**AE4ASM005: Fatigue of Structures & Materials**
**Answers**

1. An aircraft fuselage with 3.95m diameter and 1 mm thin fuselage skin is loaded as result of internal pressure by a pressure differential of $\Delta p$ = 45 kPa.
   a. Calculate the circumferential and longitudinal stress in the skin of this fuselage.
   b. Will the ratio circumferential strain over longitudinal strain, be higher than/equal to/lower than the ratio circumferential stress over longitudinal stress? Explain your answer.

If you're unable to answer these questions, you are strongly advised to study Chapters 6 and 7 from the reader *Introduction to Aerospace Engineering – Structures and Materials* by R.C. Alderliesten available from ocw.tudelft.nl

**Ans a:**

$$\sigma_{circ} = \frac{\Delta pR}{t}$$

$$\sigma_{long} = \frac{\Delta pR}{2t}$$

**Ans b:**

$$\varepsilon_{circ} = \frac{\sigma_{circ}}{E} - \nu\frac{\sigma_{long}}{E}$$

$$\varepsilon_{long} = \frac{\sigma_{long}}{E} - \nu\frac{\sigma_{circ}}{E}$$

2. An aircraft fuselage contains a little crack. Visual inspections reveal that this crack increases every flight with $1{\cdot}10^{-4}$ µm. Do you consider this possible?

If you're uncertain about the correct answer to this question, you are advised to study *Mechanics of Materials* by R.C. Hibbeler (any edition) or another Mechanics of Materials text book.

**Ans: No, because $1{\cdot}10^{-4}$ µm is the length scale of an atom, which you can't see visually.**

3. Non-digital clocks (which are becoming rare these days) have a second hand that rotates around in a regular and repeating fashion, and sometimes they even have a pendulum.
   a. What is the frequency of rotation of a second hand on these clocks?
   b. What is the mean value of the movement of the pendulum?

If you're uncertain about the correct answer to this question, you are advised to study Dynamics by R.C. Hibbeler (any edition) or another Engineering Dynamics textbook.

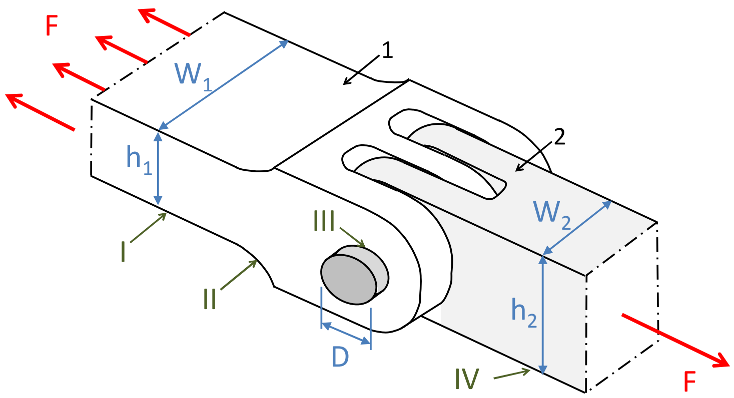
**Ans a: 1/60**
**Ans b: 0**

4. Consider a composite plate made of longitudinal fibres and a polymer matrix. The following properties of these fibres and matrix are given: $E_f$ = 180 GPa, $\rho_f$ = 2500 kg/m$^3$, $E_m$ = 3 GPa, $\rho_m$ = 1200 kg/m$^3$.
   a. Calculate the Young's modulus of a unidirectional laminated plate in longitudinal direction if the fibre-volume fraction is 58%
   b. Calculate the Young's modulus of a cross-ply laminated plate in longitudinal direction if the fibre-volume fraction is 58% and the amount of plies in longitudinal and transverse direction are the same
   c. Calculate the density of the cross-ply laminate from question b).

If you're unable to answer these questions, you are strongly advised to study Chapters 1 and 3 from the reader *Introduction to Aerospace Engineering – Structures and Materials* by R.C. Alderliesten available from Ocw.tudelft.nl

**Ans a: 105.6 GPa**
**Ans b: 54.3 GPa**
**Ans c: 1954 kg/m$^3$**

5. Consider a pin loaded joint (lug) as illustrated below. Both aluminium components are made of Aluminium 2024-T3 and the pin is made of a medium carbon steel. The measurements of the components are $W_1$ = 100 mm, $W_2$ = 60 mm, $h_1$ = 50 mm and $h_2$ = 70 mm. The properties of both materials are given in the table below the figure.




| Material | Aluminium 2024-T3 | Medium carbon steel |
|---|---|---|
| Ultimate tensile strength $\sigma_{ult}$ | 483 | 800 |
| Tensile yield strength $\sigma_{yield}$ | 345 | 640 |
| Ultimate shear strength $\tau_{ult}$ | 283 | 470 |

   a. What is the smallest possible diameter of the medium carbon steel pin if pin failure should be avoided at a load F = 1000 kN?
   b. Rank the locations I to IV in the order of increasing stress at each location (low to high).

If you're unable to answer these questions, you are strongly advised to study Chapters 1, 9 and 11 from the reader *Introduction to Aerospace Engineering – Structures and Materials* by R.C. Alderliesten available from Ocw.tudelft.nl

**Ans a: 26.0 mm**
**Ans b: I – IV – II – III**

6. Consider the case where structural analyses revealed that at a certain point in a structure the following stress could be determined: $\sigma_x$ = 3 MPa, $\sigma_y$ = 1 MPa, and $\tau_{xy}$ = 2 MPa.
   a. Using Mohr's circle, what is the principal direction and principal normal stresses?
   b. What is the maximum shear direction and the maximum shear stress?
   c. What are the stresses at an angle of 15°?

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| If you're not familiar with Mohr's circle, and you're unable to answer these questions, you are strongly advised to study *Mechanics of Materials* by R.C. Hibbeler (any edition) or another Mechanics of Materials text book. |

$$\tan\left(2\theta_p\right) = \frac{2}{3-2} = 2 \quad ; \quad \theta_p = 31.72^\circ$$

**Ans a:** $\sigma_1 = \sigma_{av} + r = 2 + 2.24 = 4.24\,MPa$

$\sigma_2 = \sigma_{av} - r = 2 - 2.24 = -0.24\,MPa$

**Ans b:** $$\tan\left(-2\theta\right) = \frac{3-2}{2} = 0.5 \quad ; \quad \theta = 13.28^\circ$$

$\tau_{max} = r = 2.24\,MPa$

$\sigma_x = \sigma_{av} + r\cos\left(2\theta_p - 2\theta\right) = 2 + 2.24\cos\left(63.44 - 30\right) = 3.87\,MPa$

**Ans c:** $\sigma_y = \sigma_{av} - r\cos\left(2\theta_p - 2\theta\right) = 2 - 2.24\cos\left(63.44 - 30\right) = 1.34\,MPa$

$\tau_{xy} = r\cos\left(2\theta_p - 2\theta\right) = -2.24\cos\left(63.44 - 30\right) = 12.32\,MPa$

7. Consider the steel, titanium and aluminium given in the table.

| Metal | Alloy | E | $\sigma_y$ | $\sigma_{ult}$ | $\varepsilon_{ult}$ |
|---|---|---|---|---|---|
| | | [GPa] | [MPa] | [MPa] | [%] |
| Steel | AISI 4340 | 205 | 470 | 745 | 22 |
| Aluminium | AA7475-T761 | 70 | 448 | 517 | 12 |
| Titanium | Ti6Al-4V (5) | 114 | 880 | 950 | 14 |

   a. Which metal is most rigid?
   b. Which metal is most brittle?
   c. Which metal is most flexible?
   d. Which metal has the highest toughness?
   e. Which metal is the softest?
   f. Which metal will exhibit the most plasticity at a given load?

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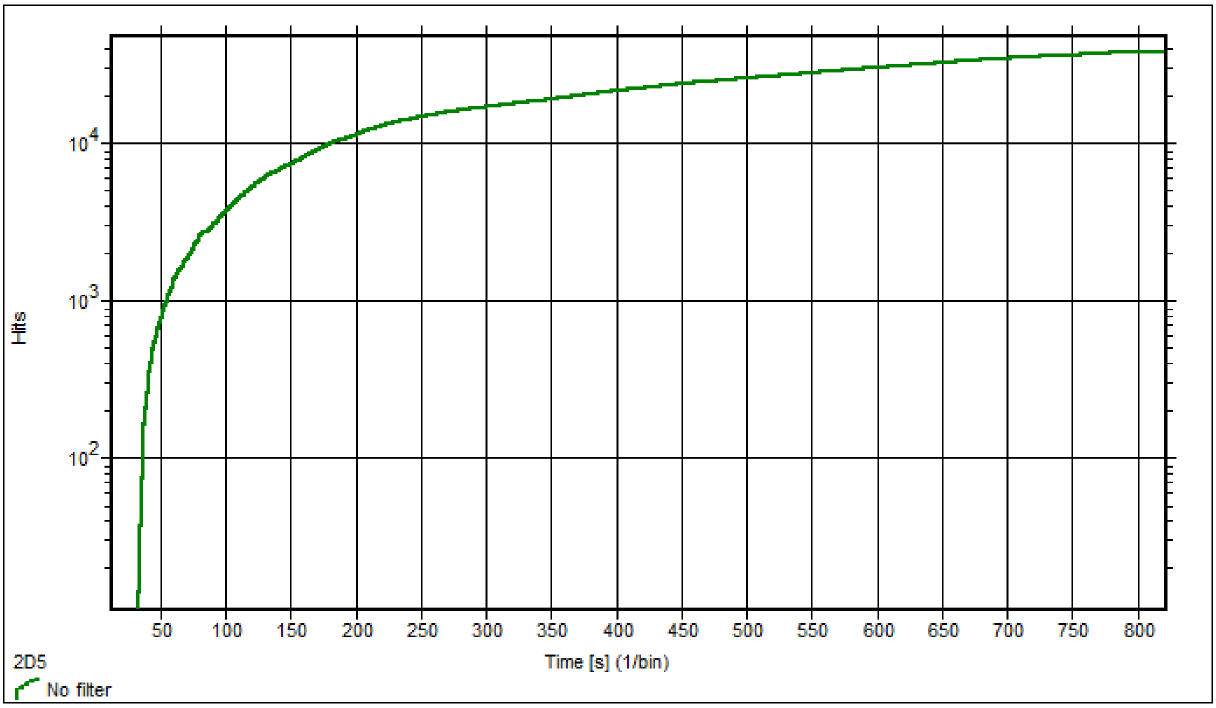
| If you're unable to answer these questions, you are strongly advised to study Chapters 1 from the reader *Introduction to Aerospace Engineering – Structures and Materials* by R.C. Alderliesten available from ocw.tudelft.nl |

**Ans a: Titanium**
**Ans b: Aluminium**
**Ans c: Aluminium**
**Ans d: Titanium**
**Ans e: Aluminium**
**And f: Aluminium**

8. Consider the graph below plotting the cumulative number of Acoustic Emission signal hits per time. At what time has the accumulated number of hits reached the five-fold of the number at 100 seconds?




| If you're unable to answer this question, you are strongly advised to study Physics for Scientists & Engineers (with Modern Physics)" by Douglas C. Giancoli or another academic physics text book |
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**Ans: Time = 300 s**