$$\begin{aligned} \cos(2x) &= \cos^2(x) - \sin^2(x) && (8)\\ &= 2\cos^2(x) - 1 && (9)\\ &= 1 - 2\sin^2(x) && (10) \end{aligned}$$

$$\sin(2x) = 2\sin(x)\cos(x) \tag{11}$$

$$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)} \tag{12}$$

Integrals

$$\int a^x \ \mathrm{d}x = \frac{a^x}{\ln(a)} + C \qquad (a \neq 1) \tag{13}$$

$$\int \ln(x) \ \mathrm{d}x = x\ln(x) - x + C \tag{14}$$

$$\int \log_a(x) \ \mathrm{d}x = \frac{1}{\ln(a)}(x\ln(x) - x) + C \qquad (a > 0 \,\text{and}\, a \neq 1) \tag{15}$$

$$\int \sin(x) \ \mathrm{d}x = -\cos(x) + C \tag{16}$$

$$\begin{aligned} \int \frac{1}{1+x^2} \ \mathrm{d}x &= \arctan(x) + C \\ &= \tan^{-1}(x) + C \end{aligned} \tag{17}$$

$$\begin{aligned} \int \frac{1}{\sqrt{1-x^2}} \ \mathrm{d}x &= \arcsin(x) + C \\ &= \sin^{-1}(x) + C \end{aligned} \tag{18}$$