

Syllabus Academic Test - v2021.1

The academic test consists of three digital subtests which contain questions related to fundamental topics of mathematics, physics, and about the selected first-year material. Below the minimum of expected knowledge for mathematics and physics is presented. Note that the questions on the respective tests might consist of a combination of multiple topics. The content in this syllabus is based on the material covered in Dutch VWO (i.e. pre-university education) schools.

Mathematics

For all questions of the Academic Test there will be an indication as to whether an exact or a rounded answer is required (e.g. $1/3$ as opposed to 0.33). The use of a graphing/programmable calculator is not allowed for any of the sections of the test.

1. Functions and Graphs

- i The candidate is able to recognize and construct compositions of standard functions. Standard functions include polynomial functions x^n , n -root functions ($\sqrt[n]{x}, x^{\frac{1}{n}}$), the power functions a^x , and its inverse the logarithm $\log_a(x)$, the exponential function e^x , and its inverse the natural logarithm $\ln(x)$, trigonometric functions $\sin(x)$, and $\cos(x)$.
- ii The candidate is able to analyze, draw and transform (compositions of) these standard functions, and to determine limits, domain, range, asymptotes and symmetry points or lines.
- iii The candidate understands the concept of inverse functions, and can find the inverse of (compositions of) standard functions.

2. Algebraic Solving

- i The candidate can rewrite expressions to isolate a variable and can substitute expressions into a given function.
- ii The candidate is able to rewrite expressions into simplified form and recognizes special products, and can use this knowledge to manipulate equations and inequalities composed of standard functions (see 1i) to find solutions of the form $y = f(x)$ or $y \leq f(x)$.
- iii The candidate is able to find roots of a function ($f(x) = 0$) using factorization techniques. The candidate is able to use the quadratic formula to find roots of quadratic equations ($ax^2 + bx + c = 0$).
- iv The candidate can solve systems of linear equations, $\begin{cases} ax + by = c \\ dx + ey = f \end{cases}$, with a, b, c, d, e, f constants.

3. Differential Calculus

- i The candidate knows the derivatives of standard functions, and is able to apply the product rule, quotient rule, and chain rule to determine derivatives of functions composed of standard functions.
- ii The candidate is able to determine the first derivative ($f'(x)$, $\frac{dy}{dx}$, $\frac{d}{dx}f(x)$) and second derivative ($f''(x)$, $\frac{d^2y}{dx^2}$, $\frac{d^2}{dx^2}f(x)$) of functions and to use these to determine locally increasing and decreasing behavior, extreme values, locally concave and convex behavior, and inflection points.
- iii The candidate is able to apply differentiation to determine the slope of a graph and the local tangent and normal lines to a function, to construct and solve an optimization problem, and to solve problems concerning distance, velocity and acceleration.

4. Integral Calculus

- i The candidate understands the concept of integration and related terms (including limits of integration, definite/indefinite integrals and the constant of integration).
- ii The candidate is able to determine the antiderivative or primitive of standard functions ($F(x) = \int f(x)dx$), and is able to use this to calculate definite and indefinite integrals of functions of the form $cf(ax + b) + d$, with a, b, c, d constants.
- iii The candidate is able to apply integration to determine the surface area, volume of a solid of revolution, arc length and the mean value of a function.

5. Trigonometry

- i The candidate understands the trigonometric functions $\sin(x)$, $\cos(x)$ and $\tan(x)$ and concept of the unit circle. The candidate understands the terms amplitude, phase, period, and frequency, and is able to convert degrees to radians and vice-versa.
- ii The candidate knows the exact values of $\sin(\theta)$, $\cos(\theta)$ and $\tan(\theta)$ for integer multiples of the following angles θ in the first quadrant, $\{0, \frac{1}{6}\pi, \frac{1}{4}\pi, \frac{1}{3}\pi, \frac{1}{2}\pi\}$, and to use periodicity and symmetry properties of $\sin(\theta)$, $\cos(\theta)$ and $\tan(\theta)$.
- iii The candidate is able to find all solutions of equations $\sin(x) = c$, $\cos(x) = c$ and $\tan(x) = c$, and of $\sin(f(x)) = \sin(g(x))$, $\cos(f(x)) = \cos(g(x))$ and $\tan(f(x)) = \tan(g(x))$, where c is a constant and $f(x)$ and $g(x)$ are linear functions of x .
- iv The candidate is able to apply the Pythagorean identity $\sin^2(x) + \cos^2(x) = 1$, sum and difference identities and double angle formulae.

6. Geometry

- i The candidate is able to determine the surface and perimeter of two-dimensional shapes including triangles, rectangles, circles, etc. The candidate is able to determine the volume and surface area of three-dimensional figures including cubes, pyramids, cylinders, cones, etc.
- ii The candidate can use properties of lines, triangles, circles, and quadrilaterals to determine lengths and angles. The candidate is familiar with the properties of a right-triangle, isosceles triangle, and equilateral triangle.
- iii The candidate can use sine, cosine, and tangent relations, the Pythagorean theorem, and the law of sines and the law of cosines to determine lengths and angles in triangles.
- iv The candidate can formulate the equation for lines and circles, and knows the relations between the slopes of parallel and tangent lines.
- v The candidate is able to find the intersections between lines and circles.

7. Vectors

- i The candidate understands the concept of a vector, and can determine the length and direction of a vector.
- ii The candidate can decompose vectors in components, can multiply a vector with a scalar, and can add and subtract vectors. The candidate can calculate the dot product of two vectors, and can use it for the calculation of angles and distances.
- iii The candidate can calculate velocity and acceleration of a moving point whose path is described by a time-dependent vector representation.

Physics

1. Fundamentals

- i The candidate can use the SI base units: meter [m] for length, kilogram [kg] for mass, second [s] for time, ampere [A] for electric current, both kelvin [K] and degrees Celsius [$^{\circ}C$] for temperature, and mole [mol] for amount of substance. The candidate is able to perform a dimensional analysis to determine the dimension of physical quantities.
- ii The candidate understands the concept of vector quantities (e.g. velocity \vec{v} , acceleration \vec{a} , force \vec{F} , etc.) in physics, i.e. quantities that have a direction and magnitude. The candidate can draw, add, subtract and decompose vector quantities.
- iii The candidate is familiar with metric prefixes as micro- μ , milli- m , kilo- k , mega- M , giga- G , etc., and can transform from one to another.
- iv The candidate is familiar with mathematical expressions including $\log(x)$, $\ln(x)$, e^{-ax} , e^{ax} , a^x , x^a , $\sin(x)$ and $\cos(x)$.
- v The candidate is familiar with linear diagrams, logarithmic diagrams and double-logarithmic diagrams.
- vi The candidate understands the concept of significant figures and is able to round a number to a specified amount of significant figures.

2. Mechanics

- i The candidate knows the relations between distance (x), velocity (v), and acceleration (a), and is able to analyze (x, t) -, (v, t) - and (a, t) -diagrams. The candidate can use these relations to analyze linear motions (e.g. movement with constant acceleration or deceleration, movement with constant friction force, free fall).
- ii The candidate is able to analyze and (de)compose forces \vec{F} , and understands and is able to apply the laws of Newton. The candidate is familiar with gravitational forces, friction forces, tension and spring force, and is able to calculate these forces and to draw them in a free-body-diagram in order to set up a balance of forces.
- iii The candidate understands the principles of work W , energy E , power P and efficiency η and knows how to relate these quantities to one another. The candidate understands the principle of conservation of energy and is able to set up an energy balance using potential energy, kinetic energy, and elastic/spring energy.
- iv The candidate can determine period, frequency, and angular velocity of circular motion, and can use these terms in the context of planetary orbits. The candidate can analyze circular motions, understands centripetal and centrifugal force and can construct a balance of gravitational force and centripetal force.

3. Electricity and Magnetic Fields

- i The candidate is familiar with the principles of electricity and understands concepts as conduction, isolation, electrons, ions, current I (Ampere), charge Q (Coulomb), electric potential U (Volt) and resistance R (Ohm). The candidate knows and is able to apply the relations for current $I = \frac{Q}{t}$, potential $U = \frac{\Delta E}{Q}$, and Ohm's law $U = IR$.
- ii The candidate is able to draw and interpret simple circuit diagrams, knows how to apply rules for parallel and series circuits, and is able to apply the laws of Kirchhoff to analyze a circuit.
- iii The candidate can use the concepts of electric power and efficiency.
- iv The candidate understands Coulomb's law and can apply it to calculate the force between two electrically charged particles.
- v The candidate is familiar with the principle of magnetism and understands concepts as induction, flux, homogeneous and inhomogeneous magnetic fields. The candidate can calculate the magnetic flux of a homogeneous magnetic field passing through a flat surface.
- vi The candidate is familiar with the Lorentz force exerted by a magnetic field on a charged particle moving through it. The candidate is able to determine the magnitude and direction of this Lorentz force based on the direction of the field, the direction of motion of the particle, and the sign of its charge.

4. Properties of Fluids and Solids

- i The candidate is familiar with the macroscopic and microscopic view of gases, and knows how to use the ideal gas law, $pV = n\mathcal{R}T$ or $p = \rho RT$. The candidate is familiar with temperature T , pressure p , volume V , the universal gas constant \mathcal{R} , amount of substance n , mass density ρ , and molar mass M . The candidate knows the definitions of density $\rho = \frac{m}{V}$, of the specific gas constant $R = \frac{\mathcal{R}}{M}$, and of pressure $p = \frac{F}{A}$ and pressure in a fluid $p = \rho gh$.
- ii The candidate can convert temperature in degrees Celsius to Kelvin and vice versa. The candidate can calculate heat (thermal energy) $Q = C\Delta T = cm\Delta T$, with C the (molar) heat capacity, and c the specific heat capacity (per unit mass).

5. Vibrations and Waves

- i The candidate is familiar with vibrations and corresponding terms like period, frequency, amplitude, phase, resonance, and damping.
- ii The candidate recognizes simple harmonic motions, and is able to analyze harmonic motion of a simple mass-spring system.
- iii The candidate is familiar with wave phenomena and corresponding terms like longitudinal and transverse waves, wavelength $\lambda = vT$, wave speed, phase, speed of sound and speed of light.

6. Modelling

- i The candidate is able to interpret simple model studies, the context of which will not exceed the concepts introduced above.
- ii The candidate is able to adjust or complete steps in the schemes for these model studies. No knowledge of programming languages is required for this.